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Simplified Method to Evaluate Upper Limit Stress Intensity Factor Range of an Inner-Surface Circumferential Crack under Steady State Thermal Striping

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ABSTRACT

Simplified method to evaluate the upper limit stress intensity factor (SIF) range of an inner-surface circumferential crack in a thin- to thick-walled cylinder under steady state thermal striping was considered in this paper. The edges of the cylinder were rotation-restrained and the outer surface was adiabatically insulated. The inner surface of the cylinder was heated by a fluid with constant heat transfer coefficient whose temperature fluctuated sinusoidally at constant amplitude ΔT. By combining our analytical temperature solution for the problem and our semi-analytical numerical SIF evaluation method for the crack, we showed that the desired maximum steady state SIF range can be evaluated with an engineering accuracy after ΔT, the mean radius to wall thickness ratio r_m/W of the cylinder, the thermal expansion coefficient and Poisson's ratio are specified. By applying are method, no transient SIF analysis nor sensitivity analysis of the striping frequency on the SIF range is necessary. Numerical results showed that our method is valid for cylinders in a range of r_m/W = 10 to 1.

1. INTRODUCTION

A coolant leakage in a nuclear power plant due to thermal fatigue induced by sinusoidal coolant temperature fluctuation (thermal striping) was reported recently (Hoshino, 2000). Since thermal striping is usually uncontrollable and since the damage accumulates quickly due to rapid sequences of cycles, we thought it important to derive stress intensity factor (SIF) solutions for various crack configurations under thermal striping. One of our recent successes was an analytical temperature solution for a long cylinder heated inside by a fluid with sinusoidal temperature change ΔT and SIF solutions for an inner-surface circumferential crack with rotation-restrained edges (Fig. 1) under this temperature change (Meshii, 2004a). Since the SIF for the problem is affected by various factors such as cylinder configuration, heat transfer conditions and thermal striping frequency, etc., we presented the SIF solutions with a minimum number of non-dimensional parameters (Meshii, 2004b).

One typical application of our SIF solution is the fatigue crack growth analysis combined with the Paris law. Considering the fact that the transient SIF reaches a steady state in the early stage, we thought it important to know the upper limit of the steady state SIF range under thermal striping for a specific cylinder and crack configuration. Though we have minimized the dominant parameters of the SIF range under thermal striping, and though we assume a steady state, we still have to find a combination of heat transfer coefficient and striping frequency that maximizes the SIF range of interest. The effort necessary for this maximization is huge. Thus in this paper, we proposed a simplified method to evaluate the upper limit of the steady state thermal striping SIF range of a circumferential crack (Fig. 1), by utilizing the fact that our detailed SIF evaluation method (Meshii, 1999) is a semi-analytical-numerical method. By the proposed simplified method, an approximate upper limit SIF range with engineering accuracy can be obtained without transient SIF analysis or sensitivity analysis for the heat transfer coefficient and striping frequency over the SIF range.

2. INTRODUCTION OF OUR SEMI-ANALYTICAL-NUMERICAL SIF EVALUATION METHOD FOR A CIRCUMFERENTIAL CRACK UNDER THERMAL STRIPING

First, we briefly introduce our semi-analytical-numerical SIF evaluation method for a circumferential crack in a cylinder under thermal striping. The heat conduction problem of a hollow and long cylinder whose inner and outer radii are r_i and r_o, respectively, is considered (Fig. 1). The cylinder temperature T_m is assumed to be uniform at time $t < 0$ and the temperature change from the initial state is defined as $u(r, t)$. The cylinder is adiabatically insulated on the outer surface and the inner surface is axisymmetrically heated by a fluid with a constant heat transfer coefficient h whose temperature is $T_m+T_f(t)$. The material constants of the cylinder, such as thermal conductivity Λ and thermal diffusivity κ are assumed to be temperature independent. If $u(r, t)$ is obtained, we can evaluate the desired transient SIF $K_{cyl}(t)$ by applying it to the following set of equations (Meshii, 1999):

