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# Enforcement Missions: Budgets vs. Targets\*

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## Abstract

Enforcement of policy is typically delegated. What sort of mission should the head of an enforcement program be given? When there is more than one firm being regulated their compliance decisions – otherwise completely separate – become linked in a way that depends on that mission. Under some sorts of missions firms compete to avoid the attention of the enforcer by competitive reductions in the extent of their non-compliance. Under others the interaction pushes in the opposite direction. We develop a general model of enforcement spillovers that allows for the ordering of some typical classes of missions. We find that in plausible settings ‘target-driven’ missions (that set a hard emissions target and flexible budget) achieve the same outcome at lower cost than ‘budget-driven’ ones (that fix budget). Inspection of some fixed fraction of firms is never optimal.

*Keywords:* enforcement; regulatory objectives, policy delegation

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# 1 Introduction

The cop can only pull over one car at a time. To avoid a ticket you don't have to obey the speed limit - you just have to be going slower than the guy in the next lane. (Anon, Lifehacker.com (2007)).

The enforcement of government policy is typically delegated. At an aggregate level, for example, enforcement of environmental legislation is delegated to an environment agency.<sup>1</sup> At the intra-agency level the enforcement of a particular area of legislation (say noise control) will generally have associated with it a dedicated enforcement program.

We ask the following question: When establishing such an enforcement program what mission should be given to the program leader?<sup>2</sup>

A variety of missions are in common use and our model will be flexible enough to embody any of them. For the purposes of the initial part of the paper, however, we will focus attention and discussion around two types. The first type requires that the enforcer achieve a particular rate of compliance at least cost. We will refer to such missions as *target-driven*. The second type (which we will refer to as *budget-driven*) requires him achieving the highest possible compliance level subject to a budget constraint. Both are observed in practice.

It is natural to suppose that the target-driven and budget-driven approaches are dual to one another and therefore that the choice between them does not matter.<sup>3</sup>

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<sup>1</sup>More generally crime control is delegated to a police force, tax collection to a revenue service and so on. We will refer to the 'enforcer' as if an individual.

<sup>2</sup>A lot of attention has been paid to the problems that arise when the interests of the principal and the delegate (as agent) are imperfectly aligned. Gailmard (2002), Hopenhayn and Lohmann (1996) are two examples amongst many. These are applications of well-understood principal-agency problems and we ignore them here. We assume, in other words, that incentives can be put in place to ensure that the enforcer pursue his mission diligently. Our model fits better in the delegation as strategic commitment strand of the literature (Spulber and Besanko (1992)).

<sup>3</sup>By dual we mean that if under a budget-driven mission involving a budget  $Y$  leads to a realized emissions rate  $X$ , then specifying  $X$  as the target under a target-driven mission would lead to realized enforcement costs  $Y$ . This may explain why scholars setting up economic models of enforcement have paid little attention to the objective assigned to the enforcer. Between the two outlined here the most commonly incorporated in theoretical models of enforcement is probably

We show that in any setting involving more than one firm such a supposition is wrong. Mission matters.<sup>4</sup>

The essence of the story we are going to tell is as follows: firms facing a common enforcer find themselves in a game not just with the enforcer, but with each other. The nature of that interaction depends critically upon what the enforcer is trying to achieve (what we refer to as his ‘mission’). This paper analyses the impact of that strategic interaction on the outcomes and provides a basis for ranking alternative missions. An important conclusion is that their should be ‘horses for courses’ - the best mission to assign in a given enforcement setting will depend in predictable ways upon the nature of the enforcement environment and technology. As such the paper generates practical policy principles.

## 1.1 A Motivating Example

To understand the sort of effect that we are looking to focus on in the paper it is useful to have an example in mind. The story here will not precisely fit the analytic model presented later, but captures the spirit of what we are trying to do.<sup>5</sup>

Consider a setting in which there are some fixed number of firms and each makes a binary decision either to emit or not emit a unit of some forbidden pollutant. Emit corresponds with ‘violate’, not emit with ‘comply’. The enforcer costlessly observes the aggregate level of emissions (has an ambient measure of pollution flow) and so knows how many firms have chosen to violate, but not which ones. Finding that out - and rectifying it - requires a two stage inspection/enforcement program. First the enforcer visits firms sequentially. Each visit reveals whether or not that firm is compliant. If it is not the enforcer exerts some additional time/money/resource that the enforcer acts to maximize non-compliance subject to a budget constraint (examples include Heyes and Rickman (1999), Harrington (1988)). Some also assume a welfare objective (for example Malik (1990)). Heyes (2000) provides a survey of environmental enforcement models.

<sup>4</sup>Without providing a formal model Firestone notes this in his admirable study of USEPA enforcement practices: “An agency’s organizational mission ... may influence how and why the agency behaves and to what effect. These influences play out not only in the development and implementation of regulations, but in enforcement decisions as well” (Firestone 2002: 409).

<sup>5</sup>In the formal model firm’s emissions will be a continuous variable - we will allow for different degrees of non-compliance.

pursuing the matter - collecting evidence, litigating, administering a fine, returning the firm to compliance and so on.

Within this setting consider the implications of the alternative missions:

**Example 1** *A target-driven mission would tell the enforcer to achieve a specified level of compliance (ensure that no more than  $k$  firms are in violation) at least cost. So the enforcer visits firms – putting violators back into compliance – until his compliance target is achieved. A decision by one firm (call it firm 1) to violate **increases** the chance that a violation by firm 2 will be detected and penalized.<sup>6</sup> We say that non-compliance by firm 1 has a **positive** enforcement spillover on firm 2. Under standard assumptions this increased risk of detection makes non-compliance less attractive to firm 2, the compliance decisions of firms are **strategic substitutes**.*

**Example 2** *A budget-driven mission would tell the enforcer to minimize non-compliance subject to a budget constraint. So the agent visits firms at random - pursuing those that it finds to be in violation - until its enforcement budget is exhausted. The higher the proportion of inspections that lead to enforcement activity, the lower the probability that any particular firm will be subject to inspection. A decision by firm 1 to violate therefore **decreases** the probability that a violation by firm 2 will be detected. We say that non-compliance by firm 1 has a **negative** enforcement spillover on firm 2. Under standard assumptions this reduced probability of detection makes non-compliance more attractive to firm 2, so that the compliance decisions of firms are **strategic complements**.*

In these examples, a switch in mission alters qualitatively the nature of the strategic interaction amongst the firms, even though the underlying technology of compliance, inspection and enforcement remains unchanged.

In Example 2 each non-compliant firm benefits from safety in numbers. Others' non-compliance means that they can be expected to 'absorb' more enforcement

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<sup>6</sup>This should be obvious. Suppose the target is to ensure that only  $k$  of, say,  $N$  firms are left non-compliant. If there are initially  $v$  violators then the probability that any given violator will be caught is  $\frac{v-k}{v}$ , which is increasing in  $v$ .

resource, lowering the chance that the enforcer will get around to uncovering its wrongdoing. In Example 1, on the other hand, there is danger in numbers. The mission dictates that only a certain number of violators can be left in violation, so an increase in the number who choose initially to violate reduces the likelihood that any particular one of them will be one of the lucky ones.<sup>7</sup>

The existing literature on enforcement has neglected this strategic interaction. It is universally assumed in the existing literature that the enforcement objective is fixed. It is also very common to assume that the enforcer is interacting with a single firm. Either assumption effectively dismisses the issues that we investigate here.<sup>8</sup>

Our model can be seen as fitting into the wider set of models in which behavior is incentivized by rewarding on the basis of relative performance. These include tournaments (Lazear and Rosen (1981)) and contests (Tullock (1978)). A mission implicitly embodies a particular structure of expected pay-offs, sensitive to my performance but also the performance of others, and so puts regulated parties in a pseudo-tournament situation.

