Real-Time Determination of Power System Frequency

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Abstract—The main frequency is an important parameter of an electrical power system. The frequency can change over a small range due to generation-load mismatches. Some power system protection and control applications, e.g., frequency relay for load shedding, load-frequency controller, require accurate and fast estimation of the frequency. Most digital algorithms for measuring frequency have acceptable accuracy if voltage waveforms are not distorted. However, due to nonlinear devices, e.g., semiconductor rectifiers, electric arc furnaces, the voltage waveforms can include higher harmonics. The paper presents a new method of measurement of power system frequency, based on digital filtering and Prony’s estimation method. Simulation results confirm, that the proposed method is more accurate than others, e.g., than the method based on the measurement of angular velocity of the rotating voltage phasor.

Index Terms—Discrete Fourier transforms, FIR digital filters, frequency measurement, power system parameter estimation, protective relaying.

I. INTRODUCTION

The basic frequency is an important operating parameter of an electrical power system, which is required to operate at a constant frequency. Under steady-state conditions the total power generated by power stations is equal to the system load and losses. Due to sudden appearance of generation-load mismatches the frequency can deviate from its nominal value. Some power system protection and control equipment, e.g., load-frequency controller, frequency relay for load shedding, require accurate and fast estimation of the frequency. The transient response should be no longer than four to five periods of the fundamental component. A variety of methods have been proposed during recent years for measuring power system frequency and frequency deviation. There are special conditions in a power system, e.g., small frequency deviation, distortion of voltage waveforms, requirements with regards to accuracy, which have to be taken into consideration. Most digital methods have acceptable accuracy if the voltage waveforms are not distorted. However, due to nonlinear devices, e.g., semiconductor rectifiers, electric arc furnaces, the voltage waveforms can include higher harmonics.

This paper presents a new method of measurement of power system frequency based on digital filtering and Prony’s estimation method. First a voltage waveform, taken from a voltage transformer, is filtered using algorithms based on the discrete Fourier transform (DFT). Coefficients of the filters are calculated assuming a constant frequency (e.g., 50 or 60 Hz). Due to deviation of the power system frequency the filtering is not enough exact. To improve the filtration effect, the Hamming or Blackman window functions are applied. To calculate the frequency, the output signal of the filter is processed using an algorithm based on the Prony’s estimation method [5], [10]. Simulation results confirm the high accuracy of the proposed method.

II. METHOD BASED ON THE DISCRETE FOURIER TRANSFORMATION

Some methods of frequency measurement, presented in literature during recent years base on the definition of the instantaneous frequency as angular velocity of the rotating voltage phasor [2], [8], [12], [15]. The phasor of the fundamental waveform of the voltage can be calculated from the $N$ samples, using the DFT or other algorithms [6]–[10].

If the sampling window equals one cycle of the basic waveform, the phasor at the time $t_k = kT$ is given by

$$G_k = \frac{2}{N} \sum_{n=0}^{N-1} v_{k+n-N+1} e^{-j\omega n T}$$

(1)

where:

- $T$ sampling interval;
- $\omega$ fundamental frequency;
- $v_{k+n-N+1}$ sampled values of a voltage.

When implementing the method, $G_k$ is updated at every sampled value. After each sampling cycle, the newest sample is taken into the calculation, while the oldest one is neglected. For each position of the phasor, its argument can be calculated. The instantaneous frequency can be determined from the two consecutive phasors

$$\omega = \frac{\arg[G_{k+1}] - \arg[G_k]}{T}$$

(2)

where

$$\arg[G_k] = \tan^{-1} \left( \frac{\text{Im}[G_k]}{\text{Re}[G_k]} \right)$$

(3)

For comparison, the described method was also tested on computer.

