

## Soliton interaction in a fiber ring laser

D. Y. Tang, B. Zhao, and L. M. Zhao

*School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore*

H. Y. Tam

*Department of Electrical Engineering, Hong Kong Polytechnic University, Hong Kong, China*

(Received 17 December 2004; revised manuscript received 4 May 2005; published 19 July 2005)

We have experimentally investigated the soliton interaction in a passively mode-locked fiber ring laser and revealed the existence of three types of strong soliton interaction: a global type of soliton interaction caused by the existence of unstable cw components, a local type of soliton interaction mediated through the radiative dispersive waves, and the direct soliton interaction. We found that the appearance of the various soliton operation modes observed in the passively mode-locked fiber soliton lasers are the direct consequences of these three types of soliton interactions. The soliton interaction in the laser is further numerically simulated based on a pulse tracing technique. The numerical simulations confirmed the existence of the dispersive-wave-mediated soliton interaction and the direct soliton interaction. Furthermore, it was shown that the resonant dispersive-wave-mediated soliton interaction in the laser always has the consequence of causing random irregular relative soliton movement and the experimentally observed states of bound solitons are caused by the direct soliton interaction. In particular, as the solitons generated in the laser could have a profile with long tails, the direct soliton interaction could extend to a soliton separation that is larger than 5 times the soliton pulse width.

DOI: [10.1103/PhysRevE.72.016616](https://doi.org/10.1103/PhysRevE.72.016616)

PACS number(s): 42.81.Dp, 42.55.Wd, 42.60.Fc, 42.65.Re

### I. INTRODUCTION

Due to their intrinsic stability, optical solitons have been proposed to use as an information carrier for long-distance fiber optic communications and optical signal processing. A major limitation for soliton applications is the soliton interaction. To understand the nature and consequence of the soliton interaction, extensive theoretical and experimental investigations have been carried out [1–9]. In the framework of solitons described by the nonlinear Schrödinger equation (NLSE), two sorts of soliton interactions have been revealed: the direct soliton interaction and the long-range dispersive-wave-mediated soliton interaction. Gordon studied the direct soliton interaction in optical fibers [1]. He found that solitons exert forces on their neighbors if they are closely spaced. Either attractive or repulsive interactions can result depending on the relative phase difference between the solitons, and the strength of the interaction decreases exponentially with the soliton separation. Such a direct soliton interaction was first experimentally verified by Mitschke and Mollenauer [2]. The direct soliton interaction can be reduced in transmission links by the use of bandpass filters or fast saturable absorbers [3,4]. Apart from the direct soliton interaction, Smith and Mollenauer also reported the observation of a long-range soliton interaction caused by the existence of dispersive waves [5]. Compared with the direct soliton interaction, this long-range soliton interaction is soliton phase independent.

A real long-distance soliton fiber optic communication system also involves soliton losses and periodic amplifications. Therefore, the dynamics of solitons in the system is not described by the NLSE, which is integrable and describes only conservative systems, but by the Ginzburg-Landau equation (GLE), where a soliton is formed not only as a result of the balanced interaction between the fiber Kerr nonlinearity and dispersion, but also as a result of the balance

between fiber losses and gain generated by the fiber amplifiers. As a soliton propagating in the system periodically experiences loss and amplification, dispersive waves with discrete spectra are generated, which resonantly draw energy from the soliton. For a soliton to survive in the system, the optical amplifier gain must also balance this dispersive-wave emission loss of the soliton. Socci and Romagnoli have analyzed the soliton interaction induced by these resonant dispersive waves [6]. They have shown that resonant dispersive waves could result in a long-range soliton interaction as well as enhance the direct soliton interaction. Either quasibound states of solitons or random wiggling of the soliton positions could be caused by the resonant dispersive-wave-mediated soliton interaction. Afanasjev and Akhmediev [7] and Malomed [8,9] have also theoretically investigated interactions between the solitons described by the GLE. They have found that bound states of solitons with fixed, discrete soliton separations could be formed under a direct interaction between solitons.

In certain sense a passively mode-locked soliton fiber ring laser mimics a miniature soliton fiber optic communication system. Study of the soliton propagation and interaction in the laser cavity therefore gives direct insight into soliton interactions in long-haul optic communication systems. So far the soliton operation of passively mode-locked fiber lasers has been extensively investigated [10–15]. Various modes of soliton operation such as soliton bunching [10], stable randomly spaced soliton distribution [11], harmonic mode locking [12], and transient stochastic soliton evolutions have been experimentally observed. Recently, a state of twin-pulse solitons has also been experimentally revealed in the laser [13]. Theoretical studies of the possible physical mechanisms for the formation of the various soliton operation modes have also been carried out; e.g. Kutz *et al.* have theoretically investigated the effect of laser gain depletion and recovery on

the soliton distribution in the lasers [14]. Pilipetskii *et al.* studied the fiber acoustic effect on the soliton interaction in a fiber soliton laser [15]. There is no doubt that these effects always exist in a fiber soliton laser and affect the soliton dynamics. However, how strong are their influences? Do they play a dominating role on the soliton dynamics in a laser? Does any other soliton interaction exist, which plays an even stronger role in the soliton operation of a laser? And how are these different soliton operation modes formed in a laser? All these questions have so far not been clearly addressed. We believe to clarify them is not only important for understanding the soliton operation of the passively mode-locked fiber soliton lasers, but also potentially useful for future applications of solitons in ultrahigh-bit-rate optical communication systems.

We have conducted detailed experimental and numerical studies of the soliton interaction in a passively mode-locked fiber soliton ring laser. In this paper we present results of our studies. The paper is organized as following: Section II provides results of the experimental investigations. Based on our experimental results, we have identified the existence of three different types of soliton interactions that play a dominating role in the soliton dynamics in the laser: a global type of soliton interaction caused by the existence of unstable continuous-wave (cw) emission, a local type of soliton interaction caused by the dispersive waves emitted by the solitons, and the direct soliton interaction. We show experimentally that each of the three types of soliton interactions has different interaction range and strength, and depending on the concrete laser operation conditions they can exist either individually or concurrently. In Sec. III we present numerical simulations of the soliton interaction in the laser. Based on a coupled GLE model and also considering the effects of dispersive-wave emission and laser-cavity feedback, we found that the major experimental observations could be well numerically reproduced. In addition, based on the results of numerical simulations we found that the effect of the dispersive-wave-mediated soliton interaction in the laser is always to cause random irregular relative soliton movements, and the states of bound solitons observed experimentally are formed through the direct soliton interaction. Section IV is the conclusion of the paper.