Of course, the precise incentives and interaction generated by alternative missions depends on the specifics of the enforcement setting. Section 2 develops a simple model to show how differences in enforcement spillovers under target-driven and budget-driven missions affect the regulatory outcome. We acknowledge that a more general mechanism design formulation could be used to explore the char-

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<sup>7</sup>AOL-Autos has as its number one tip for avoiding a speeding ticket finding a ‘pack’ of speeding cars to travel in. “If you’re within a pack of cars all going 10 mph over the limit, you’ve automatically improved your odds of not being the one that gets pulled over for a speeding ticket, even though you’re all technically speeding. The cop has to pick one car; if you are in a pack of cars its less likely to be you.” (AOL Autos 2007).

<sup>8</sup>There are occasional exceptions. That compliance performance of one firm could affect the enforcement intensity brought to bear upon others has been noted by Lear and Maxwell (1997), but they do not consider the issue of alternative objective functions. In a different setting Erard and Feinstein (1994) characterize the interdependence of income reporting decisions in an income tax compliance-enforcement game. Our model develops these themes further, and emphasizes the fact that the strategic interaction between firms’ compliance choices is conditioned by the enforcement mission.

acteristics of an ‘optimal’ mission, whilst not (for reasons of tractability) going down that route. Section 3 generalizes the argument and derives a criterion to rank alternative missions according to their efficacy in the presence of enforcement spillovers. Section 4 concludes.

## 2 A Model Incorporating Enforcement Spillovers

An enforcer is appointed to control the level of some anti-social activity. For concreteness, we regard this activity as illegal emission of some pollutant but the model could relate to almost any anti-social activity. There are  $N$  identical firms, each chooses a level of emissions simultaneously. Firm  $i$ ’s choice – its emission in excess of the permissible limit – is given by  $x_i \in [0, \hat{x}]$ . Here  $x_i$  is a measure of the firm’s non-compliance, with  $x_i = 0$  denoting complete compliance, and  $\hat{x}$  specifies some physical limit to the level of non-compliance.

The purpose of enforcement is to influence aggregate pollutant levels. It does this in two ways. The threat of detection influences pollution choices *ex ante*, whilst pollution levels can be pushed down *ex post* by the enforced abatement activity that follows prosecution.

Enforcement is costly so there is the usual trade-off between achieved pollution levels and enforcement expenditures. As we see below, the terms of this trade-off are sensitive to the mission pursued by the enforcer. We assume that the assigned mission is common knowledge and that the enforcer pursues it diligently.

Each firm finds being clean expensive and faces a cost function  $c(x_i)$ . This function is continuous and differentiable and has standard features:  $c'(x_i) < 0$  and  $c''(x_i) > 0$ . So other things equal each firm would choose a high value of  $x_i$ .

However, non-compliance carries the risk of prosecution and penalty. That risk depends upon the enforcement regime and the decisions of other firms.

There are a variety of ways in which we might sensibly model the process of enforcement (see Heyes (2000) for a survey of the approaches taken in the literature).

In a particular setting the appropriate approach will depend upon a variety of aspects of the nature of emissions, the physical environment, mechanics and

technologies of detection, the policy and legal ‘architecture’, and so on. But we don’t want to get bogged down in the particularities here - the set-up in this section is illustrative, the framework in Section 3 is definitive and allows for much more generality.

For current purposes we assume that enforcement has two stages: the first involves conducting inspections to identify non-compliers, the second involves pursuing/prosecuting firms found to be non-compliant and forcing them back into compliance. Each inspection costs the enforcer  $\phi_0$ . The cost of pursuing/prosecuting a firm found non-compliant is  $\phi_1(x)$  where  $\phi_1(0) = 0$  (compliant firms absorb no enforcement effort),  $\phi_1' > 0$  and  $\phi_1'' < 0$ . Without providing a micro-level description of the process of enforcement, we have in mind that the effort required to generate the evidence for and prosecute a large polluter will exceed that required for a small polluter.<sup>9</sup> Nothing substantive, however, rests on this and we can imagine situations where  $\phi_1$  would take other forms. Total enforcement cost for a firm with non-compliance level  $x_i$  is, then, given by  $k(x_i) = \phi_0 + \phi_1(x_i)$ ,  $k'(x_i) > 0$ .

We are going to focus on symmetric outcomes and the relationship between number of inspections,  $n$ , and the average level of non-compliance in the population,  $\bar{x}$ . If  $n \leq N$  firms are inspected randomly the probability that *a particular firm* will be picked for inspection is simply

$$r(\bar{x}) = \frac{n(\bar{x})}{N}, \quad (1)$$

and we will refer to  $r(\bar{x})$  as the *enforcement risk* faced by that representative firm.

If  $n(\bar{x})$  is increasing in  $\bar{x}$  - so that an increase in non-compliance by other firms increases the enforcement risk - we say that the *enforcement spillover* is positive.<sup>10</sup>

If  $n(\bar{x})$  is decreasing in  $\bar{x}$  - so that an increase in non-compliance by other firms decreases the enforcement risk - we say that the *enforcement spillover* is negative.

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<sup>9</sup>Penalties for large violators are bigger than those facing small (we will introduce a penalty function below) and it may be that the court would set a higher standard on the quality of evidence that it requires, or that the larger violator would engage in more obfuscation than its smaller counterpart. Further, recall that  $\phi_1$  incorporates the cost of putting the violator back into compliance and this might reasonably be thought to be increasing in the amount of ‘movement’ needed.

<sup>10</sup>For tractability, we treat  $n$  as a continuous variable. This saves mess.



A firm found to be in violation has to pay a fine and undertake whatever abatement activity is needed to return to compliance. The costs to the firm associated with these are captured in a single, composite ‘penalty’ function  $p(x)$ .<sup>11</sup> We will make the standard assumptions that  $p(0) = 0$  whilst  $p(x) > 0$ ;  $p'(x) > 0$  and  $p''(x) > 0, \forall x > 0$ .

As usual an individual firm’s choice involves a trade-off between the costs of compliance  $c(x_i)$  and the expected penalty  $r(\bar{x})p(x_i)$  from non-compliance. But importantly a particular firm’s point of view the risk of any non-compliance being penalized varies with the choices of other firms. As we focus on symmetric equilibria, we use  $(x_i, x_{-i}) \in R_+^2$  to denote the configuration where firm  $i$  chooses  $x_i$  and all firms other than  $i$  choose the identical value  $x_{-i}$ . Let  $r(x_i, x_{-i})$  denote the enforcement risk associated with this configuration. So firm  $i$  chooses  $x_i$  to minimize

$$V(x_i, x_{-i}) = c(x_i) + r(x_i, x_{-i})p(x_i), \quad (2)$$

taking  $x_{-i}$ , the (symmetric) choice of other firms as given. Letting  $V_1$  and  $V_2$  denote the partial derivatives of this function with respect to  $x_i$  and  $x_{-i}$  respectively, an interior minimum must satisfy

$$V_1(x_i, x_{-i}) \equiv c'(x_i) + p'(x_i)r(x_i, x_{-i}) + p(x_i)\frac{\partial r}{\partial x_i}(x_i, x_{-i}) = 0. \quad (3)$$

This implicitly defines a ‘reaction function’,  $R_i(x_{-i})$ , which tells firm  $i$ ’s optimal response to  $x_{-i}$ .<sup>13</sup> At a symmetric Nash equilibrium  $\mathbf{x}^* = \{x^*, x^*, \dots, x^*\}$ , we have  $V_1(x^*, x^*) = 0$ :

$$c'(x^*) + p'(x^*)r(x^*, x^*) + p(x^*)\frac{\partial r}{\partial x_i}(x^*, x^*) = 0. \quad (4)$$

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<sup>11</sup>This might also capture reputational or so-called ‘market losses’, or any other costs to a firm being found in violation of the regulation.

