Damping estimation of joined plate structures using static contact analysis

Takao HIRAI*, Fumiyasu KURATANI*, Kazushi KOIDE* and Ichiro KIDO**
*Department of Mechanical Engineering, University of Fukui
3-9-1 Bunkyo, Fukui-shi, Fukui 910-8507, Japan
E-mail: takao.hirai0314@gmail.com
**CAE Technology Div.3, TOYOTA TECHNICAL DEVELOPMENT CO., LTD.
NORE Sakuradori Bldg., 2-28-23 Izumi, Higashi-ku, Nagoya-shi, Aichi 461-0001, Japan

Abstract
Mechanical joints such as bolted and spot welded joints are widely used to join plate structures. The existence of contact interface of the joint increases the vibration damping in the structure because the energy dissipation due to friction occurs at the interface. This paper proposes a procedure for calculating the dissipated energy at the interface using finite element static contact analysis. The structure considered consists of three steel plates bolted together where a long plate is sandwiched between two short plates. We use the mode shapes of linear FE model of the joined structure as a distributed displacement loading in the static contact analysis to estimate the damping for each mode. To accurately calculate the dissipated energy, we apply a surface load on the overlap region of the joint along with the distributed displacement and find the necessary value of surface load. To validate the proposed procedure, the modal damping ratios estimated from the dissipated energy with the necessary surface load are compared with the measured damping ratios. The results show when the mode shapes obtained from the linear FE model approximate the actual deformation, the proposed procedure enables to accurately calculate the dissipated energy of the joined plate structure while the surface load on the overlap region is appropriately set. The necessary surface load increases with increasing mode number and varies almost proportionally to the square of the angular frequency.

Keywords: Modal Analysis, Finite Element Method, Damping, Mode Shape, Contact Analysis, Energy Dissipation, Surface Load

1. Introduction

The dynamic behavior of built up structures is affected by the characteristics of mechanical joints such as bolted, riveted and spot welded joints. In particular, the existence of contact interface of the joint increases the vibration damping in the structure because energy dissipation due to friction occurs at the interface.

Friction damping in the joint has been the subject of many studies. Goodman et al. considered a cantilever beam consisting of two identical beams pressed together and proposed a formula for the energy dissipation when a beam was loaded with a shear force at the free end (Goodman and Klumpp, 1956). Bournine et al. presented an analysis of the damping behavior of two identical beams bolted together and of the energy dissipation due to friction (Bournine et al., 2011). Metherell et al. investigated the damping behavior of a lap shear joint and derived an expression for the energy dissipation when it was subjected to an axial load (Metherell and Diller, 1968). Chen et al. investigated the effect of friction on the damping response of interfaces using finite element (FE) method and compared the results with those studied by Goodman et al. and Metherell et al. (Chen and Deng, 2005). Asadi et al. and Dovstam et al. demonstrated a technique for the simulation of friction damping using linear FE modelling (Asadi et al., 2012, Dovstam et al., 2012). Almost all of the studies dealing with bending deformation focused on only the fundamental mode, i.e., the first bending mode shape for the cantilever structure. Abbadi et al. presented a nonlinear model for the energy dissipation due to friction in a single welding spot joint (Abbadi et al., 2004). In this study, the time history of the dissipated energy and the velocity response were calculated using nonlinear transient analysis (dynamic contact analysis) and the modal
damping ratios for the higher modes were identified from the simulation data. However, the effect of mode shape on the damping characteristics was not mentioned. In addition, when the plates are joined together with multiple bolts or spot welds, the dynamic contact analysis is difficult to apply because the complex contact situations between the metals in the joint often cause severe convergence difficulties.

In the previous paper, we presented the procedure for calculating the dissipated energy due to friction at the interface (Kuratani et al., 2013) for the joined structure with partial overlap region. The static contact analysis using a nonlinear FE model was used instead of the dynamic contact analysis. This reduces the analysis time and the difficulty of the contact analysis. The dissipated energy was calculated and the modal damping ratios were estimated for some modes. The estimated damping ratios did not show good agreement with the measured ratios. The major reason for this was to ignore the clamping pressure of the bolts.

