GOETHE AND SCHOPENHAUER ON MATHEMATICS.

BY ARNOLD EMCH.

IS it a mere accidental coincidence that Goethe and Schopenhauer in some of their writings should both express themselves more or less adversely towards mathematics and mathematical methods in the study of natural phenomena?

The fact that Schopenhauer in 1813, when twenty-five years of age, went to Weimar and became acquainted with Goethe, under whose powerful influence he wrote a memoir *Ueber Sehen und die Farben* (published in 1816), would warrant the conclusion that their opinions on various scientific topics were a result of rather penetrating mutual discussions.

It is a proof for the universality of their intellects that they dared to enter into a discussion on the merits of a science of which both had only a very rudimentary knowledge. There is a kernel of truth in some of their statements, while others are dilettantic and still others erroneous or at least warped.

As is well known, Goethe was deeply interested in problems of natural philosophy during his later life, and his fundamental discoveries justly entitle him to be classed as a pioneer of Darwinism. That Goethe was fully aware of his handicap in attacking certain scientific problems appears from the following extract from "Mathematics and its Abuse"[1]: "Considering my inclinations and conditions I had to appropriate to myself very early the right to investigate, to conceive nature in her simplest, most hidden origins as well as in her most revealed, most conspicuous creations also without the aid of mathematics. . . . I was accused of being an opponent, an enemy of mathematics in general, although nobody can appreciate it more highly than I, as it accomplishes exactly those things which I was prevented from realizing."

[1] *Naturwissenschaftliche Schriften,* 2d part, Vol. II, p. 78, Weimar, 1893.

Further on, however, when the thought turns again upon mathematics and mathematicians, we find this curious statement:[2] "It is a wrong conception to think that a phenomenon could be explained by calculus or words" and "mathematicians are like Frenchmen; if one speaks to them they translate it into their own language, and then it will be very soon something entirely different." On page 138 when writing about natural science in general, Goethe expresses his idea of the mathematician as he ought to be in the following striking manner: "The mathematician is perfect only in so far as he is a perfect man, as he feels the beauty of truth; only then does he become thorough, penetrating, pure, clear, graceful and even elegant. All this is necessary to become like Lagrange."

What particular individual he had in mind when he wrote: "There are pedants who are at the same time thieves, and these are by far the worst," is not revealed. It is a partial consolation for the modern scientists, however, to find that Goethe already had to contend with such types.

It is extremely interesting that Goethe should quote d'Alembert as an authority on mathematics. We see here the influence of the encyclopedists upon European thought of that great period. There probably never lived a more brilliant and influential circle of philosophers and scientists that shaped the destiny of nations. Diderot and d'Alembert as co-editors of the great *Encyclopédie ou dictionnaire raisonné des sciences, des arts et des métiers,* Helvetius in his famous work *De l'esprit,* Voltaire by his piercing satire and Rousseau by his educational philosophy, La Mettrie as the author of *L'homme machine*[3] and Holbach in his *Système de la nature,* were all teaching that a new time had arrived.

With the exception of Kant, the great intellectual giant at Königsberg, Germany had during that whole period no philosophers and scientists of her own to boast of. From 1741 to 1766 it was the Swiss Euler and from 1766 to 1787 Lagrange, who gave lustre to the Academy at Berlin. Others, like the poet-scientist Haller, as appears from the dedication[4] of *L'homme machine,* were intellectually not even a match with such men as La Mettrie. Towards the end of the eighteenth and the beginning of the nineteenth century Gauss began his epoch-making discoveries and thereby placed Ger-

[2] *Loc. cit.* p. 98.

[3] English translation by Gertrude C. Bussey, published by the Open Court Publishing Co.

[4] This is not included in the above-mentioned English edition, but may be found in *The Open Court* of July, 1913, p. 427.

many in mathematics on a level with France, where men like d'Alembert, Lagrange, Monge, Laplace, Legendre and Fourier had won international reputation.

