# Acoustic Holography with a Phase Plate

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#### Abstract

A simple method of a fully faithful holographic reconstruction is proposed for the case, such as in acoustic holography, where the wavelength of the recording radiation is greater than that of the reconstructing radiation. The simple method involves reconstruction with the introduction of a suitably constructed phase plate in the reconstruction process and does not require scaling down the hologram as in the usual case.

### Inhalt

Akustische Holographie mit einer Phasenplatte. Vorgeschlagen wird eine einfache Methode zur holographischen Rekonstruktion für den Fall, daß – wie bei der akustischen Holographie – die Wellenlänge bei der Hologrammaufnahme größer ist als die Wellenlänge der rekonstruierenden Strahlung. Bei dieser Methode wird eine geeignet dimensionierte Phasenplatte in den Strahlengang der Rekonstruktion eingefügt. Dadurch wird die sonst übliche Verkleinerung des Hologramms vermieden.

In acoustic holography where the wavelength of the recording radiation,  $\lambda_0$ , is greater than that of the reconstructing radiation,  $\lambda_c$ , a fully faithful reconstruction can be obtained only when the hologram is scaled inversely as the ratio of recording to reconstructing wavelength [1, 2]. When the ratio becomes so large as to  $10^3 (= \lambda_0/\lambda_c)$  or more than this value this, in turn, causes the considerable experimental difficulties of scaling down the hologram to the above ratio and then the necessity of microscopic viewing [3]. The purpose of this letter is to propose a new method for obtaining the perfect reconstruction without the hologram reduction. The method involves reconstruction with the introduction of a suitably constructed phase plate in the reconstruction process in addition to the usual processes. The idea of this method is based on the work by *R. W. Meier* [2] that, if the recording, the

reference and the reconstructing radiations all originate from a point source and have constant intensity across the hologram, optical properties of the reconstructed image can be calculated only from a phase of a reconstructed wavefront. A condition of the phase plate for the perfect reconstruction is derived in the case of plane reference and reconstructing waves from the practical point of view in this letter.

Fig. 1 shows the holographic geometry. Following the Champagne's field description [4], the spatial variation of the scattered wave from the object,  $\varepsilon_0$ , may be written in the form

$$\varepsilon_{\mathbf{o}} = (\mathbf{A}_{\mathbf{o}}/\mathbf{r}_{\mathbf{o}}) \cdot \exp\{i(2\pi/\lambda_{\mathbf{o}})\,\mathbf{r}_{\mathbf{o}}\}\,. \tag{1}$$

The same expressions hold for the reconstructing wave,  $\varepsilon_c$ , as well as for the reference wave,  $\varepsilon_r$ . The hologram is recorded by exposure of the photographic plate with simultaneous illumination from both the object and the reference source,  $\lambda_o = \lambda_r$ . The recording plate is then processed so that it has a transmittance proportional to the intensity distribution of the interference pattern as in the usual case. When the hologram is illuminated with the reconstructing wave,  $\varepsilon_c$ , as shown in Fig. 1(b) the waves of the form

$$\varepsilon_{i} = A \cdot \exp \{i(2\pi/\lambda_{c}) r_{c}\} \cdot \exp \{\pm i 2\pi/\lambda_{o} (r_{o} - r_{r})\} \cdot \exp (i\Phi), \qquad (2)$$

emerge from the phase plate assuming that the 1/r variations in the waves of the various point sources are negligible. The term,  $\exp(i\Phi)$ , is a complex amplitude transmittance of the phase plate introduced here for obtaining the



Fig. 1. Holographic geometry (a) recording geometry, (b) reconstructing geometry.

fully faithful reconstruction without hologram reduction. The + sign and the - sign in (2) refer to the virtual and the real image terms, respectively. The fully faithful reconstruction requires the expression (2) to be of the form

$$\varepsilon'_{i} = \mathbf{A}' \cdot \exp\left\{\pm \mathbf{i}(2\,\pi/\lambda_{c})\,\mathbf{r}_{0}\right\}.$$
(3)

The reference and the reconstructing waves are restricted to the plane waves from the practical point of view that the phase plate is obtainable by some ways as discussed later. The phase plate requirement is then

$$\Phi = \pm \left(2 \pi / \lambda_{\rm c} - 2 \pi / \lambda_{\rm o}\right) r_{\rm o}. \tag{4}$$

It is not possible, of course, to construct the phase plate fully capable of satisfying (4) everywhere. However, if we limit ourselves to paraxial rays, and to reconstructed object subtending small angles at the hologram, then (4) can be satisfied to a good approximation simply by a Fresnel zone plate of focal length  $r_0$  which can be obtained using a coherent point source having a wavelength  $\lambda(=\lambda_c\lambda_0/\lambda_0 - \lambda_c)$  located at  $r_0$  from the recording plane.

It is found from the above consideration that, if the phase plate satisfying (4) can be constructed within a reasonable accuracy, the reconstruction process can be carried out without the hologram reduction and then the primary distortion due to the wavelength change can be eliminated. The analogous idea can be used when the wavelength of the recording radiation is shorter than that of the reconstructing radiation.

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