Limitations for the use of the spectral function in the semiexclusive ^{3}He$(e,e'N)$ process

J. Golak, H. Witała, and R. Skibiński
M. Smoluchowski Institute of Physics, Jagiellonian University, PL-30059 Kraków, Poland

W. Glöckle
Institute für Theoretische Physik II, Ruhr Universität Bochum, D-44780 Bochum, Germany

A. Nogga
Institute for Nuclear Theory, University of Washington, Box 351550 Seattle, Washington 98195, USA

H. Kamada
Department of Physics, Faculty of Engineering, Kyushu Institute of Technology, 1-1 Sensuicho, Tobata, Kitakyushu 804-8550, Japan
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The limitations for the use of the spectral function S in the process ^{3}He$(e,e'N)$ has been investigated in a kinematical regime constrained by the conditions that the three-nucleon (3N) center-of-mass energy $E_{3N}^{\text{c.m.}} \leqslant 150$ MeV and the magnitude of the three-momentum transfer, $|\vec{Q}| \leqslant 600$ MeV/c. Results based on a full treatment of the final state interaction are compared to the spectral function approximation. In the case of proton knockout in the direction of the photon kinematical conditions have been identified where both response functions, R_L and R_T, can be well approximated by S. These conditions occur for certain low missing momenta and missing energies but not in all cases. So care is required. In case of neutron knockout only R_T is a candidate for an approximate treatment by S. In the case of R_L the concept of using S is not valid in the studied kinematical regime. This does not exclude the possibility that beyond that regime it might be useful. Possible applications using S for the extraction of electromagnetic form factors of the nucleons are pointed out.

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I. INTRODUCTION

The $(e,e'N)$ reactions have been widely analyzed in the past using the concept of the spectral function. This quantity has been introduced for instance in the work of Refs. [1,2] in the context of inclusive electron scattering on ^{3}He. In the following it has been intensively investigated by Ciofi degli Atti and collaborators [3–7] and Sauer and collaborators [8–10] as well as other groups. For heavier systems there is a rich literature where that tool has been also extensively used [11]. More recent work can be found in Refs. [12–14]. In [12] effects of polarizations are included.

The description of $(e,e'N)$ reactions using the spectral function is an approximation. It is based on the simplifying assumption that the nucleon is knocked out as a free particle and only the remaining nucleons interact among themselves. Thus for a ^{3}He target only a final state interaction between two nucleons is considered. Also the antisymmetrization of the knocked out nucleon with the other two nucleons is neglected. This picture appears to be reasonable if the knocked out nucleon receives all or essentially all of the photon three-momentum, which moreover should be not too small. Of course that simplification of the description was also enforced in the past by the simple fact that the complete final state interaction could not be controlled.

Over the years it has become possible to take FSI among the three nucleons completely into account in the case of ^{3}He [15]. We present such a solution and critically investigate the simplified picture leading to the spectral function. Our framework, however, is still nonrelativistic, which forces us to stay below the pion threshold, thus below about 150 MeV 3N c.m. energy. In order not to induce too high nucleon momenta, which also would require a relativistic treatment, we restricted the three-momentum of the photon to the maximally allowed values of 600 MeV/c. Though this is already a too high value, we used it to get a first indication whether there will be a tendency that at the higher momenta the final state interaction might decrease. Also we expect that this violation will not be too severe to prevent a reasonable insight into the failure or validity of the assumptions underlying the simplistic picture of the spectral function.

The paper is organized as follows. Section II is a brief reminder of the definition of the spectral function and of the complete formulation for the final state interaction in case of ^{3}He. The two relevant pairs of kinematical variables for $(e,e'N)$ processes are the missing momentum and missing energy, k and E, and the virtual photon momentum and its energy, Q and ω. So in Sec. II we also illustrate the mappings of the two related regions in the k-E and Q-ω planes. In Sec. III we compare the spectral function under various kinematical conditions to results taking the full final state interaction into account. This investigation is performed for proton and neutron knockout from ^{3}He. We summarize in Sec. IV.

II. THEORETICAL FRAMEWORK

We regard the semiexclusive process ^{3}He$(e,e'N)$ in parallel kinematics, where the nucleon N is knocked out with the momentum $\vec{p}_1$ parallel to the virtual photon momentum

$\vec{Q}$. In the unpolarized case the cross section is simply given as

$$\frac{d^6\sigma}{dE_{e'}\, d\Omega_{e'}\, d\Omega_1\, dE_1} = \sigma_{\rm Mott}\int d\hat{p}[v_L R_L + v_T R_T]\frac{m^2 p p_1}{2}, \tag{1}$$

since the response functions R_{TT} and R_{TL} vanish under the parallel condition [16,17]. The functions v_L and v_T are standard kinematical factors. The two response functions R_L and R_T are expressed in terms of the nuclear matrix elements N_0 and $N_{\pm 1}$ as

$$R_L \equiv \frac{1}{2}\sum_M \sum_{m_1,m_2,m_3} |N_0(\vec{p}_1,\vec{p}_2,\vec{p}_3;M,m_1,m_2,m_3;\nu_1,\nu_2,\nu_3)|^2,$$

$$R_T \equiv \frac{1}{2}\sum_M \sum_{m_1,m_2,m_3} (|N_1(\vec{p}_1,\vec{p}_2,\vec{p}_3;M,m_1,m_2,m_3;\nu_1,\nu_2,\nu_3)|^2 + |N_{-1}(\vec{p}_1,\vec{p}_2,\vec{p}_3;M,m_1,m_2,m_3;\nu_1,\nu_2,\nu_3)|^2), \tag{2}$$

where M, m_1, m_2, m_3 are the initial ^{3}He and final 3N spin magnetic quantum numbers, and ν_1,ν_2,ν_3 are isospin magnetic quantum numbers needed to identify the nucleons in the final state. The direction (magnitude) of the relative momentum of the two undetected nucleons is denoted by $\hat{p}(p)$ and the nucleon mass by m. The matrix elements N_0 and $N_{\pm 1}$ are driven by the charge density operator and spherical components of the transverse current operator, respectively. In general the nuclear matrix element has the form

$$N^\mu \equiv \langle \Psi_f^{(-)}|j^\mu(\vec{Q})|\Psi_{^3\rm He}\rangle, \tag{3}$$

where f comprises the momenta and the magnetic spin and isospin quantum numbers of the three final nucleons. We shall concentrate here on the complete break up and refer the reader for the case of the pd breakup to Ref. [18]. As has been shown in Ref. [18], N^μ can be represented as

$$N^\mu = \langle \phi_0|(1+P)j^\mu(\vec{Q})|\Psi_{^3\rm He}\rangle + \langle \phi_0|(1+P)|U^\mu\rangle, \tag{4}$$

where the auxiliary state $|U^\mu\rangle$ obeys the Faddeev-type integral equation

$$|U^\mu\rangle = tG_0(1+P)j^\mu(\vec{Q})|\Psi_{^3\rm He}\rangle + tG_0P|U^\mu\rangle. \tag{5}$$

The ingredients in Eq. (5) are the free 3N propagator G_0, the NN t-operator generated via the Lippmann-Schwinger equation from any modern NN interaction, and a suitably chosen permutation operator P [19]. The state ϕ_0 in Eq. (4) is a plane wave, antisymmetrized in the two-body subsystem, where t acts. For a generalization of Eqs. (4) and (5) including a three-nucleon force we refer the reader to Ref. [18]. In the present study we restrict ourselves to NN forces and allow only for one-body currents $j_\mu(\vec{Q})$. It is illustrative to present the physical content of the expressions (4) and (5) in the following way. If one iterates the integral equation and inserts the resulting terms into (4) one arrives at the infinite sequence of processes shown in Fig. 1.