<sup>12</sup>The associated second-order condition for a minima is  $V_{11}(x_i, x_{-i}) > 0$  and we restrict attention to cases where that is met.

<sup>13</sup>Whether firms’s choices are strategic substitutes or complements, in the sense of Bulow, Geanakoplos and Klemperer (1985), depends on the sign of the cross-partial  $V_{12}$ . It is easy to show that choices are strategic substitutes if  $V_{12} > 0$ , and strategic complements if  $V_{12} < 0$ .

The equilibrium will be unique and ‘stable’ if the absolute value of the slope of the reaction function is less than 1.<sup>14</sup>

Clearly, there exists a wide range of models of compliance, with *ad hoc* assumptions about the objective function of the enforcer, that are consistent with the above enforcement setting. What we want to do now is explore the relationship between enforcement mission and outcomes and identify the characteristics of a “good” enforcement mission.

## 2.1 Target-driven mission

Suppose, first, that the enforcer is asked to get aggregate emissions down to some level  $\tau > 0$ . The aggregate level of pollution in any symmetric equilibrium is given by  $N\bar{x}$ . So if this exceeds  $\tau$ , the enforcer must prosecute enough firms to reduce the total level of pollution to  $\tau$ . Since each prosecution brings a firm back into compliance and reduces pollution by an amount  $\bar{x}$ , the required reduction in aggregate pollution,  $N\bar{x} - \tau$  necessitates

$$n^t(\bar{x}; \tau) = \frac{N\bar{x} - \tau}{\bar{x}} \quad (5)$$

inspections (we use the superscript  $t$  to denote that the target-setting mission is in play). At any symmetric outcome  $x_i = x_{-i} = \bar{x}$ , the probability of inspection for *any particular* firm is

$$r^t(x_i, x_{-i}; \tau) = \frac{N\bar{x} - \tau}{N\bar{x}} \quad (6)$$

if  $N\bar{x} \geq \tau$ , and zero otherwise. Using  $r_1^t$  and  $r_2^t$  to denote the partial derivatives of this enforcement risk with respect to  $x_i$  and  $x_{-i}$  we have

$$r_1^t = \frac{\tau}{(N\bar{x})^2} > 0, \quad (7)$$

and  $r_2^t = (N - 1)r_1^t > 0$ .

The last relation describes the nature of the spillover for this mission. From the perspective of firm  $i$ , an increase in non-compliance by other firms increases average

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<sup>14</sup>For this we require that  $|V_{12}| < |V_{11}|$  which corresponds to the standard stability assumption routinely made in models of this sort.

non-compliance. This compels the enforcer to increase the number of inspections in order to preserve the target  $\tau$ . In terms of our earlier terminology, the enforcement spillover is positive. In a two firm setting, for example, an increase in violation by firm A increases the enforcement risk faced by firm B and makes violation less attractive to the latter.

Under conditions that are easy to specify each firm's optimal choice is well-defined.<sup>15</sup> The unique symmetric equilibrium under this mission can be represented as  $(x^t, x^t)$ , where  $x^t$  satisfies

$$c'(x^t) + p'(x^t)r^t(x^t, x^t) + p(x^t)r_1^t(x^t, x^t) = 0. \quad (8)$$

Note that the equilibrium outcome varies with  $\tau$ , so that we have  $x^t = x^t(\tau)$ . It is easy to verify that  $x^t$  is increasing in  $\tau$ . In words, a less stringent target leads to an increase in the ex ante rate of non-compliance.<sup>16</sup>

## 2.2 Budget-driven mission

Consider an alternative mission in which the enforcer is given a fixed budget  $\beta > 0$  and told to get the level of pollution as low as possible subject to that budget constraint.<sup>17</sup>

Once again, consider a symmetric outcome. Given the average level of non-compliance  $\bar{x} = x$  at this outcome, the budget can finance at most

$$n^b(\bar{x}, \beta) = \frac{\beta}{k(\bar{x})} \quad (9)$$

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<sup>15</sup>For the optimal choice to be a minimum, it is sufficient that the elasticity of the penalty function exceeds  $\frac{1}{N}$  at the relevant point. Firms' choices are strategic substitutes if the elasticity of the penalty function at exceed  $\frac{2}{N}$  at any symmetric outcome. Details are in the Appendix.

<sup>16</sup>We have to distinguish between ex ante non-compliance, the level defined by this first-order condition, and the ex post non-compliance that will prevail after the enforcement program has run its course and some subset of violators have been pushed back into compliance by regulatory effort. The ex post rate of compliance here will be  $\tau$ .

<sup>17</sup>To keep things interesting we assume that the budget is binding - in other words that inspecting all firms is not feasible. Analytically  $\beta < Nk(0)$ .

inspections (and resulting pursuits/prosecutions). The implied probability of prosecution at this symmetric outcome is

$$r^b(x_i, x_{-i}) = \frac{\beta}{Nk(\bar{x})}. \quad (10)$$

The partial derivatives of this function are given by

$$r_1^b = -\frac{\beta k'(\bar{x})}{[Nk(\bar{x})]^2} < 0 \quad (11)$$

and  $r_2^b = (N - 1)r_1^b < 0$ . Again this second term is of interest. An increase in non-compliance by other firms increases the average level of non-compliance. Why is this so? Recall that prosecution cost is increasing in  $\bar{x}$ , so the increased expected burden-per-inspection on the enforcer's limited enforcement budget results in a reduction in the expected number of inspections. With this budget-driven mission, the enforcement spillover is negative. In a two firm setting, for example, an increase in violation by firm A makes violation more attractive to firm B.

Once again, under moderate conditions, each firm's optimal response is given by well-behaved reaction functions. The firm's choices can be described by a reaction function (which is well-behaved under moderate conditions - see Appendix) and are strategic complements if the elasticity of the penalty function is sufficiently high. The symmetric equilibrium under this mission,  $(x^b, x^b)$ , must satisfy

$$c'(x^b) + p'(x^b)r^b(x^b, x^b) + p(x^b)\frac{\partial r^b}{\partial x}(x^b, x^b) = 0. \quad (12)$$

The equilibrium outcome in this case depends on the enforcement budget. It is straight-forward to verify that  $x^b(\beta)$  is decreasing in  $\beta$ , so that a higher enforcement budget achieves greater ex ante compliance.

## 2.3 Comparing outcomes

How do the equilibria under these two missions compare in terms of compliance decisions and enforcement expenditure? Note that outcomes  $x^t(\tau)$  and  $x^b(\beta)$  vary with the chosen target  $\tau$  and budget  $\beta$  respectively, so that any comparison makes sense only for suitably calibrated pairs of values of these parameters.

One possible approach may be to choose values of these parameters so that the two alternative missions are somehow similar in terms of their enforcement pressure. Such calibration is not straightforward because the enforcement pressure functions also varies with firms' choices, which may differ across missions.

