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Improvement on the Accuracy of the Polynomial Form Extrapolation Model in Distributed Virtual Environment

Abstract In this paper, we studied the relationship between the accuracy of the extrapolating data and the update interval in a distributed virtual environment (DVE). Based on the properties of the polynomial models, we proposed the new method to extrapolate the attribute data which arrives at discrete time period. Theoretical models were formulated and showed that the average error of proposed method is less than that of current method. Finally, we confirmed that the proposed method can improve the accuracy in comparison with current method by conducting experiments with the pen motion of a series of letters written by human.

Keywords distributed virtual environment · update interval · extrapolation · error model · numerical difference

1 Introduction

In recent years, much research has been focused on distributed virtual environments (DVE) and on their applications. DVEs allow users to share the common virtual space, regardless of geographical location. DVEs have been used in many areas, including industry, medical treatment, and entertainment. In DVE systems, network latency such as the update interval, transmission lag, and jitter are serious problems, since both consistent state and system throughputs can not be kept in high quality. Especially the discrete-time property of information updating due to limitations in transmission capacity of the data affects the interactivity, operability, and performance of systems [4,5,13,24,25].

A prediction method called dead reckoning is one of the methods to solve these problems and has been applied in various types of DVEs[3]. A number of studies on the effects of using a dead reckoning method in real-time systems to solve the issue caused by the discrete-time property of information updating have previously been reported[1,2,7,8,13,16, 19,22,23]. Various types of interpolation polynomials are often used for the prediction of dead reckoning, since the task performance, operability of the user, and so on, can be improved[7,13,19,23,22]. Dead reckoning in DVE consists of three kinds of functions. The first function is *lag compensation*, that is, time lag compensation in transmitting information due to the network delays. The second function is *extrapolation*, that is, the real-time prediction of the data between the update intervals. The third function is *convergence*, that is, the smooth correction of error caused by the result of the above two functions. In these functions, extrapolation is important function when we construct a DVE which requires high realtime interactivity.

We proposed the new method to extrapolate data generated from received data in the discrete temporal axis in a DVE[12]. In references[9,26], we found qualitative characteristics on the lag compensation using polynomial models in a DVE. Also in references [10,11], we made theoretical studies of the current data extrapolation method in a DVE. Based on the results of our previous studies, we studied the relationship between the update interval on a DVE and accuracy of extrapolation, in order to find the factors that degrade the accuracy of data extrapolation using polynomial models. We also compared our method with the current method and evaluated the validity of our method theoretically and experimentally.

2 An overview of DVE with extrapolation

We define a DVE as an imitation of space with physical features, and a physical space for performing task such as col-

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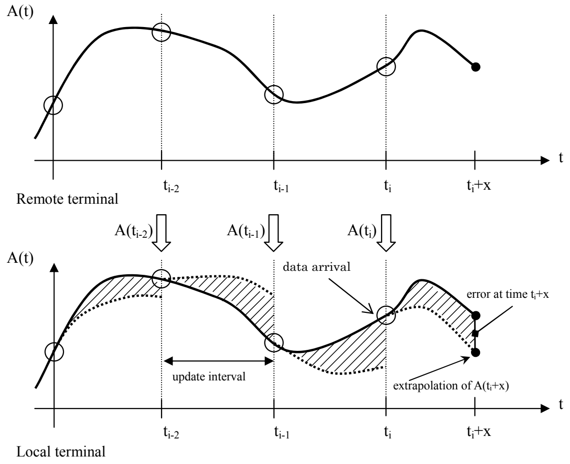


Fig. 1 Model of observation and generated error using extrapolation

laboration, training, racing game, etc. via the network is constructed on a DVE. The attributes (e.g. physical information, such as position, direction, etc.) of each user's avatar (an object of manipulation in the DVE) undergo change continuously over time due to manipulation performed by the user. When we construct such DVE systems, motion of all avatars should be display on each terminal in realtime while each user can manipulate its own avatar. There are two kinds of factors which cause lack of consistency or throughput on the current networks. One of the factors is transmission lag and another is update interval (each terminal must wait a fixed time interval for sending data to the other terminals). In a server-client model, it is difficult to keep high throughputs since all data of avatar are transmitted through the server and calculation time is required to manage consistency. In a peer-to-peer model, it is difficult to manage consistency since a terminal manages the attributes of each avatar in a uniform fashion dose not exist.

