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Decentralized of licensing of complementary patents: comparing royalty, fixed fee and two part tariff.

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Abstract

This paper analyzes how an inventor should fix the licensing terms to license a standard in complying with a non-discrimination requirement. Using a model incorporating imperfect competition between a finite number of users and product differentiation, we compare three different regimes: fixed fee (also known as royalty free), per unit royalty and two-part tariff. We highlight the different effects of each design on prices and number of varieties. We identify which one dominates with respect to the licensor's profit and total welfare. Finally we extend our model to a setting where the standard is protected by several licenses owned by non-cooperating owners.

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1   Introduction

Cooperative technology standards frequently embody complementary patents belonging to various owners. During the last two decades, the number of such patents has increased dramatically (Bekkers et al., 2002; Simcoe, 2005), thereby raising a royalty stacking problem, also coined "double marginalization" or "Cournot-Shapiro" issue. Each patent owner indeed enjoys a monopoly position and can therefore charge high royalties to its licensees. By doing so, patent owners however do not take into account that they also reduce the demand for licenses on other complementary patents. The resulting royalty stacking induces lesser demand for standard-compliant technologies, but also lower profits for the patent owners themselves. In that context it would be more profitable for patent owners to form "patent pools" in order to grant a unique package license for the bundle of their patents, and share the resulting licensing revenue (Shapiro, 2001).

We explore in this paper whether the coordination failure featuring the "Cournot-Shapiro" issue may apply to licenses based on other schemes than royalty, namely fixed fee or a combination of fixed fee and royalty (two-part tariff). We then identify which scheme is more efficient when numerous complementary patents reading on a technology standard are licensed in a decentralized way. Using the best licensing scheme is an interesting alternative to patent pools since the latter frequently fail to form in practice. Indeed patent owners have strong incentives to stay out of a pool in order to free ride on the low price of the package license by charging a higher price for their own patents (Aoki & Nagaoka, 2004).

In order to analyze and compare the different types of licenses, we develop a model in which $\chi \geq 1$ owners of complementary patents reading on a new standard sell licenses to manufacturers of standard compliant products. Manufacturers have to buy a license on each patent in order to enter the product market. Consistently with the requirements usually imposed on owners of patents reading on standards[3], we assume throughout the paper that patent owners must grant a license to each manufacturer who is willing to pay the same terms as the other licensees. While the "Cournot-Shapiro" issue has been characterized in the case of homogenous products (Shapiro, 2001), we allow for horizontal differentiation in the product market.

A first finding is that royalty and fixed fee are more or less appealing for licensors depending on the degree of product differentiation. Fixed fees impose an entry cost to manufacturers and are therefore a way for patent owners to control the number of licensees and to extract all their profits. By contrast, patent owners cannot use per-unit royalties to control entry, neither to extract all profits. Yet royalties increase marginal manufacturing costs, and therefore allow patent owners to monitor market prices for a given number of manufacturers. If there is only one patent owner, we show that royalty generates more

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[3]The Intellectual Property policies of most standard setting organizations indeed require that patent owners license their patents under Fair Reasonable and Non-discriminatory terms.

(respectively less) licensing profit than fixed fee when products are strongly (respectively weakly) differentiated. Charging a high fixed fee is indeed a way to allow one licensee only and reap the monopoly profit when products are homogenous. As products become more differentiated, new entrants generate additional consumer willingness to pay. Then it eventually becomes more profitable to use royalties, thereby letting a large number of manufacturers enter the market and mitigating price competition by increasing their marginal costs. Unsurprisingly, we find that a single licensor will always prefer two-part tariff to fixed fee or royalty alone, since it combines the best of both worlds.

An important finding is that introducing additional patent owners (e.g., $\chi > 1$) in this setting can generate coordination failures with each type of license. As a general rule, total licensing revenue is non-increasing in χ whatever the licensing scheme that is considered. Although fixed fees do not distort competitive prices, their stacking has indeed a negative entry deterrence effect. In turn, two-part tariffs combine coordination failures pertaining to both royalty and fixed fee stacking.

Comparing the three regimes with $\chi > 1$, we find that each type of license may maximize total licensing revenue depending on the degree of product differentiation. Fixed fee only and two-part tariff are more profitable respectively for weakly and strongly differentiated products. Interestingly, we show that pure royalties dominate both fixed fee and two-part tariff for intermediate levels of differention. Indeed they prevent the negative entry deterrence effect attached to fixed fee stacking in the two other licensing schemes. Total welfare analysis in turn highlights a convergence between the patent owners' incentives to innovate and the social value of the standard. While fixed fee and two-part tariff still dominate for respecively weakly and strongly differentiated products, we find that royalties only would indeed maximize social surplus for medium degrees of product diffenretiation.

This paper provides an original contribution to the analysis of licensing of complementary patents. The coordination failure resulting of decentralized licensing of complementary patents has been pinpointed by Shapiro (2001) in the case of royalty-based licenses only. Other papers have then demonstrated the pro-competitive effects of patent pools (Lerner & Tirole, 2004; Lerner et al., 2005), but also their limitations due to strategic incentives for patent owners not to join them (Aoki & Nagaoka, 2004). To our knowledge potential coordination failures with other licensing schemes had not been explored, although it may have interesting implications in terms of selection of license. Our approach also relates to the literature on optimal licensing. A large strand of papers (See Sen (2005) for a good review) compare auctions, royalty and fixed fee licensees in the trail of the seminal contribution of Kamien and Tauman (1986, 1992), who concluded on the superiority of fixed fee. In particular, Muto (1993) shows that royalty may be superior when products are differentiated in a Bertrand competition setting. Erutku and Richelle (2007) also consider two-part tariff in a model derived from Kamien and Tauman (1986) and show that it always dominates the other schemes. Our analysis, which is consistent with their results, addresses similar questions in a context of multiple licenses on complementary

patents.