## II. EXPERIMENTAL SETUP AND RESULTS

The fiber soliton laser we used for our experimental studies is schematically shown in Fig. 1. It has a ring cavity of about 5.5 m long, which comprises of a 3.5-m 2000 ppm erbium-doped fiber (EDF) whose group velocity dispersion is about  $-10$  ps/(km nm), a piece of 1-m-long single-mode dispersion-shifted fiber whose group velocity dispersion is about  $-2$  ps/(km nm), and another piece of 1-m standard telecom fiber (SMF28). The nonlinear polarization rotation technique is used to achieve the self-started mode locking of the laser. For this purpose a polarization-dependent isolator together with two polarization controllers—one consists of two quarter-wave plates and the other of two quarter-wave plates and one half-wave plate—is used to adjust the polarization of light in the cavity. The polarization-dependent iso-

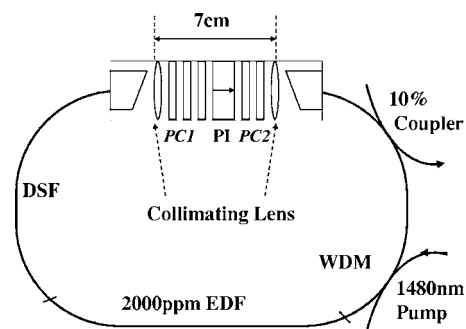


FIG. 1. A schematic of the fiber laser setup. PI: polarization dependent isolator. PC: polarization controller. WDM: wavelength division multiplexer. EDF: erbium-doped fiber. DSF: dispersion-shifted fiber.

lator and the polarization controllers are mounted on a 7-cm-long fiber bench to easily and accurately control the polarization of the light. The laser is pumped by a pigtailed InGaAsP semiconductor diode of wavelength 1480 nm. The output of the fiber laser is taken via a 10% fiber coupler and analyzed with an optical spectrum analyzer (ANDO AQ6317B) and a commercial optical autocorrelator (Inrad 5-14-LDA). A 50-GHz-wide bandwidth sampling oscilloscope (Agilent 86100A) and a 25-GHz photodetector (New Focus 1414FC) are used to study the soliton evolution in the laser cavity.

We are interested in finding out the physical mechanisms for the formation of the various modes of the multiple-soliton operation, in particular how they are related to the soliton interaction in the laser. To this end we have designed various experiments to identify and separate the various natures of soliton interactions in the laser. In all our experiments we have simultaneously monitored the laser output by using an optical spectrum analyzer, a 50-GHz high-speed sampling oscilloscope, and a commercial autocorrelator. Through carefully analyzing the experimental results, we could clearly identify the existence of three different types of strong soliton interactions in our laser. These are a global type of soliton interaction caused by the existence of unstable cw components, a local type of soliton interaction mediated through the dispersive waves, and the direct soliton interaction. Figure 2 shows, for example, a series of experimentally measured oscilloscope traces that demonstrate the existence of the global type of soliton interaction in the laser. The solitons observed in our laser have an average pulse width [full width at half maximum (FWHM)] of about 340 fs, and the cavity round-trip time of our laser is about 26 ns. In the state shown in Fig. 2, there are five solitons coexisting in the cavity. Two solitons have a very small soliton separation ( $\sim 6$  ps) and are bound together. Details about the properties and binding mechanism of this bound soliton will be discussed later. As our detection system cannot resolve these two solitons, it appears as a large pulse on the oscilloscope trace. We used the bound soliton as the trigger signal for our measurements (it therefore has a fixed position in the oscilloscope traces) and experimentally investigated the relative movement of the other three solitons in the cavity. Figures 2(a) and 2(b) show the relative position changes

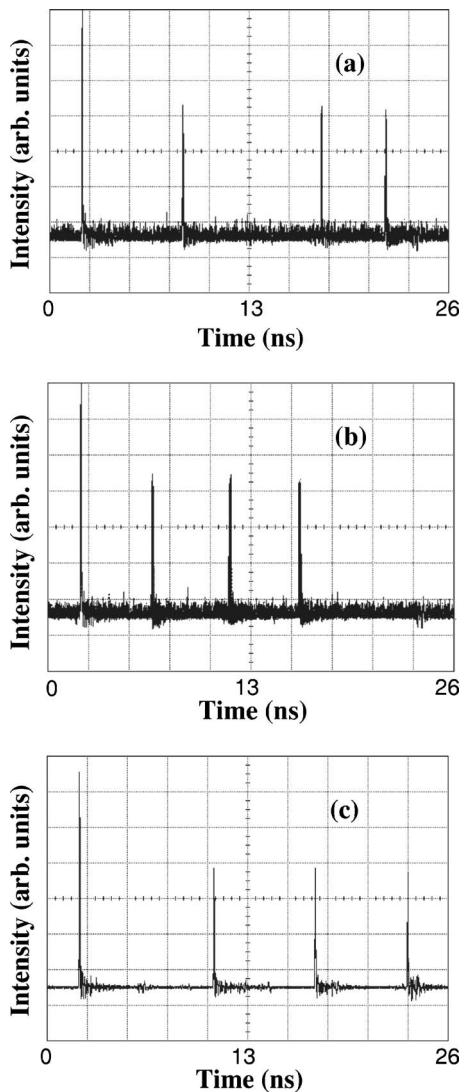


FIG. 2. Oscilloscope traces demonstrating an unstable cw-lasing-induced global soliton interaction. (a) and (b) Two consecutive instants under the existence of a noisy background. Solitons change their positions in the cavity. (c) Noisy background is suppressed. Solitons form a stable irregular pattern.

of the three soliton pulses under fixed laser parameters. We observed that the three solitons ceaselessly moved with respect to each other with an average speed of about half a division per second. Their movements were purely random. The solitons could come close or move apart from each other. In such a state of laser operation, the movement of one soliton affects all the other solitons in the cavity no matter how far apart they are. We therefore believe that there is a global type of soliton interaction between them. We further investigated the physical mechanism of the global soliton interaction and found that it was caused by the existence of an unstable cw component in the laser. To demonstrate this, we show in Fig. 3 the experimentally measured soliton spectrum of the state. It can be seen that there is actually cw lasing coexisting with the soliton operation. Through purely adjusting the pump power, the strength of the cw lasing can be controlled. As the laser is operating in the anomalous

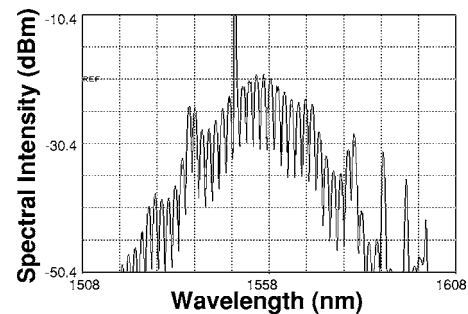


FIG. 3. Optical spectrum of the laser output under the same condition of Figs. 2(a) and 2(b). The strong line in the spectrum is the cw emission.

dispersion region, a cw component is intrinsically unstable due to the modulation instability (MI). We observed that when the strength of the cw component was strong, it became unstable as characterized by the appearance of the modulation instability sidebands in the spectrum [16]. In the oscilloscope trace, instability of the cw component is also represented by the appearance of background noise, as can be clearly seen in Figs. 2(a) and 2(b). Reducing the strength of the cw component, its instability becomes less obvious, and consequently the noise background disappears from the oscilloscope trace as shown in Fig. 2(c). Without the noise background all solitons then stay stable in the cavity, forming a stable irregular distribution pattern.