III. FILTERING

In the proposed approach a voltage waveform taken from a voltage transformer, is first filtered using algorithms based on the DFT. For further processing, we need only the time function of the fundamental component of voltage equals to the
real part of the phasor \([6], [7], [9], [10]\). The filter algorithm is described as

\[
g_k = \frac{2}{N} \sum_{n=0}^{N-1} y_{k+n} \cos(n\omega T) \quad (4)
\]

However, when the frequency changes, the rectangular window inherent in the DFT has some disadvantages. To improve the filter properties, applying of a smoothing window is proposed. The investigation was carried out for two most common window functions: Hamming window or Blackman window [16]. The Hamming window is described by

\[
w_H = 0.54 - 0.46 \cos \left( \frac{2\pi n}{N-1} \right) \quad (5)
\]

and the Blackman window by

\[
w_B = 0.42 - 0.5 \cos \left( \frac{2\pi n}{N-1} \right) + 0.08 \cos \left( \frac{4\pi n}{N-1} \right) \quad (6)
\]

The aim of the prefiltering is to improve the accuracy of the frequency determination.
IV. ALGORITHM BASED ON THE PRONY’S ESTIMATION METHOD

At the output of the filter algorithm we obtain samples of the fundamental component of a voltage as in (4). Due to deviation of the frequency the filtering is not exact. For the calculation of the frequency we propose an algorithm based on the Prony’s estimation method [5], [11].

The method is based on the assumption that given a series of samples \( g_1, g_2, \cdots, g_M \), a filtered voltage waveform can be approximated by one sinusoid

\[
y_m = A \cos(m \omega T + \psi)
\]

for \( m = 1, 2, \cdots, M \), (7)

where \( M \) is the number of samples taken into the approximation. In the complex exponential form, this may be written as

\[
y_m = b z_1^m + b^* z_1^m
\]

(8)

where

\[
z_1 = e^{j\omega T}
\]

(9)

\[
b = \frac{A}{2} e^{j\psi}.
\]

(10)

The estimation problem is, to find the values of \( b \) and \( z_1 \) so that the error

\[
\delta_m = y_m - y_n
\]

(11)

will be minimized.

The key idea of the Prony’s estimation method is to transform this nonlinear problem into a linear fitting problem by minimizing the error \( E \) defined as

\[
E = \sum_{m=0}^{M-1} (\varepsilon_m)^2
\]

(12)

where \( p \) is the number of exponents and \( \varepsilon_m \) is defined by

\[
\varepsilon_m = \sum_{k=0}^{p} a_k \delta_{k+m-1}.
\]

(13)

The parameters \( a_k \) are initially unknown, and are related to the frequency of the sinusoid. The key step to the estimation is to recognize that the (8) is the solution to some linear constant-coefficient difference equation. In order to find the form of the difference equation, the polynomial \( F(z) \) is defined for \( p = 2 \)

\[
F(z) = a_0 (z - z_1)(z - z_1^*) = 0.
\]

(14)

The exponents \( z_1 \) and \( z_1^* \) are roots of the polynomial. Now, using (8) we obtain

\[
\sum_{k=0}^{2} a_k y_{m+k-1} = a_0 y_{m-1} + a_1 y_m + a_2 y_{m+1} = 0.
\]

(15)
From (11) and (13) it follows that

\[ \varepsilon_m = \sum_{k=0}^{2} a_k (y_{k+m-1} - y_{k+m-1}) = a_0 g_{m-1} + a_1 g_m + a_2 g_{m+1}. \]  

(16)

The desired roots \( z_1 \) of the polynomial \( F(z) \) have unit modulus. If \( z_1 \) is a root, then \( z_1^{-1} \) is also. So the coefficients \( a_k \) are symmetric about \( a_1 \), i.e., \( a_0 = a_2 \). It is convenient to choose \( a_0 \) so that \( a_1 = 1 \). For \( a_0 = a_2, a_1 = 1 \)

\[ \varepsilon_m = g_m + a_0 (g_{m-1} + g_{m+1}). \]  

(17)