Gauss, however, never published anything for a general scientific public on his early meditations on the nature of mathematical reasoning and in particular on what we call now non-Euclidean geometry, so that naturally Goethe, even in his old age, was not able to learn anything about the new views in the science of space.

The passage of d'Alembert to which Goethe refers may be found in the famous *Discours préliminaire de l'encyclopédie*:[5]

"As regards mathematical sciences, which constitute the second of the limits of which we have spoken, their nature and their number must not startle us. What are most of the axioms of which geometry is so proud, if not the expression of the same simple idea by two different signs or words? The man who says that *two times two is four*, does he know more than somebody that contents himself by saying *two times two is two times two*? The ideas of the whole, the part, of greater and less, properly speaking, are they not the same simple and individual idea; since one cannot have one of them without the others presenting themselves all at the same time? As some philosophers have observed we owe many errors to the abuse of words; it is perhaps to the same abuse that we owe the axioms."

This is as far as Goethe quotes, so that without the rest of d'Alembert's argument one might look upon the latter as a rather one-sided critic. From d'Alembert's achievements as a mathematician and those portions of his *Discours* that treat of the various divisions of mathematics it is plain what great intrinsic value he placed upon mathematics and the mathematical spirit in scientific investigations in general. When he speaks of the abuse of words he simply states those truths which later his famous compatriot Poincaré, on various occasions, advanced against some claims of the modern logisticians.

Concerning logic d'Alembert has the following to say:[6] "It is the reduction to an art of the manner in which knowledge is gained and in which we conmmunicate reciprocally our own thoughts to each other. It teaches to arrange ideas in the most natural order and to link them by the most direct chain of thoughts, to resolve those that contain too large a number of simpler ideas, to look at them from all sides, in order to present them to others in a form

[5] *Œuvres de D'Alembert*, Vol. I, pp. 30-32, Paris 1821.

[6] *Loc. cit*, pp. 33-34.

in which they can be easily grasped. It is in this that this science of reasoning consists and which is justly considered as the key to all our knowledge. One must not believe, however, that it occupies the first place in the realm of invention. The art of reasoning is a gift presented by nature of her own accord to good intellects (*bons esprits*); and it may be said that the books which treat of logic are hardly of any use except to those who can get along without them.[7] Those that are familiar with Poincaré's style might easily mistake the last humerous remark as one of his famous sallies.

In this connection it is interesting to see what a modern writer, Mr. H. C. Brown, thinks about "the problem of method in mathematics and philosophy." He writes:[8] "The fact which seems to have been neglected by mathematicians is that the proof of consistency, by demanding an exhibition of something already known, puts a check on the "free creation" theory of mathematical systems and places them logically on a level with the concepts of all other sciences which all aim at hypothetico-deductive procedure.—A merely deductive mathematics would be of as little value as a 'freely created' philosophy.—All sciences must turn upon some existence, and a science which turns to a merely imagined world is dream-play."

D'Alembert returns with great detail to a discussion of the principles of the various branches of human knowledge and of scientific methods in his *Essai sur les élémens de philosophie*.[9] For the mathematicians and philosophers that make a study of the foundations of science, chapters fourteen to twenty are of particular interest. On pp. 278-280, for instance, we find a very clever discussion of the difficulties that arise in connection with the parallel-axiom. The "Elements" were published in 1759, at a time when hardly anybody thought of a critical examination of Euclid's Elements.[10]

Schopenhauer's remarks on mathematical questions were on the whole less personal than Goethe's. From his principal work *Die Welt als Wille und Vorstellung*,[11] whose first volume appeared in 1819 (a second edition increased by a second volume did not appear till 1844) we translate the following lines on Euclid's method:

[7] See a recent article by J. Charpentier: "Diderot et la science de son temps," in *La Revue du Mois*, Vol. 8, pp. 537-552 (Nov. 1913).

[8] *Essays Philosophical and Psychological*, p. 427.

[9] *Loc. cit.*, pp. 115-348.