In this paper, we propose a procedure for calculating the dissipated energy at the interface of the joint of a plate structure with partial overlap region. The structure considered in this paper consists of three steel plates bolted together where a long plate is sandwiched between two short plates. The basic concept is the same as in the previous paper. The energy dissipation due to friction at the interface is calculated using the static contact analysis and the modal damping ratios are estimated. The distributed displacement loading is adopted instead of force loading that is usually used in many studies and as a distributed displacement, the scaled mode shapes obtained from a linear FE model are adopted. This enables to obtain the energy dissipation depending on each mode shape for the fundamental and higher modes. Especially, a surface load on the overlap region between the plates, which was not considered in previous paper, is applied. We examine the effect of surface load on the energy dissipation and find the necessary value of surface load for estimating the measured damping ratio for each mode. To validate the proposed procedure for calculating the dissipated energy, the modal damping ratios estimated from the dissipated energy are compared with the ratios measured in the modal testing.

2. Friction damping due to bending deformation

In this paper, we consider a structure consisting of three plates with partial overlap region where a long plate is sandwiched between two short plates as shown in Fig. 1. When the plates deflect, a relative slip at the interface occurs. This produces the energy dissipation due to friction, providing damping in a built up structure. For the structure considered, the three plates do not have the same deflection, i.e., they have different radii of curvature. This causes the plates to contact each other locally along the interface and a slip will occur at a portion of the interface. In this case, it is necessary to calculate the dissipated energy due to friction using finite element analysis because of the inapplicability of analytical methods. By calculating the energy dissipation at the interface depending on each mode shape, we can estimate the modal damping ratios and evaluate the damping characteristics for each mode.

![Fig. 1 Joined plate structure with partial overlap region. This shows a typical deformation shape of the partial overlap region.](image)

3. Calculation procedure for energy dissipation

In this section, we propose a simplified method for calculating dissipated energy for each mode. The basic concept of the procedure is the same as in the previous paper (Kuratani et al., 2013). Static contact analysis is carried out to calculate the dissipated energy where distributed displacement loading is used instead of force loading. As a distributed displacement for each mode, the scaled mode shapes obtained by the vibration analysis are used. This enables to obtain the energy dissipation depending on each mode shape.
3.1 Flow chart of calculation procedure for energy dissipation

Figure 2 shows a flow chart of the calculation procedure for the dissipated energy due to friction. First, a linear FE model is built and the mode shapes are obtained by the vibration analysis. Second, contact elements are defined on the overlap region between the long plate and the two short plates, changing the linear FE model to a nonlinear FE model. Third, the static contact analysis is carried out using the modified FE model. In the contact analysis, a surface load corresponding to the clamping pressure at the interface is applied on the overlap region. Then, the distributed displacement is applied to only the long plate, which is the scaled mode shape different for each mode. The distributed displacement is changed from zero deflection to the maximum deflection while the constant surface load continues to be applied. The analysis results such as the contact pressure, the contact area and the slip displacement are obtained. Finally, the dissipated energy due to friction is calculated from the contact analysis results.

Fig. 2  Flow chart of the procedure for calculating dissipated energy. First, a linear FE model is built and the mode shapes are obtained. Second, contact elements are defined on the overlap region. Third, the surface load and the distributed displacement are applied in the static contact analysis and the contact results such as the contact pressure, contact area and slip displacement are obtained. As the distributed displacement, the scaled mode shape is used for each mode. Finally, the dissipated energy due to friction is calculated from the contact analysis results.