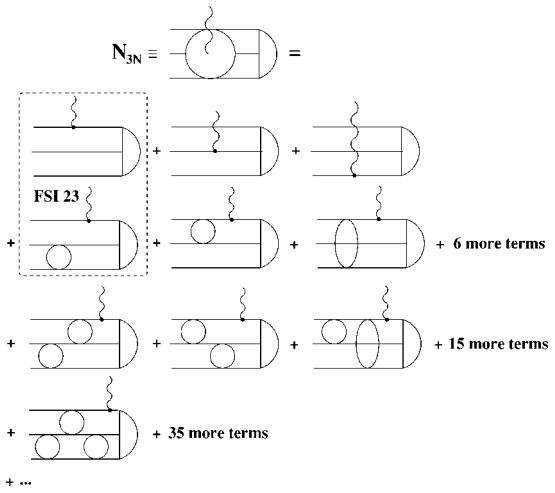

FIG. 1. Diagrammatic representation of the nuclear matrix element for the three-body electrodisintegration of ^{3}He. The open circles and ovals represent the two-body t-matrices. Three horizontal lines between photon absorption and forces, and between forces describe free propagation. The half-moon symbol on the very right stands for ^{3}He.

In the second row there is no final state interaction and the photon is absorbed by nucleons 1, 2, and 3. The next three rows include rescattering processes of first order in the NN t-operator (denoted by a circle). Then follow processes of second order in t, third order, etc. That complete sum of processes is generated by solving the integral equation (5). Now taking only the first diagrams in rows 2 and 3 into account underlies the concept of the spectral function S. The corresponding expression is

$$N^\mu = \langle \phi_0|(1+t_{23}G_0)j^\mu(\vec{Q};1)|\Psi_{^3\rm He}\rangle, \tag{6}$$

where the argument 1 in the current explicitly indicates that the photon is absorbed only on one nucleon, numbered 1 in our notation. That approximation, the two encircled diagrams in Fig. 1, will be called in the following FSI23 for short and stands for final state interaction in the spectator pair (23). Related to that nuclear matrix element is the spectral function S. It is defined as

$$S(k,E) = \frac{mp}{2}\frac{1}{2}\sum_M \sum_{m_1,m_2,m_3}\int d\hat{p}|\sqrt{6}\langle \nu_1\nu_2\nu_3|\langle m_1m_2m_3|\langle \vec{p}\vec{k}|(1+t_{23}G_0)|\Psi_{^3\rm He}\rangle|^2. \tag{7}$$

The arguments of S are the magnitude k of the missing momentum

$$k \equiv |\vec{Q} - \vec{p}_1| \tag{8}$$

and the excitation energy E of the undetected pair. Nonrelativistically,

$$E \equiv \frac{p^2}{m}, \tag{9}$$

where p is the relative momentum of the undetected nucleons. Comparing the expression (7) for S to the ones for R_L and R_T under the FSI23 approximation one finds

$$\begin{aligned} S(k,E) &= \frac{1}{2} mp \frac{1}{(G_E)^2} \int d\hat{p} R_L(\mathrm{FSI23}) \\ &= \frac{1}{2} mp \frac{2m^2}{Q^2 (G_M)^2} \int d\hat{p} R_T(\mathrm{FSI23}). \end{aligned} \tag{10}$$

This inserted into (1) yields the well-known relation between the cross section and the spectral function

$$\begin{aligned} &\frac{d^6\sigma}{dE_{e'}\, d\Omega_{e'}\, d\Omega_1\, dd\, E_1} \\ &\quad = \sigma_{\mathrm{Mott}} \left[v_L (G_E)^2 + v_T \frac{Q^2 (G_M)^2}{2m^2} \right] S(k,E) m p_1 \\ &\quad \equiv \sigma_{eN} S(k,E) \rho_f. \end{aligned} \tag{11}$$

Here the nonrelativistic phase space factor ρ_f is simply

$$\rho_f = m p_1 \left(1 + \frac{2E_e}{m} \sin^2 \frac{\theta_e}{2} \right) \tag{12}$$

and the unpolarized electron-nucleon cross section in the nonrelativistic approximation reads

$$\sigma_{eN} = \sigma_{\mathrm{Mott}} \left[v_L (G_E)^2 + v_T \frac{Q^2 (G_M)^2}{2m^2} \right] \frac{1}{1 + \frac{2E_e}{m} \sin^2 \frac{\theta_e}{2}}. \tag{13}$$

(Note we always keep the kinematical factors related to the electron relativistically). The central question we want to answer in this paper is, how reliable that approximation is. Clearly, this will depend on the kinematic regime. Here we shall restrict ourselves to photon energies ω and momenta $Q = |\vec{Q}|$ such that the 3N c.m. energy in the final state is essentially below the pion mass m_π,

$$E_{3\mathrm{N}}^{\mathrm{c.m.}} = \omega - \frac{\vec{Q}^2}{6m} + \epsilon_3 \leqslant m_\pi \tag{14}$$

(to be exact we consider cases with $E_{3\mathrm{N}}^{\mathrm{c.m.}} \leqslant 150$ MeV) and $Q \leqslant 600$ MeV/c That Q value is in fact already somewhat too high to use strictly nonrelativistic kinematics and to neglect relativistic corrections in the current and the dynamics. But we consider this small excursion to be justified to acquire a first insight into a decline of FSI with increasing Q values. Qualitatively, we do not expect a change of our results if relativistic structures will be incorporated. We shall, however, not enter into the kinematic regime with even higher Q values and/or $E_{3\mathrm{N}}^{\mathrm{c.m.}}$ significantly greater than m_π.