Consider an arbitrary budget-driven mission  $(b, \beta)$  and an arbitrary target-driven one  $(t, \tau)$ . Suppose under the former mission, the symmetric outcome  $x^b(\beta)$  obtains with  $n^b(x^b, \beta)$  prosecutions. We ask the following question: does there exists a value of  $\tau$  (call it  $\tau^*$ ) which, if set under target-driven mission  $(t, \tau)$  results in precisely  $n^b(x^b, \beta)$  prosecutions when firms choose  $x^b(\beta)$ ? For ranges of  $(x^b, \beta)$  where such  $\tau^*$  can be found, we have a functional relationship  $\tau^*(\beta)$ , with

$$r^t(x^b, \tau^*(\beta)) \equiv r^b(x^b, \beta). \quad (13)$$

The central question is, how does the outcome  $x^t(\tau^*)$  under mission  $(t, \tau^*(\beta))$  compare with that under mission  $(b, \beta)$ ? We have the following proposition.

**Proposition 1** *Let  $x^b(\beta)$  denote the outcome under budget-driven mission  $(b, \beta)$ . The same outcome can be achieved at lower enforcement cost under an appropriately calibrated target-driven mission  $(t, \tau)$ .*

A formal proof is provided in the Appendix, but a comparison of (8) and (12) is suggestive. The two missions differ in the nature of the enforcement externality. The target-driven mission generates a positive enforcement spillover which serves to enhance the incentive impact of any particular level of enforcement pressure. A budget-driven mission dilutes incentives, so ends up with higher realized enforcement costs for any particular compliance outcome.

### 3 A Generalization

While this comparison in the last Section illustrated the significance of enforcement spillovers for outcomes, the analysis was limited by the specificity of the missions and the particularities of the enforcement setting and the missions considered. Our aim in this Section is to make things much more general, and to establish as a

general result what we have just noted by example - namely that a ‘good’ mission in a particular setting will be one that generates, when interacted with the other elements of that setting, positive enforcement externalities.

As above we consider an enforcer with a mission to control the level of some anti-social activity. There are  $N$  identical firms and again each firm’s non-compliance choice is denoted by a real-valued variable  $x_i \in [0, \hat{x}]$ . Aggregate non-compliance is given by the  $N$ -dimensional vector  $\mathbf{x} = \{x_1, x_2, \dots, x_N\}$ . The purpose of enforcement is to influence  $\mathbf{x}$ .

The enforcer is issued with a mission to pursue. We consider any mission of the form  $(M, \mu)$ , where  $M$  describes any broad objective and  $\mu$  is a real-valued parameter associated with that objective. To relate this to the missions compared in the previous section,  $M$  might refer to ‘maximize compliance with given enforcement budget’, or ‘minimize enforcement cost of achieving some target level of compliance’, while  $\mu$  is the assigned target level or allocated budget. Going beyond those missions  $M$  might, for example, call for inspecting a fixed fraction  $\mu$  of the population of firms. It might also be capture some hybrid version of the target-driven/budget-driven cases, with a parameter capturing the ‘softness’ of the budget constraint.

As before firms face a choice between spending on compliance and the risk of being penalized for non-compliance. The enforcement environment faced by each firm can be described by an *enforcement pressure function*, which captures the probability that non-compliance will be detected and penalized. This depends on the mission in place (as well as the behavior of other firms). For firm  $i$ , write the enforcement pressure function under mission  $(M, \mu)$  as:

$$r_i^M(x_i, \mathbf{x}_{-i}; \mu).$$

In this formulation, the enforcement pressure on the firm depends on its own choice  $x_i$  but also on  $\mathbf{x}_{-i}$ , the vector of choices made by the  $N - 1$  other firms. We make no prior assumption about the effect of changes in  $\mathbf{x}_{-i}$ , as this can differ across missions. A mission generates negative spillovers if the enforcement pressure on

firm  $i$  is decreasing in another firm's – call it firm  $j$  – level of non-compliance:

$$\frac{\partial r_i^M}{\partial x_j} < 0.$$

The opposite sign describes a positive spillover.

We restrict attention again to symmetric cases, allowing us to drop the firm-specific subscript (so that  $r_i^M = r_j^M = r^M$ ). Two, that the effect of individual compliance choices on the enforcement pressure function is symmetric across firms (so that  $\frac{\partial r^M}{\partial x_i} = \frac{\partial r^M}{\partial x_j}$  for all  $i$  and  $j$ .) The latter assumption is natural in environments where, as in the previous section, individual choices affect enforcement intensity through the average level of non-compliance. Lastly, the enforcement pressure function is assumed to be smooth and differentiable.

First consider an individual firm's choices in such enforcement environments. The firm aims to maximize expected profits, given by a function of the form

$$\pi(x_i, r^M(x_i, \mathbf{x}_{-i})). \quad (14)$$

As greater enforcement intensity is associated with higher expected value of financial penalties, we assume this function is decreasing in its second argument,  $r$ . Each firm's profit varies with other firms' choices because of enforcement spillovers.<sup>18</sup>

To study the strategic interaction in firms' compliance choices, we make the standard assumption that firms choose their own compliance choices taking other firms' choices as given and consider symmetric Nash equilibria in the level of non-compliance. Define  $W(x_i, x_{-i})$  as the profit function for the typical firm when it chooses  $x_i$  and all other firms make a symmetric choice  $x_{-i}$ .<sup>19</sup> As defined,  $W$  is a function of just two arguments, the firm's own choice  $x_i$  and the symmetric choice  $x_{-i}$  made by other firms. We consider a firm's profit-maximizing choice of  $x_i$  given any arbitrary  $x_{-i}$ , focusing on environments in which the optimal choice is interior. Let  $W_1$  and  $W_2$  denote the partial derivatives of the profit function with respect to

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<sup>18</sup>In order to focus on the regulatory spillover, we abstract from any other linkages between firms. We do not, for example, consider the possibility that firms might interact in an imperfectly competitive product market such that they might have incentive to 'raise rivals costs' (Salop and Scheffman (1983)).

<sup>19</sup>That is, with a slight abuse of notation, we have  $\mathbf{x}_{-i} = \{x_{-i}, x_{-i}, \dots, x_{-i}\}$ .

these arguments. An interior solution must satisfy the first-order condition:

$$W_1(x_i, x_{-i}) = \frac{\partial \pi}{\partial x_i} + \frac{\partial \pi}{\partial r} \frac{\partial r^M}{\partial x_i} = 0. \quad (15)$$

If  $W_{11}$  is negative at this solution, the solution characterizes firm  $i$ 's best response to  $x_{-i}$ . The optimal choice defines a reaction function. As the enforcement spillover is sensitive to the enforcement mission  $(M, \mu)$ , so is the reaction function: we write  $R_i^M(x_{-i})$ . A Nash equilibrium is given by  $\{x_1^M, x_2^M, \dots, x_N^M\}$  where

$$x_i^M = R_i^M(x_{-i}^M; \mu) \quad \text{for all } i. \quad (16)$$

The superscript  $M$  highlights the feature that equilibrium outcome varies with the enforcement mission.

Given the assumed interiority of the optimal choices at the symmetric equilibrium, we have

$$W_1(x^M, x^M) = 0. \quad (17)$$

A sufficient condition for uniqueness is that the absolute value of slope of firms' reaction function is less than unity at any symmetric equilibrium. Formally, letting  $W_{12}$  be the second-order cross-partial of the function  $W$  then  $|W_{12}| < |W_{11}|$  is sufficient to ensure uniqueness, and we assume that this condition is satisfied.

