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Research articles

Design of integrated systems for the control and detection of actuator/sensor faults

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Abstract

Considers control systems operating under potentially faulty conditions. Discusses the problem of designing a single unit which not only handles the required control action but also identifies faults occurring in actuators and sensors. In common practice, units for control and for diagnosis are designed separately. Attempts to identify situations in which this is a reasonable approach and cases in which the design of each unit should take the other into consideration. Presents a complete characterization for each case and gives systematic design procedures for both the integrated and non-integrated design of control and diagnosis units. Shows how a combined module for control and diagnosis can be designed which is able to follow references and reject disturbances robustly, control the system so that undetected faults do not have disastrous effects, reduce the number of false alarms and identify which faults have occurred.

Introduction

In the control of industrial systems, it is rare that a control system functions continuously without shutdown throughout the scheduled life cycle of the plant and controller hardware. Owing to wear of mechanical and electrical components, both actuators and sensors can fail in more or less critical ways. For safety-critical processes, it is of paramount importance to detect when faults are likely to happen and then to identify these faults as fast as possible once they have occurred.

To meet such industrial needs, a number of schemes for fault detection and isolation (FDI) have been put forward in the literature on automatic control. Much of the research has dealt with the design of filters which monitor a process and generate alarms when faults have occurred. In most cases, the filters are model-based devices which act independently of the computer-implemented digital controllers. In this paper, however, the advantages of combining the control algorithm and the FDI filter in a single module will be discussed, and a relatively simple methodology to design such combined modules will be described.

It will be shown that a combined module will be beneficial in terms of implementation and reliability, but it will also be shown that the quality of control and the quality of detection will not improve by using the integrated design – compared with the individual designs of two components – provided that a good nominal model is available. This result is shown to be very general. A special case using an algebraic Riccati equation approach was presented in Tyler and Morari (1994).

On the other hand, if the quality of the available model is poor, the design of the control system and of the diagnosis system has to be undertaken simultaneously in order to improve overall functionality.

Useful surveys about early work on FDI can be found in Frank (1990) and Patton *et al.* (1989). Many of these techniques are observer-based, such as Magni and Mouyon's (1991). These methods have since been refined and extended. A more recent work in this line of research is that of Frank and Ding (1994). The original idea of using the information already available in the "observer" part of a controller for diagnostic purposes was given in Nett *et al.* (1988).

Early work on FDI experienced problems owing to modelling uncertainties. In some cases, false alarms were common due to imperfect modelling. This called for issues of robustness to be incorporated into the FDI design algorithm. Specific robustness solutions to FDI problems were discussed in Bokor and Keviczky (1994); Mangoubi *et al.*, (1995); Murad *et al.*, (1996); Patton and Chen (1991); Qiu and Gertler (1993); and Wang and Wu (1993). All these methods used frequency domain techniques in contrast to Ajbar and Kantor's (1993), which used l_∞ techniques.

An interesting application of FDI techniques was presented in works by Blanke *et al.* (1995), Jørgensen *et al.* (1995), Grainger *et al.* (1995) and Garcia *et al.* (1995), which suggested using a diesel-engine actuator as an FDI benchmark problem.

Problem formulation

Figure 1 illustrates a control problem in the standard system configuration (see, for example, Zhou *et al.* (1996) for an introduction to the standard configuration paradigm). Here, w_d can be thought of as a collection of undesired signals (disturbances) entering the system $G(s)$ or as set-points. The signals y_c are the measurements used by the controller $K(s)$, generating the control signals u_c in order to make the outputs to-be-controlled z_c sufficiently small.

The system in Figure 1 can be described in either the state space formulation:

$$\begin{pmatrix} \dot{x} \\ z_c \\ y_c \end{pmatrix} = \begin{pmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{pmatrix} \begin{pmatrix} x \\ w_d \\ u_c \end{pmatrix}$$

or, alternatively, in transfer matrix function form:

$$\begin{pmatrix} z_c \\ y_c \end{pmatrix} = G(s) \begin{pmatrix} w_d \\ u_c \end{pmatrix} = \begin{pmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{pmatrix} \begin{pmatrix} w_d \\ u_c \end{pmatrix} = \begin{pmatrix} C_1(sI - A)^{-1} B_1 + D_{11} \\ C_2(sI - A)^{-1} B_1 + D_{21} \\ C_1(sI - A)^{-1} B_2 + D_{12} \\ C_2(sI - A)^{-1} B_2 + D_{22} \end{pmatrix} \begin{pmatrix} w_d \\ u_c \end{pmatrix} \quad (1)$$

For the standard problem shown in Figure 1, a controller $K(s)$, making the transfer function from w_d to z_c small, can be found by standard control optimization tools. Popular control design methods that support standard problem optimization comprise: LQG (or \mathcal{H}_2) methods (Zhou *et al.*, 1996), \mathcal{H}_∞ methods (Zhou *et al.*, 1996), \mathcal{L}_1 methods (Dahleh and Pearson, 1987) and μ methods (Zhou *et al.*, 1996).

Usually, the model $G(s)$ will contain the plant model itself, but it can also contain models of disturbances, measurement noise, time variations, non-linearities and unmodelled dynamics. Hence, making the transfer function from w_d to z_c small ensures a number of performance and robustness properties.

The everyday operation of such a feedback system depends on reliable actuators and sensors. However, in most industrial environments actuators and sensors can fail. One way to model this is depicted in Figure 2.

Here, the measurements used by the controller are $y = y_c + f_s$ rather than y_c and the controls acting on the plant are $u_c + f_a$ rather than u_c . For example $y_c + f_s \equiv 0$ or $u_c + f_a \equiv 0$ can represent completely defective sensors or actuators, respectively.

For safety-critical processes in particular, faults must be identified and action taken immediately. Two main paths of action can be taken: either the control design algorithm can be modified to tolerate minor errors; or, using

Figure 1 Control system in standard control configuration

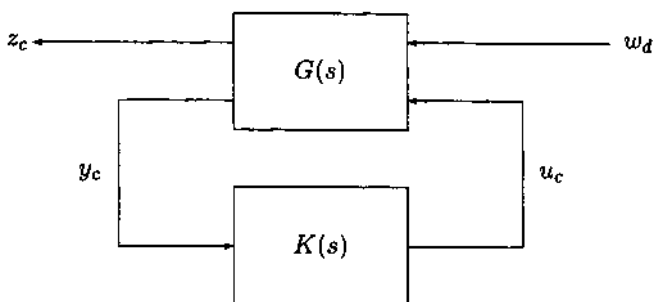
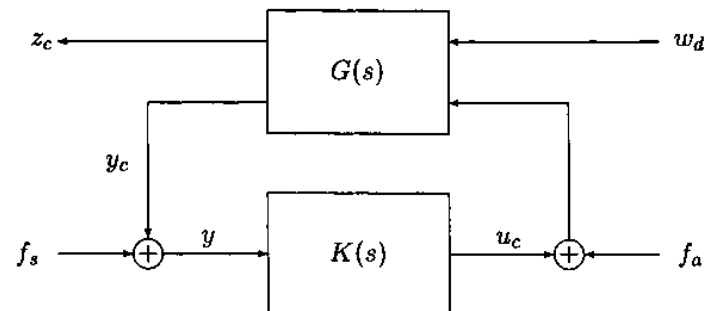


Figure 2 Control system with actuator and sensor faults



an estimator, the faulty signal can be identified and action taken by the operator or a supervisory system. In most applications, the latter would be preferable.