We focused on one local terminal in a DVE and examined the circumstances under which the user of that terminal observes another avatar which is controlled by remote terminal. Suppose that $A(t)$ is a function, whose parameter is a continuous time variable t. $A(t)$ is assumed to be a continuous and differentiable function of variable t in its domain. Specifically, in the DVE, attribute data, such as the spatial position of an avatar, is presumed as $A(t)$. Here, $A(t)$ undergoes change continuously over time due to interactive manipulation performed by the user. Suppose that the frame update interval of $A(t)$ is limited by the limitation of network bandwidth, only function value $A(t_i)$ in the discrete time $t_i (i = 0,1,2,3...)$ can be observed as the update data in the client. In this case, if frame update interval u is not assumed sufficiently small, the smoothness of a change in $A(t)$ is lost in proportion to the update interval. Extrapolation is a method for extrapolating data between update intervals by predicting in clients. A model of the observation of temporally discrete data using extrapolation is shown in Fig.1. In this figure, circle marks show the update data. Solid lines and dotted lines show the path of actual and extrapolating data, respectively. Dashed areas show the cumulative error of extrapolation between two frames in a sequence. As shown in Fig.1, since the system extrapolates data between the latest frame and the next frame, data in a client can be presented in a small time interval. Today, extrapolation is used in many real-time systems [24].

3 Modeling of error of extrapolation

3.1 Extrapolation using polynomial models

A method for the analysis of the error based on interpolation polynomial model was seen in this section[10,11]. As a model of common data extrapolation method, we used the Lagrange polynomial model that uses several frames of received data as parameters [18,20].

$$
L_u(A(t),t_i)_n = \begin{cases} A(t_i) & (n=0) \\ \displaystyle\sum_{j=0}^{n} \left(A(t_{i-j}) \prod_{k=0(k\neq j)}^{n} \frac{t-t_{i-k}}{t_{i-j}-t_{i-k}} \right) & (n \geq 1) \end{cases} \tag{1}
$$

Here, t, t_i, u, and n denote the time ($t \geq t_i$), the time of the latest data, the update interval and order of the polynomial, respectively.

We considered polynomial models of orders 0, 1, and 2 (a polynomial model of order n is noted below as an "*nth*-order model"). Furthermore, for simplicity, we considered $A(t)$ to be a one dimension variable and u to be a fixed temporal interval. These equations are described as follows.

$$
L_u(A(t_i+x),t_i)_0 = A(t_i) \qquad (n=0) \tag{2}
$$

$$
L_u(A(t_i+x),t_i)_1 = A(t_i) + \frac{A(t_i)-A(t_{i-1})}{u}x \qquad (n=1) \tag{3}
$$

$$
\begin{aligned} L_u(A(t_i+x),t_i)_2 &= A(t_{i-1}) + \frac{A(t_i)-A(t_{i-2})}{2u}(x+u) \\ &+ \frac{A(t_i)-2A(t_{i-1})+A(t_{i-2})}{2u^2}(x+u)^2 \qquad (n=2) \end{aligned} \tag{4}
$$

3.2 Approximation models for extrapolation error

When we analyzed the extrapolation error mentioned above using error analysis on numerical differentiation [10,11], we let N denote the total number of observed frames.

The cumulative numerical error from the time t_i to t_{i+1}, is equivalent to the cumulative numerical error of $|A(t_i+x) - L_u(A(t_i+x),t_i)_n|$ generated between frames. Accordingly, denoting the average of the cumulative error per unit time as $error_L(A(t),u,n)$,

$$\begin{aligned} & error_L(A(t),u,n) \\ & = \frac{1}{Nu}\sum_{i=0}^{N-1}\int_0^u |A(t_i+x) - L_u(A(t_i+x),t_i)_n| dx. \end{aligned} \tag{5}$$

Since $A(t)$ is assumed to be a continuous and differentiable function of the variable t, the Taylor expansion of $A(t)$ around $t = t_i$ can be considered. The magnitude of that error $|A(t_i + x) - L_u(A(t_i+x),t_i)_n|$ by $0th$-order model can therefore be calculated as

$$\begin{aligned} & |A(t_i+x) - L_u(A(t_i+x),t_i)_0| \\ & = \left| \frac{A^{(1)}(t_i)}{1!}x + \frac{A^{(2)}(t_i)}{2!}x^2 + \cdots \right| \end{aligned} \tag{6}$$

By assuming that the second or higher term of the right side of eq.(6) can be neglected, $error_L(A(t),u,0)$ can be expressed with the update interval, u, and the average of $|A^{(1)}(t_i)|$ as $\overline{|A^{(1)}|}$.

$$error_L(A(t),u,0) = \frac{\overline{|A^{(1)}|}}{2}u. \tag{7}$$

The error generated as a result of extrapolation using the $1st$-order model is equivalent to the divergence from uniform motion, i.e., to acceleration or higher temporal differential terms that occur between two frames in a sequence. By using the Taylor expansion of $A(t_{i-1})$ around $t = t_i$,

$$\frac{A(t_i) - A(t_{i-1})}{u} = \frac{A^{(1)}(t_i)}{1!} - \frac{A^{(2)}(t_i)}{2!}u + \cdots, \tag{8}$$

where $A^{(m)}(t_i)$ is the mth order differentiation at $t = t_i$. From this, the magnitude of the error $|A(t_i+x) - L_u(A(t_i+x),t_i)_1|$ using the $1st$-order model can therefore be calculated as follows,

$$\begin{aligned} & |A(t_i+x) - L_u(A(t_i+x),t_i)_1| \\ & = \left| \frac{A^{(2)}(t_i)}{2!}x(x+u) + \frac{A^{(3)}(t_i)}{3!}x(x^2-u^2) + \cdots \right| \end{aligned} \tag{9}$$