The article is organized in 6 Sections. Section 2 introduces the model. We solve in Section 3 the licensing equilibria for each type of licensing scheme, and highlight different coordination failures when there are more than one patent owner. We then compare the three licensing schemes in the next two Sections, with respect respectively of licensing revenue (Section 4) and social value of the standard (Section 5). Section 6 concludes.

2   The model

We introduce in this Section the general model used throuhgout the paper. We consider a market for products compliant with a technology standard. The standard incorporates χ patented technologies belonging to χ independent owners. The χ patents are essential, so that each manufacturer of standard compliant product must license all patents to enter the product market. Patent owners are not involved in product manufacturing, and simply seek to maximize their profit through licensing.

Manufacturers of standard compliant products must buy a license on each patent in order to enter the market. We assume that at most n ($n \in N$ and $n \geq 1$) symmetric firms are capable of using the standard to produce differentiated outputs. We consider that there is imperfect competition on the product market and assume that manufacturers compete à la Cournot. Let the demand function for product i, produced by firm i when k ($1 \leq k \leq n$) firms sell substitute products be given by:

$$p_i(q_i, Q_{-i}) = a - q_i - \alpha Q_{-i},$$

where $i = 1, ..., k$ and $Q_{-i} = \sum_{j \neq i} q_j$. The parameter $\alpha \in [0, 1]$ measures product substitutability. The total cost function is linear and such that $TC(q) = cq$. Assume that for any k we have $a > c$, meaning that production of each product is worthwhile.

We consider three possible licensing regimes: fixed fee only, royalty only or two-part tariff. The timing is the following. First the patent holder announces the patent policy stating the fixed fee $l \geq 0$ and royalty $r \geq 0$. By definition the fixed fee regime has $r = 0$, the royalty regime has $l = 0$ while two-part tariff has $r > 0$ and $l > 0$. Second, the firms decide whether to buy the licence. We consider that they get a 0 profit without a license. When $l > 0$ we consider that entry is determined by a 0 profit condition since profits are decreasing in the number of firms. We let $k \leq n$ denote the number of firms who purchase the license. Third, the firms compete a la Cournot knowing k.

Although the number k of licensees is by definition an integer, we study it as a real number in the remaining of the paper. However, we consider that k must be at least equal to 1 for the patent owners to make any profits and that $k \leq n$. By considering any $k \in [1, n]$ we skip the comparison of the closest upper and lower integer bounds of k, which simplifies the analysis.

2.1 Output, price and profits.

We search for a sub-game perfect Nash equilibrium and solve the game backwards. Let $L = \sum_{i=1,\dots,\chi} l_i$ and $R = \sum_{i=1,\dots,\chi} r_i$. The fixed fee L is a cost paid up-front. It affects the Cournot outcome by determining the number of licensees k who compete. Indeed, let $\pi(k, R)$ denote the Cournot profit when k firms compete and the royalties sum up to $R \geq 0$. Since $\dfrac{\partial \pi}{\partial k} < 0$ for any $L > 0$ there is a unique k^* such that

$$\pi(k^*, R) - L = 0. \tag{1}$$

In a free entry equilibrium k firms purchase the license where $k = min\{k^*, n\}$.

Each firm then solves

$$\max_{q_i} \left[a - q_i - \alpha Q_{-i} \right] q_i - (c + R) q_i,$$

$$\text{where } Q_{-i} = \sum_{\substack{j=1,\dots,k \\ j \neq i}} q_j.$$

Oberve that the parameter α denoting product differentiation confers a local market power to each firm. The unique symmetric equilibrium is such that

$$q(k, R) = \begin{cases} \left(\dfrac{a - c - R}{2 + \alpha(k - 1)} \right) & \text{if } R < a - c, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

The resulting symmetric price, provided there is production is such that

$$p(k, R) - (c + R) = q(k, R).$$

Observe that the output per firm and the margin per unit of output are increasing in the degree of product differentiation $1/\alpha$. Therefore the equilibrium profit is also increasing with product differentiation:

$$\pi(k, R) = \begin{cases} \left(\dfrac{a - c - R}{2 + \alpha(k - 1)} \right)^2 & \text{if } R < a - c, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

3 Licensing strategies at equilibrium

So far the separate licensing of complementary innovations have been studied only in the case of royalty-based licenses. We solve this case in this Section and extend the analysis to licenses based on fixed fee and on two-part tariff. We characterize the profit maximizing licensing contract for each of the three different regimes. To identify possible coordination failures, we analyze in each case how total licensing revenue varies with the number of patent owners.

3.1 Royalty regime

We consider first the usual royalty-based licensing contract with χ licensors. Let $(k^R, r_1^R, ..., r_\chi^R)$ denote the profit maximizing number of licensees and royalty under this regime. The profit of licensor i $(i = 1, ..., \chi)$ is $k.r_i.q(k, r_i, r_{-i})$, where r_i denote the royalty charged by licensor i, $r_{-i} = \sum_{j \neq i} r_j$, and $q(k, R)$ is given by (2). We can deduce from (2) and (3) that n manufacturers will enter if $R > a - c$, and none otherwise. Consequently licensor i maximizes the following expression:

$$\Pi^R(r_i, r_{-i}) = \begin{cases} r_i.n.\dfrac{a - c - r_i - r_{-i}}{2 + \alpha(n - 1)} & \text{if } r_i \leq a - c - r_{-i} \\ 0 & \text{otherwise.} \end{cases}$$

It can be checked easily that individual royalties charged by patent owners are strategic substitutes. Un-surprisingly, solving for the symmetric equilibrium shows that all n manufacturers enter the product market.