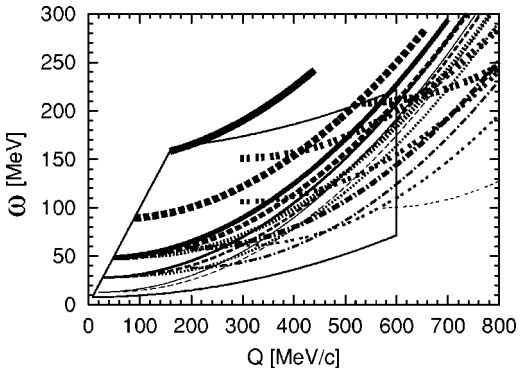

FIG. 2. The domain D in the Q-ω plane for $E_{3\mathrm{N}}^{\mathrm{c.m.}} \leqslant 150$ MeV and $Q \leqslant 600$ MeV/c. The additional lines correspond to fixed (k,E) values. Solid lines are for $k=0.1$ fm^{-1}, dashed for $k=0.25$ fm^{-1}, dotted for $k=0.5$ fm^{-1}, dashed-dotted for $k=1$ fm^{-1}, double-dashed for $k=1.5$ fm^{-1}, and triple-dashed for $k=2.7-2.9$ fm^{-1}. The thickness of the lines increases with increasing E; it is minimal for $E=5$ MeV and maximal for $E=140$ MeV. Note that we restrict ourselves to the "less relativistic" case in Eq. (15) (the minus sign), for which $|\vec{p}_1| \leqslant |\vec{Q}|$.

The kinematical restriction imposed above leads to the domain D in the Q-ω plane shown in Fig. 2.

Using the energy and momentum conservation in nonrelativistic kinematics leads to the following connection between the variables ω, Q and k, E:

$$\omega + \epsilon_3 = \frac{(Q \pm k)^2}{2m} + \frac{k^2}{4m} + E, \tag{15}$$

where in (14) and (15) ϵ_3 is the negative ^{3}He binding energy. The sign $-(+)$ refers to $0 \leqslant p_1 \leqslant Q$ $(p_1 \geqslant Q)$, respectively. Thus taking a pair Q-ω in D provides a relation between E and k. It is a simple matter to map the domain D into the domain D' in the k-E plane. This is shown in Fig. 3 encircled by the roughly horizontal line around $E=140$ MeV and the vertical line at $k=3$ fm^{-1}.

To illustrate the mappings we also display in Fig. 3 a few examples for the continuously distributed k-E pairs to each fixed Q-ω out of D. We see that for fixed Q the sequence of curves shifts upwards and to the left with increasing ω. Once the bended curves hit the $k=0$ axis there appears a branch related to the other sign of k in Eq. (15) reaching again to nonzero k values. As will be clear below we are especially interested in the Q-ω pairs which lead to curves in the k-E plane ending up near $E \approx 0$ and $k \approx 0$.

As is obvious from Eq. (15) that mapping from D to D' is not one-to-one.

Thus, for each k-E pair, only a relation between Q and ω is determined. Again quite a few examples are displayed in Fig. 2. In this paper we investigate only the Q-ω pairs within the domain D. For the pairs outside D a relativistic treatment is obligatory and therefore outside the scope of this paper. For a better orientation of the reader, the chosen k-E pairs are among the ones displayed in Fig. 4.

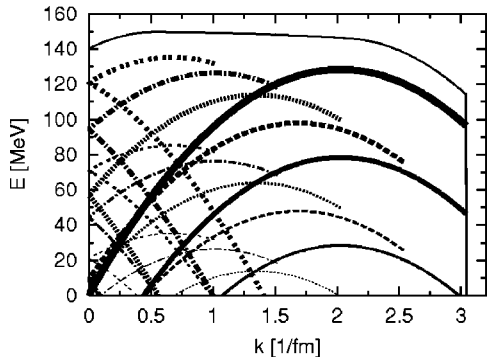

FIG. 3. The domain D' in the k-E plane for $E_{3N}^{c.m.} \leqslant 150$ MeV and $Q \leqslant 600$ MeV/c. The solid lines correspond to $Q \leqslant 600$ MeV/c, dashed lines to $Q \leqslant 500$ MeV/c, dotted lines to $Q \leqslant 400$ MeV/c, dashed-dotted lines to $Q \leqslant 300$ MeV/c, double-dotted lines to $Q \leqslant 200$ MeV/c. The lines thickness increases with ω: the thinest line stands for $\omega = 50$ MeV, then come the thicker and thicker lines for $\omega = 100$, 150, and 200 MeV, respectively.

In order to investigate the usefulness of S one can use Eq. (10) and replace the response functions R_L, R_T evaluated under the simplifying assumption FSI23 by the full response functions taking FSI completely into account. This is required for the cross section given in Eq. (1). Let us call the resulting expressions S_L(Full) and S_T(Full), respectively. It is also of interest to neglect any FSI but keep all three terms in row 2 of Fig. 1. This we call the symmetrized plane wave impulse approximation, PWIAS, since then antisymmetrization is fully taken into account, and the resulting quantities will be denoted as S_L(PWIAS) and S_T(PWIAS). Finally, one can assume only the very first process in Fig. 1 to be present, leading to S_L(PWIA) and S_T(PWIA). In this manner we can compare $S(k,E)$ to the other three choices of dynamical input. Each k-E pair fixes according to Eq. (15) ω if Q is given. Thus we shall plot the four S's for fixed (k,E) as a function of Q; in other words as a function of the electron kinematics. By construction S_L(PWIA), S_T(PWIA) and S are functions of k and E only and do not depend on Q. This, however, does not hold for S_L(PWIAS) and S_T(PWIAS) and the results based on full treatment of FSI, S_L(Full) and S_T(Full).

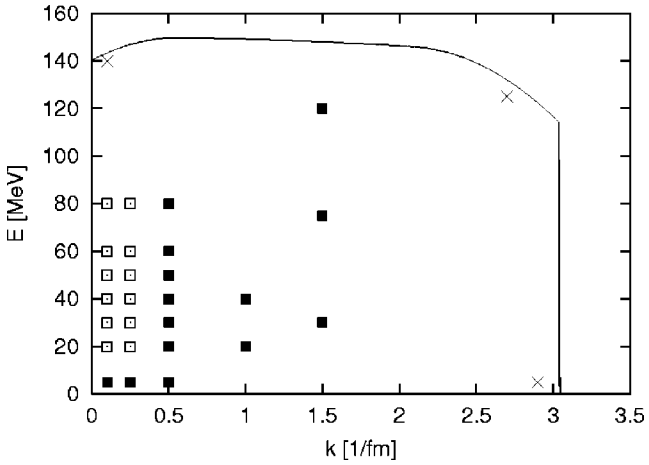

FIG. 4. The same domain D' shown in Fig. 3 in the k-E plane resulting from $E_{3N}^{c.m.} \leqslant 150$ MeV and $Q \leqslant 600$ MeV/c together with (k,E) points for which the related Q-ω curves are displayed in Fig. 2. For the k-E pairs corresponding to solid squares the full result in the case of proton knockout has a tendency to approach the spectral function S with increasing Q, whereas this is not the case for the k-E pairs corresponding to the open squares. For the three points marked with $\times$'s the allowed Q range is too small to describe the trend.