Similar to eq.(7), $error_L(A(t),u,1)$ can be expressed, with the average of $|A^{(2)}(t_i)|$ as $\overline{|A^{(2)}|}$,

$$error_L(A(t),u,1) = \frac{5\overline{|A^{(2)}|}}{12}u^2. \tag{10}$$

Moreover, the error generated as a result of extrapolation using the $2nd$-order model is equivalent to the divergence from stable acceleration motion i.e. the jerk or higher temporal differential terms that occur between two frames in a sequence. By using the Taylor expansion of $A(t_{i-2})$ around $t = t_{i-1}$, that of $A(t_{i-1})$ around $t = t_{i-1}$, and that of $A(t_i)$ around $t = t_{i-1}$,

$$\begin{aligned} & \frac{A(t_i) - A(t_{i-2})}{2u} \\ & = A^{(1)}(t_{i-1}) + \frac{A^{(3)}(t_{i-1})}{3!}u^2 + \cdots. \end{aligned} \tag{11}$$

$$\begin{aligned} & \frac{A(t_i) - 2A(t_{i-1}) + A(t_{i-2})}{u^2} \\ & = A^{(2)}(t_{i-1}) + \frac{2A^{(4)}(t_{i-1})}{4!}u^2 + \cdots \end{aligned} \tag{12}$$

The magnitude of that error $|A(t_i+x) - L_u(A(t_i+x),t_i)_2|$ by the $2nd$-order model can therefore be calculated as,

$$\begin{aligned} & |A(t_i+x) - L_u(A(t_i+x),t_i)_2| \\ & = \left| \frac{A^{(3)}(t_{i-1})}{3!}x(x+u)(x+2u) \right. \\ & \left. + \frac{A^{(4)}(t_{i-1})}{4!}x(x+u)^2(x+2u) + \ldots \right| \end{aligned} \tag{13}$$

Similarly in eq.(7), $error_L(A(t),u,2)$ can be expressed as follows, with the average of $|A^{(3)}(t_{i-1})|$ as $\overline{|A^{(3)}|}$,

$$error_L(A(t),u,2) = \frac{3\overline{|A^{(3)}|}}{8}u^3. \tag{14}$$

From the above results, the statistical error in the extrapolation using Lagrange Polynomial model (referred to below as current method) can be regarded as the statistics of approximation error, since Lagrange polynomial models do not include enough information. The error in current method is supposed to converge on zero if the update interval is sufficiently small and the order of the model n is sufficiently large. We can find two kinds of factor which grows the error in current method.

Factor(1) The update interval of DVE has a certain temporal size. Thus, accuracy of the calculation of the difference is degraded (eq.(8,(11),(12))[18,20].

Factor(2) Terms of $(n+1)$th or higher order differentiation are neglected. Thus accuracy of the approximation by polynomial is degraded.

4 A proposal of extrapolation protocol

We proposed new extrapolation method with the improvement of the factors of the error in the previous section. According to the Factor (1), nth order difference calculated from backward difference becomes as accurate as temporal size u is small. Therefore it is desirable that the differences are calculated in the data sending terminal. In the current DVE system, sampling interval of the avatar in local terminal can be set smaller than transmission interval of the updating data. From the data over the last several sampling frames, a terminal can calculate the differences (velocity, acceleration, and jerk) of its own avatar more accurately than the other remote terminals. By using backward difference [18], the nth order differentiation of $A(t)$ at t based on the sampling interval can be calculated from

$$A_\varepsilon^{(n)}(t) = \frac{A_\varepsilon^{(n-1)}(t) - A_\varepsilon^{(n-1)}(t-\varepsilon)}{\varepsilon}, \tag{15}$$

where ε denotes the local sampling interval of $A(t)$.

Since nth order differentiation at $t = t_i$ calculated from eq.(15) received in the remote terminals, we can consider the Taylor expansion of $A(t_i + x)$ around $t = t_i$ [26]. From this, we considered applying the Taylor expansion based on nth and lower order differentiations as the data extrapolation models. These equations are described as follows.

$$T_\varepsilon(A(t_i+x),t_i)_n = \sum_{k=0}^{n} \frac{A_\varepsilon^{(k)}(t_i)}{k!}x^k \tag{16}$$

The magnitude of that error $|A(t_i+x) - T_\varepsilon(A(t_i+x),t_i)_n|$ can therefore be calculated as,

$$\begin{aligned}&|A(t_i+x) - T_\varepsilon(A(t_i+x),t_i)_n|\\ &= \left|\frac{A^{(n+1)}(t_i)}{(n+1)!}x^{n+1} + \frac{A^{(n+2)}(t_i)}{(n+2)!}x^{n+2} + \dots\right| \end{aligned}\tag{17}$$