[10] *La geometria del compasso* by Mascheroni appeared in 1797 in Pavia.

[11] *Werke*, Vol. I, p. 75 (Leipsic, F. A. Brockhaus, 1901).

"It is true in mathematics, according to Euclid's treatment, that the axioms are the only undemonstrable premises, and all demonstrations are successively subordinated to them. This treatment, however, is not essential, and, indeed, every theorem begins with a new construction in space which in itself is independent of the preceding ones and which in reality can be recognized also in entire independence of them, in itself, by pure intuition of space, in which in reality also the most complicated construction is immediately as evident as the axiom itself."

This remarkable statement interpreted by an inventive geometrician or intuitionist of the present day would of course not stand serious criticism. How, for instance, should Steiner's famous solution of Malfatti's problem to construct three circles each tangent to the other two and to two sides of the triangle, or the Steinerian problem of closure in connection with cubics and quartics be obvious even to the most acute geometrician? From a more general standpoint the only reasonable meaning which may be placed on Schopenhauer's idea is that an intrinsic geometric truth is independent of any particular set of axioms.

Schopenhauer denies the creative power of logistic geometry when he says "that intuition is the first source of evidence and that the immediate and intermediate relations derived from it are the only absolute truth, furthermore that the shortest path to truth is always the surest and that the transmission through concepts is subject to many illusions....We demand the reduction of every logical proof to one of an intuitional nature; Euclid's mathematics, however, makes great efforts to cast off wantonly its intuitional evidence everywhere near at hand, in order to substitute in place of it a logical proof. We must find that this is as if somebody would cut his legs off in order to go on crutches....That what Euclid proves is true we have to acknowledge through the principle of contradiction; but we do not learn the reason why it is true. We experience therefore almost the same unpleasant sensation that is caused by a sleight-of-hand performance, and, indeed, most of Euclid's proofs singularly resemble such tricks. The truth almost always appears through the back door, since it results by accident from some minor condition. An apagogical proof often closes one door after another and leaves open only one through which to pass. According to our opinion, therefore, Euclid's method in mathematics appears as a very brilliant perversity (*Verkehrtheit*)."

Schopenhauer maintains that the reason for the Euclidean system could be traced back to the prevailing philosophic system of that time. The Eleatics were the first to discover the difference, and frequently the contradiction, between the things observed and the same things thought of. The sophists and skeptics drew attention to illusions, i. e., to the deception of the senses. It was recognized that intuition through the senses was not always reliable. For this reason they came to the conclusion that only logical reasoning could establish truth. Plato and Pyrrhon, on the other hand, showed by examples how definitions and conclusions in agreement with the laws of logic were likewise apt to mislead and to produce sophisms which were much more difficult to solve than deceptions of the senses. Rationalism in opposition to empiricism however became the dominant philosophy, and, according to Schopenhauer, it is under its influence that Euclid wrote his "Elements," in which he felt compelled to regard only the axioms as based upon intuitional evidence (*φαινόμενον*) while the remainder follows from conclusions (*νοούμενον*). In a highly refined form the controversy which separated the Greeks is still present. As Carus[12] says: "In philosophy we have the old contrast between the empiricist and transcendentalist." Concerning the origin or the starting-point of mathematical system the same author remarks "that the data of mathematics are not without their premises; they are not, as the Germans say, *voraussetzungslos,* and though mathematics is built up from nothing, the mathematician does not start with nothing. He uses mental implements and it is they that give character to his science."[13]

Schopenhauer's conception of the domain that should be characteristic of mathematics is that the existence of a mathematical truth should be equivalent with the reason for it. It would of course be a tremendous advantage if this equivalence could always be established in the most simple manner by pure intuition, even when conceived in a higher sense. This method followed by the inventive mathematician as conceived by Poincaré is of a superior type and has presumably led to the greatest mathematical discoveries. The process of coordination with other branches and of rigorous analysis of the elements that constitute the truth is subsequently a problem of the mathematical logician. In a noted lecture[14] on humanistic education and exact science Poincaré said:

[12] *The Foundations of Mathematics, a Contribution to the Philosophy of Geometry,* p. 36. Chicago, Open Court Publishing Co., 1908.