Obviously Eq. (15) can also be written as

$$\omega + \epsilon_3 = E_1 + \frac{k^2}{4m} + E \tag{16}$$

with $E_1 = p_1^2/2m$. Therefore E_1 can be equally used as the abscissa. Note, however, that the different E_1's belong to different electron kinematics. This is one way to represent our results starting from fixed (k,E) values.

We shall also provide examples using a fixed electron kinematics and plot the results as a function of E_1, which is more natural in relation to the experiment.

III. RESULTS

In all calculations the AV18 nucleon-nucleon potential [20] has been used without its electromagnetic parts. It is plausible to assume for the parallel kinematics considered in the paper that meson exchange currents (MEC) do not play any essential role. Thus we concentrate on the FSI effects and neglect any contribution from MEC.

Under the simplifying assumptions represented by the two encircled diagrams in Fig. 1, the response functions R_L and R_T are directly linked to the spectral function S, as shown in Eq. (10). In order to achieve insight under which conditions this form has validity, we shall cover the domain D' in Fig. 3 by a representative grid of (k,E) points chosen in Fig. 2 and marked by squares in Fig. 4.

To each such pair corresponds a quadratic relation between the photon energy ω and its three-momentum Q, as given in Eq. (15). This traces out a curve and examples thereof are shown in Fig. 2. We shall now choose those curves inside the domain D and compare the spectral function S to the expressions S_L(Full) and S_T(Full) evaluated under the full sequence of rescattering processes, and further compare S to S_L(PWIAS) and S_T(PWIAS) taking the correct antisymmetrization into account but neglecting any final state interaction and finally we compare S to S_L(PWIA) and S_T(PWIA) keeping only the very first process in Fig. 1. The results are displayed in Figs. 5–13.

Let us first concentrate on the full calculation represented by a solid line in comparison to the spectral function S given as a dotted line in case of the proton knockout process (lower panels). We show only examples from the k-E pairs in Fig. 4 since the patterns are similar. For the k-E pairs (0.1 fm^{-1}, 5 MeV) and (0.5 fm^{-1}, 5 MeV) the two curves approach each other with increasing Q values in our domain. Turning to larger E values like $E = 20$ and 40 MeV, we see that this is true at $k = 0.5$ fm^{-1} but not at $k = 0.1$ fm^{-1}. We looked at further k-E pairs as shown in Fig. 4 and found a region in the k-E plane indicated by the open squares, where the full result deviates more and more from S with increasing Q. At k

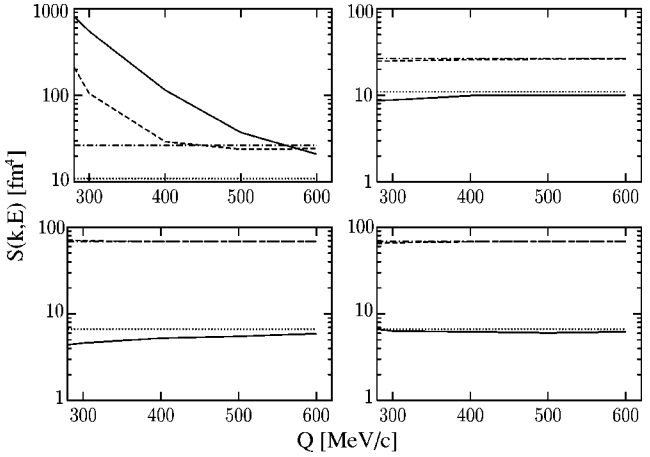

FIG. 5. The spectral function $S(k,E)$ and results based on the form given in Eq. (10) but using different dynamical assumptions for the response functions R_L and R_T as a function of the momentum transfer Q for a fixed (k,E) pair: $k=0.1$ fm^{-1}, $E=5$ MeV. Top figures describe the neutron knockout and bottom ones the proton case. The longitudinal (left figures) and transverse (right figures) response functions are employed. PWIA (dashed-dotted line), PWIAS (dashed) and Full results (solid line) are shown. The FSI23 result (dotted) is the spectral function $S(k,E)$, which is independent of Q like the PWIA result.

$=0.5$ fm^{-1}, however, for all studied E values the full result has a tendency to approach S with increasing Q. The reason for that different behavior is not known to us. Contributions to the full result arise in addition to the two processes underlying S from the absorption of the photon by the other two nucleons, like in the second row of Fig. 1 and from rescattering processes among all nucleons and to all orders. All those complex amplitudes of similar magnitude strongly interfere with each other. A careful consideration of the terms in Fig. 1 beyond the ones leading to S appears therefore to be advisable to stay on the safe side.

That behavior is qualitatively similar for R_L and R_T. For the other k-E pairs the FSI23 approximation leading to the spectral function is by far not sufficient and the full rescattering takes place. But in all those cases at least we see a tendency that the full result comes closer to S for larger and

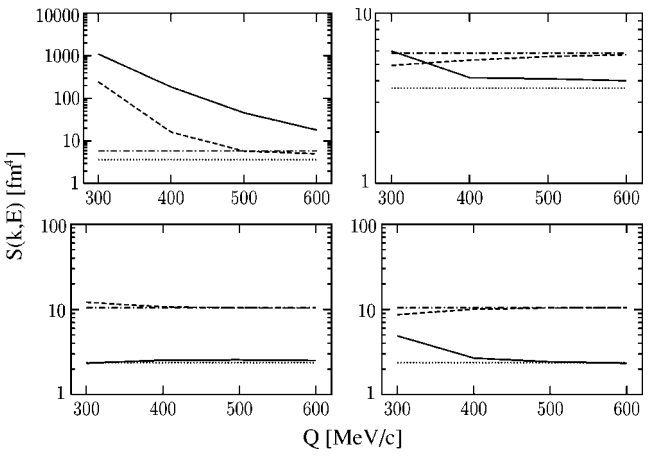

FIG. 6. The same as in Fig. 5 for $k=0.5$ fm^{-1}, $E=5$ MeV.