$$K_{\text{cyl}}(t) = K_{\text{free c}}(t) + K_{\text{fbr}}(t) \quad \text{(1)}$$

$$K_{\text{freec}}(t) = \int_{r_\text{i}}^{r_\text{i}+a} \frac{r\left[-\sigma_M(r,t)\right]}{r_\text{i}+a} \cdot w(r - r_\text{i}; a)\,dr \quad \text{(2)}$$

$$K_{\text{fbr}}(t) = F_{\text{tfbr}} \cdot \left\{ \frac{-6M_\text{t}(t)}{W^2} \sqrt{\pi a} \cdot F_M(a/W) \right\} \quad \text{(3)}$$

$$\sigma_M(r, t) = E\alpha\{u(r, t) - u_{\text{avg}}(t)\}/(1-\nu) \quad \text{(4)}$$

$$u_{\text{avg}}(t) = \left\{ \int_{r_\text{i}}^{r_\text{o}} 2\pi r \cdot u(r,t)\,dr \right\} / \left\{ \pi (r_\text{o}^2 - r_\text{i}^2) \right\} \quad \text{(5)}$$

$$M_\text{t}(t) = \int_{r_\text{i}}^{r_\text{o}} \sigma_M \cdot (r_\text{m} - r)\,dr \quad \text{(6)}$$

where E, ν and α are Young's modulus, Poisson's ratio and coefficient of thermal expansion, respectively. F_M is the correction factor of finite width for a single edge cracked strip under pure bending. w and F_{tfbr} are functions of structural parameters for a circumferential crack, given concretely in references (Meshii, 1999 and 2001a).

To evaluate a specific case of thermal striping, we derived $u(r, t)$ for the case $T_\text{f}(t) = \Delta T \sin \omega t$ (Meshii, 2004a). Considering a steady state (suffix s stands for steady state), it can be expressed as follows:

$$\frac{u(r,t)}{\Delta T} = \sin \omega t + \sum_{n=1}^{\infty} T_{\text{s}n}(t) R_n(r) \quad \text{(7)}$$

$$R_n(r) = \frac{2\alpha_0 r_\text{i}}{\rho_n^2} \frac{\{J_0(\rho_n r_\text{i}) - b_n Y_0(\rho_n r_\text{i})\}\{J_0(\rho_n r) - b_n Y_0(\rho_n r)\}}{r_\text{o}^2 \{J_0(\rho_n r_\text{o}) - b_n Y_0(\rho_n r_\text{o})\}^2 - r_\text{i}^2 \{1 + (\alpha_0/\rho_n)^2\}\{J_0(\rho_n r_\text{i}) - b_n Y_0(\rho_n r_\text{i})\}^2} \quad \text{(8)}$$

$$T_{\text{s}n}(t) = \frac{-\omega(\kappa\rho_n^2 \cos \omega t + \omega \sin \omega t)}{(\kappa\rho_n^2)^2 + \omega^2} \quad \text{(9)}$$

where $b_n \equiv J_1(\rho_n r_\text{o})/Y_1(\rho_n r_\text{o})$, and J and Y represent the Bessel functions of first and second kind, respectively. Suffixes 0 and 1 for J and Y represent the order of these functions. The eigenvalues ρ_n are the positive roots of the following equation ($\rho_n \le \rho_{n+1}$),

$$J_1(\rho_n r_o)\{Y_0(\rho_n r_i)\alpha_0 + Y_1(\rho_n r_i)\rho_n\} = Y_1(\rho_n r_o)\{J_0(\rho_n r_i)\alpha_0 + J_1(\rho_n r_i)\rho_n\} \quad \cdots\cdots(10)$$

where $\alpha_0 = h/\Lambda$.

3. UPPER LIMIT SIF RANGE OF A CIRCUMFERENTIAL CRACK UNDER STEADY STATE THERMAL STRIPING

3.1 Key ideas of Simplified Method to Evaluate Upper Limit SIF Range of a Circumferential Crack under Steady State Thermal Striping

In this section, we introduce 6 key ideas or assumptions in our simplified method to evaluate the upper limit SIF range of a circumferential crack under steady state thermal striping. Then we proceed to summarize the concrete procedure of the proposed simplified method.