Lastly, within a particular mission, the equilibrium outcome is sensitive to the choice of the parameter  $\mu$ . Implicit differentiation of set of first-order conditions suggests that  $x^M(\mu)$  is increasing (decreasing) in  $\mu$  if and only if  $W_{1\mu}$  is positive (negative).

### 3.1 Comparing outcomes under alternative missions

We say that two missions are equivalent in terms of their enforcement pressure if the implied risk of being penalized is equal under the two missions. To formalize this, consider any two missions,  $(A, \alpha)$  and  $(B, \beta)$ . The enforcement pressure under these missions depends parametrically on  $\alpha$  and  $\beta$ , and also varies with firms' choices  $\mathbf{x}$ . We ask if, for a given configuration of firms choices,  $\mathbf{x}$ , there exist



values  $\alpha$  and  $\beta$  such that the enforcement pressure functions are equi-valued,<sup>20</sup> and propose the following definition.

**Definition 1** *The enforcement pressure under two missions  $(A, \alpha)$  and  $(B, \beta)$  is equivalent for some profile of firms' choices,  $\mathbf{x}$ , if*

$$r^A(\mathbf{x}, \alpha) = r^B(\mathbf{x}, \beta).$$

We aim to compare outcomes under alternative missions that are equivalent in terms of their enforcement pressure but differ in their enforcement spillover. To elaborate, let  $\mathbf{x}^A(\alpha)$  denote the unique equilibrium outcome under mission  $(A, \alpha)$ . Consider another mission  $(B, \beta)$ , where by suitable choice of parameter value,  $r^A(\mathbf{x}^A, \alpha) = r^B(\mathbf{x}^A, \beta)$ . Now if  $\mathbf{x}^B(\beta)$  is the unique equilibrium under mission  $(B, \beta)$ , how does outcome  $\mathbf{x}^B(\beta)$  compare with  $\mathbf{x}^A(\alpha)$ ? Indeed, as we consider only symmetric equilibria, each outcome can be characterized by the choice of the typical firm under that mission. Our question reduces to: How do we rank  $x^A(\alpha)$  and  $x^B(\beta)$ ?

If the equilibrium outcome is sensitive to enforcement spillovers, it should not surprise us that missions that differ in enforcement spillovers generate distinct outcomes even when the enforcement pressure is equivalent. Our aim, then, is to examine if outcomes vary with the nature of the spillover in a systematic fashion. We have the following proposition.

**Proposition 2** *Consider two missions  $(A, \alpha)$  and  $(B, \beta)$ , with unique symmetric outcomes  $\mathbf{x}^A(\alpha)$  and  $\mathbf{x}^B(\beta)$ . If these missions are equivalent in terms of enforcement pressure at outcome  $\mathbf{x}^A$ , then if*

$$\frac{\partial r^A(\mathbf{x}^A, \alpha)}{\partial x_{-i}} > \frac{\partial r^B(\mathbf{x}^A, \beta)}{\partial x_{-i}} \quad (18)$$

*it must be that  $x^B(\beta) > x^A(\alpha)$ .*

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<sup>20</sup>Of course, two arbitrarily chosen missions could differ so much that such equivalence never holds, regardless of the values of  $\alpha$ ,  $\beta$  and  $\mathbf{x}$ . We confine attention to mission-pairs that are not inconsistent in this sense. Such a restriction should not trouble the reader.

A formal proof of this proposition is in the Appendix.<sup>21</sup>

The Proposition says that, relatively speaking, if a mission generates strong (positive) enforcement spillover, it serves to enhance the compliance incentives associated with a given level of enforcement pressure. In the comparison described in the previous section target-driven missions, which generate positive spillovers, induced more compliance than budget-driven alternatives with negative spillovers. Proposition 2 allows for greater generality: it is not restricted to the case where missions under comparison generate spillovers of differing signs. It is the relative ordering of enforcement spillovers that is key in determining the relative efficacy of alternative enforcement missions. As long as enforcement spillovers can be ranked, so can the outcome: any given level of spending on enforcement will generate a correspondingly higher level of compliance through missions that have stronger enforcement spillovers.

In general the size and sign of spillovers will depend upon the combination of the fundamental elements of the enforcement setting, and the mission according to which the agency embedded in that setting behaves. This provides for the notion that particular missions may be particularly suited (be ‘good’) in particular contexts. Further exploration of these context-specificities provides the basis for further work.

Alternatively we can fix performance for the purpose of comparison. Corollary 1 highlights the fact that the expected enforcement cost of achieving a particular compliance outcome is lower for missions that induce positive enforcement spillovers.

**Corollary 1** *Consider two missions  $(A, \alpha)$  and  $(B, \beta)$  that satisfy the inequality in (18). Then any given outcome  $\mathbf{x}$  can be achieved at lower enforcement cost under mission A than under mission B.*

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<sup>21</sup>Note that the proposition requires us to compare the value of the derivatives only at a specific points  $(x^A, \alpha)$  and  $(x^A, \beta)$  respectively. This provides the weakest necessary condition for the proposition to hold. In the preceding examples one of these derivatives was positive and the other negative, so the required inequality held everywhere.

Proposition 2 and its corollary allow us to assess the efficacy of other missions too. Consider, for instance, a mission that calls for an inspection of an exogenously-fixed fraction of firms. By design such missions imply no enforcement spillovers. However, our argument tells us that this will be dominated, in term of efficacy, by missions that create positive enforcement spillovers.

Various elements of the enforcement ‘setting’, combined with the mission, will serve to determine the size and sign of the spillovers (recall Example 1 and Example 2 set out in the Introduction). Are inspections sequential? What is the order of moves between the agency and firms, and amongst firms? Is it inspection that is costly, or is it the enforcement against a firm shown to be non-compliant absorb extra resource? Is inspecting a non-compliant firm more costly than inspecting a compliant one? Does the agency have access to a measure of aggregate compliance rates in the population (such as an ambient measure of pollution in an environmental setting) before deciding on the intensity with which to progress a firm-by-firm inspection/enforcement programme? But amongst this wide set of ways in which particular enforcement settings might vary the analysis here allows us to understand the principles according to which particular combinations of missions and enforcement settings can be evaluated – the basis on which we can distinguish ‘good’ ones from less good ones *in a particular context*.

When characterizing strategic interaction it is natural to think in terms of strategic complementarity or substitutability, so it is natural to ask how they fit in with the analysis and results here. (The terminology of strategic substitutes and complements was introduced by Bulow, Geanakoplos and Klemperer (1985)). Strictly speaking strategic complementarity and spillover are not the same thing. Spillovers describe interactions in payoffs, while strategic complementarity refers to interactions in strategies. Mathematically the difference is straightforward: spillovers refer to the sign of the partial derivative of one firm’s objective function with respect to a rival’s choice, while strategic complementarity is determined by the sign of the second cross-partial derivative of the objective function. In the particular cases that we have explored – analyzing enforcement/compliance

games underpinned by stylized inspection ‘technologies’ of various different types<sup>22</sup> – we have found that negative (positive) spillovers invariably go together with the non-compliance game played between firms being one in strategic complements (substitutes). It is intuitive why this should typically be the case, and whilst we cannot rule out the possibility of the perverse pairing it is straightforward to develop conditions that ensure a correspondence between the two. The Appendix does so for the examples discussed in Section 2.

## 4 Conclusions

We have shown that outcomes - actual patterns of compliance achieved - depend not just on the amount of money spent on enforcement but also on the specific mission given to the enforcer.