Proposition 1: *In a royalty regime, there is a unique symmetric equilibrium in which n manufacturers enter. The individual royalties and licensing profits are then given by:*

$$r^* = \frac{a - c}{1 + \chi}$$

$$\Pi^R = \frac{n(a - c)^2}{(2 + \alpha(n - 1))(1 + \chi)^2}.$$

Proof: See Appendix.

Observe that both individual royalties and profits are decreasing in the number χ of licensors. Lemma 2 below in turn displays the effect of χ on cumulative royalties $(R^* = \chi r^*)$ and licensing profits $(\chi \Pi^R)$.

Lemma 1: *In a royalty regime, the cumulative royalty paid by manufacturers (i.e. χr^*) is increasing in χ, while the cumulative profit of licensors is decreasing in χ.*
Proof: Obvious and thus omitted.

This result captures the double marginalization problem that arises when complementary patents are licensed separately (Shapiro, 2001). Each licensor charges a mark-up without taking into account that this reduces the demand addressed to other licensors. In the end, cumulative royalties are too high at equilibrium, and reducing them would increase the total licensing profits. In this context, a merger between patent owners or, which may be more realistic, the creation of a patent pool would entail more profits for patent owners.

3.2 Fixed fee regime

We now study the same problem when licences are based on a fixed fee. Let $\left(k^F, l_1^F, ..., l_\chi^F\right)$ denote the profit maximizing number of licensees and fixed fees under this regime. Under free entry the fixed fees permit full extraction of the firms' profit. Observe indeed that the cumulative fee $L = \sum_{i=1,...,\chi} l_i$ determines the number k^F of competing manufacturers at equilibrium. All n firms will enter if $L \leq \pi(n, 0)$. For any $L > \pi(n, 0)$ there is a unique $k^F < n$ such that (1) holds and we have $\pi\left(k^F, 0\right) = L$.

Proposition 2: *Fixed fee regime with χ licensors.*

1-For $\alpha \in \left[\frac{1}{\chi}, 1\right]$, there are a multiplicity of equilibria defined by:

$$
\begin{aligned}
k^F &= 1 \\
\sum_{i=1,...,\chi} l_i^* &= \pi(1, 0)
\end{aligned}
$$

2-For $\alpha \in \left]\frac{2}{(2\chi-1)n+1}, \frac{1}{\chi}\right[$, there is a unique symmetric equilibrium defined by:

$$
\begin{aligned}
k^F &= \left(\frac{2}{\alpha} - 1\right)(2\chi - 1)^{-1} \\
l_i^* &= \frac{\pi(k^F, 0)}{\chi}, \quad \forall i = 1, ..., \chi
\end{aligned}
$$

3-For $\alpha \leq \frac{2}{(2\chi-1)n+1}$, there are a multiplicity of equilibria defined by:

$$
\begin{aligned}
k^F &= n \\
\sum_{i=1,...,\chi} l_i^* &= \pi(n, 0)
\end{aligned}
$$

Proof: See appendix.

Before discussing these results, it is useful to consider the case in which there is only one patent owner. Consider thus the Corrolary below. Three types of licensing equilibria may take place with a unique patent owner, depending on the number of potential entrants and the degree of product differentiation. If $\alpha = 1$, products are homogenous. Then monopoly is clearly the profit maximizing market structure and the licensor allows only one entrant. As products become more differentiated $(2/(n+1) < \alpha < 1)$, the patent owners allows a limited number of entrants $k^F < n$. Here the licensors' incentive to allow entry is due to the additional licensing profits generated by product variety, and k^F is thus increasing in the degree of differentiation $1/\alpha$. The number of licensees results from a trade-off between product variety and price competition. Any increase of the fee indeed entails an increase in the profit per licensee, but also a decrease

in the number of entrants, and thus less variety. As product differentiation increase, the second effect becomes more important and more licensed are issued. Beyong a certain threshold of product differentiation ($\alpha \leq 2/(n+1)$), it is profitable to allow all candidates into the market.

Corrolary: *Fixed fee regime with a single licensor:*
1-For $\alpha = 1$, the licensor allows one entrant only,
2-For $\alpha \in \left] \frac{2}{n+1}, 1 \right[$, the licensor allows a limited number of entrants $k^F < n$,
where $k^F = \left(\frac{2}{\alpha} - 1 \right)$.
3-for $\alpha \leq \frac{2}{n+1}$, the licensor grants a license to all n candidates.

We can now turn again to Proposition 2, where $\chi \geq 1$. We can still observe the three equilibria identified with a unique licensor, but the number of patent owners now modifies the intervals over which the equilibria take place, and the number of entrants in the second equilibrium. Consider first the second equilibrium ($k^F < n$ entrants). We can see now that the number of entrants is decreasing in the number of licensors. This is due to an entry deterrance effect of the fixed fees, which stracking makes it more difficult for licensees to recover their entry costs. Since more licensors induce less entrants, it follows that equilibria 1 and 3 now take place for a wider (respectively narrower) range of product differentiation. In other terms entry is now restricted to one licensee only when products are weakly differentiated, and products need to be more differentiated for all n candidates to be allowed into the market.

Lemma 2: *Let $\chi\Pi^F = \sum_{i=1,\ldots,\chi} l_i^*$ denote the cumulative licensing profit in a fixed fee regime:*
1-For $\alpha \leq \frac{2}{(2\chi-1)n+1}$, and $\alpha \in \left[\frac{1}{\chi}, 1 \right]$, the cumulative licensing profit $\chi\Pi^F$ does not depend on χ.
2-For $\alpha \in \left[\frac{2}{(2\chi-1)n+1}, \frac{1}{\chi} \right]$, the cumulative licensing profit $\chi\Pi^F$ is decreasing in χ.
Proof: Obvious in cases 1 and 3. See appendix for case 2.