A method is now described which allows for either, or both, approaches to be incorporated into a single design step that also comprises the controller design. This is achieved using a single module which generates both the control action and the fault estimates.

To identify individual faults successfully, it is essential to have reliable fault models. One way to describe fault models is to introduce frequency weightings on the fault signals:

$$f_a = W_a(s)w_a \text{ and } f_s = W_s(s)w_s$$

where w_a and w_s are signals that are anticipated to have flat power spectra (white noise). These are imaginary signals with the sole purpose of generating the frequency-coloured signals f_a and f_s . The module to be designed should, in addition to the control signal u_c , also generate a signal containing estimates of potential faults:

$$u_f = \begin{pmatrix} \hat{f}_a \\ \hat{f}_s \end{pmatrix}$$

This situation is depicted in Figure 3.

The final step is to define a fault estimation error z_f as:

$$z_f = \begin{pmatrix} f_a \\ f_s \end{pmatrix} - u_f$$

Using these signals, a new augmented standard problem can be established as shown in Figure 4.

Defining:

$$\xi = \begin{pmatrix} x \\ x_a \\ x_s \end{pmatrix}, w = \begin{pmatrix} w_d \\ w_a \\ w_s \end{pmatrix}, u = \begin{pmatrix} u_c \\ u_f \end{pmatrix}, z = \begin{pmatrix} z_c \\ z_f \end{pmatrix}, y = y_c + f_s \quad (2)$$

the following “new” standard problem is obtained in state space form:

$$\begin{aligned} \dot{\xi} &= \tilde{A}\xi + \tilde{B}_1w + \tilde{B}_2u \\ z &= \tilde{C}_1\xi + \tilde{D}_{11}w + \tilde{D}_{12}u \\ y &= \tilde{C}_2\xi + \tilde{D}_{21}w + \tilde{D}_{22}u \end{aligned} \quad (3)$$

or in transfer function matrix form:

$$\begin{pmatrix} z \\ y \end{pmatrix} = \tilde{G}(s) \begin{pmatrix} w \\ u \end{pmatrix} = \begin{pmatrix} \tilde{G}_{11}(s) & \tilde{G}_{12}(s) \\ \tilde{G}_{21}(s) & \tilde{G}_{22}(s) \end{pmatrix} \begin{pmatrix} w \\ u \end{pmatrix}$$

(The explicit formulae are given below.)

Using standard control optimization methods, a generalized controller $u = \tilde{K}(s)y$ for the diagram shown in Figure 5 can now be computed, which will be able to generate both control signals and fault estimates.

In the following, the solution to the standard problem depicted in Figure 5 will be given, and the interpretation of that solution will be discussed. \mathcal{H}_∞ optimization is well suited to this problem, because this method provides valuable clues to the proper selection

Figure 4 Standard model; for integrated control and FDI

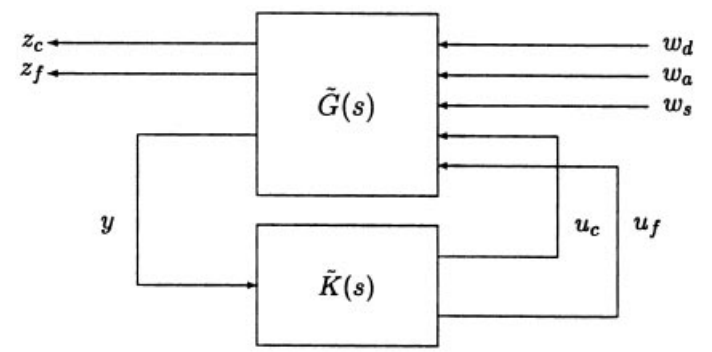


Figure 5 Standard problem for control optimization

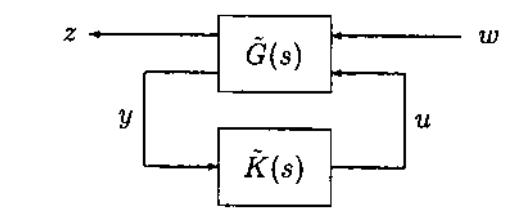
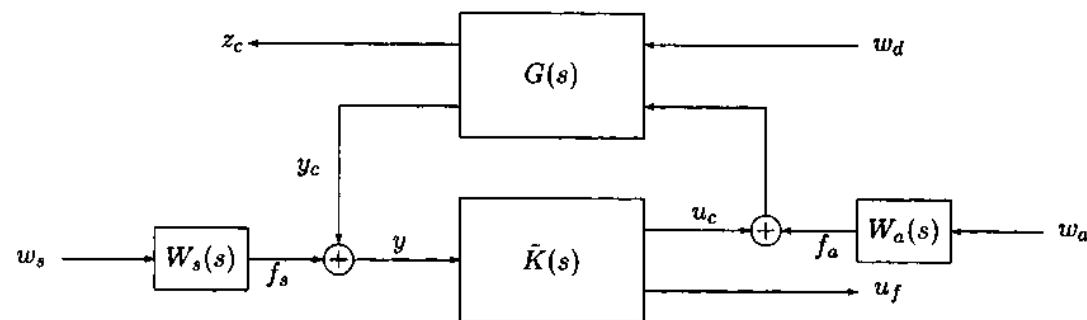


Figure 3 Control system with faults and diagnostics



of weighting matrices, which is crucial for the problems considered. However, the main observation, which is a type of separation principle, will hold for any criteria of the form:

$$\|z_c\| < 1. \quad \|z_f\| < 1$$

subject to bounded sets of disturbances and fault signals.