Different missions can generate qualitatively different types of strategic interaction amongst firms. Those that generate positive enforcement spillovers are preferable to those that generate negative – or positive but smaller – spillovers.<sup>23</sup> In plausible settings this suggests a preference for target-driven missions over budget-

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<sup>22</sup>Not just those reported here, but the numerous others we have experimented with in developing the framework in this paper.

<sup>23</sup>The ambiguity of the direction of the spillover – and its sensitivity to the agency’s objective function – has been noted in the context of a model of tax reporting and verification by Heyes (2001). He notes (page 224) that: “In Erard and Feinstein (1994) the tax agency is assumed to have a fixed monitoring budget. Optimal policy involves concentrating verification on low-income reports, which have a greater chance of being under-reports. An increase in the proportion of honest taxpayers reduces the fraction of low income reports and makes any such report more likely to be audited, so an increase in the proportion of taxpayers who are honest has the effect of encouraging dishonest taxpayers to cheat by less. The honest impose a form of (negative) externality on the dishonest.” In contrast, in his own model the agency chooses an optimal level of resource to devote to verification. That level is decreasing in the number of dishonest in the population (since a reduction in the propensity to dishonest reduces the likelihood that an inspection will score a ‘hit’) such that “... the presence of an additional honest firm induces an incremental cut in monitoring intensity which advantages the dishonest. Growth in the propensity to honesty in the population will cause the equilibrium behavior of the dishonest to get worse.” (Heyes (2001: p. 227)).

driven ones.

While we have explored strategic linkages through the mission, other features of enforcement regimes might generate linkages too. Heather Eckert at Alberta University is using GIS methods to investigate spatial correlations in inspection patterns. One stylized story to hold in mind there is that an inspector who has reason to drive to locale  $X$  to visit some firm may have a tendency to visit other firms nearby “whilst he is in the neighborhood” (Eckert, personal correspondence).<sup>24</sup>

The model suggests clear policy principles. In determining missions think about how alternative missions interact with the particularities of the enforcement setting to generate strategic inter-dependence amongst regulatory parties. Opt for one that generates positive such inter-dependences - generate what we might refer to as ‘races to the top’ (high performance) amongst firms, rather than ‘races to the bottom’.

The spirit of our enquiry suggests a more fundamental mechanism design problem: the issue of an optimal mission, and indeed whether delegation of enforcement activity is optimal.<sup>25</sup> We do not address this larger problem in this paper, taking as given that most enforcement activity is delegated to specialist agencies.

The extent to which better-designed missions can improve the outcome will, of course, depend upon the setting. It is reasonable to conjecture that the benefits will be greatest where the number of regulated parties is comparatively small. Indeed the strategic interaction matters less as the number of firms becomes large (or as each firm becomes ‘small’ in the formal sense) – the type of mission matters more in oligopolistic than more competitive sectors. This may further the case for compartmentalizing the activities of enforcement to a more local level to allow, with appropriate choice of mission given to local enforcers, firms to be put ‘in

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<sup>24</sup>Decker and Pope (2005) provide empirical evidence from the US that the compliance behavior of firms is increasing in the compliance behavior of other firms in their sector. This would be consistent with the notion of strategic complementarity between firms’ compliance behavior when the enforcement agency has a fixed budget.

<sup>25</sup>Though it is reasonable to think that in the sorts of setting we are considering delegation is inevitable – the Prime Minister cannot police every section of highway and every effluent pipe on his own!

competition' with one another.<sup>26</sup>

In order to focus on the main message we have made a number of simplifying assumptions along the way, and there are a number of directions in which the analysis might usefully be extended. Firms might be heterogenous in compliance costs (implying asymmetric equilibria), fine levels might be endogenised, inspections might be less-than-fully informative of the true compliance state of the inspected firm, and so on. Whilst these would complicate the modeling we would not expect them to change the basic lessons from the paper.

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<sup>26</sup>'Local' could refer to the usual geographical notion or to, for example, a tighter delineation of enforcement activities by industry or activity. The debate about the appropriate boundaries to place around the activities of the various regulatory agencies (state versus federal, for example) has been particularly keen in the US and EU.

## A Appendix - probably not for publication

### A.1 Details of formal arguments in Section 2

Firm  $i$  chooses  $x_i \in [0, \hat{x}]$  to minimize the continuous function

$$V(x_i, x_{-i}) = c(x_i) + p(x_i)r(x_i, x_{-i}).$$

The solution is interior for a given  $x_{-i}$  as long as  $\lim_{x_i \rightarrow 0} V_1(x_i, x_{-i}) < 0$  and  $\lim_{x_i \rightarrow \hat{x}} V_1(x_i, x_{-i}) > 0$ , where  $V_1$  is the partial derivative with respect to  $x_i$ . A sufficient condition for the first inequality is that  $c'(0) + p'(0) < 0$ . In what follows, we assume this to hold. An interior minimum must satisfy the first-order condition

$$V_1(x_i, x_{-i}) \equiv c'(x_i) + p'(x_i)r(x_i, x_{-i}) + p(x_i)\frac{\partial r}{\partial x_i}(x_i, x_{-i}) = 0.$$

As  $c' < 0$  the first-order condition requires that  $p'r + pr_1 > 0$  at the optimum where  $r_1$  is the partial derivative of  $r$  with respect to  $x_i$ . If  $r_1 > 0$  this requirement is straightforward. If  $r_1 < 0$ , we require that

$$\frac{p'}{p} > -\frac{r_1}{r},$$

or that the elasticity of the penalty function exceed the (absolute value of) elasticity of the enforcement pressure function with respect to a firm's own choice.

A sufficient condition for this critical point to be a minima is that the second derivative  $V_{11}(x_i, x_{-i})$  be positive. Formally,

$$V_{11} = c'' + pr_{11} + 2r_1p' + rp'' > 0.$$

As long as these conditions are satisfied, there exists a well-defined reaction function  $R_i(x_{-i})$  that represents firm  $i$ 's optimal response to the symmetric choices made by other firms.

Define

$$V_{12} = pr_{12} + p'r_2.$$

The slope of a firm's reaction function (to the symmetric choice of others) is given by

$$\frac{dx_i}{dx_{-i}} = \frac{-V_{12}}{V_{11}}.$$

As  $V_{11} > 0$  at the minima, the reaction function is upward sloping (a case of strategic complementarity) if  $V_{12} < 0$ , and downward sloping (strategic substitutes) if  $V_{12} > 0$ .

At a symmetric Nash equilibrium  $\mathbf{x}^* = \{x^*, x^*, \dots, x^*\}$ , we have  $V_1(x^*, x^*) = 0$ , or equivalently

$$c'(x^*) + p'(x^*)r(x^*, x^*) + p(x^*)\frac{\partial r}{\partial x_i}(x^*, x^*) = 0.$$

The equilibrium will be unique and 'stable' if the absolute value of the slope of the reaction function is less than 1. For this we require that  $|V_{12}| < |V_{11}|$ .

### A.1.1 Target-driven mission

For this case

$$r^t(x_i, x_{-i}; \tau) = \frac{N\bar{x} - \tau}{N\bar{x}}$$

if  $N\bar{x} \geq \tau$ , and zero otherwise. Recall that firm  $i$  minimizes

$$c(x_i) + p(x_i)r^t(x_i, x_{-i}).$$

Consider any positive target  $\tau < N\hat{x}$ . If aggregate pollution is within the target  $\tau$ , the probability of inspection is zero, and with  $c' < 0$  there is an incentive for every firm to pollute more. In the aggregate, emissions must rise to exceed the target. Abatement activity on prosecution then restores pollution to the target level. This establishes interiority of the optimum.