The first assumption is that the upper limit steady state SIF range is obtained for the case of heat transfer coefficient $h \to \infty$. To distinguish this case from a general one, we use the suffix ∞.

Second, we consider the fact that the absolute values of the maximum and minimum SIF for the crack in Fig. 1 under steady state thermal striping are the same. Thus we focus our attention on the maximum SIF K_{cylmax} and evaluate the SIF range by $\Delta K_{\text{cylmax}} = 2\,K_{\text{cylmax}}$. Here we implicitly assumed that the SIF K_{cyl} under consideration is a deviation from the constant SIF due to a load such as internal pressure, so that the crack does not close.

Third, we assume that the cylinder is long, as usually assumed for theoretical solutions. Considering the fact F_{tfbr} in Eq. (3) approaches 0 when cylinder length $H \to \infty$ (Meshii 1999 and

2001a), $K_{fbr}(t) \approx 0$ is deduced for long cylinders. Thus we can assume that $K_{cyl}(t) \approx K_{freec}(t)$ holds.

Fourth, we consider the fact that once $\{-\sigma_M(r, t)\}$ on the crack surface is approximated by a polynomial, $K_{freec}(t)$ can be evaluated with SIF solutions summarized in the published tables (Meshii, 2001b). That is, introducing $x = r - r_i$, we approximate $\{-\sigma_M(r, t)\}$ by a quadratic function $\sigma(x)$ in Eq. (11) by determining coefficients σ_j (j = 0~2) appropriately. Then, we refer to coefficients K_j in the tables and evaluate K in Eq. (11) as $K_{freec}(t)$.

$$\sigma(x) = \sum_{j=0}^{2} \sigma_j (x/W)^j ;\ K = \sqrt{W} \sum_{j=0}^{2} \sigma_j K_j \qquad (11)$$

Fifth, we assume that the $K_{cyl}(t)$ attains its maximum at the same time when the inner-outer wall stress difference $\Delta\sigma(t) \equiv \sigma_M(r_i, t) - \sigma_M(r_o, t)$ reaches the maximum $\Delta\sigma_{max}$. In addition, considering the fact that the average stress across the wall should be zero, only two of σ_j are independent, so we change the Eq. (11) so that we can use σ_0 and $\Delta\sigma_{max}$ as these two independent parameters. Thus, we can evaluate the maximum SIF K_{max} from Eq. (11) approximately as follows:

$$K_{max} = \frac{\sqrt{W}}{2} \left\{ \left[K_1 (4 + \frac{W}{r_m}) - K_2 (6 + \frac{W}{r_m}) \right] \Delta\sigma_{max} + 2 \left[K_0 - 6K_1 + 6K_2 \right] \sigma_0 \right\} \qquad (12)$$

Finally, we assume for the unknown coefficient $\sigma_0 = -\sigma_M(r_i, t)$ an upper limit of $E\alpha\Delta T/(1-\nu)$. Then we can approximate the maximum SIF K_{max} by Eq. (12), once we know $\Delta\sigma_{max}$.

3.2 Simplified Method to Evaluate the Upper Limit SIF Range under Steady State Thermal Striping

In this section, we proceed to propose a method to evaluate the upper limit SIF range under steady state thermal striping without running a sensitivity analysis on ω or t. From the six key ideas or assumptions in the previous section, we can now evaluate the desired maximum SIF range approximately as 2 $K_{\max}$ from Eq. (12), once we become able to evaluate the $\Delta\sigma_{\max}$ (maximum of the inner-outer wall stress difference $\Delta\sigma(t)$) appropriately. By substituting Eq. (4) into the definition of $\Delta\sigma(t)$ and by using Eq. (7), we find that $\Delta\sigma_{s,\infty}(t)$ can be evaluated by