The nominal case

Using the partition (1), the following expressions for the standard problem that is equation (3) (depicted in Figures 4 and 5) can be derived:

$$\begin{aligned} \begin{pmatrix} z \\ y \end{pmatrix} &= \begin{pmatrix} z_c \\ z_f \\ y \end{pmatrix} = \tilde{G}(s) \begin{pmatrix} w \\ u \end{pmatrix} = \begin{pmatrix} \tilde{G}_{11}(s) & \tilde{G}_{12}(s) \\ \tilde{G}_{21}(s) & \tilde{G}_{22}(s) \end{pmatrix} \begin{pmatrix} w \\ u \end{pmatrix} \\ &= \left(\begin{array}{ccc|cc} G_{11}(s) & G_{12}(s)W_a(s) & 0 & G_{12}(s) & 0 \\ 0 & \begin{pmatrix} W_a(s) \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ W_s(s) \end{pmatrix} & 0 & -I \\ \hline G_{21}(s) & G_{22}(s)W_a(s) & W_s(s) & G_{22}(s) & 0 \end{array} \right) \begin{pmatrix} w_d \\ w_a \\ w_s \\ u_c \\ u_f \end{pmatrix} \end{aligned}$$

Introducing the control law $u = \tilde{K}(s)y$, the following closed-loop formula can be obtained:

$$\begin{pmatrix} z_c \\ z_f \end{pmatrix} = T_{zw}(s) \begin{pmatrix} w_d \\ w_a \\ w_s \end{pmatrix}$$

where

$$\begin{aligned} T_{zw}(s) &= \tilde{G}_{11}(s) + \tilde{G}_{12}(s)\tilde{K}(s)(I - \tilde{G}_{22}(s)\tilde{K}(s))^{-1}\tilde{G}_{21}(s) \\ &= \begin{pmatrix} G_{11}(s) & G_{12}(s)W_a(s) & 0 \\ 0 & \begin{pmatrix} W_a(s) \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ W_s(s) \end{pmatrix} \end{pmatrix} \\ &\quad + \begin{pmatrix} G_{12}(s) & 0 \\ 0 & -I \end{pmatrix} \tilde{K}(s) (I - (G_{22}(s)0)\tilde{K}(s))^{-1} \\ &\quad \quad \quad (G_{21}(s)G_{22}(s)W_a(s)W_s(s)). \end{aligned}$$

The transfer matrix $G(s)$ will often be stable owing to inner loops which are included in the standard control model. The following analysis can be carried out for unstable standard models too, but for simplicity $G(s)$ will be assumed stable below. In this case, the YJBK parameterization (Youla *et al.*, 1971) of all stabilizing controllers can be obtained simply by making the substitution:

$$\begin{aligned} Q(s) &= \tilde{K}(s)(I - \tilde{G}_{22}(s)\tilde{K}(s))^{-1}, \\ \tilde{K}(s) &= Q(s)(I + \tilde{G}_{22}(s)Q(s))^{-1}. \end{aligned}$$

Partitioning the control sensitivity function $Q(s)$ as

$$Q(s) = \begin{pmatrix} Q_1(s) \\ Q_2(s) \end{pmatrix}$$

the following expression is obtained:

$$T_{zw}(s) = \begin{pmatrix} G_{11}(s) + G_{12}(s)Q_1(s)G_{21}(s) & G_{12}(s)(I + Q_1(s)G_{22}(s))W_a(s) \\ -Q_2(s)G_{21}(s) & \begin{pmatrix} W_a(s) \\ 0 \end{pmatrix} - Q_2(s)G_{22}(s)W_a(s) \\ & G_{12}(s)Q_1(s)W_s(s) \\ & \begin{pmatrix} 0 \\ W_s(s) \end{pmatrix} - Q_2(s)W_s(s) \end{pmatrix}$$

Now, the crucial observation in this expression is that each of the two rows of the block-partitioned matrix depends on only one of the Q_i s, $i \in \{1, 2\}$. This has the following two consequences:

- (1) Making the closed-loop transfer function associated with the control objectives small and making the closed-loop transfer function associated with the FDI objectives small can be achieved independently.
- (2) Optimizing independently eliminates some of the conservatism often introduced in optimization methods that optimize the norm of a transfer matrix built by stacking transfer matrices corresponding to different criteria.

This possibility of a separation principle shall be exploited in the design procedure below. A separation principle similar in spirit to this is described in Stoustrup and Niemann (1997).

Since the upper row partition of $T_{zw}(s)$ depends only on $Q_1(s)$ and the lower row partition depends on $Q_2(s)$, the transfer function T_{zw} can be optimized by individually optimizing the different block terms. Hence, after separating the optimizations for z_c and z_f we are faced with the problem of optimizing the following two transfer matrices independently:

$$\begin{pmatrix} G_{11}(s) + G_{12}(s)Q_1(s)G_{21}(s) & G_{12}(s)(I + Q_1(s)G_{22}(s)) \\ W_a(s) & G_{12}(s)Q_1(s)W_s(s) \end{pmatrix} \quad (4)$$

and

$$\begin{pmatrix} -Q_2(s)G_{21}(s) & \begin{pmatrix} W_a(s) \\ 0 \end{pmatrix} - Q_2(s)G_{22}(s)W_a(s) \\ \begin{pmatrix} 0 \\ W_s(s) \end{pmatrix} - Q_2(s)W_s(s) \end{pmatrix} \quad (5)$$

The standard control problems corresponding to equations (4) and (5) are in a form known as the model-matching problem, which is a simpler, special case of the so-called general four-block controlled problem (see, for example, Zhou *et al.*, 1996).

The standard problem formulation corresponding to equation (4) is:

$$\begin{pmatrix} z_c \\ y_{Q1} \end{pmatrix} = \left(\begin{array}{ccc|c} G_{11}(s) & G_{12}(s)W_a(s) & 0 & G_{12}(s) \\ G_{21}(s) & G_{22}(s)W_a(s) & W_s(s) & 0 \end{array} \right) \begin{pmatrix} w_d \\ w_a \\ w_s \\ u_{Q1} \end{pmatrix} \quad (6)$$

where u_{Q1} is the output of the $Q_1(s)$ partition and y_{Q1} is the input to the $Q_1(s)$ subsystem.