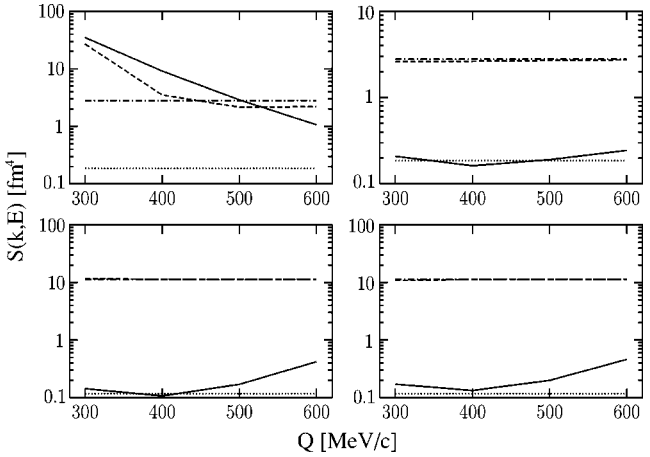

FIG. 7. The same as in Fig. 5 for $k=0.1$ fm^{-1}, $E=20$ MeV.

larger Q. We also show the very first process in Fig. 1 denoted by PWIA and a second case where the correct antisymmetrization is kept but no rescattering process is allowed. This we denote by PWIAS. Figures 5–13 exhibit different situations in relation of the PWIAS versus the PWIA results and the PWIA versus the FSI23 results. In nearly all cases shown symmetrization in plane wave approximation (PWIAS) is quite unimportant except sometimes at the small Q values. In the cases (1.5 fm^{-1}, 75 MeV) and (2.7 fm^{-1}, 125 MeV) symmetrization, however, is quite important. All that is easily understood regarding the momentum values for the two additional processes of PWIAS. In the case of PWIA the ^{3}He wave function $\Psi_{^3\mathrm{He}}(\vec{p},\vec{q})$ is evaluated for $\vec{p}=\frac{1}{2}(\vec{p}_2-\vec{p}_3)$ and $\vec{q}=\vec{p}_1-\vec{Q}$. For the two additional processes present in PWIAS the corresponding arguments are $\left[\vec{p}=\frac{1}{2}(\vec{p}_1-\vec{p}_2),\vec{q}=\vec{p}_3-\vec{Q}\right]$ and $\left[\vec{p}=\frac{1}{2}(\vec{p}_3-\vec{p}_1),\vec{q}=\vec{p}_2-\vec{Q}\right]$. Interestingly in case of Figs. 12 and 13 the PWIA and FSI23 results agree very well. Thus the final state interaction among the two spectator nucleons is negligible. If additionally the symmetrization and all of the final state interaction were negligible, one would have a perfect view right away into the ^{3}He wave function, since S evaluated under PWIA condition displays directly the magnitude of the ^{3}He wave function. [This is

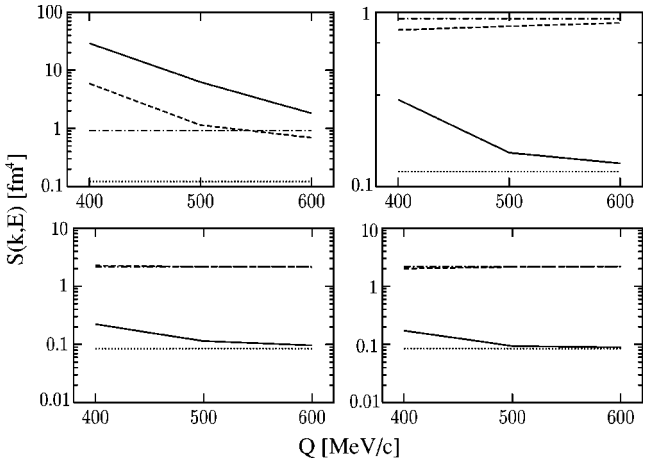

FIG. 8. The same as in Fig. 4 for $k=0.5$ fm^{-1}, $E=20$ MeV.

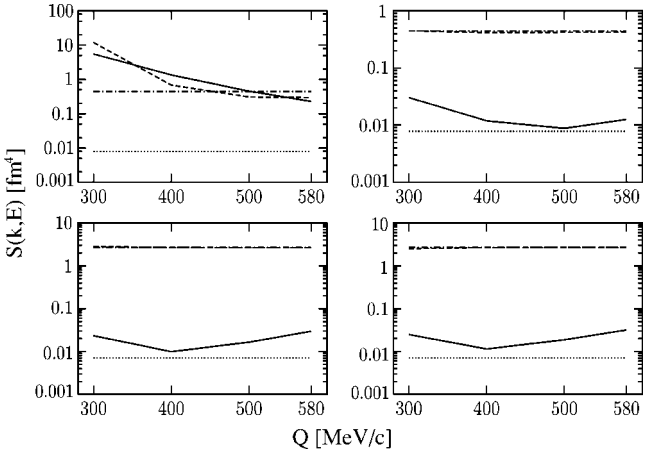

FIG. 9. The same as in Fig. 5 for $k=0.1$ fm^{-1}, $E=40$ MeV.

obvious from Eq. (7) if one drops the contribution proportional to $t_{23}G_0$.] That neglect is, however, not justified as documented in Figs. 12 and 13. Already the correct antisymmetrization, which is independent of FSI changes the results totally.

In Ref. [13] $S(k,E)$ is displayed together with S evaluated under the PWIA condition. They essentially agree for $k \geqslant 1.5$ fm^{-1} along $E=k^2/(4m)$. As examples one could take $k=2$ fm^{-1} corresponding to $E\approx 40$ MeV or $k=3$ fm^{-1} with E about 90 MeV. This suggested direct insight into the ^{3}He wave function. In view of our results shown in Figs. 12 and 13 this suggestion is not valid if the electron kinematics belongs to the domain D.

This does, of course, not exclude that outside of D the situation might be more favorable to such an ideal situation. In Fig. 2 one can see that for those pairs of k-E values there are continuous ω-Q pairs, where such an ideal situation might exist. This requires, however, above all a relativistic treatment and taking all the additional dynamical ingredients into account, which is outside the scope of the present study. Please also note that for c.m. 3N energies above the pion threshold no nuclear forces comparable in quality to the ones below are available.

Regarding now the neutron knockout even at low k-E values the spectral function in case of R_L is insufficient. For

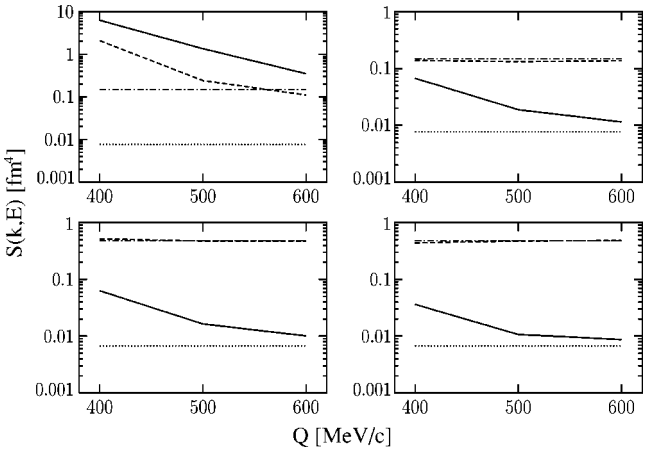

FIG. 10. The same as in Fig. 5 for $k=0.5$ fm^{-1}, $E=40$ MeV.