Coexistence of cw lasing and soliton operation was frequently observed in passively mode-locked fiber soliton lasers [16–18]. The interaction between solitons in the presence of a frequency-shifted cw was also investigated by Loh *et al.* [19]. It was found that the existence of a weak frequency-shifted cw caused the soliton central frequency to shift. So far we have not been able to fully understand how the solitons interact with an unstable cw in the laser cavity. However, considering that both the cw component and the solitons coexist in the laser cavity and share the same laser gain, the existence of mutual influence between them seems easy understandable. Hence we may explain the experimental observations in the following way. When a stable cw coexists in the cavity, all the solitons would experience the same central frequency shift, and overall the relative soliton velocity will not change. While if the cw component becomes unstable, each soliton would experience different local perturbations; therefore, they would have different central frequency shifts. Passive harmonic mode locking of soliton pulses has been frequently observed in our laser [18]. Experimentally we noticed that all the harmonically mode-locked states were actually obtained under the existence of unstable cw components. This experimental finding suggests that the unstable cw-component-induced global soliton interaction must have played an important role in the formation of the harmonic mode locking. Intuitively one can imagine that under the global soliton interaction, solitons will gradually adjust their positions. The only stable state they could eventually reach would be a harmonically mode-locked state as only in such a state are all solitons subject to the same influence caused by an unstable cw.

Obviously, in a state like that shown in Fig. 2—the soliton interaction induced by the gain depletion and recovery—the

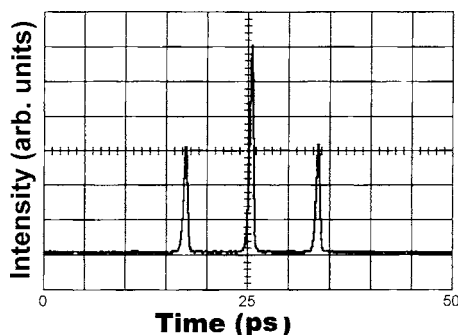


FIG. 4. Autocorrelation trace of another bound solitons obtained. Soliton separation is about 8.4 ps.

fiber acoustic effect should also exist. However, experimentally we observed that once the noise background was suppressed, the solitons then stayed in the cavity stably. They did not change their positions until the unstable cw component was created again. With this method we had actually controlled the soliton distribution in the cavity and obtained various stable soliton distribution patterns. This experimental result clearly demonstrated that the effects of soliton interactions caused by gain competition and/or fiber acoustics are too weak to be measured in the laser.

In Fig. 2 there are two solitons binding together, forming a state of bound solitons. From the autocorrelation trace we found that the two solitons have a fixed pulse separation of about 6 ps. In our experiment, as we increased or decreased the background noise, neither the state of bound solitons was destroyed nor the pulse separation was changed, which indicates that the binding force between the two solitons is stronger than the force of the global soliton interaction induced by the unstable cw. Bound states of solitons with various soliton separations are frequently obtained in the laser. Figure 4 shows, for example, the autocorrelation trace of another state of bound solitons with a larger soliton separation. Coexistence of bound solitons with different pulse separations has also been observed [20]. Experimentally we found that the larger the pulse separation, the weaker the binding energy between the solitons and, therefore, the easier the state of bound solitons to be destroyed by the perturbations.

A common feature of these states of bound solitons is that soliton separations have fixed, discrete values, which are independent of the laser operation conditions. Whenever such a bound state of solitons is obtained, it will have one of the values of the soliton separation. As the soliton separations of the bound solitons are fairly large compared with the soliton pulse width, which in the case of our laser is only about 340 fs, based on the theory of soliton interactions of NLSE solitons [1], it seems unlikely that these states of bound solitons are formed through the direct soliton interaction, as according to the theory, when the separation between the solitons are larger than 5 times of their pulse widths, there is practically no overlap between the soliton profiles. Therefore, the existence of these bound solitons in our laser suggests that either the theory of direct soliton interaction of the NLSE solitons does not apply to the solitons of the laser or there exists an unknown soliton interaction that binds the solitons together. However, this needs to be further studied.

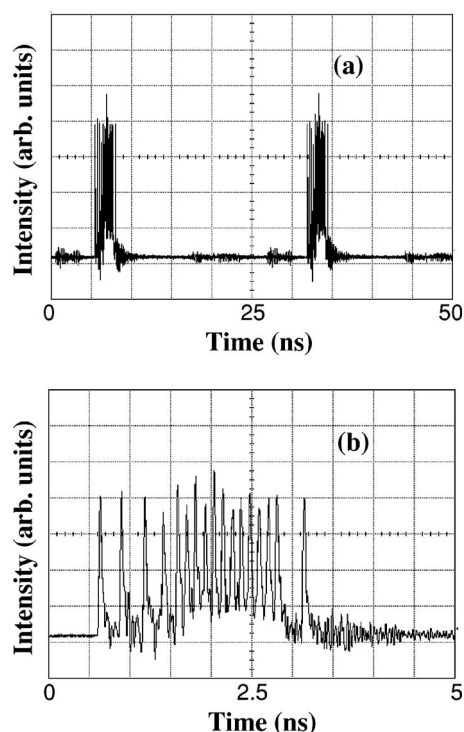


FIG. 5. Oscilloscope traces of a soliton bunching state. (a) The bunch repeats with the cavity repetition rate. (b) Detailed structure of the bunch.

We note that such a kind of bound solitons has also been observed by other authors in other passively mode-locked soliton fiber lasers [21].