For equation (5), the associated standard problem is:

$$\begin{pmatrix} z_f \\ y_{Q2} \end{pmatrix} = \left(\begin{array}{ccc|c} 0 & \begin{pmatrix} W_a(s) \\ 0 \end{pmatrix} & 0 & -I \\ G_{21}(s) & G_{22}(s)W_a(s) & W_s(s) & 0 \end{array} \right) \begin{pmatrix} w_d \\ w_a \\ w_s \\ u_{Q2} \end{pmatrix} \quad (7)$$

where u_{Q2} is the output of the $Q_2(s)$ partition and y_{Q2} is the input to the $Q_2(s)$ subsystem. Given Q_1 and Q_2 , the solution to the standard problem of equation (3) is:

$$\tilde{K}(s) = \begin{pmatrix} K_1(s) \\ K_2(s) \end{pmatrix}$$

where $K_1(s)$ and $K_2(s)$ – the feedback control part and the FDI part, respectively – can be computed as:

$$K_1(s) = Q_1(s)(I + G_{22}(s)Q_1(s))^{-1} \quad (8)$$

and

$$\begin{aligned} K_2(s) &= Q_2(s)(I + G_{22}(s)Q_1(s))^{-1} \\ &= Q_2(s)\left(I + G_{22}(s)Q_1(s)(I + G_{22}(s)Q_1(s))^{-1}\right) \\ &= Q_2(s)(I - G_{22}(s)K_1(s)). \end{aligned} \quad (9)$$

Remark 1

It is important to note that the expression that is equation (8) for K_1 does not depend on Q_2 but only on Q_1 which is found by an optimization which also does not depend on Q_2 . This means that, in this formulation of the problem, the control action does not directly depend on the fault estimator dynamics. Still, the regulating controller can be detuned compared with a set-up in which faults are not allowed for, since the control design algorithm regards the faults as disturbances and noise (as can be seen from equation (6)). Where this is not desirable, some attention must be paid to the weighting selection scheme to avoid detuning. Alternatively, the optimization problem of equation (4) can be completely reformulated by virtue of the separation principle described above. Equation (9) for K_2 depends on Q_1 . This is physically obvious, since the fault detection and isolation filter has to use the observer part of the controller to identify the faults.

Relationships to the four-parameter controller

The four-parameter controller was introduced by Nett *et al.* (1988) in connection with fault detection. The four-parameter controller can be considered as an extension of the two-parameter controller introduced above.

Let the plant still be given by equation (2). The four-degree-of-freedom controller also has access to a reference signal t as well as the measurement signal y , and the controller returns both a control signal u and a diagnostic signal a :

$$\begin{pmatrix} u \\ a \end{pmatrix} = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \begin{pmatrix} y \\ t \end{pmatrix}$$

The design set-up for the four-parameter controller can also be formulated using the standard system description given in Figure 4. The generalized system $G_{ncffp}(s)$ is then given by:

$$G_{ncffp}(s) = \left(\begin{array}{ccc|cc} G_{11} & G_{12}W_a & 0 & 0 & G_{12} & 0 \\ 0 & \begin{pmatrix} W_a \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ W_s \end{pmatrix} & 0 & 0 & -I \\ G_{21} & G_{22}W_a & W_s & 0 & G_{22} & 0 \\ 0 & 0 & 0 & I & 0 & 0 \end{array} \right) \quad (10)$$

As above, assume that the system $G_{ncffp}(s)$ is open-loop stable. Then we can again use the following parameterization of all stabilizing controllers:

$$Q(s) = K(s)\left(I - \begin{pmatrix} G_{22} & 0 \\ 0 & 0 \end{pmatrix}K(s)\right)^{-1}$$

$$K(s) = Q(s)\left(I + \begin{pmatrix} G_{22} & 0 \\ 0 & 0 \end{pmatrix}Q(s)\right)^{-1}$$

Again, let $Q(s)$ be partitioned as:

$$Q(s) = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix} \quad (11)$$

Using $Q(s)$, the closed-loop transfer function T_{ncffp} is then given by:

$$T_{ncffp}(s) \begin{pmatrix} G_{11} + G_{12}Q_{11}G_{21} & G_{12}(I + Q_{12}G_{22}) \\ W_a - Q_{21}G_{21} & \begin{pmatrix} W_a \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ W_s \end{pmatrix} - Q_{21}W_s & -Q_{22} \end{pmatrix} \quad (12)$$

Again, note that there is a separation. Q_{11} and Q_{12} appear only in the first row of T_{ncffp} and Q_{21} and Q_{22} appear only in the second row. Based on the Q controller, we can calculate the K controller by using equation (11). The controller K then takes the following form:

$$K(s) = \begin{pmatrix} Q_{11}(I + G_{22}Q_{11})^{-1} & (I + Q_{11}G_{22})^{-1}Q_{12} \\ Q_{21}(I + G_{22}Q_{11})^{-1} & Q_{22} - Q_{21}(I + G_{22}Q_{11})^{-1}G_{22}Q_{12} \end{pmatrix} \\ = \begin{pmatrix} Q_{11}(I + G_{22}Q_{11})^{-1} & (I - K_{11}G_{22})Q_{12} \\ Q_{21}(I - G_{22}K_{11}) & Q_{22} - Q_{21}K_{11}G_{22}Q_{12} \end{pmatrix}$$

As in the previous section, there is a separation between control and fault-detection objectives. Note that in the implementation, the pure control part K_{11} depends on Q_{11} only, K_{12} depends both on Q_{11} and Q_{12} , K_{21} depends on Q_{11} and Q_{21} and K_{22} depends on all elements of Q .

This four-parameter controller set-up has been analysed by Nett *et al.* (1988) in the nominal case. The set-up applied in Nett *et al.* (1988) is slightly different from that used in this paper. The design set-up for the four-parameter controller has not been formulated in the standard set-up as in Figure 4. One consequence is that the separation in the controller design does not appear in the parameterization used in Nett *et al.* (1988). A design scheme has been carried out by Nett *et al.* (1988) based on the so-called single controller principle roles.

An \mathcal{H}_∞ solution to the nominal problem

To obtain explicit design formulae, the criterion of optimization needs to be more specific. For a number of purposes, \mathcal{H}_∞ optimization is a good choice, since it constitutes a flexible loop-shaping tool.

By appropriately selecting the weightings, we can assume without loss of generality that we are faced with normalized \mathcal{H}_∞ constraints, in which case equations (4) and (15) take the form:

$$\left\| \begin{pmatrix} G_{11}(s) + G_{12}(s)Q_1(s)G_{21}(s) & G_{12}(s)(I + Q_1(s)G_{22}(s))W_a(s) \\ G_{12}(s)Q_1(s)W_s(s) \end{pmatrix} \right\|_\infty < 1$$

and

$$\left\| \begin{pmatrix} -Q_2 \begin{pmatrix} G_{21}(s) & W_a(s) \\ 0 & 0 \end{pmatrix} - Q_2(s)G_{22}(s)W_a(s) \\ \begin{pmatrix} 0 \\ W_s(s) \end{pmatrix} - Q_2(s)W_s(s) \end{pmatrix} \right\|_\infty < 1. \quad (15)$$

The only remaining step in devising an algorithm for the computation of the combined control and FDI device is to solve the two inequalities of equations (14) and (15), which have the two standard formulations of equations (6) and (7). Using polynomial \mathcal{H}_∞ theory (see Kwakernaak, 1993), the following results are obtained.