3.2 Dissipated energy due to friction

The dissipated energy due to friction is calculated from the contact pressure, the contact area and the slip displacement obtained in the static contact analysis at each step when the distributed displacement is changed step by step, that is

$$
E_r^d = \sum_{i=1}^{N} d_{ij} \mu A_{ij} r_{s_{ij}}
$$

where $E_r^d$ is the dissipated energy at $j$th step for $r$th mode, $\mu$ is the friction coefficient. The slip displacement at each step $s_{ij}^{(j)}$ is calculated as the relative displacement between two analysis steps. The total dissipated energy for the half cycle $E_r^d$ is obtained from the sum of the dissipated energy at each step $E_r^d$ ($j=1, \ldots, 200$).

3.3 Modal damping ratio

The modal damping ratio is approximately calculated from the vibration energy and the dissipated energy by (The Japan Society of Mechanical Engineers ed., 1998)

$$
\xi_r \approx \frac{1}{4\pi} \left( \frac{2E_r^d}{E_v^r} \right)
$$

(2)
where $\zeta_r$ is the modal damping ratio of $r$th mode, $E_r^d$ is the total dissipated energy during the half cycle calculated in section 3.2, $E_r^v$ is the vibration energy which is calculated from the modal kinetic energy $\bar{E}_r^v$ obtained by the vibration analysis, that is

$$E_r^v = R_r^2 \bar{E}_r^v$$

(3)

The modal kinetic energy $\bar{E}_r^v$ is a relative value and defined by

$$\bar{E}_r^v = \frac{1}{2} \omega_r^2 \{\phi_r\}^T [M] \{\phi_r\}$$

(4)

where $[M]$ is the mass matrix of full model, $\omega_r$ and $\{\phi_r\}$ are the natural angular frequency and the mode shape for $r$th mode. $R_r$ is the ratio of the value of distributed displacement at the node of the FE model corresponding to the measurement point in the modal testing and the value of mode shape at the measurement point.

4. Measuring damping ratio

4.1 Test structure

Figure 3(a) shows the plate structure considered in this paper where a long plate was sandwiched between two short plates. The long plate has the dimensions of 480×80×3 mm and the two short plates have the dimensions of 80×80×2 mm. The long plate and short plates are made of SS400 steel. The three plates have 8 holes (5.5 mm diameter) while the four holes in the bolt pattern shown in Fig. 3(b) were used to bolt the plates together with M5 bolts and nuts with a tightening torque of 3 Nm. This bolt pattern had the largest damping ratio among the three bolt patterns in the previous paper, which are the center joint (Fig. 3(b)), the edge joint and the all joint. To compare the effect of the short plate length on the damping characteristics, the structure with the 60 mm short plate length (60×80×2 mm) was also tested. Two sets of test structures were prepared for each short plate lengths.

The friction coefficient used in the static contact analysis (section 5.2) was measured by the tribo-test machine (Tribostation Type 32, Shinto Sci. Co., Ltd.). Since the mean of measured data was 0.248 and the standard deviation was 0.036, we used the value of 0.25 in the contact analysis.

![Fig. 3 Test structure where a long plate is sandwiched between two short plates.](image)

(a) Whole view

(b) Bolt pattern

4.2 Experimental modal damping ratio

Figure 4 shows the modal testing setup. The test structure was suspended using strings at the nodal lines (zero magnitude) of mode shape to realize the free-free boundary condition. The impact testing was carried out to measure the frequency response functions (FRFs). To extract only the bending mode (in this paper, only the bending modes are considered), the impact point was located at the middle of the edge of the test structure and the accelerometers were placed at the middle of each edge (One of the accelerometers was placed on the other side of the impact point). The excitation force and the response acceleration were measured and the FRFs for the two measurement points were
calculated. From the two measured FRFs, the natural frequencies and the modal damping ratios were extracted using the SDOF curve-fitting algorithm.