Apart from the above bound states of solitons, depending on the experimental conditions frequently the state of so-called soliton bunching was also observed in the laser. This state was described as another mode of the multiple-soliton operation of the passively mode-locked soliton fiber lasers [10]. For the purpose of comparison, we have shown again in Fig. 5 a typical case of such a state observed in our laser. Figure 5(a) shows that the soliton bunch moves stably in the laser cavity with the fundamental cavity repetition rate. Figure 5(b) is the expansion of the bunch. Seventeen solitons coexist in the bunch with fixed relative soliton separations. Note that the soliton separations in the bunch vary randomly, and the separations between the solitons are also fairly large as compared with those of the bound solitons. It is to point out that although a state of the soliton bunch is somehow like a state of bound solitons with large soliton separation, the state is far less stable than the bound solitons. Frequently only slightly changing the experimental conditions—e.g., the pump power—or even environmental perturbations could destroy the state. We also note that the state shown in Fig. 5 is just one typical case of soliton bunching; in the experiment depending on the concrete experimental conditions, especially depending on the history of the soliton operation as also noticed by Grellu and Soto-Crespo [22], other forms of soliton bunching, such as the one shown in Fig. 6 where three solitons bunch in the cavity and the formed bunch coexists with other solitons, can also be observed.

Experimentally we found that if two solitons were spaced closely, even after the suppression of the global type of soli-

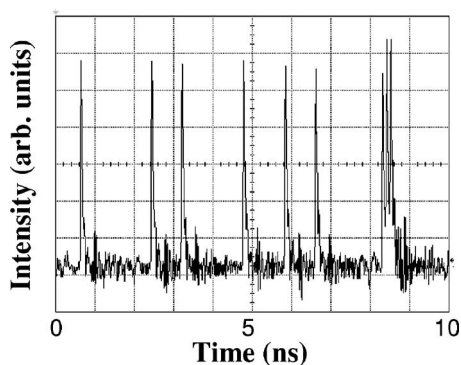


FIG. 6. Oscilloscope trace showing coexistence of solitons with bunched solitons.

ton interaction as discussed above, occasionally they could still oscillate relatively to each other, and their oscillations had no effect on the other solitons in the cavity. While when solitons were far apart, they always have stable relative separations. This experimental result suggests that there must exist a local type of interaction between the solitons. We have experimentally checked the maximum range of the local type of soliton interaction. The range varies from laser to laser depending on the laser cavity design and properties. For lasers with long cavity length, this kind of soliton interaction is very obvious and the interaction could extend to more than 200 ps, while for our current laser we could just identify this kind of relative soliton oscillations with our detection systems occasionally. This interaction range reminded us of the well-known long-range soliton interaction mediated through dispersive waves first discovered by Smith and Mollenauer [5]. Dispersive-wave emission is a generic property of solitons under perturbations. The dispersive-wave-mediated long-range soliton interaction has been extensively investigated for the case of soliton propagation [5,6]. Socci and Romagnoli have theoretically investigated the properties of soliton interactions in periodically amplified fiber links and predicted the existence of quasibound solitons and an oscillation type of soliton interaction [6]. However, to our knowledge, no experimental studies of the effect on the fiber soliton lasers have been reported. Different to the cases of soliton propagation the strength of dispersive waves in a fiber laser can be controlled either through carefully adjusting the pump strength or through changing the linear cavity phase delay bias. An different design of the laser cavity can also either suppress or enhance the resonant dispersive-wave generation in a laser. All these features of the dispersive waves in a laser make the soliton interaction in the system more complicated.

Theoretical studies of the interaction between the NLSE solitons have shown that when two solitons have a separation of less than about 5 times of their pulse widths, the direct soliton interaction must be considered [1,4]. Extensive theoretical studies have been done of the direct soliton interaction, in particular, in the case of solitons described by the GLE, the formation of bound solitons under the direct soliton interaction was predicted. Nevertheless, only a few experimental studies have focused on the direct soliton interaction in passively mode-locked fiber lasers [23,24]. Partially be-

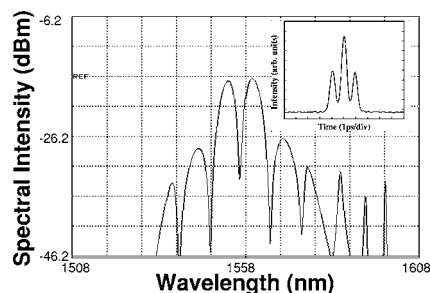


FIG. 7. Optical spectrum of a state of bound solitons under existence of the direct soliton interaction. The inset is the corresponding autocorrelation trace.

cause the direct soliton interaction depends sensitively on the relative soliton phase difference, in practice, due to the unavoidable environmental perturbations, the relative soliton phase always varies. Even if the environmental perturbations could be suppressed, the existence of the long-range soliton interaction mediated through dispersive waves as mentioned above could complicate the dynamics of the direct soliton interaction in a laser.

In our experiments we could frequently observe that solitons moved across or collided with one another during a transient process, mostly when a laser parameter was suddenly significantly changed. It doubtless shows the existence of a direct soliton interaction in the system. However, this process occurs normally very fast. It is difficult to experimentally investigate their interaction dynamics. Nevertheless, under certain special laser operation conditions we could experimentally easily obtain stable states of bound solitons with pulse separation less than 5 times of the soliton pulse width. Figure 7 shows, for example, the optical spectrum and the corresponding autocorrelation trace of one of the states. The autocorrelation trace shows that two solitons of equal pulse height bind tightly together with a pulse separation of merely 920 fs. Obviously a direct soliton interaction exists between the two solitons. We should point out that these bound solitons also exhibit fixed, discrete soliton separations. In addition, as the solitons are so close, from their optical spectra we can clearly identify that the relative phase difference between the two bound solitons is always about  $\pi$ ; as can also be seen in Fig. 7, there is always a dip in the center of their optical spectra.

