The explanation of the deformed Schild string

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Abstract

The author comments on [1]. One of the deformed actions can express the Neveu-Schwarz-Ramond superstring under three gauge conditions. One of these depends on a matrix induced by the string coordinate.

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1 The review of the Schild string and the deformation

We summarize the properties of the Schild string and the results in [1]. In next section we will find the relation between the deformed Schild string in [1] and the Neveu-Schwarz-Ramond superstring.

The Schild string is given by the following action

$$S_{Schild} = \int d^2x \left[\frac{\Gamma}{e} \{X^{\mathcal{A}}, X^{\mathcal{B}}\}^2 + \Delta e \right], \tag{1}$$

where $\mathcal{A} = 0, 1, \cdots, D-1$ and $X^{\mathcal{A}}$ are coordinates for a string propagating in D space-time dimensions [2][3]. e, which is a scaler density, is an auxiliary field. Γ and Δ are arbitrary constant. The bracket of $\{X^{\mathcal{A}}, X^{\mathcal{B}}\}$ is defined by

$$\{X^{\mathcal{A}}, X^{\mathcal{B}}\} = \epsilon^{mn} \partial_m X^{\mathcal{A}} \partial_n X^{\mathcal{B}}, \tag{2}$$

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where ϵ^{mn} is $\epsilon^{mn} = -\epsilon^{nm}$ and $\epsilon^{01} = 1$.

"What are the impediments in constructing new Schild type superstring action from the Neveu-Schwarz-Ramond superstring ?" is the honest motivation of [1]. Supersymmetry can be made obvious by formulating the theory in the super-space. There we deformed the action of the Schild string. It is based on the super-space formulation of the Neveu-Schwarz-Ramond superstring. The Neveu-Schwarz-Ramond superstring is described in N=1 superconformal flat superspace as

$$S_{NSR} = -\frac{1}{4\pi}\int d^2x d^2\theta E D_\alpha Y^{\mathcal{A}} D^\alpha Y_{\mathcal{A}}, \tag{3}$$

where D_α is the covariant derivative for the local supersymmetry and the index α denote the chiralities of two dimensional spinor [5]. $Y^{\mathcal{A}}$ is the superfield

$$Y^{\mathcal{A}} = X^{\mathcal{A}} + i\bar{\theta}\psi^{\mathcal{A}} + \frac{i}{2}\bar{\theta}\theta B^{\mathcal{A}}, \tag{4}$$

where $B^{\mathcal{A}}$ is an auxiliary field which is introduced to close the gauge algebra of the supersymmetry on the field $\psi^{\mathcal{A}}$. E is the superdeterminant of the supervierbein. The supervierbein has been found in [4],

$$\begin{aligned} E_m{}^a &= e_m{}^a + i\bar{\theta}\gamma^a\chi_m + \frac{1}{4}\bar{\theta}\theta e_m{}^a A \\ E_m{}^\alpha &= \frac{1}{2}\chi_m{}^\alpha + \frac{1}{2}\theta^\mu(\gamma_5)_\mu{}^\alpha\omega_m - \frac{1}{4}\theta^\mu(\gamma_m)_\mu{}^\alpha A \\ &\quad -\frac{3i}{16}\bar{\theta}\theta\chi_m{}^\alpha A - \frac{1}{4}\bar{\theta}\theta(\gamma_m)^{\alpha\beta}\phi_\beta, \\ E_\mu{}^\alpha &= i\theta^\lambda(\gamma^\alpha)_{\lambda\mu}, \\ E_\mu{}^\alpha &= \delta_\mu{}^\alpha - \frac{i}{8}\bar{\theta}\theta\delta_\mu{}^\alpha A, \end{aligned} \tag{5}$$

where $e_m{}^a$ is the vierbein and $\chi_m{}^\alpha$ is the Rarita-Schwinger field. The supergauge transformation of the $\chi_m{}^\alpha$ contains an auxiliary field A. By integrating the super-coordinates, we have the action of the Neveu-Schwarz-Ramond superstring,