$$\Delta\sigma_{s,\infty}(t) = \frac{E\alpha\Delta T}{1-\nu}\sum_{n=1}^{\infty} T_{s,\infty n}(t)\cdot\left(R_{\infty,n}(r_i) - R_{\infty,n}(r_o)\right) \equiv \frac{E\alpha\Delta T}{1-\nu}\sum_{n=1}^{\infty} T_{s,\infty n}(t)\cdot\Delta R_{\infty n} \quad \cdots\cdots(13)$$

without integral calculation (evaluation of $u_{avg}(t)$) required for obtaining $\sigma_M(r, t)$. Here we added suffixes s and ∞ to clarify that the quantities in Eq. (13) are for steady state and infinite heat transfer coefficient, as described in the previous section.

First, we consider the eigenfunction. By applying fundamental relationships of Bessel functions (Abramowitz, 1972), the eigen function in Eq. (8) for a case $h \to \infty$ can be simplified as follows:

$$\Delta R_{\infty n} = \frac{2}{r_o\rho_{\infty n}}\frac{Y_1(\rho_{\infty n}r_o)Y_0(\rho_{\infty n}r_i)}{\{Y_1(\rho_{\infty n}r_o)\}^2 - \{Y_0(\rho_{\infty n}r_i)\}^2} = \frac{2}{\psi\, x_{\infty n}}\frac{Y_1(\psi\, x_{\infty n})Y_0(x_{\infty n})}{\{Y_1(\psi\, x_{\infty n})\}^2 - \{Y_0(x_{\infty n})\}^2} \quad \cdots\cdots(14)$$

where $\psi \equiv r_o / r_i$ and $x_{\infty n} \equiv \rho_n r_i$ ($0 < x_{\infty n} \leq x_{\infty n+1}$) are the non-dimensional eigenvalues which satisfies the following equation.

$$J_1(\psi x_{\infty n})/Y_1(\psi x_{\infty n}) = J_0(x_{\infty n})/Y_0(x_{\infty n}) = b_{\infty n} \quad \cdots\cdots(15)$$

Note from Eq. (15) that $x_{\infty n}$ is determined by a single parameter ψ, which is related to the cylinder configuration.

When we consider the fact that the non-dimensional eigenvalues $x_{\infty n}$ are a monotonically increasing series, and apply the integer-order Bessel function's principal asymptotic forms for large variables (Abramowitz, 1972) to Eq. (14), we obtain

$$\Delta R_{\infty n} \approx \frac{2}{\sqrt{\psi}\, x_{\infty n}} \frac{(\cos x_{\infty n} - \sin x_{\infty n})(\cos \psi x_{\infty n} - \sin \psi x_{\infty n})}{(1-\psi+\psi \sin 2x_{\infty n} + \sin \psi x_{\infty n})} \qquad \therefore |\Delta R_{\infty n}| \leq |\Delta R_{\infty 1}|, \cdots\cdots(16)$$

where we expect that $|\Delta R_{\infty n}|$ is a monotonically decreasing series, because $x_{\infty n}$ is a monotonically increasing series. Here we naturally assumed that the denominator is not 0. We note that the numerical studies we made on $|\Delta R_{\infty n}|$ (n = 1~96) for ψ= 3~1.11(r_m/W =1, 6, 10) validated our assumption.

Subsequently, we consider the time-related function in Eq. (9) and rewrite it as follows for the case $\rho_n = \rho_{\infty n}$.

$$T_{s,\infty n}(t) = -\frac{1}{\sqrt{(\kappa\rho_{\infty n}^2/\omega)^2+1}} \sin(\omega t+\varphi_n);\ \cos\varphi_n = \frac{1}{\sqrt{(\kappa\rho_{\infty n}^2/\omega)^2+1}} \cdots\cdots(17)$$

Since the eigenvalues $\rho_{\infty n}$ are a positive monotonically increasing series, we expect that $|\Delta T_{s,\infty n}|$ is a positive monotonically decreasing series. In addition, we see that

$$\frac{\partial}{\partial\omega}\left\{\frac{1}{\sqrt{(\kappa\rho_{\infty 1}^2/\omega)^2+1}}\right\} > 0\ for\ \omega > 0,\ \lim_{\omega\to 0}\frac{1}{\sqrt{(\kappa\rho_{\infty 1}^2/\omega)^2+1}} = 0,\ \lim_{\omega\to\infty}\frac{1}{\sqrt{(\kappa\rho_{\infty 1}^2/\omega)^2+1}} = 1, \cdots\cdots(18)$$

thus, we conclude that $|\Delta T_{s,\infty n}| \leq 1$.