Coexistence of such multiple bound solitons was also observed experimentally. Surprisingly, different to the bound solitons described above, which can coexist with the unbound solitons in the cavity, the bound solitons as shown in Fig. 7 have never experimentally found to coexist with other bound solitons or unbound solitons. The such multiple bound solitons can form various modes of operation, such as bunching of the bound solitons [13] and passive harmonic mode locking of the bound solitons [18]. Figure 8 shows, for example, a case of laser operation with such multiple bound solitons in the cavity, where ten bound solitons coexist with stable irregular separations in the cavity. Here again, limited by the resolution of the detection system, the separation between the two solitons within a bound soliton cannot be resolved; however, note that each spike in the oscillation trace

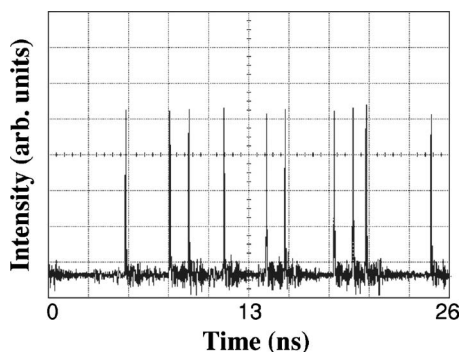


FIG. 8. Oscilloscope trace of the laser output showing the coexistence of multiple identical bound solitons. Note that each pulse in the trace consists of two solitons with a separation of about 930 fs.

is actually a bound soliton with two soliton binding together as detected from their autocorrelation traces and optical spectra. The bound solitons behave as an entity. They can also interact with each other and form bound-bound solitons [20]. All these features suggest that the bound solitons formed under the direct soliton interaction have very strong binding energy. It could be treated virtually as a new form of soliton in the laser. We note that Paré and Bélanger have theoretically found that a dispersion-managed system can support an antisymmetric stationary pulse consisting of two close-spaced dispersion-managed (DM) solitons with the  $\pi$  phase shifted. They explained the new stationary pulse as a higher-order mode of the fundamental DM soliton [25]. Recently Maruta *et al.* have also numerically found the existence of stable symmetric (in-phase) and antisymmetric (antiphase) bisolitons as they called in dispersion-managed systems [26,27]. Their numerical simulations also revealed that the bisolitons have fixed discrete soliton spacings. Indeed, the bound solitons of our laser shown in Fig. 7 exhibit close similarity in properties to the antisymmetric bisoliton of Paré and Bélanger and Maruta *et al.*, which further supports our conjecture that it could be a new form of soliton in the laser.

### III. NUMERICAL SIMULATIONS

To have a better understanding of the soliton interaction in the laser—in particular, to find out the effect of the dispersive waves on the soliton interaction—we have also numerically simulated the multiple-soliton operation of our laser. Again our simulations are based on the pulse tracing technique reported previously [28]. The basic idea of the technique is to numerically determine the eigenstate of a laser under a certain laser operation condition by circulating light in the laser cavity to a steady state and then to compare the steady-state result with the experimental observations. Concretely, we start from an arbitrary weak light input and let the light circulate in the laser cavity according to the exact laser-cavity configuration. As the nonlinear polarization rotation technique was used to achieve mode locking in our laser, where both the linear and nonlinear light polarization rotations within a one-cavity round trip play an important role in the laser dynamics, to accurately simulate the light polariza-

tion effect, we have used the coupled complex Ginzburg-Landau equations to describe the light propagation in the optical fibers,

$$\begin{aligned} \frac{\partial u}{\partial z} &= i\beta u - \delta \frac{\partial u}{\partial t} - \frac{i}{2} \kappa'' \frac{\partial^2 u}{\partial t^2} + i\gamma \left( |u|^2 + \frac{2}{3} |v|^2 \right) u + \frac{i\gamma}{3} v^2 u^* + \frac{g}{2} u \\ &\quad + \frac{g}{2\Omega_g^2} \frac{\partial^2 u}{\partial t^2}, \\ \frac{\partial v}{\partial z} &= -i\beta v + \delta \frac{\partial v}{\partial t} - \frac{i}{2} \kappa'' \frac{\partial^2 v}{\partial t^2} + i\gamma \left( \frac{2}{3} |u|^2 + |v|^2 \right) v + \frac{i\gamma}{3} u^2 v^* \\ &\quad + \frac{g}{2} v + \frac{g}{2\Omega_g^2} \frac{\partial^2 v}{\partial t^2}, \end{aligned} \quad (1)$$

where  $u$  and  $v$  are the two normalized slowly varying pulse envelopes along the slow and fast axes of the optical fiber, respectively.  $2\beta = 2\pi\Delta n/\lambda$  is the wave-number difference, and  $2\delta = 2\beta\lambda/2\pi c$  is the inverse group-velocity difference.  $\kappa''$  is the dispersion parameter.  $\gamma$  is the nonlinearity of the fiber.  $g$  is the gain coefficient for the EDF, and  $\Omega_g$  is the gain bandwidth. For the undoped fiber,  $g$  is taken as zero. The EDF gain is saturated by the total light energy in the cavity described by

$$g = g_0 \exp \left( - \frac{\int_{-\infty}^{\infty} (|u|^2 + |v|^2) dt}{E_s} \right), \quad (2)$$

where  $g_0$  is the small signal gain and  $E_s$  is the saturation energy.

We simulated the effects of other cavity components, such as the polarization controllers and the polarization-dependent isolator, by multiplying their transform matrixes to the light field whenever the light field met them. After the light circulated one round trip in the cavity, we used the final light field as the input for the next round of circulation, until a steady state was reached. If no steady state could be obtained, we simply calculated to 5000 rounds and took the final results for our analysis. Note that our simulations are different from the conventional master equation method widely used to simulate the operation of soliton lasers [29,30]. It covers all results obtained by the master equation. In addition, it also has the following advantages: Since the calculation is made following pulse propagation in the cavity, pulse evolution within one round trip can be studied with the model. As within each step of the calculation the pulse variation is always small, even if the overall change of the pulse within one round trip were big, the model can still give stable solutions. Therefore, the simulation has no limitation on the pulse change within one round trip, and the dynamical process of soliton evolution, such as soliton interaction, soliton generation, etc., can be investigated. In addition, the effect of discrete cavity components, the influence of the dispersive waves, and the different order of cavity components on the soliton property are automatically included in the calculation. To make the simulated results comparable with our experimental results, we have used the following parameters for our simulations: the laser cavity length  $L=6$  m, the linear

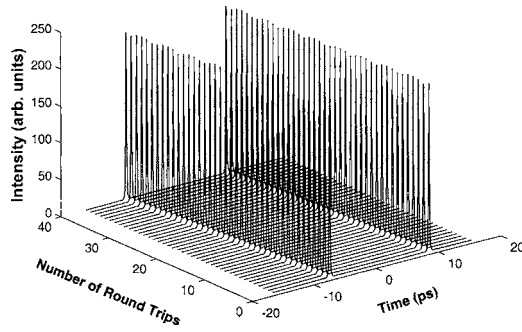


FIG. 9. Numerically calculated soliton evolution in a cavity when the solitons are far apart. The cavity linear phase delay bias  $\varphi=1.4\pi$ .  $g_0=350$ .

cavity beat length  $L_b=L/4$ , the effective gain bandwidth  $\Delta\Omega_g=20$  nm, the polarization of the intracavity polarizer to the fast axis of the birefringent fiber  $R=\pi/8$ , the group-velocity dispersion (GVD) of the dispersion shifted fiber, the EDF, and the standard single mode fiber  $\kappa''=-2$  ps/(nm km),  $\kappa''=-10$  ps/(nm km), and  $\kappa''=-18$  ps/(nm km), respectively. The normalized gain saturation energy of the erbium-doped fiber  $E_s=1000$ . We have only varied the values of the linear cavity phase delay between the two polarization components within one cavity round trip and the small signal gain coefficient of the fiber; these are to simulate the effects of rotating the wave plates and changing the pump, respectively. By using exactly the same numerical model we have previously successfully simulated other effects of the soliton fiber lasers such as the sideband asymmetry and subsidenceband generation [31] and periodic soliton peak power modulation [32].