$$\begin{aligned} S_{NSR} = -\frac{1}{2\pi}\int d^2x e(&g^{mn}\partial_m X^{\mathcal{A}}\partial_n X_{\mathcal{A}} + i\bar{\psi}^{\mathcal{A}}\gamma^m\partial_m\psi_{\mathcal{A}} - B^{\mathcal{A}}B_{\mathcal{A}} \\ &-i\bar{\chi}_n\gamma^m\gamma^n\psi^{\mathcal{A}}\partial_m X_{\mathcal{A}} + \frac{1}{8}\bar{\psi}\psi\bar{\chi}_m\gamma^n\gamma^m\chi_n), \end{aligned} \tag{6}$$

where $e = \det e_m{}^a$. We proposed two actions by using the supervierbein. One of these is expressed as

$$S_2 = \int d^2x d^2\theta \left\{ \frac{\Gamma}{E}\det h_{\alpha\beta} + \Delta E \right\}, \tag{7}$$

where

$$h_{\alpha\beta} = D_\alpha Y^{\mathcal{C}} D_\beta Y_{\mathcal{C}}. \tag{8}$$

The determinant of $h_{\alpha\beta}$ is represented as a bracket

$$\det h_{\alpha\beta} = -\frac{1}{2}\left[Y^{\mathcal{A}}, Y^{\mathcal{B}}\right]^2, \tag{9}$$

where

$$\left[Y^{\mathcal{A}}, Y^{\mathcal{B}}\right] = \epsilon^{\alpha\beta} D_\alpha Y^{\mathcal{A}} D_\beta Y^{\mathcal{B}}. \tag{10}$$

Integrating θ we obtained

$$\begin{aligned} S_2 = \int d^2x \frac{\Gamma}{e} [\, & B_{\mathcal{A}} B^{\mathcal{B}} \bar{\psi}_{\mathcal{B}} \psi^{\mathcal{A}} - B^2 \bar{\psi}\psi + \frac{3i}{2} B^{\mathcal{A}} \bar{\psi}\psi \bar{\chi}_m \gamma^m \psi_{\mathcal{A}} - 2B^{\mathcal{A}} \bar{\psi}_{\mathcal{A}} \gamma^n \psi^{\mathcal{B}} \partial_n \chi_{\mathcal{B}} \\ & + \left(\bar{\psi}\psi\right)^2 \left(\frac{3i}{8} A + \frac{1}{2} g^{mn} \bar{\chi}_n \chi^m + \frac{1}{16e} \epsilon^{nm} \bar{\chi}_m \gamma^5 \chi_n\right) \\ & + \bar{\psi}\psi \,(\, i\bar{\psi}^{\mathcal{B}} \gamma^m \partial_m \psi_{\mathcal{B}} + g^{nm} \partial_n X_{\mathcal{A}} \partial_m X^{\mathcal{A}} - \frac{5i}{2} g^{nm} \bar{\chi}_n \psi^{\mathcal{A}} \partial_m X_{\mathcal{A}} \\ & + \frac{i}{2e} \epsilon^{ml} \partial_m X^{\mathcal{A}} \bar{\psi}^{\mathcal{A}} \gamma^5 \chi_l \,) - \bar{\psi}^{\mathcal{A}} \psi^{\mathcal{B}} g^{mn} \partial_m X_{\mathcal{A}} \partial_n X_{\mathcal{B}} \\ & \frac{1}{e} \{X_{\mathcal{A}}, X^{\mathcal{B}}\} \bar{\psi}_{\mathcal{A}} \gamma^5 \psi^{\mathcal{B}} \,] - \int d^2x \frac{\Delta}{2} \left(\frac{1}{2} \epsilon^{mn} \bar{\chi}_m \gamma^5 \chi_n + ieA\right). \end{aligned} \tag{11}$$