In this context, Lemma 2 states that the total licensing revenue is non increasing in the number of patent owners. More precisely, adding patent owners reduces total licensing revenue when it has an effective deterrence effect on entry (equilibrium 2). When differentiation is very strong so that n licenses are granted (equilibrium 3), increasing χ will not affect licensing revenues until the threshold $\alpha = \frac{2}{(2\chi-1)n+1}$ is reached. When equilibrium 1 emerges, there is only one licensee left and increasing the sum of fixed fees would kill the market. Hence patent owners adjust their fees and their total revenue remains constant.

3.3 Two-part tariff

We study, as a third step, the licensing equilibrium when patent owners can use both per unit royalties and fixed fees. Such two-part tariffs allow the licensors

to control the number of manufacturers through the fixed fee, and the product prices through the royalties. Proposition 4 below shows the resulting licensing equilibrium with χ patent owners.

Proposition 3: *The following fixed fee and royalty form the unique symmetric Nash equilibrium.*

1-*For* $\alpha \in \left]\frac{1}{\chi}, 1\right]$ *we have* $r^* = 0$ *and* $l^* = \dfrac{(a-c)^2}{4\chi}$ *and* $k(L^*, R^*) = 1$.

2-*For all* $\alpha \leq \frac{1}{\chi}$ *we have*

$$r^T = \frac{\alpha(n-1)\,(a-c)}{2 + \alpha(n-1)(\chi+1)} \text{ and } l^T = \frac{1}{\chi}\left(\frac{a-c}{2 + \alpha(n-1)(\chi+1)}\right)^2.$$

and

$$k\left(l^T, r^T\right) = n$$

Proof: See appendix.

As with the fixed fee license, it is useful to consider first the case of a unique patent owner (see Corrolary below). A single licensor will have two strategies depending on product differentiation. If products are homogenous ($\alpha = 1$), it will set a pure fixed fee license and allow one entrant only. Two-part tariff is then equivalent to fixed fee. When products are differentiated ($\alpha < 1$), the licensor will however allow all n entrants into the market. The two-part tariff indeed allows to derive the maximum benefit of product variety while avoiding rent dissipation through competition between licensees. To do so, the licensor uses the royalty to monitor competitive prices, and the fixed fee to extract all market profits. It can be checked easily that the per unit royalty is then decreasing in the degree of product differentiation (since differentiation increasingly mitigates price competition) while the fixed fee increases in parallel (because the market profit per licensee increases with differentiation).

Corrolary: *Two-part tariff with a single licensor:*
1-*For* $\alpha = 1$ *the licensor grant a pure fixed fee contract to a unique licensee,*
2-*If* $\alpha < 1$, *the licensor allows* n *entrants and sets the following license*

$$r^T = \frac{\alpha(n-1)\,(a-c)}{2 + 2\alpha(n-1)} \text{ and } l^T = \left(\frac{a-c}{2 + 2\alpha(n-1)}\right)^2.$$

We can now go back to Proposition with $\chi > 1$ patent owners. Although we still have the two types of equilibria, observe they are now affected by the number of licensors. Considering first the equilibrium n licensees (differentiated products), we can check easily that total royalties are increasing in χ for a given degree of differentiation α, which clearly denotes a double marginalization issue. By contrast, the total fixed fee is decreasing in χ, which reflects the fact that the royalty stacking problem reduces the total market profit generated by

the licensees. We can finally see that the number of licensors also affects the ranges of parameters for which the two equilibria take place. *Ceteris paribus*, an increase of the number of patent owners will reduce the range of parameters for which n licensees are allowed in the market, and increase beyond $\alpha = 1$ the levels of diffenrentiation for which only one firm is sold a license.

Lemma 3: *The cumulative licensing profit with two-part tariff is:*

$$\chi\Pi^T = \frac{n(a-c)^2\left(1 + \alpha\chi(n-1)\right)}{\left[2 + \alpha(n-1)(\chi+1)\right]^2} \text{ for } \alpha \leq \frac{1}{\chi}$$

and

$$\chi\Pi^T = \frac{(a-c)^2}{4} \text{ for } \alpha > \frac{1}{\chi}.$$

It is non-increasing in χ.
Proof: See appendix.

It is therefore not surprising that, as in the other licensing regimes, the cumulative profits of the patent owners is decreasing in their number when the equilibrium allows n licensees. Beyond a certain number of licensors $\chi = 1/\alpha$, the equilibrium changes and only one entrant is sold a fixed fee license. Any increase in royalties and/or fixed fee would then kill the market and the total licensing revenue becomes thus unaffected by χ.

4 Comparing the different regimes

Having characterized the equilibria and coordination issues with each licensing regimes, we now compare these regimes with respect to total licensing revenues of patent owners. For clarity of exposition we firstly consider the case of a unique patent owner, and then extend the analysis to multiple patent owners.

4.1 Comparing profits under monopolistic license ownership.

Assume that $\chi = 1$, which may mean that there is a unique patent owner or else that all patent owners form a patent pool. Before considering two-part tariffs, it is interesting to compare as a first step the fixed fee and royalty regimes. We establish in Proposition 5 each regime may maximize total licensing revenue depending on

Proposition 5: *The royalty regime leads to higher profits than the fixed fee regime provided $n \geq 6$ and $\alpha \in [\underline{\alpha}, \overline{\alpha}]$, where*

$$\underline{\alpha} = \frac{n+1-\sqrt{n^2+1-6n}}{2n} \text{ and } \overline{\alpha} = \frac{n+1+\sqrt{n^2+1-6n}}{2n}$$

For any other combinations of α and n the fixed fee regime dominates the royalty regime.