With appropriate linear cavity phase delay selection and through increasing the pump power, which in our model corresponds to increasing the small signal gain coefficient, self-started mode locking of the laser can be automatically obtained in our simulations. Exactly like the experimental case, immediately after the mode locking coexistence of multiple-soliton pulses in the simulation window is always obtained. Decreasing the pump power soliton can be destroyed one by one from the window, and the soliton operation of the laser exhibits clear pump power hysteresis [33,34]. With already solitons in the cavity, increasing the pump power soliton can also be generated one by one in the cavity. Depending on the laser operation parameter settings, the new solitons can be formed either through the soliton shaping of dispersive waves [31], of the unstable cw components, or through the pulse splitting [35]. All these numerical results are well in agreement with the experimental observations, which demonstrate that our simulation can faithfully reproduce the features of the laser. Here we will focus on the soliton interaction in the laser and the comparison between the numerically calculated results to those of the experimental observations.

To simulate the soliton interaction in the laser, we first input two arbitrary weak light inputs in the cavity and let them evolve into solitons with a desired separation; we then changed the laser operation condition to study their interaction. Figure 9 shows the case of soliton interaction with a large soliton separation. In the steady state it can be seen that

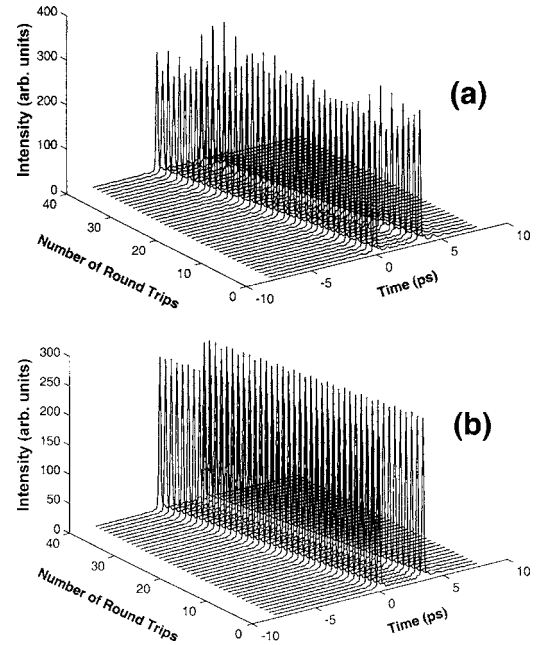


FIG. 10. A numerically calculated bound state of solitons. The cavity linear phase delay bias  $\varphi=1.4\pi$ . (a)  $g_0=370$ . (b)  $g_0=365$ .

both solitons have identical soliton parameters such as the peak power and pulse width. Changing the laser operation conditions, both solitons respond in exactly the same way, showing that except sharing the laser gain they have practically no other interactions. In this case even as we increased the pump strength to just before a new soliton was generated, the relative soliton separation remained constant. We then gradually reduced the separation between the solitons. To a separation of about 12 ps we found that as we increased the pump to a certain strength, the solitons started to change their relative separation and in the meantime the soliton peak powers started to fluctuate randomly, indicating that there was an interaction between them and the interaction is pump strength related. As compared with the soliton pulse width the solitons are still fairly far apart, we attribute the soliton interaction as caused by dispersive waves. With soliton separations setting below 8 ps, we observed that, depending on the pump strength, the solitons either formed a state of bound solitons or oscillated relatively. Figure 10 shows, for example, a case of the bound states of solitons formed. The solitons evolved to the separation and stabilized there. The interaction between the solitons is obvious. The state is called a state of bound solitons because, as we varied the pump strength within a certain range, despite the fact that the average soliton energy increased and there is an interaction between the solitons, the soliton separation remained the same. Note that the soliton peak powers fluctuate randomly. If we carefully reduced the pump, the peak power fluctuations could be suppressed and the solitons then had identical profiles as shown in Fig. 10(b). Figure 11 shows the case of soliton oscillations and collisions. Even solitons are initially in a state of bound solitons, when the pump power is increased beyond a certain value, which varies with the soliton separations. The bound solitons can break their binding and start to oscillate relatively. If the pump power is further in-

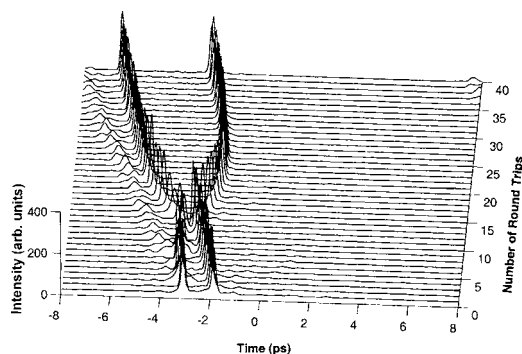


FIG. 11. A numerically calculated state of soliton oscillations and collisions. The cavity linear phase delay bias  $\varphi = 1.4\pi$ .  $g_0 = 375$ .

creased, the soliton can even collide with each other as shown in Fig. 11. Nevertheless, the relative soliton oscillations and collisions can be suppressed by simply reducing the pump strength. We emphasize that like the experimental observations, the separations between the calculated bound solitons shown in Fig. 10 are obviously larger than 5 times of the soliton pulse widths.