This is the second deformation in [1]. It is invariable under the volume-preserving diffeomorphism in the superspace, in which the infinitesimal parameter ξ^M of the super gauge transformation satisfies

$$\partial_M \xi^M = 0. \tag{12}$$

By expanding (12) in terms of θ we found three differential equations,

$$\partial_m f^m + \frac{i}{2} \bar{\chi}_m \gamma^m \zeta = 0, \tag{13}$$

$$i\partial_m (\gamma^m \zeta)_\alpha + \frac{i}{2} (\gamma_5 \gamma^m \zeta)_\alpha \omega_m + \frac{1}{4} \chi_{m\alpha} \bar{\zeta} \gamma^n \gamma^m \chi_n = 0, \tag{14}$$

$$\partial_m (\bar{\zeta} \gamma^n \gamma^m \chi_n) = 0, \tag{15}$$

where f^m and ζ^μ are the parameters of the two dimensional general coordinate transformation and the local supersymmetry, respectively. The solution of (15) is

$$\bar{\zeta} \gamma^n \gamma^m \chi_n = C'^m, \tag{16}$$

where C'^m is an arbitrary constant. Notice that in the virtue of the two dimansional gamma-matrix, $\gamma^n\gamma_m\gamma_n = 0$, (16) is invariable under the local fermionic transformation which is defined by the shift of the Rarita-Shwinger field as

$$\chi_m \to \gamma_m \eta_W, \tag{17}$$

where η_W, which is Majorana spinor, is the infinitesimal parameter. We easily found

$$\frac{\delta S_2}{\delta A} = \frac{3i\Gamma}{8e}(\bar{\psi}\psi)^2 - \frac{\Delta i}{2}e = 0, \tag{18}$$

in (11). In the next section we rearrange the terms in (11) by using (18).

2 The relation between the deformed Schild string and the Neveu-Schwarz-Ramond superstring

The deformed action (11) is expressed as the $e = 1$ gauge fixed Neveu-Schwarz-Ramond superstring action and additional terms as follows. The result is

$$S_2 = \pm k\Gamma S_{NSR(e=1)} + K_\pm + U, \tag{19}$$

where k is expressed by Γ and Δ as

$$k^2 \equiv \frac{4\Delta}{3\Gamma}. \tag{20}$$

We have introduced k as

$$(\bar{\psi}\psi)^2 \;=\; \frac{4\Delta}{3\Gamma}e^2. \tag{21}$$

As the result of (18) (19) is explicitly written by

$$\begin{aligned} S_{NSR(e=1)} \;=\; & \int d^2x\,[\, g^{nm}\partial_n X^{\mathcal{A}}\partial_m X^{\mathcal{A}} + i\bar{\psi}^{\mathcal{B}}\gamma^m\partial_m\psi_{\mathcal{B}} - B^2 \\ & -i\bar{\chi}_l\gamma^m\gamma^l\psi^{\mathcal{A}}\partial_m X_{\mathcal{A}} + \frac{1}{8}\bar{\chi}_m\gamma^n\gamma^m\chi_n\bar{\psi}\psi\,]\,. \end{aligned} \tag{22}$$

$$K_\pm = \int d^2x\left[\frac{\Delta}{2}e\bar{\chi}_n\gamma^n\gamma^m\chi_m \mp \frac{3}{2}ik\Gamma\partial_m X^{\mathcal{A}}\bar{\psi}^{\mathcal{A}}\gamma^m\gamma^n\chi_n\right] \tag{23}$$

$$\begin{aligned} U = \int d^2x\frac{\Gamma}{e}\{ & B_{\mathcal{A}}B^{\mathcal{B}}\bar{\psi}_{\mathcal{B}}\psi^{\mathcal{A}} \pm \frac{3i}{2}B^{\mathcal{A}}e\bar{\chi}_m\gamma^m\psi_{\mathcal{A}} - 2B^{\mathcal{A}}\bar{\psi}_{\mathcal{A}}\gamma^n\psi^{\mathcal{B}}\partial_n X_{\mathcal{B}} \\ & -\bar{\psi}^{\mathcal{A}}\psi^{\mathcal{B}}g^{mn}\partial_m X_{\mathcal{A}}\partial_n X_{\mathcal{B}} + \frac{1}{e}\{X_{\mathcal{A}}, X_{\mathcal{B}}\}\bar{\psi}_{\mathcal{A}}\gamma_5\psi_{\mathcal{B}}\}. \end{aligned} \tag{24}$$

