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The Hypernuclei ${}^4_\Lambda$He and ${}^4_\Lambda$H: Challenges for Modern Hyperon-Nucleon Forces

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The hypernuclei ${}^4_\Lambda$He and ${}^4_\Lambda$H provide important information on the hyperon-nucleon interaction. We present accurate Faddeev-Yakubovsky calculations for the Λ separation energies of the 0^+ ground and the 1^+ excited states based on the Nijmegen SC *YN* interactions. We explicitly take the Σ admixture into account. Mass differences of the baryons and the charge dependence of the interaction are considered. The results show that the Nijmegen models cannot predict all separation energies simultaneously hinting to failures of the current interaction models. It is pointed out that the differences of the Λ separation energies of ${}^4_\Lambda$He and ${}^4_\Lambda$H are interesting observables to probe the *YN* interaction models.

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Several nucleon-nucleon (*NN*) interaction models have been adjusted successfully to the rich set of *NN* scattering data [1–3]. Deviations of the predictions of these models to the experimental data (such as the underbinding of the 3*N* bound states) might be traced back to the action of 3*N* or higher order forces [4,5] and are presumably no hints to failures of these *NN* interactions. Therefore the hyperon-nucleon (*YN*) system provides interesting and important new insights into the interaction mechanisms [6,7], because the predictions of today's interactions are sensitive to the used model [8]. Unfortunately there are hardly any scattering data available for the *YN* system. Therefore interaction models are generally constrained by flavor-SU(3) symmetry, which, however, is considerably broken. In this spirit one-meson exchange [9–13] and quark-cluster [14] forces have been developed, which are all consistent with the scarce *YN* database. In this Letter we present results based on the Nijmegen soft core interactions SC89 [11] and SC97*a-f* [12] and study their predictions for the Λ separation energies (SE's) of the four-body hypernuclei.

Both Nijmegen models are based on one-meson exchange and take pseudoscalar, vector, and also scalar meson nonets into account. The models are augmented by Pomeron and tensor meson exchange. The coupling constants within these nonets are related by flavor-SU(3) symmetry and are mostly determined by fits to *NN* scattering data. The physical masses of the mesons and mixing between mesons and between baryons introduce a sizable charge symmetry breaking (CSB). Additionally, the new models include more flavor-SU(3) breaking mechanisms (for details, see [12]). Tuning the magnetic $F/(F + D)$ ratio for the vector mesons, the Nijmegen group provided a series of models SC97*a-f*, which give a very different spin-spin interaction, to enable research on the strength of this part of the force.

As most of the *YN* models, the Nijmegen group incorporates the strong conversion process of the ΛN to a ΣN system explicitly. This is very important, because the conversion process is strongly affected by the nuclear medium. The *YN* interaction is generally weaker than the *NN* interaction leading to a core-hyperon structure of the hypernuclei. In some cases Λ-Σ conversion is suppressed, if a change of the isospin of the core nucleus requires excitations [15,16]. The contribution of this process is much stronger than Δ-N conversion in ordinary nuclei, because it accounts for the long-range part of the interaction, it is not suppressed in *s* waves, and because the $\Lambda - \Sigma$ mass difference is much smaller. Because of this, effective ΛN interactions require strong 3-baryon and even higher order forces, which are quite unknown. Predictions for few-baryon systems based only on two-body effective ΛN interactions are meaningless. On the other hand, an understanding of the medium dependence of the ΛN force provides insights into the interaction mechanisms, which cannot be obtained studying the *NN* or even the *YN* system.

We already emphasized that the *YN* data are not sufficient to probe the interaction models. Therefore the few-body hypernuclear bound states are very important benchmarks for these interactions [6], because exact solutions based on the full interaction models can now be obtained and because the SE's are experimentally known. There is no *YN* two-body bound state. The lightest bound system, the ${}^3_\Lambda$H, has already been solved by Miyagawa and co-workers in [17,18]. We confirmed that from the set of Nijmegen *YN* forces only SC89, SC97*e*, and SC97*f* bind ${}^3_\Lambda$H. For those models we summarize our results in Table I. While SC97*e* underbinds ${}^3_\Lambda$H considerably, SC89 and SC97*f* are in agreement with the experiment. The results are converged within 2 keV. We also confirmed the independence of the results from the used *NN* force. The results shown are based on the Nijm 93 [3] or Bonn *B* [19] *NN* interaction. We emphasize that our values are based on the full Nijmegen interaction models. We also applied the Gaussian approximation SC97-sim, which recently has been developed to simulate the SC97*f* interaction [20]. We found that this interaction overestimates the original result by 109 keV and that conclusions on the interaction based on approximated potentials should be taken with caution.