Figure 5 shows the mean and the standard deviation of the modal damping ratios extracted from the measured FRFs. Although the damping ratios for the 80 mm short plate length are larger than those for the 60 mm short plate length, both the damping ratios have the same tendency, i.e., the 1st, 3rd and 5th modes are larger than the 2nd and 4th modes. As seen from Fig. 7, the mode shapes of the 1st and 3rd modes (and 5th mode) are symmetric while the mode shapes of the 2nd and 4th modes are antisymmetric. As mentioned in the previous paper, the difference between the mode shapes strongly affects the damping characteristics. For all the modes, the variation in the damping ratio is small.

In the figure, the damping ratios measured from only the long plate without the short plates are shown. The value obtained by subtracting the value for the long plate from the value for the joined structure is used to estimate the necessary surface load because the dissipated energy $E_d$ in Eq. (2) is assumed to be due to friction and other damping sources such as material damping of the plate are ignored.

Bournine et al. investigated the effect of bolt clamping torque on the damping characteristics and indicated the clamping torque that maximizes the damping. We also measured the damping ratio at tightening torques of 2.0, 3.0 and 4.0 Nm to examine the effect of the deviation from the standard bolt tightening torque. The results indicated very little variation in the damping ratio. The reason is probably that for the structure and the bolt pattern considered here, the increase in the tightening torque does not lead to the increase in the clamping pressure on the overlap region and as a result, the stick phase of a stick slip does not occur. Consequently, there is very little effect of bolt tightening torque on the damping.
5. Vibration analysis and static contact analysis

Vibration analysis and static contact analysis are carried out using the finite element program ANSYS.

5.1 Vibration analysis

Figure 6 shows a linear finite element (FE) model for the 80 mm short plate length for vibration analysis. The three steel plates were modeled with solid-shell elements (Solsh 190), the bolt heads and nuts were modeled with solid elements (Solid 185), and the bolt shafts were modeled with beam elements (Beam 188, 5 mm diameter). In the FE model, the coincident nodes on the interface between the bolt head/nut and the short plate are merged (share degrees of freedom). The coincident nodes on the interface between the short plate and long plate at the periphery of the bolt shafts are coupled (have the same displacement) while the shaft and the long plate have no common nodes. The material properties for the plates, the bolts and nuts are Young’s $E=206$ GPa, Poisson’s ratio $\nu=0.3$ and mass density $\rho=7800$ kg/m$^3$.

Figure 7 shows the mode shapes for the first four bending modes for the 80 mm short plate length. The 1st and 3rd modes are symmetric about the $y$-$z$ plane shown in Fig. 7(a) while the 2nd and 4th modes are antisymmetric. Fig. 8 shows the enlarged view of join part for the 3rd and 2nd mode shapes in Fig. 7, respectively. For the 3rd mode, the deflection of the long plate is large and the deflection of the short plate is small. This is the same as for the 1st mode. On the other hand, for the 2nd mode, the long plate and short plates do not move away from each other and rotate together for the 2nd mode. This is the same as for the 4th mode.

![Excitation and response node](image)

Fig. 6  Linear FE model for vibration analysis for 80 mm short plate length. The three steel plates were modeled with solid-shell element (Solsh 190), the bolt heads and nuts were modeled with solid element (Solid 185) and the bolt shafts were modeled with beam element (Beam 188).

![y-z symmetry plane](image)