Proof: See Appendix.

When the standard either supports sufficiently differentiated products (α low) or appeals to very few users (n low), a fixed fee license granted to all is superior to a royalty-based license because it allows to extract all of the close to monopoly profits from each licensee. Aside from these two situations, the fixed fee regime does not systematically maximize profits. For any given k sold licenses the revenue from either regime decreases as product become more homogeneous ($\alpha \to 1$). Yet, because the revenue from the royalty is proportional to quantity only, it does not suffer as much from an increase in α as the fixed fee revenue which depends on both, price and quantity. Under fixed fee the licensor can nonetheless balance the losses from an increased α via direct control over competition. Notice in particular that as $\alpha \to 1$, we find that the fixed fee is superior because the licensor limits entry to only one firm. This corresponds to the findings in Kamien and Tauman (1986). However, this ability to limit entry is not always sufficient for the fixed fee to dominate. As it appears, the royalty prevails for some range of product differentiation.

Lemma 4: *The two part tariff regime yields a higher profit than fixed fee and royalty. (Proof obvious and thus omitted.)*

The above result is obvious since a monopolistic owner can always replicate the outcome of both the royalty and fixed fee regimes using a two-part tariff. We now restrict attention to the royalty and fixed fee regimes. Let us define the following variables:

Proposition 5: *The royalty regime leads to higher profits than the fixed fee regime provided $n \geq 6$ and $\alpha \in [\underline{\alpha}, \overline{\alpha}]$, where*

$$\underline{\alpha} = \frac{n + 1 - \sqrt{n^2 + 1 - 6n}}{2n} \, and \, \overline{\alpha} = \frac{n + 1 + \sqrt{n^2 + 1 - 6n}}{2n}$$

For any other combinations of α and n the fixed fee regime dominates the royalty regime.

Proof: See Appendix.

The fact that two-part tariff dominates is obvious. Indeed the other two regimes are similar to a restricted two-part tariff regime. We now explain the second result.

4.2 Comparing cumulative profits when $\chi \geq 2$.

Proposition 6:

1-For any $\alpha \in \left[0, \frac{2}{(2\chi-1)n+1}\right]$, we have $\chi\Pi^T > \chi\Pi^F > \chi\Pi^R$.

2-For any $\alpha \in \left]\frac{2}{(2\chi-1)n+1}, \frac{1}{\chi}\right]$ the two-part tariff dominates both the fixed-fee and royalty regimes. Moreover, there exists a unique $n_1(\alpha, \chi)$ such that $\chi\Pi^R > \chi\Pi^F$ if and only if $n > n_1(\alpha, \chi)$.

3-Consider any $\alpha \in \left]\frac{1}{\chi}, 1\right]$. Over that interval we have $\chi\Pi^T = \chi\Pi^F$. Moreover, we can establish the following:

(i) for $\alpha \in \left]\frac{1}{\chi}, \frac{4\chi}{(1+\chi)^2}\right[$ there exists a unique $n_2(\alpha, \chi)$ such that $\chi\Pi^R > \chi\Pi^s$ (with $s = F, T$) if and only if $n > n_2(\alpha, \chi)$.

(ii) for $\alpha \in \left[\frac{4\chi}{(1+\chi)^2}, 1\right]$ we have $\chi\Pi^R < \chi\Pi^s$ (with $s = F, T$).

Proof: See Appendix.

The above result highlights the fact that the royalty regime dominates the other two regimes only when there are a sufficiently large number of downstream firms and $\alpha \in \left[\frac{1}{\chi}, \frac{4\chi}{(1+\chi)^2}\right[$. In any other cases, the two-part tariff is a better option. The table below depicts, in greater details, the cases for which a royalty regime would dominate.

		$n_2(\frac{1}{\chi}, \chi)$	$n_2(\frac{4\chi}{(1+\chi)^2} - \varepsilon, \chi)$	$n_2(\frac{1}{2}(\frac{1}{\chi} + \frac{4\chi}{(1+\chi)^2}), \chi)$
$\chi = 2$	$\alpha \in [0.50, 0.88[$	4	11112	7
$\chi = 3$	$\alpha \in [0.33, 0.75[$	4	12501	7
$\chi = 4$	$\alpha \in [0.25, 0.64[$	5	13601	8
$\chi = 5$	$\alpha \in [0.20, 0.38[$	6	14445	9

Finally, for a given value of n, we have conducted some simulations comparing the joint revenue from fixed fee and the royalty regimes for different values of α. In the tables below we write which regime maximizes the patent owners profit. R stands for royalty and F for fixed fee.

As before, for a given χ, the range for which royalty is favoured is wider as n increases. For a given n we can see that royalty becomes less popular as the number of patent owners increases.

5 Total surplus

5.1 Comparing product prices and variety

... highlights a trade-off for consumers between prices and variety...

Proposition 4: *Prices and variety on the product market are ranked such that*

$$\begin{aligned} p^F &\leq p^T < p^R \\ k^F &\leq k^T \leq k^R \end{aligned}$$

Proof: See Appendix.

Proposition 4 would be straightforward under an equal numbers of licensees. Indeed royalties increase the cost and thus the price. Yet proposition 4 extends this intuition to the seemingly more ambiguous case where the number of competitors is restricted below n due to a positive fixed fee. We have here an interesting application of the Cournot (1836) double marginalization theorem to an industry where the monopoly is replaced by competitors selling differentiated items.