The minimization of \( E \) with respect to the unknown \( a_0 \) will be achieved if

\[ \frac{\partial E}{\partial a_0} = \sum_{m=2}^{M-1} 2 \left[ g_m + a_0 (g_{m-1} + g_{m+1}) \right] (g_{m-1} + g_{m+1}) = 0. \]  

(18)

As solution of (18) we obtain

\[ a_0 = \frac{\sum_{m=2}^{M-1} g_m (g_{m-1} + g_{m+1})}{\sum_{m=2}^{M-1} (g_{m-1} + g_{m+1})^2}. \]  

(19)

The polynomial \( F(z) \) (14) can be expressed as

\[ z^2 + \frac{1}{a_0} z + 1 = 0. \]  

(20)

The roots of the polynomial are

\[ z_{1,2} = \frac{-1}{2a_0} \pm j \sqrt{1 - \frac{1}{4a_0^2}}. \]  

(21)

Since the roots are defined as (8)

\[ z_{1,2} = e^{\pm j \omega T} = \cos(\omega T) \pm j \sin(\omega T) \]  

(22)

the angular frequency \( \omega \) is given by

\[ \omega = \frac{1}{T} \cos^{-1} \left\{ \frac{\sum_{m=2}^{M-1} (g_{m-1} + g_{m+1})^2}{2 \sum_{m=2}^{M-1} g_m (g_{m-1} + g_{m+1})} \right\}. \]  

(23)

V. COMPUTER SIMULATION RESULTS

The developed method was investigated on computer and compared to the method based on the DFT. The program generates a voltage which is sampled at preselected rate. These samples were processed according to (1) to calculate
the phasor, and according to (4) to calculate the time function of the main waveforms. The frequency was calculated either using the (2) or using the new method, described by (23). The voltage waveforms were distorted by higher harmonics.

When implementing the methods, the calculated frequency is updated at every sampled value: After each sampling cycle, the newest sample is taken into the calculation, while the oldest one is neglected.

Fig. 1 shows results of frequency estimation for heavy distorted voltage waveform $g(t) = \cos(\omega t) + 0.2 \cos(5\omega t) + 0.1 \cos(7\omega t)$. Fig. 1(a) shows results when applying the Hamming window, Fig. 1(b) and (c) the Blackman window. For comparison, the results when applying the DFT method (Section II) have also been shown. The best accuracy has been achieved using the Blackman smoothing window. Fig. 2 shows results for the voltage waveform with realistic distortion in high voltage networks $g(t) = \cos(\omega t) + 0.02 \cos(5\omega t) + 0.01 \cos(7\omega t)$. The computer investigation disclosed a high accuracy of the developed method. For realistic voltage distortion and frequency deviation the error was less than 1 mHz. The dynamic behavior of the method was also investigated. The results showed in Fig. 3 confirm a good tracking capability of the method.

The other methods presented in the referenced literature[3], [4], [12]–[14] were also investigated[17]. Owing to limited space in this paper we cannot show the results. The proposed method deliver more accurate results than others.

VI. CONCLUSION

The paper describes a new algorithm for accurate and fast determination of the main frequency of a power system in the presence of higher harmonics. Most digital methods for measuring frequency have acceptable accuracy if the waveforms are not distorted. In the developed method the distorted voltage waveform is first filtered using a Fourier algorithm with the Blackman smoothing window. The output signal of the filter algorithm is then processed by an algorithm based on the Prony’s estimation method. The proposed method was tested on computer, assuming the frequency deviation up to 2 Hz (4%). The investigations results disclosed its high accuracy. For realistic voltage distortion and frequency deviation the calculation error was less than 1 mHz. The results are more accurate than when applying the method based on the measurement of angular velocity of the rotating voltage phasor. The dynamic investigations confirm a good tracking capability of the proposed methods. The response time of the new method is equal to three to four periods of the fundamental component.

REFERENCES


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