Lemma 1

Consider the following J -spectral factorization:

$$\Pi_1 = \begin{pmatrix} I & 0 \\ 0 & -G_{12} \end{pmatrix} \begin{pmatrix} -G_{21}G_{21} - G_{22}W_aW_aG_{22} - W_sW_s \\ -G_{11}G_{21} - G_{12}W_aW_aG_{21} \\ -G_{21}G_{11} - G_{21}W_aW_aG_{12} \\ I - G_{11}G_{11} - G_{12}W_aW_aG_{12} \end{pmatrix}^{-1} \\ \times \begin{pmatrix} I & 0 \\ 0 & -G_{12} \end{pmatrix} \\ = Z_1 J_1 Z_1$$

where $Z_1(s)$ is a square matrix which is invertible as an element of $\mathcal{R}\mathcal{H}_\infty$, and J_1 is a constant matrix of the form

$$J_1 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

with a suitable number of 1s and -1 s. J_1 is called the signature matrix of Π_1 . The model-matching problem equation (6) has a solution if and only if the following controller is stabilizing:

$$Q_1^c = \begin{pmatrix} 0 & I \end{pmatrix} Z_1^{-1} \begin{pmatrix} I \\ 0 \end{pmatrix} \left(\begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} Z_1^{-1} \begin{pmatrix} I \\ 0 \end{pmatrix} \right)^{-1}. \quad (16)$$

Moreover, in that case, all solutions are given by:

$$Q_1 = Y_1 X_1^{-1} \quad (17)$$

where

$$\begin{pmatrix} X_1 \\ Y_1 \end{pmatrix} = Z_1^{-1} \begin{pmatrix} A_1 \\ B_1 \end{pmatrix}$$

and A_1 and B_1 are (free) stable rational matrices, $\det A_1$ having all its roots in the open left half complex plane, satisfying:

$$A_1^\dagger A_1 \geq B_1^\dagger B_1.$$

Lemma 2

Similarly, for equation (7), consider the following J -spectral factorization:

$$\Pi_2 = \begin{pmatrix} -G_{21}G_{21} - G_{22}W_aW_aG_{22} - W_sW_s \\ -W_aW_aG_{22} \\ -W_sW_s \end{pmatrix} \\ \begin{pmatrix} -G_{22}W_aW_a & -W_sW_s \\ I - W_aW_a & 0 \\ 0 & I - W_sW_s \end{pmatrix}^{-1} \\ = Z_2 J_2 Z_2$$

where $Z_2(s)$ is a square matrix which is invertible as an element of $\mathcal{R}\mathcal{H}_\infty$ and J_2 is the signature matrix of Π_2 . The model-matching problem equation (7) has a solution if and only if the following controller is stabilizing:

$$Q_2^c = \begin{pmatrix} 0 & I \end{pmatrix} Z_2^{-1} \begin{pmatrix} I \\ 0 \end{pmatrix} \left(\begin{pmatrix} I & 0 \end{pmatrix} Z_2^{-1} \begin{pmatrix} I \\ 0 \end{pmatrix} \right)^{-1}. \quad (18)$$

Moreover, all solutions are given by:

$$Q_2 = Y_2 X_2^{-1} \quad (19)$$

where:

$$\begin{pmatrix} X_2 \\ Y_2 \end{pmatrix} = Z_2^{-1} \begin{pmatrix} A_2 \\ B_2 \end{pmatrix}$$

and A_2 and B_2 are (free) stable rational matrices, $\det A_2$ having all its roots in the open left half complex plane, satisfying:

$$A_2^{-1} A_2 \geq B_2^{-1} B_2.$$

Employing the separation principle described above, and combining lemmas 1 and 2, the main result can be stated.

Theorem 3

Consider the set-up depicted in Figure 3 where $\tilde{K}(s)$ is a combined controller and FDI module. The following two statements are equivalent:

- (1) There exists a transfer matrix $\tilde{K}(s)$ making the transfer function from disturbances to controlled outputs smaller than 1, and making the transfer function from actuator and sensor faults to the fault estimation error smaller than 1.
- (2) The controller Q_1^c given by equation (16) stabilizes the standard problem given by equation (6) and, likewise, the controller Q_2^c given by equation (18) stabilizes the standard problem given by equation (7).

Moreover, when these conditions are satisfied, a possible choice of $\tilde{K}(s) = \begin{pmatrix} K_1 \\ K_2 \end{pmatrix}$ is given by (8) and (9) where $Q_1(s)$ and $Q_2(s)$ are given by (17) and (19), respectively.

In conclusion, this section has shown how an algorithm can be used for designing a single module which comprises both feedback control action and fault diagnosis and isolation. The design method is very flexible. Manipulating weights, the following four objectives can be explicitly designed for:

- (1) following references and rejecting disturbances robustly;
- (2) controlling the system so that undetected faults do not have disastrous effects;
- (3) reducing the number of false alarms;
- (4) identifying which faults have occurred.

These objectives are discussed in more detail below.

The algorithm was based on a type of separation principle which facilitates transparency in the design process with respect to the fundamental trade-offs related to diagnosing and controlling a system.

Not only have the processes of designing a filter and a controller been separated, but also the design criteria. This shows that the controller does not need to be detuned to implement a sound fault detection mechanism. Moreover, this statement holds for optimization with respect to any choice of (norm-based) design criteria, formulated as one criterion for the controller and another for the filter.

Design of filters for uncertain systems

In the previous sections, the interdependence of the controller and the filter design in the nominal case have been examined. It was discovered that a separation exists between controller design and filter design. In this section, the way in which the presence of uncertainty affects the results will be considered.

First, consider the standard control configuration in Figure 1 where the system G is a function of an uncertain block Δ , i.e. $G = G(s, \Delta)$. The Δ block represents the uncertain or unmodelled part of the system. The uncertain part of the system is normally described as additive model uncertainty given by:

$$G(s) = G_0(s) + \Delta(s)$$

or as multiplicative model uncertainty given by:

$$G(s) = G_0(s) (I + \Delta(s))$$

where $G_0(s)$ is the nominal system (see Skogestad and Postlethwaite (1996) for more on model uncertainties).