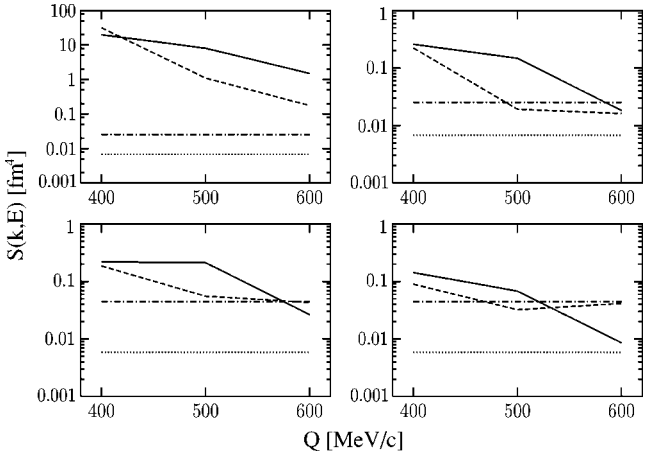

FIG. 11. The same as in Fig. 5 for $k=0.1$ fm^{-1}, $E=40$ MeV.

R_T, however, the situation is quite similar to the proton knockout. Thus neutron knockout for ^{3}He without separation of R_L and R_T is not suitable for that application of the spectral function S.

We must conclude that for most of the Q-ω values in the domain D the use of the spectral function is quantitatively not justified and identifying experimentally extracted S functions after integration over E with the ^{3}He momentum distribution is not correct. It is only for a certain group of very small k-E values (both) and for proton knockout that S_L(Full) and S_T(Full) approach S at the higher Q values in the domain D.

It is at least that "corner" of the k-E domain where the theoretical prediction should be valid since only the NN t-matrix together with the ^{3}He wave function enter at low momenta. Therefore precise data there would be quite important to validate at least that expectation. For other regions inside D the full dynamics is acting.

In actual experiments it is natural to present the data for the process ^{3}He$(e,e'N)$ for a given Q-ω pair as a function of E_1, the energy of the knocked out nucleon. In this case the k-E values trace out a curve in the domain D' as shown in Fig. 3. Of course investigating such a scenario there will be no new information beyond the one we already displayed.

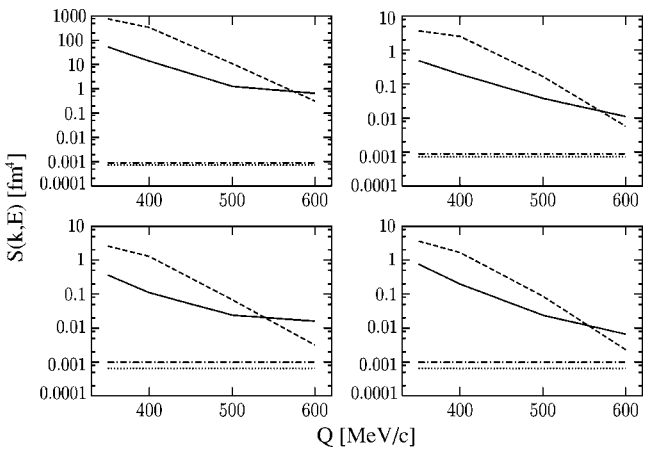

FIG. 12. The same as in Fig. 5 for $k=1.5$ fm^{-1}, $E=75$ MeV.

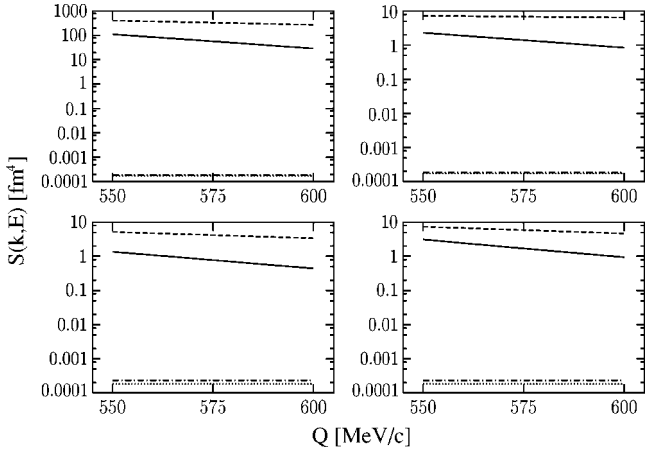

FIG. 13. The same as in Fig. 5 for $k=2.7$ fm^{-1}, $E=125$ MeV.

Nevertheless since this is what appears naturally in an experiment we would like to show the corresponding S curves now as a function of E_1. First we choose proton knockout and take Q-ω values which in the k-E plane lead to curves ending up in the "corner," where both k and E are rather small. As seen from Eq. (15) and displayed in some examples in Fig. 3 suitable cases are $\omega=100$ MeV, $Q=400$ MeV/c; $\omega=100$ MeV, $Q=500$ MeV/c; $\omega=150$ MeV, $Q=600$ MeV/c.

In all these cases k and E get very small when E_1 approaches its maximal value. This is illustrated in Figs. 14–16. We restrict ourselves to the upper end of the energy E_1 since only there S and S(Full) approach each other.

We see a very nice coincidence of S with the full results at the upper end of E_1, both for R_L and R_T. Thus the full cross section can be rather well represented by the spectral function approximation.

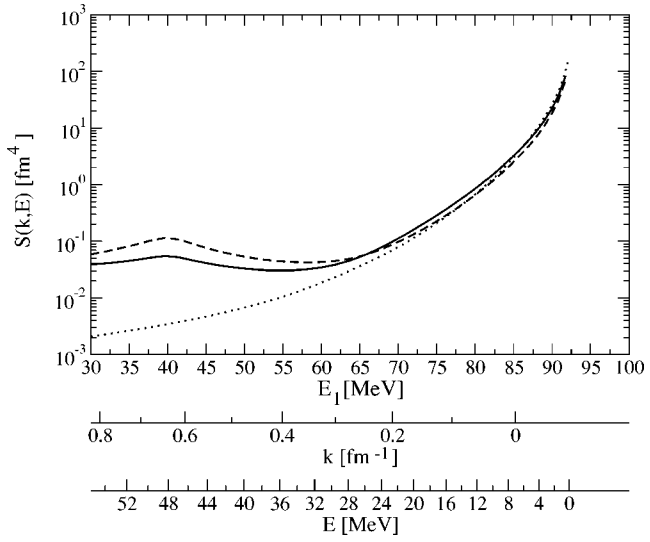

FIG. 14. The spectral function $S(k,E)$ for the proton knockout (dotted line), full results based on the form given in Eq. (10) for the response functions R_L (dashed line) and full results for the response functions R_T (solid line) for a fixed (Q-ω) pair: $\omega=100$ MeV, $Q=400$ MeV/c as a function of the ejected proton energy E_1 for the parallel kinematics $\vec{p}_1||\vec{Q}$. The corresponding values of k and E are also indicated.