(a) Whole view  (b) Joint part

(a) $y$-$z$ symmetry plane

(b) 1st mode: 68.0 Hz  (c) 2nd mode: 194.2 Hz

(d) 3rd mode: 376.1 Hz  (e) 4th mode: 601.7 Hz

Fig. 7  First four bending mode shapes for 80 mm short plate length. The mode shapes of the 1st and 3rd modes are symmetric about the $y$-$z$ plane while the mode shapes of the 2nd and 4th modes are antisymmetric.
The FE model for the vibration analysis is validated by experimental results. Fig. 9 shows a comparison of the driving point frequency response functions (FRFs) measured in the modal testing and predicted from the FE model. Fig. 9(a) is for the 80 mm short plate length and Fig. 9(b) is for the 60 mm short plate length. In the predicted FRFs, the damping ratios measured in the modal testing were used. The solid line represents the predicted FRF and the dashed line represents the measured FRF. For the 80 mm short plate length, the predicted FRF is in good agreement with the measured FRF below 800 Hz and the 1st to 4th modes are in this frequency range. For the 60 mm short plate length, the predicted FRF is in good agreement with the measured FRF below 1500 Hz and the 1st to 6th modes are in this frequency range. Thus it is possible to calculate the dissipated energy depending on each mode shape using the 1st to 4th modes for the 80 mm short plate length and the 1st to 6th modes for the 60 mm short plate length.

5.2 Static contact analysis

The contact elements (Conta 173 and Targe 170 with Coulomb friction law) are defined on the overlap region between the long plate and the two short plates of the linear FE model, changing the linear FE model to a nonlinear FE model as shown in Fig. 10. The coincident nodes on the interface between the short plate and long plate at the periphery of the bolt shafts are decoupled since these coincident nodes were coupled in the linear FE model. In addition, to prevent rigid body motions, the nodes on the symmetric planes of the long plate and the short plates in the y direction (width direction) are constrained in the y direction and also the nodes on the intersection of the symmetric plane in the x direction (longitudinal direction) and the middle surface of the long plate are constrained in the x direction. In the contact analysis, first, a surface load is applied on the lower surface of the upper short plate and the upper surface of the lower short plate. Then, the distributed displacement, i.e., the z direction (normal to surface) components of the scaled mode shape is applied to the nodes on the middle surface of the long plate to obtain the contact analysis results such as the contact pressure, contact area and slip displacement at the interface. In the analysis, the augmented Lagrangian method with automatic contact stiffness adjustment was used as the algorithm of contact analysis to improve the convergence and reliability. The friction coefficient was set to 0.25 as explained in section 4.1 and the penetration tolerance was set to 1 µm.
The distributed displacement was increased from zero deflection to the maximum deflection of the long plate (a quarter cycle of vibration) in 100 steps and then decreased to zero deflection (a half cycle of vibration) in 100 steps. Consequently, the contact pressure, the contact area and the slip displacement at the total 200 steps for a half cycle of vibration were obtained. The surface load continued to be applied with the constant during all the steps.

![Nonlinear FE model for contact analysis. The contact elements were defined on the overlap region between the long and the two short plates, changing the linear FE model to a nonlinear FE model.](image)

**Fig. 10** Nonlinear FE model for contact analysis. The contact elements were defined on the overlap region between the long and the two short plates, changing the linear FE model to a nonlinear FE model.

### 6. Effect of surface load on energy dissipation

In this section, the effect of surface load on energy dissipation is discussed and the necessary value of the surface load is determined. For this purpose, the dissipated energy (modal damping ratio) was calculated as the surface load was varied and the relationship between the dissipated energy and the surface load for the 60 mm and 80 mm short plate length was obtained.

#### 6.1 Necessary value of surface load

Figure 11 shows the relationships for the 1st to 4th modes for the 80 mm short plate length. The dashed lines indicate the measured modal damping ratios in Fig. 5, where the measured damping ratio is the result of subtracting the ratio of only the long plate from the ratio of the joined structure. This is because the dissipated energy in Eq. (2) is assumed to be due to friction and other damping sources such as material damping of the plate are ignored. The surface load at the intersection of the solid and dashed lines indicates the necessary surface load that is required to obtain the same damping ratio as measured in the modal testing. From the necessary surface loads for each mode, we find that the necessary surface load increases with increasing mode number while the modal damping ratio does not increase with increasing mode number. This means that the necessary surface load varies for each mode and each mode has a proper surface load to accurately estimate the measured damping ratio. The result for the 60 mm short plate length denotes the same tendency.