By including a model uncertainty in the set-up of Figure 1, the standard control configuration for robust control as shown in Figure 6 is obtained.

The system $G_0(s)$ in Figure 6 can be described in either the state space formulation or in transfer function form. The state space description is given by:

$$\begin{pmatrix} \dot{x} \\ e \\ z_c \\ y_c \end{pmatrix} = \begin{pmatrix} A & B_1 & B_2 & B_3 \\ C_1 & D_{11} & D_{12} & D_{13} \\ C_2 & D_{21} & D_{22} & D_{23} \\ C_3 & D_{31} & D_{32} & D_{33} \end{pmatrix} \begin{pmatrix} x \\ d \\ w_d \\ u_c \end{pmatrix}.$$

Let us consider the set-up from Figure 4 and include a model uncertainty. Before the standard set-up for integrated control and FDI for systems with model uncertainties are given, a

Figure 6 Control system in standard control configuration for systems with model uncertainty

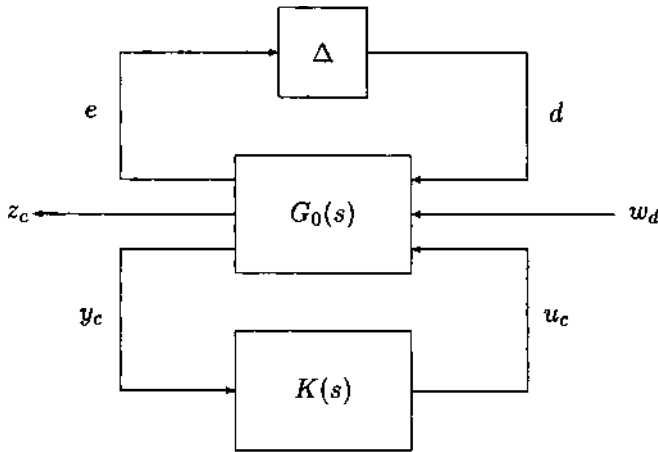
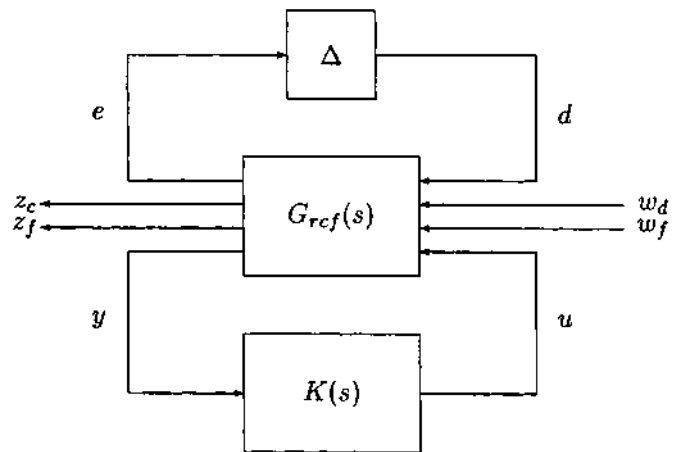


Figure 7 Generalized set-up for robust control and fault detection with a 2 parameter controller



compact notation is introduced for the fault signals in order to simplify the equations in the rest of this paper. Let the fault signal vector f be given as:

$$f = \begin{pmatrix} f_a \\ f_s \end{pmatrix}$$

Further, the weighted fault signal f is given by:

$$f = W_f(s)w_f = \begin{pmatrix} W_a(s) & 0 \\ 0 & W_s(s) \end{pmatrix} \begin{pmatrix} W_a \\ W_s \end{pmatrix}$$

By introducing this notation, the set-up from Figure 4 with model uncertainty is illustrated in Figure 7.

It is assumed that Δ is scaled in such a way that $\|\Delta\| \leq 1 \forall \omega$. Further, Δ can be structured or unstructured. The transfer function from w_d to z_c defines the performance of the closed-loop control system, and the transfer function from w_f to z_f defines the performance for the fault detection filter.

The generalized system $G_{ref}(s)$ in Figure 7 is given by:

$$\begin{pmatrix} e \\ z_c \\ z_f \\ y \end{pmatrix} = G_{ref}(s) \begin{pmatrix} d \\ w_d \\ w_f \\ u \end{pmatrix}$$

where:

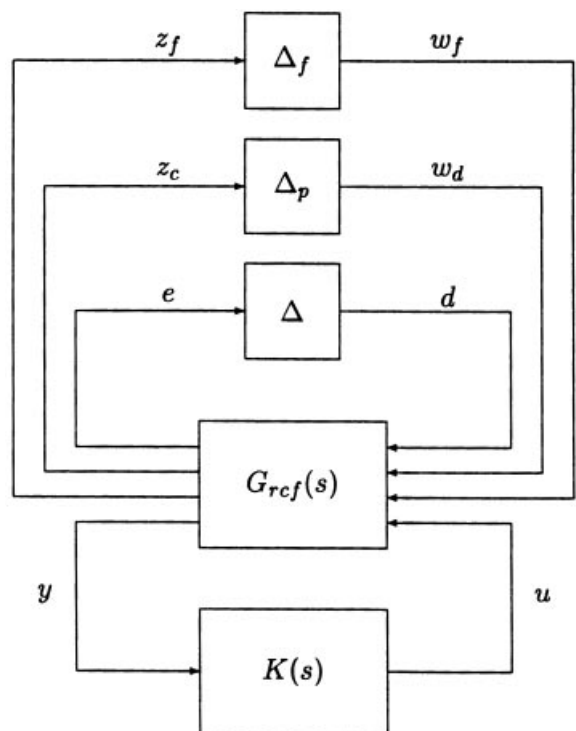
$$G_{ref}(s) = \left(\begin{array}{ccc|cc} G_{ed} & G_{ew_d} & G_{ew_f}W_f & G_{eu} & 0 \\ G_{z_c,d} & G_{z_c,w_d} & G_{z_c,w_f}W_f & G_{z_c,u} & 0 \\ 0 & 0 & W_f & 0 & -I \\ \hline G_{yd} & G_{yw_d} & G_{yw_f}W_f & G_{yu} & 0 \end{array} \right) \quad (20)$$

In comparison to the system used in “The nominal case” section earlier in the paper, the introduction of the uncertainty block Δ changes the possible design concepts considerably, as will be demonstrated.

Consider Figure 8, where the Δ_p and Δ_f blocks represent performance specifications for the closed-loop transfer function and performance for the fault detection signal. Introduction of such fictitious perturbation blocks is a standard device in μ synthesis to obtain robust performance (see, for example, Zhou *et al.* (1996)). It is assumed that weighting matrices on the performance specifications in Figure 8 are included in the generalized system $G_{ref}(s)$.