Bound states of solitons with soliton separations less than 5 times the soliton pulse width were also obtained. Figure 12 shows, for example, one of the states. In this case the direct soliton interaction can obviously not be ignored. Different from the bound states of solitons shown in Fig. 11(a), the bound solitons have no peak power fluctuations, even when we increased the pump power to the point where the bound solitons breaks, and afterwards it jumped to a state of bound solitons with a larger soliton separation. Figure 12(b) shows the optical spectrum of the bound soliton, which shows that

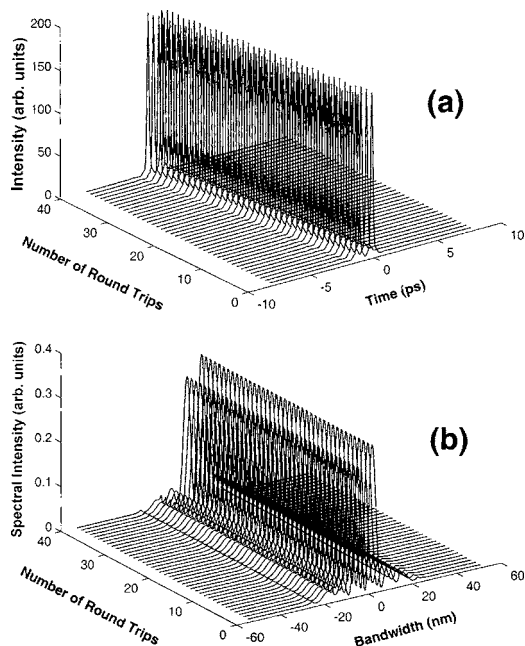


FIG. 12. A numerically calculated state of bound solitons with close soliton separation. The cavity linear phase delay bias  $\varphi = 1.4\pi$ .  $g_0 = 320$ . (a) Time evolutions of the state. (b) Optical spectral variations.

the phase difference between the bound solitons is also about  $\pi$ . Numerically we found that different bound solitons have different stability as reflected by the range of pump power change over which they are stable. The closer the solitons are separated, the larger is their binding energy. In addition, the soliton separations of the numerically calculated bound solitons also exhibit discrete values, which is in excellent agreement with the experimental observations. At last we emphasize that the calculated results are independent of the initial conditions of the simulation. By simply increasing the pump power we can also obtain multiple solitons in the cavity, and we found that their interactions exhibit exactly the same features. Particularly, in these cases states of bound-bound solitons and the formation of multiple identical bound solitons as observed experimentally were obtained, which we will report separately.

#### IV. DISCUSSIONS

Our numerical simulations have well reproduced the typical effects of the soliton interaction in the laser. Based on results of the experimental observations and numerical simulations, we can now explain the formation of the various multiple-soliton operation modes of our laser. A fundamental difference between the solitons formed in a laser and in a fiber propagation system is that the laser solitons are also subject to the influence of the laser cavity and laser action; e.g., they are not only a soliton pulse, but also a mode-locked pulse in the cavity. The mode locking enforces that all the solitons in cavity have exactly the same central frequency and constant phase differences. Therefore, in the steady state and if there is no other force on the solitons, they should have exactly the same properties. And consequently they remain static relative to each other. We believe this should be why we can observe the soliton bunches and stable irregular soliton distributions in the laser. It can be seen that these states are actually not states of bound solitons as there is no interaction between the solitons.

Solitons coexisting in a laser share the same laser gain; therefore, there is gain competition between them. Previous theoretical studies have also shown that the soliton interaction caused by gain depletion and recovery could affect the soliton distribution in the cavity [14]. However, our experimental results shown in Fig. 2 clearly demonstrated that the strength of this soliton interaction is actually even weaker than that caused by the unstable cw waves. Therefore, in practice its effect on the soliton distribution and dynamics in a laser could be ignored. This is also true for the soliton interaction induced by fiber acoustic effects; as compared with other types of soliton interactions existing in the laser, this type of soliton interaction is too weak to play a role.

Although the bound solitons calculated from the GLE also exhibit fixed, discrete soliton separations, they are only weakly stable as shown by Malomed [8,9]. The bound solitons formed in the laser are on the contrary very stable. At first sight it seems that they could be formed by a different mechanism. Nevertheless, our numerical simulations confirm that the bound solitons formed in a laser are indeed very stable. To explain this difference we believe that the mode



locking feature of the laser solitons could have contributed to it. In addition, our numerical simulation also showed the existence of bound solitons with soliton separations larger than 5 times the soliton pulse width. We have numerically checked the soliton profiles of our laser. It revealed that the profiles varied with laser operation conditions, and due to the strong influence of the cavity properties and cavity components, in some cases it is very different from those of the NLSE and GLE solitons. Although the 3-dB pulse width of the soliton is still narrow, the soliton profile could have a tail of considerable strength extending to a distance as large as about 6 ps. The direct soliton interaction is certainly soliton profile dependent. As the solitons formed in different nonlinear systems are different, there is no surprise that bound states of solitons with soliton separations larger than 5 times the soliton pulse widths could be obtained in the lasers. Therefore, these bound states of solitons observed are still formed by the direct soliton interaction. These experimental and numerical results also suggest that the criteria for identifying direct soliton interactions for solitons of different nonlinear dynamical systems should be different.

Obviously in the laser the direct soliton interaction always coexists with the dispersive-wave-mediated soliton interaction. It is therefore difficult to separate their effects. There has speculation arguing that the resonant dispersive-wave-mediated soliton interaction could synchronize the soliton phases and result in the formation of bound solitons of discrete separations [36]. However, analyzing our numerical results it seems that the dispersive-wave-mediated soliton interaction in a laser only causes random, irregular relative soliton movements. To further identify the effect of the soliton interaction, we have also numerically simulated lasers with different cavity lengths and cavity dispersions. We found that for lasers of long cavity lengths the soliton sidebands can become very strong, and in the meantime the range of soliton interactions can become very long. In this case a soliton interaction range as large as about 60 ps has been numerically obtained. At this soliton separation the direct soliton interaction could be safely ignored. However, except for random irregular relative soliton oscillations, no bound states of solitons were ever obtained. Assuming that the effect of the dispersive-wave-mediated soliton interaction is only to cause random irregular soliton movements we found that all experimental and numerical results obtained could be consistently well explained. With relatively large soliton separation there is no direct soliton interaction; therefore, solitons can only exhibit relative oscillations. With close soliton separations, depending on the relative strength

between the dispersive-wave-mediated soliton interaction and the direct soliton interaction, solitons could either bind together to form bound states of solitons or oscillate relatively. As the dispersive waves in a laser are pump strength dependent, carefully controlling the pump can reduce the strength of the dispersive-wave-mediated soliton interaction, therefore eliminating the random peak power fluctuation of the bound solitons.