Moreover, the cumulative numerical error from the time t_i to t_{i+1}, is equivalent to the cumulative numerical error of $|A(t_i+x) - T_\varepsilon(A(t_i+x),t_i)_n|$ generated between frames. Accordingly, denoting the average of the cumulative error per unit time as $error_T(A(t),u,n)$,

$$\begin{aligned}&error_T(A(t),u,n)\\ &= \frac{1}{Nu}\sum_{i=0}^{N-1}\int_0^u |A(t_i+x) - T_\varepsilon(A(t_i+x),t_i)_n|dx. \end{aligned}\tag{18}$$

Similar to eq.(7), $error_L(A(t),u,1)$ can be expressed as follow, with the average of $|A^{(n)}(t_i)|$ as $\overline{|A^{(n)}|}$, since the error by Factor(1) in previous section can be neglected.

$$error_T(A(t),u,n) = \frac{\overline{|A^{(n+1)}|}}{(n+1)!}u^{n+1}. \tag{19}$$

Comparing $error_T(A(t),u,n)$ and $error_T(A(t),u,n)$, the average error of data extrapolation using eq.(15) and eq.(16) should be smaller than that in the current method. Most of the current methods (a terminal receiving data calculates velocity, acceleration and jerk) use eq.(8), (11) and (12) in order to calculate differences of the avatar.

Based on the above study, we proposed the new data extrapolation protocol which improve the extrapolation error.

<**Data Extrapolation Protocol**>

Process in the terminal transmitting data

From the historical data of the avatar A based on the sampling interval ε, a terminal calculates $A(t_i), A_\varepsilon^{(1)}(t_i), \cdots$, and $A_\varepsilon^{(n)}(t_i)$ together. After the calculation of the differentiation, these data are transmitted to the remote terminal.

Process in the terminal receiving data

After the receiving of the above data, terminal extrapolates the motion of avatar A by substituting into eq.(16). Fig.2(a) shows data transaction diagram in the proposed data extrapolation protocol.

Other ways which can implement proposed protocol are shown in the following. If a terminal receives data over the last n frames (Fig.2(b)) or these are transmitted in the sampling interval ε at the frame update timingFig.2(c), $A_\varepsilon^{(k)}(t_i)(1 \leq k \leq n)$ can be calculated in the terminal receiving data. Total cost of calculation for extrapolating data in the proposed method is same as that in the current method. On the other hand, data size transmitting to the remote terminal in proposed method is approximately $n+1$ times as much as that in the current method. However, comparing each theoretical model (eq.(7), (10), (14) and (19)), the average error is expected to be reduce into 2/5 (n=1) and 1/9 (n=2) respectively.

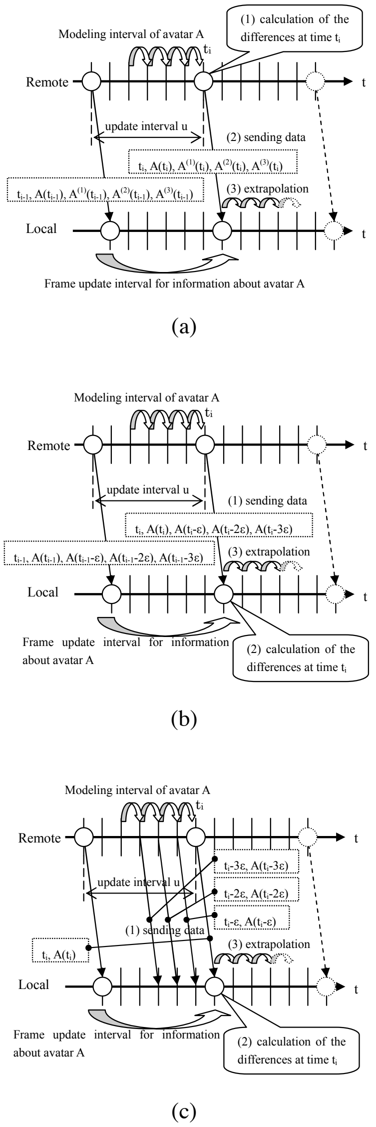


Fig. 2 Comparison of average error

5 Related works

An effective dead reckoning method in networked virtual environment is proposed in reference[23,22]. Thresholds tuning method for updating packet timing in dead reckoning is examined in reference[21,17]. Dead reckoning method are applied in virtual human animation on networked virtual environment[2], tactile communication system[13], collision detection between two objects in virtual space[19] and reduction of heterogeneity in networked virtual environment[6]. However, theoretical analysis on the properties of the dead reckoning method based on interpolation of the polynomial model has not been sufficiently done. Therefore, to find the way to reduce the error growing in proportion to the update interval in systems using dead reckoning was difficult. On the other hand, Consistency measurements in DVE using dead reckoning are proposed in reference[27]. However relationship between the accuracy of numerical difference and consistency is not analyzed. In the method such as IEEE 1278[14](a terminal transmitting data calculates velocity, acceleration and jerk)[13,19,23], there have been few reports that showed the advantage of the calculation using eq.(15). Moreover, there have been no report that formulated the extrapolation error in the way of eq.(19).