TABLE I. Λ SE's of ${}^{3}_{\Lambda}$H for different *YN* and *NN* force combinations (*YNF* and *NNF*) compared to the experimental value. All energies are in MeV.

YNF	*NNF*	E^{Λ}_{sep}
SC97*e*	Nijm 93	−0.023
SC97*f*	Nijm 93	−0.080
SC89	Nijm 93	−0.143
SC89	Bonn *B*	−0.155
Expt.		−0.130(50)

In the Faddeev equations, the interaction enters via their *t* matrices [17]. Therefore one simulates an effective ΛN interaction, which predicts the same ΛN phase shifts as the original interaction, if one takes Λ-Σ conversion for the evaluation of the *t* matrix into account, but keeps only the ΛN-ΛN elements for the three-body calculation. We found that one looses about 116 keV binding in this way (in the case of SC89). We consider this result as an important additional hint that effective ΛN interactions fail in the few- and many-body systems and that explicit Λ-Σ conversion is important. From our wave functions (WF's) we extracted that the Λ particle predominantly stays far apart from the two nucleons, with a rms distance from the *NN* center of mass for SC89 of about $r = 10.9$ fm, but that the 70 MeV higher mass of the Σ particle forces it to stay close to the nucleons with $r = 1.7$ fm.

Let us now move on to the central issue, the two hypernuclei ${}^{4}_{\Lambda}$He and ${}^{4}_{\Lambda}$H. The system of three nucleons (particles 1–3) and one hyperon (particle 4) is described by five coupled Faddeev-Yakubovsky equations [21],

$$\psi_{1A} = G_0 t_{12}(P_{13}P_{23} + P_{12}P_{23})(\psi_{1A} + \psi_{1B} + \psi_{2A}) + (1 + G_0 t_{12})G_0 V^{(3)}_{123}\Psi , \tag{1}$$

$$\psi_{1B} = G_0 t_{12}[(1 - P_{12})(1 - P_{23})\psi_{1C} + (P_{12}P_{23} + P_{13}P_{23})\psi_{2B}], \tag{2}$$

$$\psi_{1C} = G_0 t_{14}(\psi_{1B} + \psi_{1A} + \psi_{2A} - P_{12}\psi_{1C} + P_{12}P_{23}\psi_{1C} + P_{13}P_{23}\psi_{2B}), \tag{3}$$

$$\psi_{2A} = G_0 t_{12}[(P_{12} - 1)P_{13}\psi_{1C} + \psi_{2B}], \tag{4}$$

$$\psi_{2B} = G_0 t_{34}(\psi_{1A} + \psi_{1B} + \psi_{2A}), \tag{5}$$

for five independent Yakubovsky components ψ_{1A} to ψ_{2B}. G_0 is the free four-body propagator, t_{ij} is the baryon-baryon *t* matrix, P_{ij} are transposition operators, and $V^{(3)}_{123}$ is a specific part of the 3*N* force [22]. There are no models for three-baryon forces (3BF's) in the *YNN* system available. Therefore we neglect those. The five components combine to the total four-body state

$$\Psi = (1 + P)\psi_{1A} + (1 + P)\psi_{1B} + (1 - P_{12})(1 + P)\psi_{1C} + (1 + P)\psi_{2A} + (1 + P)\psi_{2B} \tag{6}$$

using the permutation $P = P_{12}P_{23} + P_{13}P_{23}$. We refer the reader to Refs. [21,23] for a detailed exposition of how to solve that set in a numerically precise manner. The 1^{+} states are more demanding; therefore we had to put a more rigid limit on the partial wave decomposition for this state. However, we checked that the SE's for both states are converged within 50 keV. The difference Δ of the SE's of both states is based on equivalent truncations for 0^{+} and 1^{+} states. Note that therefore the results for the SE's and Δ do not exactly match in the following tables, but that we could obtain a more accurate result for Δ in this way.