Applying the same technique as in the nominal case by using a parameterization of the controllers, the following closed-loop transfer function T_{ref} is obtained:

Figure 8 Generalized set-up for robust control and fault detection with performance specifications represented by fictitious perturbation blocks



$$T_{rcf} = \begin{pmatrix} G_{ed} + G_{eu}Q_1G_{yd} & G_{ew_d} + G_{eu}Q_1G_{yw_d} \\ G_{z_c,d} + G_{z_c,u}Q_1G_{yd} & G_{z_c,w_d} + G_{z_c,u}Q_1G_{yw_d} \\ -Q_2G_{yd} & -Q_2G_{yw_d} \\ G_{ew_f}W_f + G_{eu}Q_1G_{yw_f}W_f \\ G_{z_c,w_f}W_f + G_{z_c,u}Q_1G_{yw_f}W_f \\ W_f - Q_2G_{yw_f}W_f \end{pmatrix} \quad (21)$$

At first glance, it seems there is again a separation between the two parameters in $Q(s)$. $Q_1(s)$ appear only in the first two rows and Q_2 appear only in the last row of T_{rcf} . However, because the feedback loop with Δ , Δ_p , and Δ_f is considered directly in the design process, there is no separation in this case due to the model uncertainties. This can be seen quite easily by considering a separate design of a robust feedback controller Q_1 and a robust fault detection filter Q_2 (see below).

First, let us consider the design of a robust stabilizing controller followed by a design of a nominal filter. The feedback controller design problem with respect to robust stability is represented by the following closed-loop transfer function:

$$T_{isc}(s) = G_{ed} + G_{eu}Q_1G_{yd}. \quad (22)$$

The design problem is a standard control optimization problem when Δ is unstructured. Otherwise, it is a structured optimization problem like a μ design problem. The following design of a nominal filter is represented by the following closed-loop transfer function:

$$T_{ipf}(s) = \begin{pmatrix} -Q_2G_{yw_d} & W_f - Q_2G_{yw_f}W_f \end{pmatrix}. \quad (23)$$

Here, there is again separation between the two designs because Q_1 appear only in equation (22) and Q_2 only in equation (23).

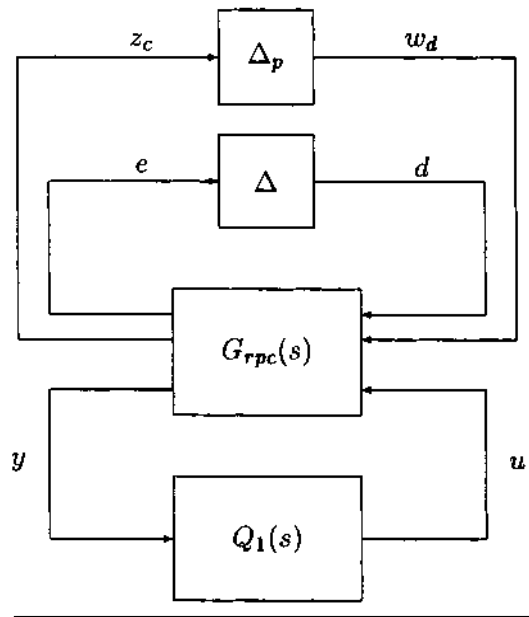
The next design case consists of a design for a feedback controller with respect to robust performance followed by the design of a nominal filter. The design problem for the feedback controller is represented by the following closed-loop transfer function:

$$T_{rpc} = \begin{pmatrix} G_{ed} + G_{eu}Q_1G_{yd} & G_{ew_d} + G_{eu}Q_1G_{yw_d} \\ G_{z_c,d} + G_{z_c,u}Q_1G_{yd} & G_{z_c,w_d} + G_{z_c,u}Q_1G_{yw_d} \end{pmatrix} \quad (24)$$

This design problem is a μ design problem owing to the structure in the perturbations (see Figure 9). As expected, only the feedback parameter Q_1 appears here.

The design of the nominal filter is still given by equation (23); there is also a separation between the two designs. In the last two design cases, the filter is designed with respect to robust performance. In the first case, the

Figure 9 Generalized set-up for robust control with performance specifications

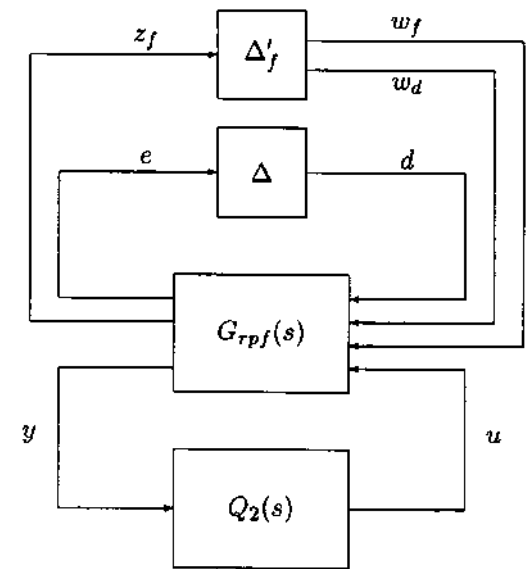


design of a feedback controller is represented by equation (22), i.e. designed with respect to robust stability. The design problem for a filter with respect to model uncertainty is represented by the following transfer function:

$$T_{ipf} + \begin{pmatrix} G_{ed} + G_{eu}Q_1G_{yd} & G_{ew_d} + G_{eu}Q_1G_{yw_d} \\ -Q_2G_{yd} & -Q_2G_{yw_d} \\ G_{ew_f}W_f + G_{eu}Q_1G_{yw_f}W_f \\ W_f - Q_2G_{yw_f}W_f \end{pmatrix} \quad (25)$$

The filter design problem is also a μ design problem owing to the structure in the (partly fictitious) perturbations (see Figure 10).

Figure 10 Generalized set-up for the design of a robust fault detection filter



From equation (25) it can be seen that the two designs are no longer separated. Both controllers appear in the design problem in equation (25). Another way of seeing that the two designs are coupled is to consider the closed-loop transfer function given by:

$$\begin{aligned} T_{z/wf}(s) &= \mathfrak{F}_u(T_{zpf}, \Delta) \\ &= W_f - Q_2 G_{yw_f} W_f - Q_2 G_{yd} \Delta \\ &\quad (I - (G_{ed} + G_{eu} Q_1 G_{yd}) \Delta)^{-1} \\ &\quad (G_{ew_f} W_f + G_{eu} Q_1 G_{yw_f} W_f). \end{aligned}$$

It is clear from the above equation that in general the design of Q_1 depends on Q_2 . Generically, it will never be possible to separate the design of the two controllers owing to the feedback with the uncertainty block Δ .