Combining the results in Eq. (16)~(18), we derive from Eq. (13) the following relationship, which gives an upper bound estimation of the inner-outer wall stress difference under thermal striping.

$$\Delta\sigma_{max} \leq \frac{E\alpha\Delta T}{1-\nu}\left|\Delta R_{\infty 1}\right| \equiv \Delta\sigma_{max,app} \cdots\cdots(19)$$

The estimated $\Delta\sigma_{max,\ app}$ and the exact upper limit of $\Delta\sigma_{max}$ (upper limit of $\Delta\sigma_{max}$ obtained by considering the angular velocity ω) for the cases of r_m/W = 10, 6 and 1 are compared in Table 1. Both stresses were normalized by $\Delta\sigma_0 \equiv E\alpha\Delta T/(1-\nu)$ and the angular velocity was normalized to $\Omega = \omega/\{2\pi(\kappa/r_i^2)\}$ and $F_o = (\kappa/r_i^2)t$. Comparison of $\Delta\sigma_{max.app}$ and $\Delta\sigma_{max}$ for wide range of r_m/W = 1~30 is shown in Fig. 2. We see from Table 1 and Fig. 2 that the upper limit of the transient inner-outer wall stress difference $\Delta\sigma_{max}$ can be conservatively evaluated by the proposed $\Delta\sigma_{max,\ app}$ and with an engineering accuracy (i.e., 5~10% larger than the exact value), without performing sensitivity analysis over time and with respect to the thermal striping angular velocity ω.

Finally, we summarize our simplified method to evaluate the upper limit SIF range ΔK_{cylmax} for a given cylinder configuration. First, we approximate the upper limit of the transient inner-outer wall stress difference $\Delta\sigma_{max}$ by the proposed $\Delta\sigma_{max,\ app}$ of Eq. (19). Then we evaluate the upper limit SIF range ΔK_{cylmax} as $2K_{max}$, where K_{max} is evaluated by Eq. (12) for the case $\Delta\sigma_{max} = \Delta\sigma_{max,\ app}$ and $\sigma_0 = E\alpha\Delta T/(1-\nu)$. Note that the necessary information for the calculation is the cylinder configuration (r_m, W), the material constants (E, α and ν) and the thermal striping condition (ΔT).

A numerical example for the case of a long cylinder (βH =10) is summarized in Table 2 for cylinders of r_m/W = 10, 6 and 1. The SIF range was normalized by $\Delta K_{\Delta T} \equiv 2E\alpha\Delta T(\pi W)^{1/2}/(1-\nu)$. Here $\beta = \{3(1-\nu^2)\}^{1/4}/(r_m W)^{1/2}$ is the characteristic for the system. We see from Table 2 that the upper limit of the transient maximum SIF range ΔK_{cylmax} can be conservatively evaluated by the proposed

$2K_{max}$ and with an engineering accuracy (i.e., approximately 20% larger than the exact value), without performing sensitivity analysis over time and with respect to thermal striping angular velocity ω.

4. CONCLUSIONS

In this paper, we proposed a simplified method to evaluate the upper limit SIF range ΔK_{cylmax} for a circumferential crack in a cylinder under thermal striping. This was done by combining our analytical temperature solution for the problem and our semi-analytical-numerical SIF evaluation method for the crack. Finally, we showed that the desired maximum SIF range could be evaluated with an engineering accuracy once the cylinder configuration (r_m, W), the material constants (E, α and ν) and the thermal striping condition (ΔT) are specified. No transient SIF analysis nor sensitivity analysis with respect to the striping frequency on the SIF range are necessary. Numerical results showed that our method is valid for thin to thick cylinders in the range of r_m/W = 10 to 1.