The first order condition for the an interior optimum for the level of emissions is

$$c' + r^t p' + p r_1^t = 0.$$

For  $N\bar{x} \geq \tau$ , the partial derivative of the enforcement pressure function is

$$r_1^t = \frac{\tau}{(N\bar{x})^2} > 0.$$



A sufficient second-order condition for a minimum is that

$$V_{11} = c'' + r^t p'' + pr_{11}^t + 2r_1^t p' > 0, \text{ where}$$

$$r_{11}^t = \frac{-2\tau}{(N\bar{x})^3} < 0.$$

As

$$pr_{11}^t + 2r_1^t p' = \frac{2\tau}{(N\bar{x})^2} \frac{p}{\bar{x}} \left[ \frac{p'}{p} \bar{x} - \frac{1}{N} \right],$$

this holds, at least in the neighborhood of the symmetric equilibrium, if the elasticity of the penalty function is not smaller than  $\frac{1}{N}$  at that point. We assume this to be the case. With symmetric choices

$$V_{12} = r_{12} p + r_2 p' = \frac{(N-1)\tau}{(N\bar{x})^2} \frac{p}{\bar{x}} \left[ \frac{p'}{p} \bar{x} - \frac{2}{N} \right],$$

using the facts that  $r_2^t = (N-1)r_1^t > 0$  and  $r_{12} = (N-1)r_{11}^t$ . Thus,  $V_{12}$  is positive – a case of strategic substitutes – as long as the elasticity of the penalty function is greater than  $\frac{2}{N}$ .

A symmetric equilibrium under this mission is given by  $(x^t, x^t)$ , where

$$c'(x^t) + p'(x^t)r^t(x^t, x^t) + p(x^t)\frac{\partial r^t}{\partial x}(x^t, x^t) = 0.$$

The requirement for uniqueness and stability, that  $|V_{11}| > |V_{12}|$ , amounts to

$$c'' + p\frac{-2\tau}{(N\bar{x})^3} + 2\frac{\tau}{(N\bar{x})^2}p' + \left(1 - \frac{\tau}{N\bar{x}}\right)p'' > p\frac{-2(N-1)\tau}{(N\bar{x})^3} + \frac{(N-1)\tau}{(N\bar{x})^2}p'.$$

This holds if  $c'' + \left(1 - \frac{\tau}{N\bar{x}}\right)p''$  is sufficiently large relative to the elasticity of the penalty function.

It is easy to check that the equilibrium level of non-compliance is increasing in  $\tau$ . A semi-formal proof runs as follows.<sup>27</sup> The shift in a reaction function  $R_i(x_{-i}, \tau)$  when  $\tau$  changes is given by

$$\frac{dR_i(\tau)}{d\tau} = -\frac{V_{1\tau}}{V_{11}}.$$

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<sup>27</sup>A formal proof requires us to solve  $N$  equations, each representing the total differential of a first-order condition. This is straightforward to check but tedious to report.

We have  $V_{11} > 0$ , so that the  $R_i^t(\tau)$  is increasing in  $\tau$  if and only if  $V_{1\tau}$  is negative.

$$V_{1\tau} = p'r_\tau + pr_{1\tau} = \frac{1}{N\bar{x}} \frac{p}{\bar{x}} \left[ -\frac{p'}{p}\bar{x} + \frac{1}{N} \right].$$

$V_{1\tau}$  is negative if the elasticity of the penalty function exceeds  $\frac{1}{N}$ , which we have assumed above. An increase in  $\tau$  shifts the reaction function outwards. At any symmetric equilibrium  $(x^t(\tau), x^t(\tau))$ , it must be that  $x^t(\tau)$  is increasing in  $\tau$ .

### A.1.2 Budget-driven mission

For this case

$$r^b(x_i, x_{-i}, \beta) = \frac{\beta}{Nk(\bar{x})}.$$

so that

$$r_1^b = -\frac{\beta k'(\bar{x})}{[Nk(\bar{x})]^2} < 0,$$

and

$$r_{11}^b = -\frac{\beta[kk'' - 2(k')^2]}{(Nk(\bar{x}))^3} > 0.$$

The firm  $i$  minimizes

$$c(x_i) + p(x_i)r^b(x_i, x_{-i}).$$

As long as we assume  $c'(0) + p'(0) < 0$ , a firm's optimal choice is bounded away from zero. The first-order condition for the an interior optimum is

$$c' + r^b p' + pr_1^b = 0.$$

As  $c' < 0$ , at the optimum we require  $r^b p' + pr_1^b > 0$ , which implies that

$$\frac{p'}{p} > \frac{1}{N} \frac{k'}{k},$$

or that the elasticity of the penalty function exceeds  $\frac{1}{N}$  times the elasticity of the average enforcement cost function.

A sufficient second order condition is that

$$V_{11} = c'' + p''r^b + pr_{11}^b + 2r_1^b p' > 0.$$

This holds if, say,  $c'' + rp''$  is large enough.

The firm's choices are strategic complements if the elasticity of the penalty function is sufficiently high. With symmetric choices

$$V_{12} = r_{12}^b p + r_2^b p'$$

must be negative. Using the facts that  $r_2^b = (N-1)r_1^b$  and  $r_{12}^b = (N-1)r_{11}^b > 0$  we require

$$\frac{p'}{p} > 2\frac{k'}{Nk} - \frac{k''}{Nk'}.$$

We assume this to be the case.

The symmetric equilibrium under this mission,  $\{x^b, x^b, \dots, x^b\}$ , must satisfy

$$c'(x^b) + p'(x^b)r^b(x^b, x^b) + p(x^b)\frac{\partial r^b}{\partial x}(x^b, x^b) = 0.$$

The requirement that  $|V_{11}| > |V_{12}|$  amounts to

$$c'' + p''r^b + 2r_1^b p' + r_{11}^b p > r_2^b p + r_{12}^b p'$$

or

$$c'' + p''r^b > (N-3)r_1^b p + (N-2)r_{11}^b p'.$$

Finally,  $R_i^t(\beta)$  is decreasing in  $\beta$  if and only if  $V_{1\beta}$  is positive.

$$V_{1\beta} = p'r_\beta + pr_{1\beta} = \frac{p}{Nk} \left[ \frac{p'}{p} - \frac{1}{N} \frac{k'}{k} \right],$$

which is positive by earlier assumptions. An increase in  $\beta$  shifts each reaction function inwards, so that at symmetric equilibria  $x^b$  is decreasing in  $\beta$ .

*Proof of Proposition 1:* If the symmetric outcome  $x^b(\beta)$  obtains under mission  $(b, \beta)$  with  $n^b(x^b, \beta)$  inspections, by construction we must have

$$n^b(x^b, \beta)k(x^b(\beta)) \equiv \beta. \tag{A.1}$$

Note that, ex-post, after prosecuted firms are brought back into compliance, aggregate pollution falls to  $[N - n^b(x^b, \beta)] x^b(\beta)$ .

Consider  $\tau^*(\beta)$ , the target level which under a target-driven mission results in precisely  $n^b(x^b, \beta)$  prosecutions, or that

$$r^t(x^b, \tau^*(\beta)) \equiv r^b(x^b, \beta), \tag{A.2}$$

and let  $x^t(\tau^*)$  be the outcome under mission  $(t, \tau^*(\beta))$ .