To gain insight into CSB effects we take the mass differences for the nucleons and the Σ's into account, also CSB of the *YN* interactions and the Coulomb forces in the pp, $\Sigma^{+}p$, and $\Sigma^{-}p$ pairs. Our calculations are, however, restricted to the total four-body isospin $T = \frac{1}{2}$, which we expect to be a very good approximation.

In Table II we document that the SE's are only moderately dependent on the used *NN* interaction and on a 3*N* force [we used Tucson-Melbourne (TM) [24]]. The *NN* interactions Nijm 93 and Bonn *B* have a very different functional form. Therefore the results can serve as an estimation of the systematic uncertainty of the SE's due to the choice of nucleon interactions. We find 120 keV uncertainty for the SE's and 50 keV for the energy splitting Δ between the 0^{+} and 1^{+} states. The following results are based on the Nijm 93 *NN* potential and the 3*N* force will be neglected.

We show in Table III the SE's of ${}^{4}_{\Lambda}$He for the 0^{+} and 1^{+} states and their difference choosing the *YN* forces, which bind ${}^{3}_{\Lambda}$H and in addition SC97*d*, since it predicts a larger triplet than singlet ΛN scattering length. We see that SC89 comes closest to the experimental value for the 0^{+} state, but it fails totally for the excited state. In the case of the SC97 potentials the SE's drop from *f* to *d* in the case of the 0^{+} state, but they increase for the excited state. In no case does one come close to the experimental values. The experimental 0^{+}-1^{+} splitting, however, is reached by SC97*f*. For all interactions the ordering of the spin states agrees with the experimental result, independent of the size of the ΛN scattering length predictions of the models. This clearly shows that direct conclusions from the four-body binding energies on the singlet and triplet scattering lengths are not possible. We add that SC97*f*-sim leads to an

TABLE II. *NN* and 3*N* interaction dependence of the ${}^{4}_{\Lambda}$He SE's E^{Λ}_{sep} and the 0^{+}-1^{+} splitting Δ. We show results for different combinations of *YN*, *NN*, and 3*N* forces (*YNF*, *NNF*, and 3*NF*). All energies are given in MeV.

YNF	*NNF*	3*NF*	$E^{\Lambda}_{\text{sep}}(0^{+})$	$E^{\Lambda}_{\text{sep}}(1^{+})$	Δ
SC97*e*	Bonn *B*	⋯	1.66	0.80	0.84
SC97*e*	Nijm 93	⋯	1.54	0.72	0.79
SC97*e*	Nijm 93	TM	1.56	0.70	0.82
SC89	Bonn *B*	⋯	2.25	⋯	⋯
SC89	Nijm 93	⋯	2.14	0.02	2.06
SC89	Nijm 93	TM	2.19	⋯	⋯

TABLE III. Predictions for the ${}^4_\Lambda$He SE's E^Λ_{sep} and the 0^+-1^+ splitting Δ of the different *YN* potential models (*YNF*) in combination with Nijm 93 in comparison to the experimental values. The 3*NF* has been neglected. All energies are in MeV.

YNF	$E^\Lambda_{\text{sep}}(0^+)$	$E^\Lambda_{\text{sep}}(1^+)$	Δ
SC89	2.14	0.02	2.06
SC97*f*	1.72	0.53	1.16
SC97*e*	1.54	0.72	0.79
SC97*d*	1.29	0.80	0.47
Expt.	2.39(3)	1.24(5)	1.15

increased 0^+ state SE of 600 keV in relation to SC97*f* and thus comes very close to the experimental value. This does, however, not reflect the properties of the full interaction. Conclusions on the Nijmegen *YN* interactions based on simulating forces are misleading.

Truncating again the *YN t* matrix to the ΛN-ΛN channel, we find for SC97*e*, as an example, a 400 keV reduction for the 0^+ state, but a slight increase of 7 keV for the 1^+ state, showing that the inclusion of the Σ channel is strongly spin dependent and can be repulsive or attractive. Again, it is mandatory to take the Λ-Σ conversion fully into account.