## V. CONCLUSIONS

In conclusion, we have both experimentally and numerically investigated the soliton interaction in a passively mode-locked fiber soliton ring laser. Our experimental studies revealed the existence of three dominating types of soliton interactions in the laser. They are a global type of soliton interaction caused by unstable cw lasing, a local type of soliton interaction caused by the resonant dispersive waves, and the direct soliton-soliton interaction. The impacts of each of the soliton interactions on the formation of the various modes of soliton operation of the laser were investigated. The global type of soliton interaction has the largest interaction range but the weakest interaction strength among the three types of soliton interactions. This type of soliton interaction can be well suppressed by carefully controlling the pump strength. Experimentally it was found that this type of soliton interaction played an important role on the formation of the passive harmonic mode locking of the laser. The interaction range of the dispersive-wave-mediated soliton interaction varies with laser operation conditions, which is determined by spatial extension of the resonant dispersive waves. This type of soliton interaction always causes random, irregular relative soliton movement. The direct soliton interaction only occurs when the soliton profiles overlap. Our numerical simulation of the soliton interaction is made based on coupled Ginzburg-Landau equations. It also takes into account the effects of resonant dispersive waves and laser-cavity properties. For the first time, to our knowledge, the details of the soliton interaction in a laser cavity can be numerically faithfully reproduced. Based on results of the numerical simulations it turned out that solitons in a laser have an intrinsic tendency of approaching synchronization. We conjecture that it is because of this feature of the solitons that the formation of states of the soliton bunch resulted. This feature of the laser solitons also enhances the stability of the bound solitons formed by the direct soliton interaction. Therefore, bound states of solitons with fixed, discrete soliton separation could be easily obtained in the laser.

[1] J. P. Gordon, *Opt. Lett.* **8**, 596 (1983).

[2] F. M. Mitschke and L. F. Mollenauer, *Opt. Lett.* **12**, 355 (1987).

[3] R. J. Essiambre and G. P. Agrawal, *Electron. Lett.* **31**, 1464 (1995).

[4] Y. Kodama, M. Romagnoli, and S. Wabnitz, *Electron. Lett.* **28**, 1981 (1992).

[5] K. Smith and L. F. Mollenauer, *Opt. Lett.* **14**, 1284 (1989).

[6] L. Socci and M. Romagnoli, *J. Opt. Soc. Am. B* **16**, 12 (1999).

[7] V. V. Afanasjev and N. Akhmediev, *Opt. Lett.* **20**, 1970 (1995).

[8] B. A. Malomed, *Phys. Rev. A* **44**, 6954 (1991).

[9] B. A. Malomed, *Phys. Rev. E* **47**, 2874 (1993).

[10] D. J. Richardson, R. I. Laming, D. N. Payne, V. J. Matsas, and

- M. W. Phillips, *Electron. Lett.* **27**, 1451 (1991).
- [11] D. J. Richardson, R. I. Laming, D. N. Payne, M. W. Phillips, and V. J. Matsas, *Electron. Lett.* **27**, 730 (1991).
- [12] A. B. Grudinin, D. J. Richardson, and D. N. Payne, *Electron. Lett.* **29**, 1860 (1993).
- [13] D. Y. Tang, B. Zhao, D. Y. Shen, C. Lu, W. S. Man, and H. Y. Tam, *Phys. Rev. A* **68**, 013816 (2003).
- [14] J. N. Kutz, B. C. Collings, K. Bergman, and W. H. Knox, *IEEE J. Quantum Electron.* **34**, 1749 (1998).
- [15] A. N. Piliipetskii, E. A. Golovchenko, and C. R. Menyuk, *Opt. Lett.* **20**, 907 (1995).
- [16] B. Zhao, D. Y. Tang, and H. Y. Tam, *Opt. Express* **11**, 3304 (2003).
- [17] S. Namiki, E. P. Ippen, H. A. Haus, and K. Tamura, *Appl. Phys. Lett.* **69**, 3969 (1996).
- [18] B. Zhao, D. Y. Tang, P. Shum, W. S. Man, H. Y. Tam, Y. D. Gong, and C. Lu, *Opt. Commun.* **229**, 363 (2004).
- [19] W. H. Loh, A. B. Grudinin, V. V. Afanasjev, and D. N. Payne, *Opt. Lett.* **19**, 698 (1994).
- [20] B. Zhao, D. Y. Tang, P. Shum, X. Guo, C. Lu, and H. Y. Tam, *Phys. Rev. E* **70**, 067602 (2004).
- [21] N. H. Seong and Dng Y. Kim, *Opt. Lett.* **27**, 1321 (2002).
- [22] Ph. Grelu and J. M. Soto-Crespo, *J. Opt. B: Quantum Semi-classical Opt.* **6**, S271 (2004).
- [23] Ph. Grelu, F. Belhache, F. Gутty, and J. M. Sota-Crespo, *J. Opt. Soc. Am. B* **20**, 863 (2003).
- [24] B. Ortac, A. Hideur, T. Chartier, M. Brunel, P. Grelu, H. Leblond, and F. Sanchez, *IEEE Photonics Technol. Lett.* **16**, 1274 (2004).
- [25] C. Paré and P. A. Bélanger, *Opt. Commun.* **168**, 103 (1990).
- [26] A. Maruta, Y. Nonaka, and T. Inoue, *Electron. Lett.* **37**, 1357 (2001).
- [27] A. Maruta, T. Inoue, Y. Nonaka, and Y. Yoshika, *IEEE J. Sel. Top. Quantum Electron.* **8**, 640 (2002).
- [28] D. Y. Tang, S. Fleming, W. S. Man, and H. Y. Tam, *J. Opt. Soc. Am. B* **18**, 1443 (2001).
- [29] H. A. Haus, J. G. Fujimoto, and E. P. Ippen, *J. Opt. Soc. Am. B* **8**, 2068 (1991).
- [30] F. X. Kärtner, I. D. Jung, and U. Keller, *IEEE J. Sel. Top. Quantum Electron.* **2**, 540 (1996).
- [31] W. S. Man, H. Y. Tam, M. S. Demokan, and D. Y. Tang, *Opt. Quantum Electron.* **33**, 1139 (2001).
- [32] B. Zhao, D. Y. Tang, L. M. Zhao, P. Shum, and H. Y. Tam, *Phys. Rev. A* **69**, 043808 (2004).
- [33] D. Y. Tang, W. S. Man, and H. Y. Tam, *Opt. Commun.* **165**, 189 (1999).
- [34] A. K. Komarov and K. P. Komarov, *Phys. Rev. E* **62**, R7607 (2000).
- [35] F. X. Kärtner, J. Aus der Au, and U. Keller, *IEEE J. Sel. Top. Quantum Electron.* **4**, 159 (1998).
- [36] J. M. Soto-Crespo, N. Akhmediev, Ph. Grelu, and F. Belhache, *Opt. Lett.* **28**, 1757 (2003).