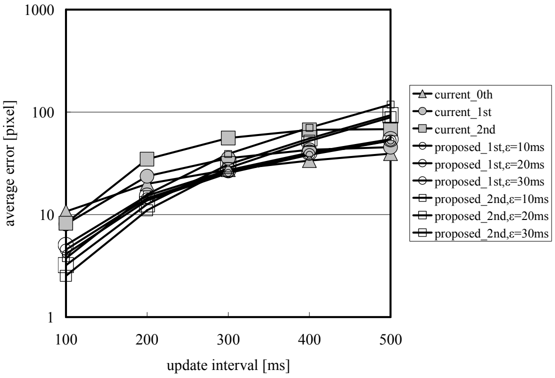
6 Experiments

6.1 Design of experiments

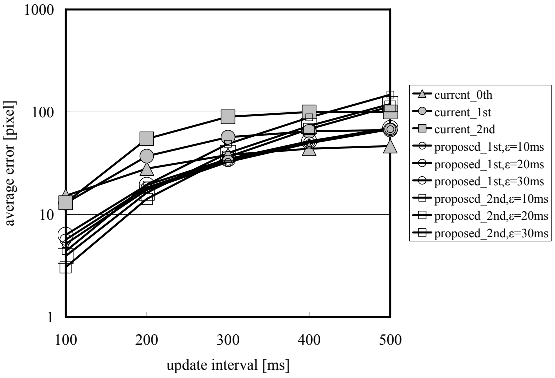
Experiments were conducted to evaluate the validity of proposed method. In the experiments, we calculates the error (the dashed area in Fig.1) based on eq.(5) and (18) from the generated pattern. This error is referred to as the "average error" in units of *pixel*. Accuracies of the extrapolation using current method and proposed method have been evaluated based on this metrics.

The pen motion of a written string of letters [15] was used as the sample of motion data. An user physical motion such as a motion of the hand or finger is often used to manipulate its own avatars in DVE[13]. In the experiments, English words which consisting five to eight letters are drawn by examinees on a touch panel screen, to avoid arbitrary movements and to make examinees understand this experiment easily. The data, which consisted of the X and Y positions of the cursor, were recorded with the time stamps. The window size is 1200×800 *pixels* and the sampling interval of the touch panel screen was 10 *ms*. Examinees were solicited to carefully write a word in the whole window.

After writing the word, the average error was calculated from the data. In the following section, the pattern of the X and Y positions are referred to as the X-axis pattern and the Y-axis pattern. Sixteen English words , “allow”, “apple”, “Column”, “Colorado”, “common”, “develop”, “Ellen”, “green”, “loose”, “Molly”, “Nevada”, “Oregon”, “paper”, “really”, “Solomon”, and ‘yellow‘” were used in the experiments. To avoid non-contiguous motion, those which contain letters such as i, x, or t were not used.



(a) X-axis



(b) Y-axis

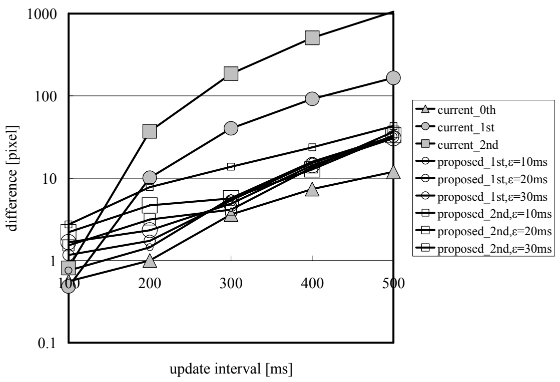
Fig. 3 Comparison of average error

6.2 Results

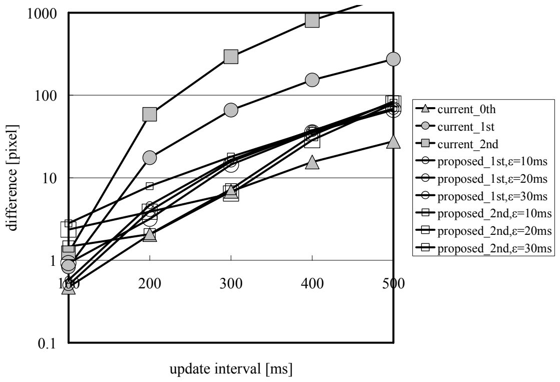
Data twelve examinees totaling 192 patterns were acquired. Experiments were performed for five update intervals (100, 200, ..., 500 *ms*) and three sampling interval (10, 20, 30 *ms*) for calculation of eq.(15). The results of these experiments are shown in Fig.3 $\sim$ **??**. The average error of the extrapolation using the current method (eq.(2), (3), and (4)) and the proposed method (eq.(16)) are shown in Fig.3. The difference between the average error and the approximation by the theoretical models is shown in Fig.4. The accuracy of the approximation using theoretical models is shown in Fig.5. The accuracy of the approximation using theoretical models was calculated by