In recent years many studies focused on the splitting of the 0^+ and 1^+ states Δ and its connection to the spin-spin interaction [25] in the *YN* system. Unfortunately, Δ is affected strongly by other parts of the interaction, namely, the Λ-Σ conversion [26–30]. Additionally, we argue that unknown 3BF's in the *YNN* system probably affect the 0^+ and 1^+ states differently and have a strong impact on Δ. The effect of those forces is known to be visible from the ordinary nuclei [4,22]. Fortunately, one can expect approximate isospin invariance for those forces. Therefore their contribution to CSB is presumably small. This makes the SE's differences Δ_{CSB} of ${}^4_\Lambda$He and ${}^4_\Lambda$H a very interesting observable to pin down properties of the *YN* two-body interaction.

To the best of our knowledge Δ_{CSB} has never been completely estimated before. It has been suggested that the Σ^+, Σ^0, and Σ^- states are not equally populated in the two hypernuclei. Because of the mass difference within the Σ multiplet this leads to a shift in the kinetic energy, which is different in ${}^4_\Lambda$He and ${}^4_\Lambda$H [15]. Further the Λ-Σ conversion creates charged Σp pairs and *pp* pairs. This leads to Coulomb force effects [15]. In addition a "core compression effect" has been studied [15,31]: the ^{3}He core in ${}^4_\Lambda$He should be slightly compressed because of the increased binding and this should lead to an increased Coulomb repulsion for the *pp* pairs. All these effects were separately estimated based on simplified models. They showed that the kinetic energy effect and the direct Coulomb effect might cause Δ_{CSB} [15]. The core compression effect was found to be less important [31].

We performed complete calculations for Δ_{CSB} in the 0^+ and 1^+ states for SC97*e*. For SC89 we restricted ourselves to the 0^+ state, because of its unrealistic SE for the 1^+ state. The results are given in Table IV. Using SC97*e* Δ_{CSB} for the 0^+ state is visibly underestimated. To our surprise it agrees with the experimental value in the case of SC89. For SC97*e* and the 1^+ state Δ_{CSB} has even the wrong sign. One has to conclude that none of the present day meson-theoretical Nijmegen *YN* forces describes the 0^+ state energies, the 0^+-1^+ spin splittings, and the differences in the SE's for ${}^4_\Lambda$He and ${}^4_\Lambda$H correctly.

Nevertheless, it is interesting to shed light on the origin of the CSB in the SE's. To this aim we performed a perturbative study of the origin of Δ_{CSB} and estimated the contributions from the expectation values for the kinetic energy ΔT^{CSB}, the strong *YN* interaction $\Delta V^{\text{CSB}}_{YN,\text{nucl.}}$, the Coulomb interaction in *pp* pairs $\Delta V^{\text{CSB}}_{NN,C}$ and in *YN* pairs $\Delta V^{\text{CSB}}_{YN,C}$, which we extracted from our realistic WF's for ${}^4_\Lambda$He (${}^4_\Lambda$H) and ^{3}He (^{3}H) as described in [21]. We study the *YN* force models SC89 and SC97*e* in the 0^+ state only and neglect the contribution of the strong *NN* interaction $\Delta V^{\text{CSB}}_{NN,\text{nucl.}}$. This appears justified, since the CSB of *NN* forces should be very similar for the pairs ${}^4_\Lambda$He/${}^4_\Lambda$H and ^{3}He/^{3}H. Our perturbative results are presented in Table V. They agree within 10 keV with the nonperturbative ones from Table IV, which shows that the perturbative estimates are sufficiently accurate. We see that the Coulomb force plays a minor role, in contrast to the estimation given in [15]. We confirm a significant contribution of the Σ-mass differences showing up in ΔT^{CSB}. There are two effects contributing to ΔT^{CSB}, a mass shift and a change in the momentum dependent part of the kinetic energy T. The first one can be shown to be $\Delta T^{\text{CSB}}_{M_R} = (P_{\Sigma^+} - P_{\Sigma^-})(m_{\Sigma^-} - m_{\Sigma^+})$. Here $P_{\Sigma^\pm}$ are the Σ probabilities in ${}^4_\Lambda$He. The Σ probabilities extracted from our wave functions lead to 80(210) keV for SC97*e*(SC89) for this quantity. The mass shift contribution is reduced by a contribution from the momentum dependent part of T. However, the sign of ΔT^{CSB} is driven by the mass shift. Therefore an increase of mass of the Σ's leads to a decrease in binding, just the opposite of the effect of increasing nucleon masses in ordinary nuclei.