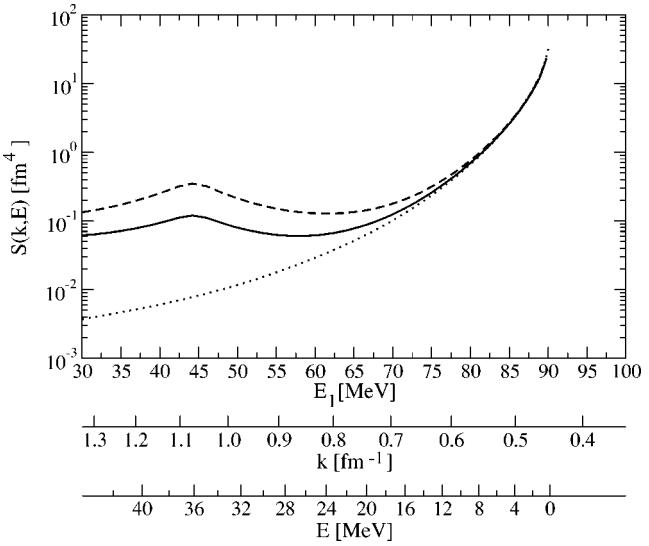

FIG. 15. The same as in Fig. 14 for $\omega=100$ MeV and $Q=500$ MeV/c.

As counterexamples one can choose $\omega=100$ MeV, $Q=200$ MeV/c; $\omega=200$ MeV, $Q=300$ MeV/c shown in Figs. 17 and 18. We see indeed, that there is no agreement of S with the full results, neither in relation to R_L nor to R_T and the approximation using S is not acceptable.

In the case of neutron knockout only R_T can be approximated by the spectral function and therefore the approximation of the full cross section is not suitable. We show in Figs. 19 and 20 only two examples, for $\omega=100$ MeV, $Q=500$ MeV/c and $\omega=150$ MeV, $Q=600$ MeV/c.

Finally one example is displayed in Fig. 21 for $\omega=100$ MeV and $Q=200$ MeV/c, where the spectral function even for R_T is not a sensible approximation.

One has to conclude that the spectral function S is not a good tool to analyze neutron knockout inside the domain D except for special Q-ω pairs at the upper end of E_1 in case of the transversal response.

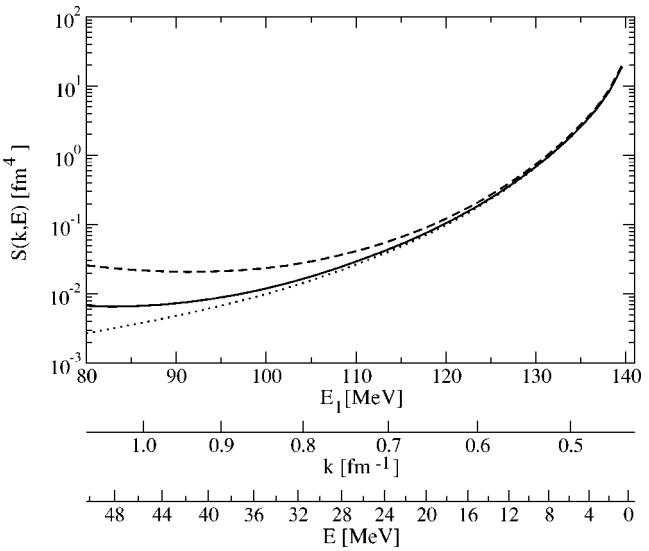

FIG. 16. The same as in Fig. 14 for $\omega=150$ MeV and $Q=600$ MeV/c.

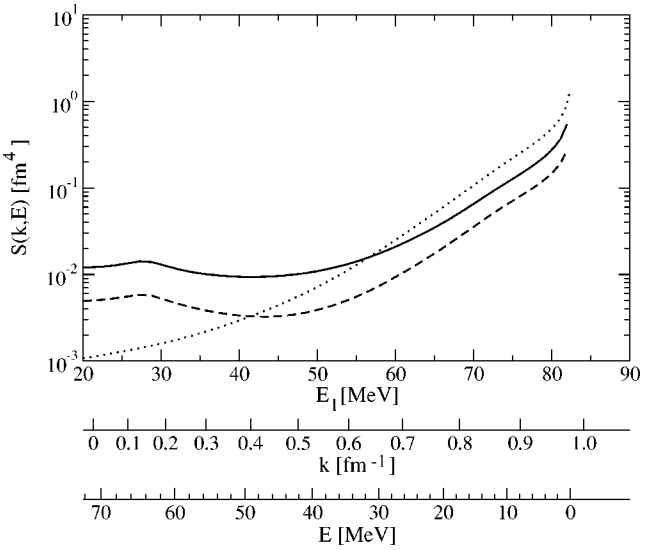

FIG. 17. The same as in Fig. 14 for $\omega=100$ MeV and $Q=200$ MeV/c.

Finally we would like to add a remark on the extraction of electromagnetic form factors of the nucleons. In the case that the FSI23 approximation is valid or in other words the use of the spectral function is justified, the electromagnetic form factors are directly accessible. As seen in Eq. (11) a L-T separation provides direct access to both, G_E and G_M. It appears interesting to check that approach first in the case of the proton knockout, where the form factors are known. In the case of the neutron knockout the transverse response function R_T can be well controlled under the kinematic conditions discussed above and therefore access to G_M^n appears possible. In the case of G_E^n it might also work at higher energy and momentum transfers, which are however outside the kinematic regime investigated in this study.

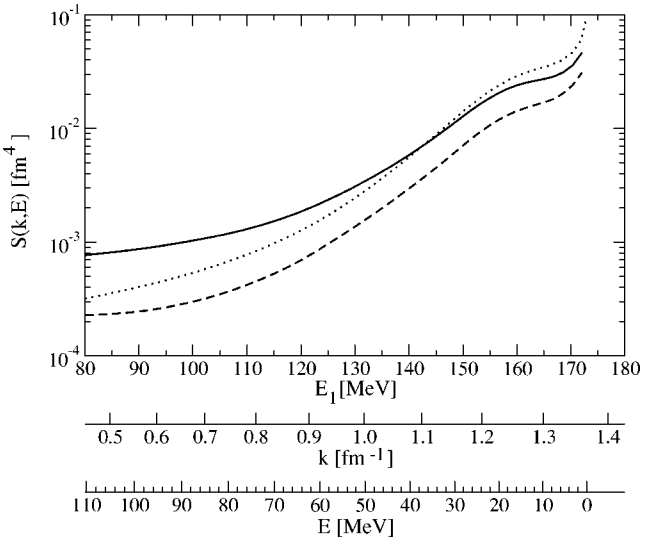

FIG. 18. The same as in Fig. 14 for $\omega=200$ MeV and $Q=300$ MeV/c.