Following Vives (1999) we know that the total surplus in an economy with differentiated item with linear cost and linear demand can be written as

$$TS^s = U\left(q_1^s, ..., q_{k^s}^s\right) - \sum_{i=1,...k^\delta} cq_i^s$$

where δ stands for the regime under consideration ($s = F, R, T$) and $U(.)$ is a quadratic utility function from a representative consumer:

$$U\left(q_1, ..., q_{k^s}\right) = a \sum_{i=1,...,k^s} q_i^s - \frac{1}{2} \sum_{i=1,...k} \left[(q_i^s)^2 + \alpha q_i^s \left(\sum_{\substack{j=1,...k^s \\ j \neq i}} q_j^s \right) \right].$$

Given that output is symmetric we can rewrite the utility as:

$$U(q, k) = akq - \frac{1}{2}kq^2 \left[1 + \alpha(k-1)\right]$$

The comparison of the total surplus is complex. The question we intend to answer is which regime maximizes total surplus. Lemma 5 and 6 give the most general result we can establish.

Lemma 5 *For any $\chi \geq 1$ we can establish that*
-For any $\alpha \in \left[0, \frac{2}{(2\chi-1)n+1}\right]$, the fixed fee regime maximizes the total surplus:

$$TS^F \geq TS^T > TS^R \text{ with } TS^F = TS^T \text{ at } \alpha = 0 \text{ only.}$$

-For any $\alpha \in \left[0, \frac{1}{\chi}\right]$ the two-part tariff regime generates a higher total surplus than the royalty regime.
Proof: See Appendix.

To be able to analyze the question further we consider the specific cases where $\chi = 1$ and $\chi = 2$.

Lemma 6: Comparing TS^F and TS^T for the specific cases $\chi = 1, 2$.
(i) Let $\chi = 1$.
   -If $n \leq 3$ then we have $TS^F \geq TS^T$ for all $\alpha \in [0, 1]$ with the equality holding only at 0 and 1.
   -If $n \geq 4$ then $TS^F = TS^T$ at $\alpha = 0, \frac{5}{n+2}$ and 1 and we have

$$TS^F > TS^T \text{ for } \alpha \in \left]0, \frac{5}{n+2}\right[\text{ and } TS^F < TS^T \text{ for } \alpha \in \left]\frac{5}{n+2}, 1\right[.$$

(ii) Let $\chi = 2$.
For all $n \geq 2$ there exists a unique $\alpha_1 \in \left[\frac{2}{3n+1}, \frac{1}{2}\right]$ such that

$$TS^T \geq TS^F \Leftrightarrow \alpha \in \left[\alpha_1, \frac{1}{2}\right].$$

We finally use simulations considering different values of n to compare TS^F and TS^R for the specific cases $\chi = 1, 2$.
(i) Let $\chi = 1$.
   -If $n \leq 16$ then we have $TS^F \geq TS^R$ for all $\alpha \in [0, 1]$.
   -If $n \geq 17$ there exists a non-empty interval $\Phi_1 \subset \left[\frac{2}{n+1}, 1\right]$, which widens with n, such that $TS^R \geq TS^F \Leftrightarrow \alpha \in \Phi_1$.
(ii) Let $X = 2$
   -If $n \leq 5$ then we have $TS^F \geq TS^R$ for all $\alpha \in [0, 1]$.
   -If $n \geq 6$ there exists a non-empty interval $\Phi_2 \subset \left[\frac{2}{3n+1}, 1\right]$, which widens with n, such that $TS^R \geq TS^F \Leftrightarrow \alpha \in \Phi_2$.

We therefore reach the following answers to our question for the cases $\chi = 1$ and $\chi = 2$:
-For $\chi = 1$ and $n \leq 4$ the fixed fee regime maximizes the total surplus. For any $n \geq 5$ then the total surplus is maximized using the fixed fee regime for $\alpha \in \left]0, \frac{5}{n+2}\right[$ and the two-part tariff for $\alpha \in \left[\frac{5}{n+2}, 1\right]$. The royalty regime should never be selected. The fixed fee regime maximizes total welfare over the white area.
-For $\chi = 2$, there exists a unique $\alpha_1 \in \left[\frac{2}{3n+1}, \frac{1}{2}\right]$ such that the fixed fee regime maximizes the total surplus for any $\alpha \in [0, \alpha_1]$.[4] The optimal regime for $\alpha \in \left[\alpha_1, \frac{1}{2}\right]$ is the two-part tariff. Regarding the remaining interval, $\alpha \in \left[\frac{1}{2}, 1\right]$, it is best to opt for the royalty regime when $n \geq 6$ and $\alpha \in \left[\frac{1}{2}, \alpha(n)\right]$, in any other cases the fixed fee and/or two-part tariff regimes maximize the total surplus. The graph below shows which regime is optimal for all combination of n and α in the case $\chi = 2$.

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[4] For the particular case $n = 1$, the fixed fee regime is optimal for all $\alpha \in [0, 1]$.