In the calculation we have used

$$\gamma^m\gamma^n = g^{mn} - \frac{1}{e}\epsilon^{mn}\gamma^5, \tag{25}$$

where $\gamma^m = \gamma^a e_a{}^m$. If we define "a matrix like gamma-matrix" as follows,

$$\begin{aligned} \Gamma_{\mathcal{A}}\Gamma_{\mathcal{B}} &\equiv G_{\mathcal{A}\mathcal{B}} - \frac{1}{e}\Upsilon_{\mathcal{A}\mathcal{B}}\gamma^5, \\ G_{\mathcal{A}\mathcal{B}} &\equiv g^{mn}\partial_m X_{\mathcal{A}}\partial_n X_{\mathcal{B}} + B_{\mathcal{A}}B_{\mathcal{B}}, \\ \Upsilon_{\mathcal{A}\mathcal{B}} &\equiv \{X_{\mathcal{A}}, X_{\mathcal{B}}\}, \end{aligned} \tag{26}$$

then we can express (24) as

$$U = \int d^2x \frac{\Gamma}{e}\{-\bar{\psi}^{\mathcal{A}}\Gamma_{\mathcal{A}}\Gamma_{\mathcal{B}}\psi^{\mathcal{B}} \pm \frac{3i}{2}B^{\mathcal{A}}e\bar{\chi}_m\gamma^m\psi^{\mathcal{A}} - 2B^{\mathcal{A}}\gamma^n\psi^{\mathcal{B}}\partial_n X_{\mathcal{B}}\}. \tag{27}$$

The construction in (26) is inspired by (25). We pay attention that the index n, m of the gamma-matrix represents the two-dimensional surface of string world-sheet and the index $\mathcal{A}, \mathcal{B}$ of $\Gamma_{\mathcal{A}}$ denotes the space-time dimension where a string is propagating. That we can neatly reexpress (11) by introducing a matrix defined in the space-time is interesting point here.

Finally we find that if we impose gauge conditions

$$\gamma^m\chi_m = 0, \tag{28}$$

$$\Gamma_{\mathcal{A}}\psi^{\mathcal{A}} = 0, \tag{29}$$

and

$$B^{\mathcal{A}} = 0, \tag{30}$$

then the deformed action (11) is equal to the $e = 1$ gauge fixed Neveu-Schwarz-Ramond superstring action. Notice that the third gauge condition is not imposed as the equation of motion $B^{\mathcal{A}}$ derived from (11).

In the last section we sum up [6] in where the author treats similar gauge conditions. Then we discuss our gauge conditions.

3 Discussion and summary

In [6] the author considers the super-Weyl invariant regularization of the two-dimmensional supergravity. He uses the superfield formulation and he finally constructs the super-Liouville action which possesses the super-Weyl invariance (the

Weyl invariance and the local fermionic symmetry) and the super-area preserving diffeomorphism. To define which part of the two-dimensional superdiffeomorphism group is compatible with the super-Weyl symmetry, he searchs the two dimensional diffeomorphism and the local supersymmetry which preserve the gauge conditions

$$e = 1, \quad \text{and} \quad \gamma^m \chi_m = 0.$$

Finally he found the constraints which the parameters of the supersummetry and the two dimensional diffeomorphism should satisfy. In our case (19) becomes the $e = 1$ gauge fixed Neveu-Schwarz-Ramond superstring action, which is invariable under the area-preserving diffeomorphism, the local fermionic transformation and the restricted supersymmetry, by virtue of (28), (29) and (30). Moreover we should notice that (13) represents the area-preserving diffeomorphism

$$\partial_m f^m = 0, \tag{31}$$

under $\gamma^m \chi_m = 0$. (31) is one of the constraints which he found in [6]. Another he found is similar to (14).

In conclusion, the superfield formulation of superstring and supergravity is quite interesting and usuful. In the superspace we can manifestly keep the supersymmmetry. However it is not trivial to define the supersymmetry which is compatible with other symmetires. We expect that the direction of our Schild string deformation which preserves the global structure of the bracket (2) would possess the dynamics of full superstring.

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