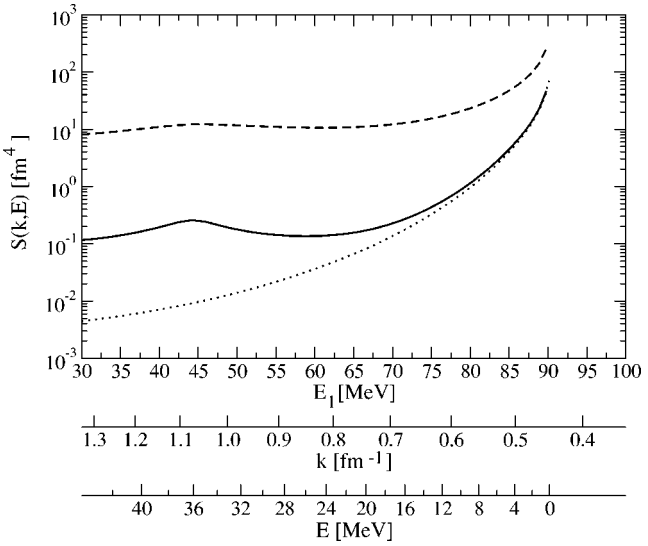

FIG. 19. The same as in Fig. 15 for the neutron knockout.

IV. SUMMARY

We reviewed briefly the formulation of the full treatment of the final state interaction for the process ^{3}He$(e,e'N)$ in the Faddeev scheme. We showed that the processes underlying the concept of the spectral function are just the very first two diagrams in an infinite series of diagrams caused by rescattering and complete antisymmetrization. The spectral function S is directly related to both response functions, R_L and R_T, under those simplifying assumptions. We used the same formal relation which leads to S but now working with the response functions which include the complete final state interaction. This leads to quantities S_L(Full) and S_T(Full), which can be compared to S. The comparison was restricted to a kinematical regime where a nonrelativistic treatment appears mostly justified. Thus we restricted $E_{3N}^{\text{c.m.}}$ to be below the pion threshold, more precisely to stay below 150 MeV and the magnitude of the photon momentum $\vec{Q}$ to be below

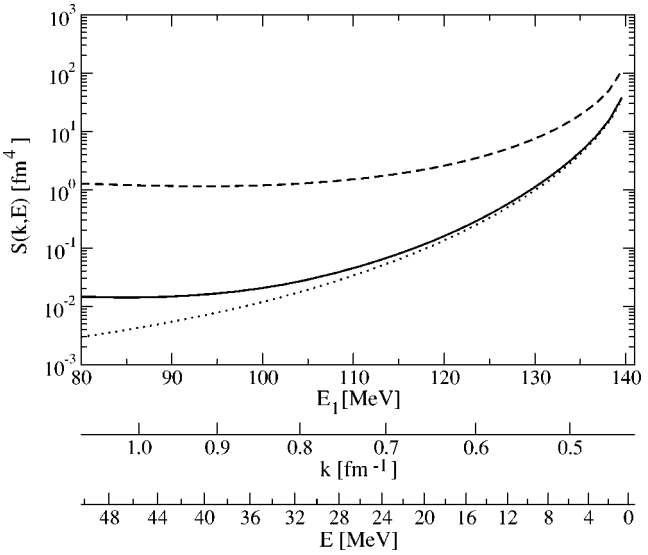

FIG. 20. The same as in Fig. 16 for the neutron knockout.

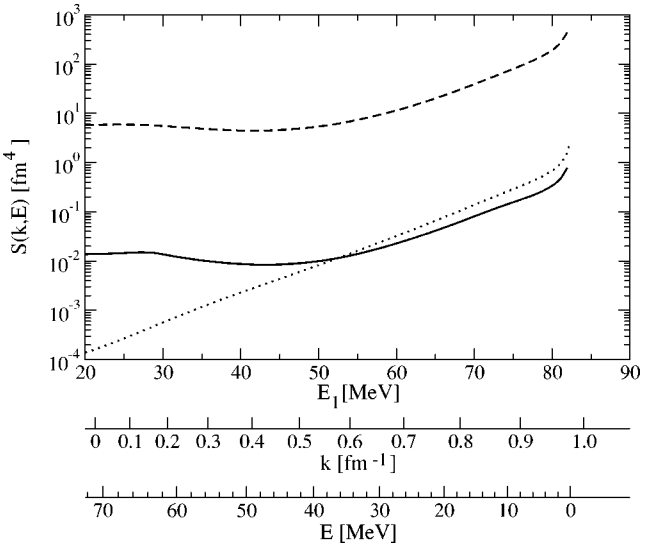

FIG. 21. The same as in Fig. 20 for the neutron knockout and $\omega = 100$ MeV, $Q = 200$ MeV/c.

600 MeV/c. This defined a domain D in the Q-ω plane. The kinematical conditions for parallel knockout lead then to a quadratic equation connecting Q-ω to k-E, the missing momentum and missing energy. Thus the domain D is mapped into a domain D' in the k-E plane and vice versa. Our results show that for proton knock out S_L(Full) and S_T(Full) agree with the quantity S (appropriately corrected by electromagnetic form factors and kinematical factors) if both k and E are very small. Unfortunately this is not always the case and we identified a certain region of small k values where with increasing Q S_L(Full) and S_T(Full) deviate more and more from S. Therefore the validity of that approximation must be checked in each case. For the rest of the domain D' in the k-E plane the use of S is not a valid approximation. Specifically there occur intriguing cases, where inside D' S coincides with the most simple approximate treatment of the process, namely pure PWIA. This suggests a direct view into the ^{3}He wave function. However, this is quite misleading under the kinematics investigated here since even the complete antisymmetrization totally destroys that simple picture not to speak of the final state interaction of the knocked out nucleon with the other two.

In the case of neutron knockout, only R_T can be approximated by S under certain kinematical conditions (low k-E values). In the case of R_L the smallness of G_E^n in relation to G_E^p leads always to an important contribution of the absorption of the photon by the two protons, which then by final state interaction knock out the neutron. So R_L in the case of neutron knockout cannot be approximated by S in the kinematic regime investigated in our study.

Finally we would like to stress that the concept of S might be useful to extract electromagnetic nucleon form factors if the kinematical conditions are suitable. In that case only the NN t-matrix and the ^{3}He wave function at low momentum values enter, where both ingredients should be fairly well under control.

ACKNOWLEDGMENTS

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