$\chi = 2$
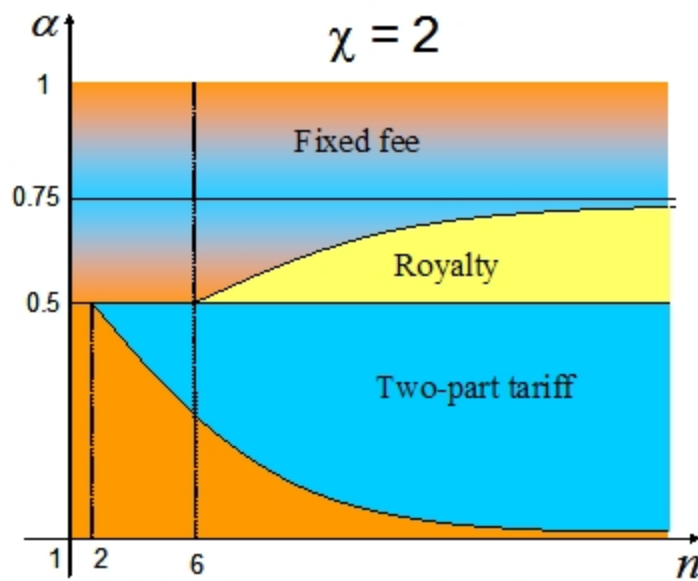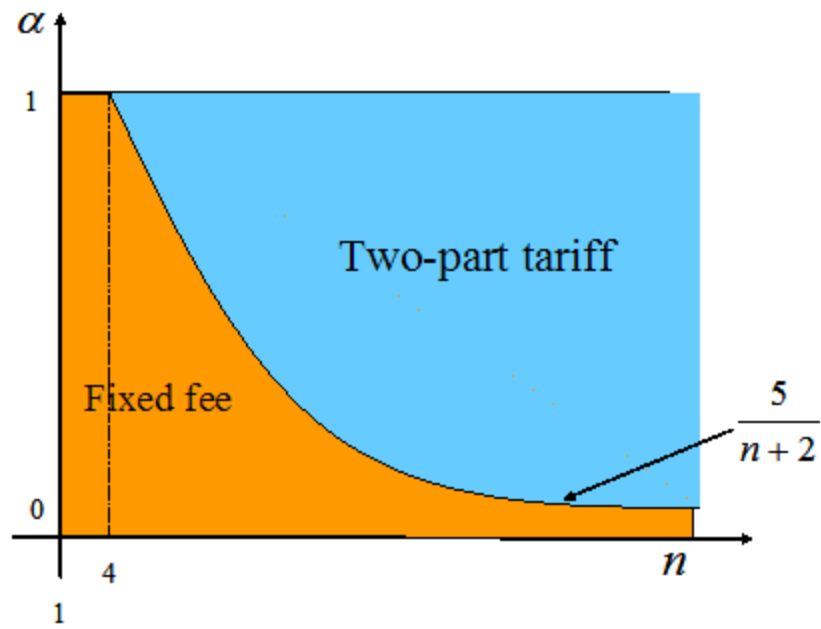
Introducing fixed fees becomes more beneficial to consumers as the number of patent owners increases.

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6 Conclusion

In this paper we compare three licensing agreements that independent patent owners can use to sell their licenses. We establish that, in many instances, the two-part tariff maximizes the joint profit as it allows to control both, entry and marginal costs. The comparison of the fixed-fee and the royalty regimes show that, as the number of downstream competitors increase, the royalty regime performs better than the fixed fee regime over a non-empty, widening, interval for the degree of product differentiation. The benefit of a royalty regime lies in the fact that it is less sensitive to a decrease in product differentiation. Thus as product become more homogeneous, the revenue to the patent owners decreases. It decreases less under royalty regime than under the fixed-fee regime. However, because the fixed-fee regime allows to control for the number of downstream producers, this regime maxmizes joint profits for the case of almost homogeneous products.

We also establish that setting a patent pool is optimal. Not only does it maximize social welfare, it also maximizes joint profits. A decentralized decision of the royalty and/or fixed fee leads to a coordination failure which either means that firms pay excessive royalties or else that entry is restricted below what would be optimal.

Appendix

Proof of lemma 1.
We solve
$$max_k k\pi(k, 0)$$

The first order condition leads to
$$\frac{(a-c)^2 [2 - \alpha - \alpha k]}{(2 + \alpha(k-1))^3} = 0 \Leftrightarrow k = min\left\{n, \frac{2-\alpha}{\alpha}\right\}.$$

It is obvious to check that the overall profit is concave.

Proof of lemma 3
The only difficulty lies in establishing that $k^T = n$. The licensor solves
$$\max_{r,k} \Pi^T(s, k)$$

where
$$\Pi^T(r, k) = kq(k, r)[q(k, r) + r].$$

Since $\dfrac{\partial q}{\partial r} = -\dfrac{1}{2 + \alpha(k-1)}$, the first order condition with respect to r leads to
$$r^T = \alpha(k-1)q(k, r^T).$$

We then have
$$\frac{\partial \Pi^T}{\partial k} = q(k, r)(q(k, r) + r) + k\frac{\partial q}{\partial k}(2q(k, r) + r)$$

Since $\dfrac{\partial q}{\partial k} = -\dfrac{\alpha}{2 + \alpha(k-1)} q(k, r)$, we have
$$\frac{\partial \Pi^T}{\partial k} = q(k, r)\left[q(k, r) + r - \frac{\alpha k}{2 + \alpha(k-1)}(2q(k, r) + r)\right].$$

Evaluated at r^T we have
$$\frac{\partial \Pi^T}{\partial k} = q^2(k, r^T)(1 - \alpha) > 0 \Rightarrow k^T = n.$$

Proof of Proposition 1.
It is obvious to show that $\Pi^T > \Pi^R$. Comparing Π^T to Π^F leads to the following
$$\Pi^T > \Pi^F \Leftrightarrow \begin{cases} \alpha^2(n-1)^2 > 0 \text{ when } \alpha \leq \frac{2}{n+1}, \\ \alpha n > 1 \text{ when } \alpha > \frac{2}{n+1}. \end{cases}$$

Both conditions always hold for the ranges over which α is defined.

We now compare fixed fee to royalty. When $\alpha \leq \dfrac{2}{n+1}$, we have $k^* = n$ and for all such cases

$$\Pi^F > \Pi^R \Leftrightarrow \alpha < \frac{2}{n-1},$$

which is systematically true.