$$Accuracy = |(Error - Model)|\,/Error. \tag{20}$$

The average error in the pattern “Colorado” and that in the pattern “really” are shown in Fig.6 and Fig.7, respectively. An example of pen motion and its extrapolating paths is shown in Fig.**??**. The velocity, acceleration, and jerk in X-axis, and Y-axis patterns were shown in Tab.1



(a) X-axis

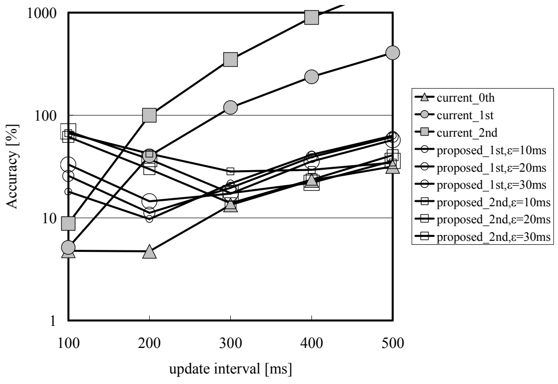


(b) Y-axis

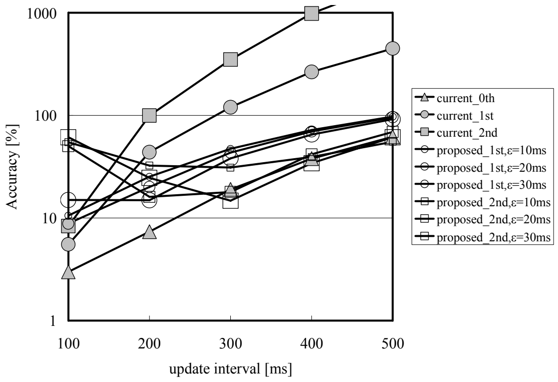
Fig. 4 Difference between average error and theoretical model

Table 1 The absolute averages of the velocity, acceleration, and jerk

pattern	velocity	acceleration	jerk
X-axis	204 $pixel/s$	2025 $pixel/s^2$	23975 $pixel/s^3$
Y-axis	297 $pixel/s$	3274 $pixel/s^2$	37841 $pixel/s^3$



(a) X-axis



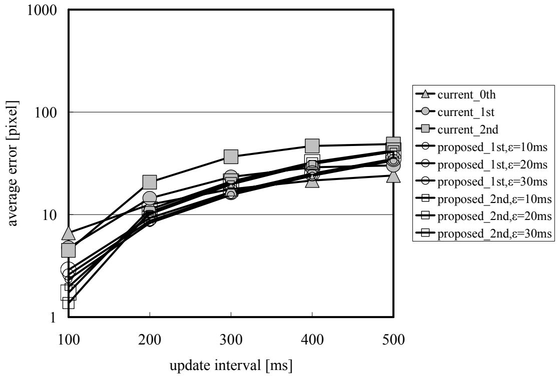
(b) Y-axis

Fig. 5 Comparison of accuracy of approximation using theoretical model

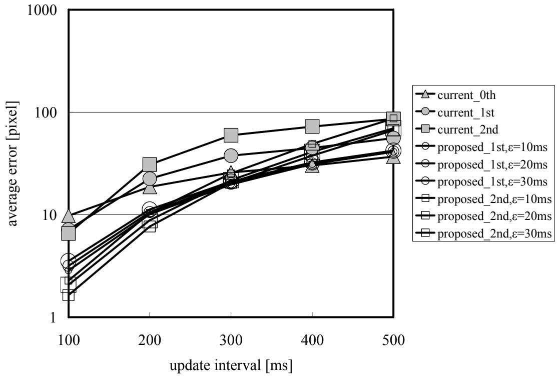
6.3 Discussion

The error of proposed method were less than that of the current method. According to Fig.3, the average error in the proposed method improved in less than half in comparison with the current method. In the case that order of the model is same, the average error of the 1st order model in the proposed method ($\varepsilon = 10ms$) was the same as about $2/5$ time as that in comparison with the current method. About the 2nd order model, the average error in the proposed method ($\varepsilon = 20ms$) was the same as about $1/9$ time as that in comparison with the current method. These results mostly agree with theoretical study in section 4. From the viewpoint of the difference of English word, the results in "Colorado" are the most accurate and the results in "really" are the most inaccurale. Much more rapid turn is containd included in the path of "really" in comparison with the path of "Colorado". These results agree with theoretical study that the accuracy becomes worth when the contribution of the higher order terms is larger. On the basis of these results, the validity of the proposed method were confirmed in the average error.