TABLE IV. CSB splitting of the ${}^4_\Lambda$He-${}^4_\Lambda$H mirror nuclei. We show SE's E^Λ_{sep} and the CSB splitting Δ_{CSB} of the SC89 and SC97*e* potential models (*YNF*). The first three rows compare 0^+ state results to the experimental values, the last two 1^+ states results. The calculations are based on Nijm 93; the 3*NF* has been neglected. All energies are given in MeV.

J^π	*YNF*	$E^\Lambda_{\text{sep}}({}^4_\Lambda\text{He})$	$E^\Lambda_{\text{sep}}({}^4_\Lambda\text{H})$	Δ_{CSB}
0^+	SC97*e*	1.54	1.47	0.07
	SC89	2.14	1.80	0.34
	Expt.	2.39(3)	2.04(4)	0.35
1^+	SC97*e*	0.72	0.73	−0.01
	Expt.	1.24(5)	1.00(6)	0.24

TABLE V. Perturbative calculation of the CSB splitting of ${}^4_\Lambda$He and ${}^4_\Lambda$H SE's in the 0^+ states. See text for explanations of the various parts. The results are based on the SC89 or SC97e *YN* force (YNF) and on Nijm 93. The 3*NF* has been neglected. All energies are given in keV.

YNF	$\Delta T^{\rm CSB}$	$\Delta V^{\rm CSB}_{NN,C}$	$\Delta V^{\rm CSB}_{YN,{\rm nucl.}}$	$\Delta V^{\rm CSB}_{YN,C}$
SC89	132	−9	255	−27
SC97*e*	47	−9	44	−7

The mass shift contribution also increases for higher Σ probabilities, which is directly governed by the Λ-Σ conversion. Thus we find again that the study of the conversion process is crucial for hypernuclear physics and that the hypernuclei provide important information on this process. We focus more on the CSB because of larger uncertainties in the predictions of the spin splitting.

The second very important part of $\Delta_{\rm CSB}$ comes from $\Delta V^{\rm CSB}_{YN,{\rm nucl.}}$. In the Nijmegen interaction models the $V_{YN,{\rm nucl.}}$ are generated by the mass differences of the baryons and mesons and by the Λ-Σ^0 mixing. If we just switch off that mixing the $\Delta V^{\rm CSB}_{YN,{\rm nucl.}}$ value reduces drastically to 52(−1) keV for SC89(SC97*e*). Thus the mixing plays an important role in these interaction models.

Finally we present some WF properties of the hypernuclei ${}^4_\Lambda$He and ${}^4_\Lambda$H. The 0^+-1^+ states differ predominantly in their spin structure and they can be seen as spin flip states. This is supported by the probability to find the total spins $S = 0$, 1, and 2 for the two states. For instance, for SC89 and ${}^4_\Lambda$He we find 90.32%, 0.13%, and 9.55% for 0^+ and 0.02%, 95.71%, and 4.25% for 1^+. It is also interesting to see the probabilities in ground states of ${}^4_\Lambda$He for the hyperon to be a Σ. The SC97*d–f* models predict $P_\Sigma = 1.49\%$ to 1.76%, whereas SC89 leads to $P_\Sigma = 4.08\%$. Therefore our $\Delta_{\rm CSB}$ results seem to support a large Σ component in the WF. Recently a similar conclusion has been drawn from the observed π^+ decay width of ${}^4_\Lambda$He [32]. Note that the Σ probabilities are also much larger than the ones for ${}^3_\Lambda$H.

Again we calculated the rms distances *r* of the hyperons to the center of mass of the nucleons. For Λ they depend strongly on the SE ranging from 20.1 fm for the 1^+ state for SC89 to 3.5 fm for the 0^+ state for SC89. The Σ particle is again forced to stay close to the nucleons (1.6 to 2.0 fm for all considered states, similar to ${}^3_\Lambda$H).

In summary we presented the first SE results based on complete meson-theoretical *YN* interactions for four-body hypernuclei. The models of the SC89 and SC97 series, when taken alone, fail. Additionally, though still unknown, *YNN* 3BF's might influence the 0^+-1^+ splitting. Therefore we focused on the CSB of the SE's and showed a strong connection to the interesting Λ-Σ conversion process.

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