[13] See also the valuable and clearly written article "De la méthode dans les sciences" by E. Picard in *De la science,* pp. 1-30, Paris, 1909.

[14] Delivered at the annual session of the *Verein der Freunde des humanistischen Gymnasiums* in Vienna, May 22, 1912.

"Before he [the mathematician] demonstrates he must invent. But nobody has ever invented anything by pure deduction. Pure logic cannot create anything; there is only one way to discovery, namely induction; for the mathematician as well as for the physicist. Induction, however, presupposes the art of divination and the ability to select; we must be satisfied with intuition and not wait for certitude. To do this, however, requires a refined intellect (*esprit de finesse*). For this reason there are two kinds of mathematicians. There are some that possess the mathematical spirit only; they may be valuable laborers who pursue successfully the paths laid out for them. We need people of this kind, we need many of them. But beside these more common mathematicians there are some that possess the *esprit de finesse*, they are the truly creative intellects."

It is true that the famous example for the evidence of the Pythagorean theorem shows the limited mathematical knowledge of Schopenhauer, or else he would have known that "evident" proofs of the general theorem are numerous. That Schopenhauer, in spite of some valuable critical remarks on mathematical methods did not understand the true meaning of Euclid's method and much less the *raison d'être* of non-Euclidean geometry[15] appears from the following characteristic passage:

"In the famous controversy over the theory of parallel lines and in the perennial attempts to prove the 11th axiom, the Euclidean method of demonstration has born from its own fold its most appropriate parody and caricature....This scruple of consciousness reminds me of Schiller's question of law:

> 'Jahre lang schon bedien' ich mich meiner Nase zum Riechen;
> Hab ich denn wirklich an sie auch ein erweisliches Recht?'
>
> [Years upon years I've been using my nose for the purpose of smelling.
> Now I must question myself: Have I a right to its use?][16]

"I am surprised that the eighth axiom: 'Figures that can be made coincident are equal,' should not be attacked. For, to coincide is either a mere tautology or else something of an entirely empirical

[15] Lobatschevsky's epoch-making work on parallels appeared between 1829 and 1840. (English translation by George Bruce Halsted under the title *Geometrical Researches on the Theory of Parallels*). *The Science Absolute of Space* by Bolyai, equally important, was published in 1826 (English translation by Dr. Halsted). *Die geometrischen Constructionen, ausgeführt mittels der geraden Linie und eines festen Kreises,* by Steiner, appeared in 1833.

[16] See Carus, *Goethe and Schiller's Xenions.*

character which does not belong to pure intuition. It presupposes movement of figures. In space, however, only matter is movable."

In *Parerga und Paralipomena*[17] Schopenhauer, discussing optical questions, strikes a personal note when he writes: "On the polarization of light the Frenchmen have nothing but nonsensical theories on undulations and homogeneous light, besides computations which are not based upon anything. They are constantly in a haste to measure and to calculate; they consider this as the main thing, and their slogan is *le calcul! le calcul!* But I say, *Où le calcul commence, l'intelligence des phénomènes cesse*: he who has only numbers in his head cannot find the trace of the connective cause."

Here again we see that Schopenhauer, like Goethe, did not appreciate at all what the French mathematical physicists had done. But how, without hardly any mathematical knowledge, could they expect to understand the Frenchmen? Nothing could show better than the foregoing statement the scientific limitations of the otherwise towering intellect of Schopenhauer. Of the real difficulties that lie at the foundation of mathematics neither Goethe nor Schopenhauer had a true conception. They were not able to anticipate even a possibility of the tremendous progress that has since been made and had been made during Schopenhauer's lifetime.

But considered from a modern standpoint their often ill-tempered remarks appear as interesting flash-lights of a great historic period.

[17] *Loc. cit,* Vol. II, p. 128.