To compare the compliance outcome and enforcement cost under these two missions, consider the first-order condition (8) with (12) in the text, setting  $\tau = \tau^*(\beta)$  in the former case. Equation (8) for the equilibrium under mission  $(t, \tau^*)$  can be written as

$$c'(x^t(\tau^*)) + p'(x^t(\tau^*))r^t(x^t(\tau^*), \tau^*) + p(x^t(\tau^*))\frac{\partial r^t}{\partial x}(x^t(\tau^*)) = 0. \quad (\text{A.3})$$

The equilibrium under mission  $(b, \beta)$  must satisfy

$$c'(x^b) + p'(x^b)r^b(x^b, \beta) + p(x^b)\frac{\partial r^b}{\partial x}(x^b) = 0. \quad (\text{A.4})$$

Given that  $p(x) > 0$ , and that  $\frac{\partial r^t}{\partial x} > 0 > \frac{\partial r^b}{\partial x}$ , these conditions are both satisfied only if

$$c'(x^b) + p'(x^b)r^b(x^b, \beta) > c'(x^t(\tau^*)) + p'(x^t(\tau^*))r^t(x^t(\tau^*), \tau^*). \quad (\text{A.5})$$

By construction,  $r^b(x^b, \beta) = r^t(x^b, \tau^*)$ , so that the last inequality requires

$$c'(x^b) + p'(x^b)r^t(x^b, \tau^*) > c'(x^t(\tau^*)) + p'(x^t(\tau^*))r^t(x^t(\tau^*), \tau^*). \quad (\text{A.6})$$

Recall that  $r^t$  is increasing in  $x$  (see (7) in the text), and both  $c'$  and  $p'$  are increasing (as we assumed  $c'' > 0$  and  $p'' > 0$ ). The above inequality can hold only if  $x^t(\tau^*) < x^b$ . In words, the calibrated target-driven mission generates better compliance.

Further, if  $x^t(\tau^*) < x^b$ , then  $n^t(x^t(\tau^*), \tau^*) < n^t(x^b, \tau) \equiv n^b(x^b, \beta)$ . Also as  $k(x)$  is increasing, we must have

$$n^t(x^t(\tau^*))k(x^t(\tau^*)) < n^b(x^b, \beta)k(x^b) = \beta. \quad (\text{A.7})$$

In words, the target-driven mission achieves greater compliance at lower enforcement cost. Given that higher enforcement budgets can only deliver better outcomes, our claim goes through.

## A.2 Formal arguments associated with the general model in Section 3

$W(x_i, x_{-i})$  denotes a typical firm profit when it chooses  $x_i$  and all other firms make the symmetric choice  $x_{-i}$ ;  $W_1^i$  and  $W_2^i$  are the partial derivatives with respect to

these arguments. The firm's profit-maximizing choice of  $x_i \in [0, \hat{x}]$  is interior if

$$\lim_{x_i \rightarrow 0} W_1(x_i, x_{-i}) > 0 \text{ and } \lim_{x_i \rightarrow \hat{x}} W_1(x_i, x_{-i}) > 0 .$$

The interior maximum must satisfy the first-order condition:

$$W_1(x_i, x_{-i}) = \frac{\partial \pi}{\partial x_i} + \frac{\partial \pi}{\partial r} \frac{\partial r^M}{\partial x_i} = 0.$$

A sufficient condition for this to be a local maximum is that

$$W_{11} \equiv \frac{\partial^2 \pi}{\partial x_i^2} + 2 \frac{\partial^2 \pi}{\partial r \partial x_i} \frac{\partial r^M}{\partial x_i} + \frac{\partial \pi}{\partial r} \frac{\partial^2 r^M}{\partial x_i^2} + \frac{\partial^2 \pi}{\partial r^2} \left( \frac{\partial r^M}{\partial x_i} \right)^2 < 0.$$

At a symmetric Nash equilibrium, where all firms choose the same  $x_i$  (call it  $x^M$ ), we have

$$W_1(x^M, x^M) = 0 \text{ for all } i.$$

Further, uniqueness holds when the cross-partial capturing the effects of a symmetric change in all rivals' choices

$$W_{12} \equiv \frac{\partial^2 \pi}{\partial r \partial x_i} \frac{\partial r^M}{\partial x_{-i}} + \frac{\partial \pi}{\partial r} \frac{\partial^2 r^M}{\partial x_i \partial x_{-i}} + \frac{\partial^2 \pi}{\partial r^2} \frac{\partial r^M}{\partial x_i} \frac{\partial r^M}{\partial x_{-i}}$$

is less than  $W_{11}$  for all firms.

*Proof of Proposition 2:* As  $\mathbf{x}^A(\alpha) = (x_i^A(\alpha), x_{-i}^A(\alpha))$  denotes the symmetric equilibrium under mission  $(A, \alpha)$ ,  $x_i^A$  must be the optimal response to the  $\mathbf{x}_{-i}^A$ . If so,  $x_i^A$  satisfies the first-order condition

$$W_1^A(\mathbf{x}^A, \alpha) \equiv \frac{\partial \pi}{\partial x_i}(x_i^A) + \frac{\partial \pi}{\partial r}(r^A) \frac{\partial r^A}{\partial x_i}(\mathbf{x}^A, \alpha) = 0. \quad (\text{A.8})$$

From the assumed equivalence of enforcement pressure for the missions at  $\mathbf{x}^A$

$$r^A(\mathbf{x}^A, \alpha) = r^B(\mathbf{x}^A, \beta). \quad (\text{A.9})$$

From the assumed ordering of spillovers, and recalling that the externality is symmetric,

$$\frac{\partial r^A}{\partial x_i}(\mathbf{x}^A, \alpha) > \frac{\partial r^B}{\partial x_i}(\mathbf{x}^A, \beta). \quad (\text{A.10})$$

Given that profit is decreasing in enforcement pressure, (A.8), (A.9) and (A.10) together imply that

$$\frac{\partial \pi}{\partial x_i}(x_i^A) + \frac{\partial \pi}{\partial r}(r^B) \frac{\partial r^B}{\partial x_i}(\mathbf{x}^A, \beta) \equiv W_1^B(\mathbf{x}^A, \beta) > 0. \quad (\text{A.11})$$

Also, as  $\mathbf{x}^B(\beta)$  is the equilibrium under mission  $(B, \beta)$ ,

$$W_1^B(\mathbf{x}^B, \beta) = 0. \quad (\text{A.12})$$

Relations (A.11) and (A.12) compare the value of the function  $W_1^B$  at two distinct points,  $\mathbf{x}^A$  and  $\mathbf{x}^B$ . This function takes the value zero at  $\mathbf{x}^B$  and is positive at  $\mathbf{x}^A$ , so its total differential at  $\mathbf{x}^B$  must be positive for  $dx_i = x_i^A - x_i^B$  and  $dx_{-i} = x_{-i}^A - x_{-i}^B$ . If so

$$W_{11}^B dx_i + W_{12}^B dx_{-i} > 0. \quad (\text{A.13})$$

The proof of the proposition is by contradiction. If  $x^A(\alpha) > x^B(\beta)$ , we have  $dx_i = x_i^A - x_i^B$  and  $dx_{-i} = x_{-i}^A - x_{-i}^B$  both positive and, by symmetry of the two Nash equilibria, equal. Recall that  $W_{11}$  is negative and that  $|W_{12}| < |W_{11}|$  by Assumption 1, so that the total differential is necessarily negative, contradicting (A.13).

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