The latter design case entails the design of a feedback controller with respect to robust performance represented by equation (24) followed by the filter design from equation (25). Once again there is no separation between the two designs.

Summary of optimal design techniques

Depending on whether robustness is considered important in the design of the control loop and of the filter, six classes of design methodology can be identified. These classes are characterized in Table I.

Remark 2

The entry for the filter's robust performance subject to the controller's nominal design presumes that the control loop will actually be stable, since the filter cannot stabilize an unstable control loop. Moreover, in this case, no optimization method exists that directly handles the coupling and gives the controller and filter in one design step. The suggested method is a reasonable sub-optimal approach.

Remark 3

The \mathcal{H}_∞ optimization for filter designs listed in Table I is based on an assumption that Δ_f is unstructured. More realistically, Δ_f will have a block diagonal structure, corresponding to individual faults. It is straightforward to incorporate this into the design procedure.

Solving the \mathcal{H}_∞ problem depicted in Figure 5 implies making six transfer functions small owing to the definitions (2) of w and z . These six transfer functions are:

- (1) $T_{z_c w_d}$ = transfer function from external disturbances to inferred outputs;

Table I The six possible design classes

	Nominal performance of filter	Robust performance of filter
Nominal performance of controller	(1) Separate designs (2) \mathcal{H}_∞ optimizations – see Remark 3 (3) Equations (14) and (15)	(1) Coupled design – Remark 2 (2) \mathcal{H}_∞ (controller) and μ (filter) optimizations (3) Equation (14) (\mathcal{H}_∞ optimization) and equation (25) (find Q_2 by μ optimization)
Robust stability of controller	(1) Separate designs (2) \mathcal{H}_∞ optimizations – see Remark 3 (3) Equations (22) and (23)	(1) Coupled design (2) μ optimization (3) Equation (25)
Robust performance of controller	(1) Separate designs (2) μ (controller) and \mathcal{H}_∞ (filter) optimizations – see Remark 3 (3) Equation (24) (μ optimizations and equation (23) (\mathcal{H}_∞ optimization)	(1) Coupled design (2) μ optimization (3) Equation (21)

Notes: For each class, item (1) describes whether the design of the controller and filter separates; item (2) specifies the method of optimization required; and item (3) specifies the relevant equations to be used for the optimization process

- (2) $T_{z_c w_a}$ = transfer function from actuator faults to inferred outputs;
- (3) $T_{z_c w_s}$ = transfer function from sensor faults to inferred outputs;
- (4) $T_{z_f w_d}$ = transfer function from external disturbances to fault estimation error;
- (5) $T_{z_f w_a}$ = transfer function from actuator faults to fault estimation error;
- (6) $T_{z_f w_s}$ = transfer function from sensor faults to fault estimation error.

The essential instrument for creating a well-functioning module for control action, fault detection and isolation is an optimization algorithm which makes these transfer functions small and trades off the individual functions by careful weighting selections.

Making each of the six transfer functions small has its own (important) interpretation:

- (1) making $\|T_{z_c w_d}\|_\infty$ small implies good disturbance rejection and robustness, i.e. the original control objectives are achieved;
- (2) making $\|T_{z_c w_a}\|_\infty$ and $\|T_{z_c w_s}\|_\infty$ small implies that undetected faults do not cause disasters;
- (3) making $\|T_{z_f w_d}\|_\infty$ small implies that disturbances are not readily interpreted as

- faults, i.e. the risk of false alarms is reduced;
- (4) making $\|T_{zfw_a}\|_\infty$ and $\|T_{zfw_s}\|_\infty$ small implies that u_f becomes a sound estimate of potential actuator and sensor faults.

From the results in this paper, not only is it clear that objective (1) has to be traded off against objective (2), and that objective (3) has to be traded-off against objective (4), but also the design process does not involve a trade-off between the two pairs of objectives, once the standard model in equation (1) has been specified.

In order not to complicate the explanation, the control weights related to control performance and control robustness have not been explicitly included, but they are, of course, present in terms of the original standard problem formulation depicted in Figure 1. It is evident that the choice of the original system's internal weightings is very significant in determining the overall performance of the combined control and FDI module.

In order for the optimization in Theorem 3 to give a useful result, it is of great importance to choose the weightings associated with the original standard problem, with the actuator faults and with the sensor faults, so that all these weightings are separated in a frequency range.

Choosing large weights for the disturbance models means that the design algorithm allows for disturbance rejection and control robustness. Alternatively, choosing large weights for the actuator- and sensor-fault models means that the quality of the fault estimates is emphasized in the design algorithm, which ensures that very few faults are undetected.

As mentioned in Remark 1, the faults are considered to be disturbances in the control sub-problem (6) and, dually, the disturbances are implicitly represented in the detection sub-problem (7) in terms of the standard problem parameters. There is no principal limitation in the design method suggested in this paper which forbids two different standard problems with different internal weightings to be applied to the control design sub-problem (6) and the FDI design sub-problem (7); nevertheless, this would complicate the design process and the weightings would have to be well motivated by the specific application.

Remark 4

It should be noted that fixed weightings for external disturbances have been considered in this paper (in fact, they have been assumed to be absorbed in the original standard formulation). This is a reasonable point of view for systems in which the frequency spectrum of external disturbances is quite well known. However, it should be mentioned that some researchers also like to consider disturbance weightings as design parameters, such that the bandwidth of the system is optimized indirectly. In this case, the decoupling results for the nominal case are misleading, since there has to be a trade-off between these weightings and those of the faults.

Conclusions

The integration of feedback controller and fault-detection filter design has been considered for systems with and without model uncertainties. It has been found that the design of the feedback controller and the fault detection filter can be separated in the nominal case. In the uncertain case, however, an optimal solution cannot be obtained by separate robust controller and robust filter designs.

In spite of the separation in the nominal case, the integrated design of both the feedback controller and the fault detection filter is still a sound option, because it is possible to use the same observer for both the feedback controller and the filter. This has been carried out by Kilsgaard *et al.* (1996), where a fourth-order SIMO system has been considered and an \mathcal{H}_∞ design based on LMI has been undertaken. The result was an integrated feedback controller and a fault detection filter of order one.

In the uncertain case, it is essential to trade-off control performance, the effects of undetected faults, the risk of false alarms and quality of fault detection carefully. In most cases, this probably requires μ synthesis methods.

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