When $\alpha > \dfrac{2}{n+1}$, we have $k^* = \dfrac{2-\alpha}{\alpha}$ and for all such cases

$$\Pi^F > \Pi^R \Leftrightarrow n < \frac{2-\alpha}{\alpha(1-\alpha)}.$$

Proof of Proposition 4:

Formally the price cost margins are given by the following expressions:

$$p^F - c = \frac{a-c}{2 + \alpha\left(k^F - 1\right)},$$

$$p^R - c = \left(\frac{a-c}{1+\chi}\right)\frac{1 + 2\chi + \alpha\chi(n-1)}{2 + \alpha(n-1)},$$

and

$$p^T - c = \begin{cases} \frac{(a-c)(1+\alpha\chi(n-1))}{2+\alpha(n-1)(1+\chi)} & \text{if } \alpha \leq \frac{1}{\chi} \\ p^F - c & \text{otherwise} \end{cases}$$

Proposition 4 follows directly from the comparison of these expressions.

Proof of Proposition 6:

Point 1 is obvious. Regarding point 2: showing that two-part tariff yields a greater income than the other 2 regimes is obvious. As for the comparison between the fixed fee and the royalty regimes, we have:

$$\Pi^F \geq \Pi^R$$

if and only if

$$(2\chi - 1)\left(1 + \chi\right)\left(2 + \alpha(n-1)\right) > 4\alpha\chi^3(2-\alpha)n.$$

Representing both the left and right hand side on a graph with n on the horizontal axis leads to the following:

Insert figure 3 here.

For any $\alpha \in \left]\frac{2}{(2\chi-1)n+1}, \frac{1}{\chi}\right]$, we have

$$4\alpha(2-\alpha)\chi^3 < 2(2\chi - 1)\left(1+\chi\right)^2.$$

Moreover, for any $\alpha \in \left]\frac{2}{(2\chi-1)n+1}, \frac{1}{\chi}\right]$,

$$\alpha\left(2\chi - 1\right)\left(1 + \chi\right) < 4\alpha(2-\alpha)\chi^3.$$

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Thus the two curves cross at most once for n sufficiently large.

Finally, we turn to point 3. We have

$$\Pi^F \geq \Pi^R$$

if and only if

$$(2 + \alpha(n-1))(1+\chi)^2 > 4\chi n.$$

Representing both the left and right hand side on a graph with n on the horizontal axis leads to the following:

Insert figure 4 here.

We have

$$2(1+\chi)^2 > 4\chi.$$

Moreover, the slopes of the left and right hand side functions of n are equal if and only if

$$\alpha = \frac{4\chi}{(1+\chi)^2}, \text{ which belongs to } \left]\frac{1}{\chi}, 1\right].$$

Provided $\alpha > \frac{4\chi}{(1+\chi)^2}$ the two curves never cross. QED.

Proof of Lemma 5

Given our results the total surplus for each regime can be expressed as follows. Under the fixed fee regime we have:

$$TS^F = \frac{k^F \left(3 + \alpha\left(k^F - 1\right)\right)}{2\left(2 + \alpha\left(k^F - 1\right)\right)^2}(a-c)^2,$$

where

$$k^F = \begin{cases} n \text{ if } \alpha \in \left[0, \frac{2}{(2\chi-1)n+1}\right], \\ \frac{2-\alpha}{(2\chi-1)\alpha} \text{ if } \alpha \in \left]\frac{2}{(2\chi-1)n+1}, \frac{1}{\chi}\right[, \\ 1 \text{ if } \alpha \in \left[\frac{1}{\chi}, 1\right]. \end{cases}$$

Under the royalty regime the total surplus is given by:

$$TS^R = \frac{n\left[3 + 4\chi + \alpha(n-1)(1+2\chi)\right]}{2(1+\chi)^2\left[2 + \alpha(n-1)\right]^2}(a-c)^2.$$

Finally, for the two-part tariff we have

$$TS^T = \frac{n\left[3 + \alpha(n-1)(1+2\chi)\right]}{2\left(2 + \alpha(n-1)(1+\chi)\right)^2}(a-c)^2 \text{ if } \alpha \in \left[0, \frac{1}{\chi}\right]$$

and $TS^T = TS^F$ for $\alpha \in \left]\frac{1}{\chi}, 1\right]$.

Proof of Lemma 6

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(i) We have $TS^F \geq TS^T$ for all $\alpha \in \left[0, \frac{2}{n+1}\right]$ with the equality holding only at 0. For $\alpha \in \left[\frac{2}{n+1}, 1\right]$ we have

$$TS^F = TS^T \Leftrightarrow (1 - \alpha)(5 - \alpha(n+2)) = 0.$$

Thus, overall, $TS^F = TS^T$ at $\alpha = 0, \frac{5}{n+2}$ and 1. Two possibilities arise depending on whether we have $\frac{5}{n+2} < 1$ (which is equivalent to $n > 3$)

(ii) The variable α_1 solves

$$4\alpha^3(n-1)(11n+9) + 4\alpha(9n-37) - \alpha^2(61n^2 + 38n - 147) + 44 = 0.$$

This third degree equation admits 3 solutions. Let

$$f(\alpha) = 4\alpha^3(n-1)(11n+9) + 4\alpha(9n-37) - \alpha^2(61n^2 + 38n - 147) + 44$$

It can be shown that $f(-1) < 0$, $f(0) > 0$, $f(1) < 0$ and $f(2) > 0$ thus two of the three solutions to $f(\alpha) = 0$ lie outside the interval $[0,1]$. A unique solution lies within $\left[\frac{2}{3n+1}, \frac{1}{2}\right]$ since we proved in the lemma 5 that $TS^F > TS^T$ at $\alpha = \frac{2}{3n+1}$ while $TS^F < TS^T$ at $\alpha = \frac{1}{2}$.

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