In Fig.5, the theoretical models of the proposed method approximated to the results of the experiments more closely than that of the current method. In the proposed method, percentage discrepancy from the experiments was smaller than that in the current method. The percentage was under 100% regardless of the order of the polynomial. The percentage in the proposed method in the case for the update interval $100ms$ was larger in comparison with the current method. However, in Fig.3, the error for small update interval was about $1 pixel$. Also in Fig.4, the difference between the error and theoretical model for small update interval was less than 3 $pixel$. In Fig.**??**, the path generated by the proposed method is more accurate than the current method. Therefore, the discrepancy in such case is not severe in comparison with case where the update interval is large. The ranking of the size of the error in each extrapolation model can be mostly

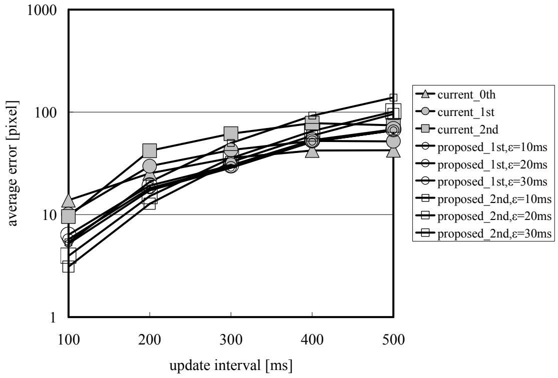


(a) X-axis

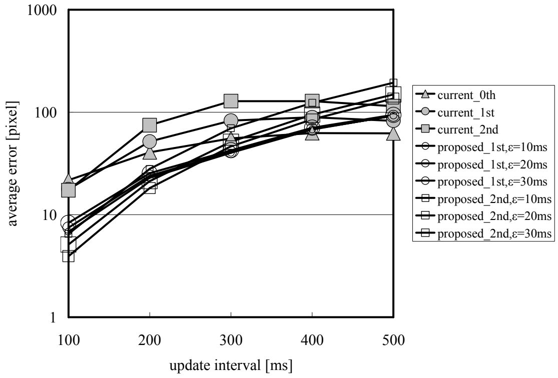


(b) Y-axis

Fig. 6 Comparison of average error ("Colorado")



(a) X-axis



(b) Y-axis

Fig. 7 Comparison of average error ("really")

estimated by the theoretical models eq.(7, (10), (14) and (19) and Tab.1. These results show that we can select the most accurate model using theoretical models and the absolute averages of the velocity, acceleration, and jerk. On the basis of these results, the extrapolation error based on the proposed method can be estimated by the theoretical models.

Next, after comparing the results between the types of patterns, average error in the X-axis pattern were smaller than that in the Y-axis pattern (Fig.3. The reason for this is thought to be that Factor(2) of the Y-axis pattern in the experiments was more larger (Tab.1). The accuracy in the Y-axis pattern was mostly same as that in the X-axis pattern (Fig.5). The percentage discrepancy is considered to increase in cases where the change of motion is more rapid, since the contribution of the $n+2$ or higher order term can not be neglected. The influence by Factor(2) in the current method consider to be larger than the proposed method.

On comparing the accuracy of the theoretical models between different order models, better results were obtained for the lower order models than the higher order models (Fig.5). The reason for this is thought to be that the contribution of *rounding-error* in higher order models is greater than that in lower order models since $1st$-order or $2nd$-order models contain the error generated from numerical differentiations. The contribution of *rounding-error* to numerical differentiations increases in proportions to the order and the update interval [18,20]. This problem becomes more serious in the proposed method since the temporal interval for the calculation of difference is smaller than that of the current method. Moreover, in the case that the update interval is larger than 500 *ms*, the error of higher order model in the proposed method is considered to become larger than that of lower order model (Fig.3). The minimum unit of spatial position transmitted from the touch panel screen was 1 *pixel*. Therefore, we believe this problem can be reduced using an input device with higher spatial resolution.

Another solution of the problem on the rounding error mentioned above is improvemnet of the calculation of differences. In Fig.3, 4 and 5, when we use $1st$-order model by proposed method, the average error, the accuracy and the difference are all the most smallest in the case of $\varepsilon = 10ms$. About $2nd$-order model in proposed method, these measurement are the most smallest in the case of $\varepsilon = 20ms$. These results show that there is an optimal size of temporal interval to calculate numerical differences acculately[18,20]. We show three kinds of methods to imprement proposed method in Section 4. Since optimal ε is different in each order difference in the most case, method in Fig.fig:ndr (a) (differences are calculated and transmitted in remote terminal) seems to be suitable. Also we think that use of centered difference seems to improve the accuracy in the proposed method. More detailed evaluation on the interval for the calculation of numerical differences and analysis on the calculation method of differences are future works. All in all, the above results reveal that the validity of the proposed data extrapolation method compared with the current method is confirmed in several viewpoints.