In order to discuss the physical meaning of the necessary surface load, we evaluate the relationship between the necessary surface load and the natural angular frequency. Fig. 12 shows the relationship between the surface load and the square of natural angular frequency for the 80 mm and 60 mm short plate length. The necessary surface load increases almost linearly to the square of angular frequency except for the 1st mode of the 60 mm short plate length.

Bournine et al. and Chen et al. examined the effect of clamping pressure on the damping. Since the studies focused on the fundamental bending mode, it has not shown that the clamping pressure is different for each mode and it varies almost proportionally with the square of the angular frequency as shown in Fig. 12.

From Fig. 12, we can see that the 80 mm short plate length needs a larger surface load compared to the 60 mm short plate length. This seems to be related to the relative displacement between the long and the short plates during vibration, for example, the gap at the edge of the short plate between the long and the short plates. When the test structure vibrates, the contact pressure becomes large according to the relative displacement. This is because the relative velocity between the long and the short plates depends on the relative displacement. We calculated the maximum gap between the long and short plates at the edge of the short plate. The results show the gap increases with an increase in the short plate length for all the modes. As a result, the difference short plate length needs the different surface load.
Fig. 11  Relationship between the damping ratio and the surface load for 80 mm short plate length. The dashed line indicates the measured damping ratio shown in Fig. 5. The surface load at the intersection of the solid and dashed lines is necessary value to obtain the same damping ratio as measured in the modal testing.

Fig. 12  Relationship between the surface load and the angular frequency. The necessary surface load varies almost proportionally to the square of the angular frequency.

6.2 Validation of surface load

In this section, we verify the validity of the necessary surface loads obtained in the previous section. A polynomial approximation of the surface loads is used to predict the surface loads and the modal damping ratios estimated are compare with the measured ratios.

The following polynomial function is used

$$p(x, y) = ax + by + cxy + d$$

(5)

where $p$ is the surface load, $x$ is the square of angular frequency, $y$ is the short plate length and coefficients $a$, $b$, $c$ and $d$ are calculated from 10 data shown in Fig. 12 using the least square method. The dashed lines in Fig. 12 represent the approximate results.
Figure 13 shows a comparison of the modal damping ratios estimated using the surface load obtained by the approximate results and the damping ratios measured in the modal testing. Fig. 13(c) is the result for the 100 mm short plate length, which was not included the approximation and for the 1st to 3rd modes, the predicted FRF was in good agreement with the measured FRF.

It is seen from Fig. 13 that the estimated damping ratios are in good agreement with the measured damping ratio for all the short plate lengths. Therefore, when the mode shapes obtained from the linear FE model approximate the actual deformation, the proposed method enables to accurately estimate the dissipated energy of the joined plate structures with the partial overlap region while the surface load at the overlap region is appropriately set.

7. Conclusion

In this paper, we proposed the procedure for calculating the dissipated energy due to friction at the interface to estimate the modal damping ratios for the fundamental and higher modes in joined plate structures with partial overlap region. We used the static contact analysis to calculate the dissipated energy due to friction instead of the dynamic contact analysis. In addition, the distributed displacement loading was adopted instead of force loading and as a distributed displacement, the scaled mode shapes obtained from a linear FE model were adopted. This enables to obtain the dissipated energy depending on each mode shape. To accurately calculate the dissipated energy, we applied a surface load on the overlap region of the joint along with the distributed displacement and determined the necessary value of surface load for estimating the measured damping ratio for each mode. To validate the proposed procedure, the modal damping ratios estimated from the dissipated energy with the necessary surface load were compared with the measured damping ratios. The results show when the mode shapes obtained from the linear FE model approximate the actual deformation, the proposed procedure enables to accurately calculate the dissipated energy of the joined plate structure while the surface load on the overlap region is appropriately set. The necessary surface load increases with increasing mode number while the modal damping ratio does not increase with increasing mode number. In addition, the necessary surface load varies almost proportionally to the square of the angular frequency.
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References