7 Conclusion

We studied the relationship between extrapolation error and the update interval of the data in a DVE using numerical analysis. First, the method of extrapolation based on the Lagrange polynomial model was used, and the relationship between the error and the update interval was numerically analyzed. As a result, in the current extrapolation method, both of the inaccuracy of difference and contribution of higher order terms are increased with the increase of the update interval. Next, new extrapolation method including the improvement of the inaccuracy of difference was proposed. Moreover, theoretical models which showed the increase in the error caused by the increase in the update interval were formalized. The theoretical models showed that our proposed models achieve accurate extrapolation. The validity of the proposed method was confirmed with experiments. Our results suggested proposed method worked effectively to extrapolate the motion of avatars such as pen motion and so on. These findings demonstrate the usefulness of the proposed method for estimating the extrapolation error based on polynomial models. By using the proposed method, an error at the any time between the last frame and the next frame by the nth-order model $(0 \leq n \leq 2)$ can be estimated using eq.(6), (9), (13), and (17) since the client can receive the differences from a server. In the proposed method, the error by the $3rd$ or more higher order model can be estimated if a remote transmits $(n+1)th$ or lower order differences. Based on these results, a local terminal can estimate and select an optimal order of the model and type (proposed or current) of the method.

In future work, we plan to study the applicability of our proposed theoretical model. For example, we plan to evaluate our method from the other viewpoints such as calculation method of differences, sampling interval tolerance to packet loss and variation of the update interval, to apply the theoretical models to the adaptive data extrapolation method[22, 23].

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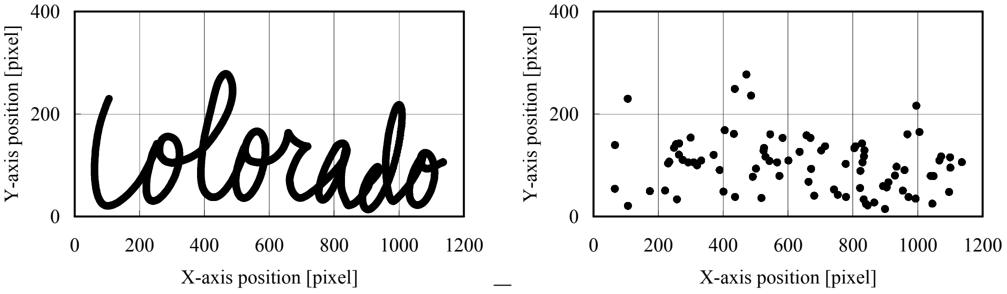
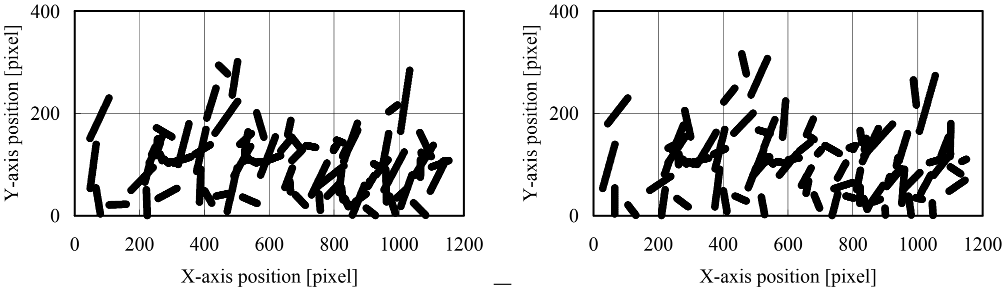


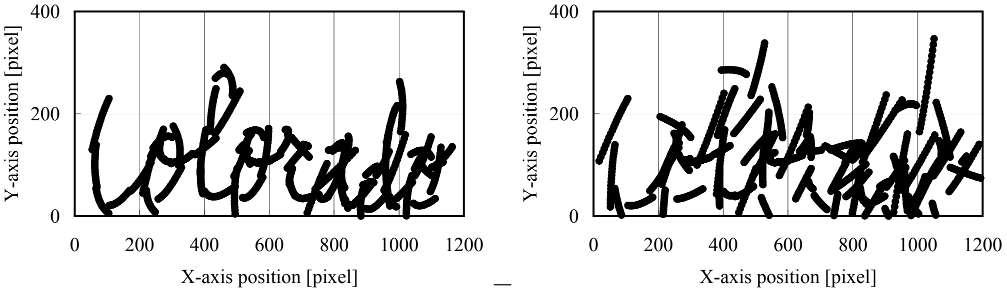
(a) pen motion

(b) current method, $0th$-order model



(c) proposed method, $1st$-order model

(d) current method $1st$-order model



(e) proposed method, $2nd$-order model

(f) current method, $2nd$-order model

Fig. 8 An example of extrapolation ("Colorado", $u = 200ms$)

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