NOMENCLATURE

a	crack depth
b_n	$= J_1(\rho_n r_o)/Y_1(\rho_n r_o)$
h	heat transfer coefficient
r	radius
r_i, r_o, r_m	inner, outer and mean radii
t	time
$u(r, t)$	temperature change from the initial state
$u_{avg}(t)$	average temperature change from the initial state
$w(r; a)$	weight function of a circumferential crack in a finite length cylinder
x	$= r - r_i$

x_n	n tht non-dimensional eigenvalue; = $\rho_n\, r_\mathrm{i}$
E	Young's modulus
F_M	correction factor of finite width for single edge cracked strip under pure bending
F_o	Fourier number; = $(\kappa/r_\mathrm{i}^2)t$
F_tfbr	function of structural parameters
H	cylinder length
J_0, J_1	first kind Bessel functions of order 0 and 1
$K_\mathrm{cyl}(t)$	SIF of a circumferential crack in a cylinder at time t
K_cylmax	maximum of $K_\mathrm{cyl}(t)$
K_max	approximation of upper limit K_cylmax at steady state
$M(t)$	thermal moment at time t
$R_n(r)$	n th eigenfunction
$T_\mathrm{f}(t)$	temperature change of fluid from the initial state
T_m	initial temperature of fluid and cylinder
$T_n(t)$	n th temperature dependent function
W	cylinder thickness
Y_0, Y_1	second kind Bessel functions of order 0 and 1
α	coefficient of thermal expansion
α_0	$= h/\Lambda$
β	characteristic of the system; = $\{3(1-\nu^2)/(r_\mathrm{m}W)^2\}^{1/4}$
κ	coefficient of diffusivity
ν	Poisson's ratio
ρ_n	n th eigenvalue
σ_0	$= -\sigma(r_\mathrm{i}, t)$; approximated as $E\alpha\Delta T/(1-\nu)$
$\sigma_M(t)$	axial component of thermal stress
ω	angular velocity
ψ	$= r_\mathrm{o} / r_\mathrm{i}$
ΔK_cylmax	maximum SIF range at steady state
ΔK_ref	reference SIF range; = $2E\alpha\Delta T(\pi W)^{1/2}/(1-\nu)$
ΔT	amplitude of fluid temperature fluctuation: $\Delta T \sin \omega t$
$\Delta\sigma(t)$	$= \sigma_M(r_\mathrm{i}, t) - \sigma_M(r_\mathrm{o}, t)$
$\Delta\sigma_\mathrm{ref}$	reference stress range; = $E\alpha\Delta T/(1-\nu)$
$\Delta\sigma_\mathrm{max}$	maximum of $\Delta\sigma(t)$
$\Delta\sigma_\mathrm{max,\ app}$	approximate of $\Delta\sigma_\mathrm{max}$
Λ	coefficient of thermal conductivity
Ω	non-dimensional angular velocity;= $\omega/\{2\pi(\kappa/r_\mathrm{i}^2)\}$

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REFERENCES

Hoshino, T., Aoki, T., Ueno, T. and Kutomi, Y., 2000. Leakage from CVS pipe of regenerative heat exchanger induced by high-cycle thermal fatigue at Tsuruga nuclear power station unit 2, *Proc. ICONE 8*.

Meshii, T. and Watanabe, K., 2004a. Stress intensity factor of a circumferential crack in a thick-walled cylinder under thermal striping. Transactions of the ASME, Journal of Pressure Vessel Technology. 126, 157-162.

Meshii, T. and Watanabe, K., 2004b. Normalized stress intensity factor range solutions of an inner-surface circumferential crack in thin-to thick-walled cylinder under thermal striping by semi-analytical numerical method. Journal of Thermal Stresses. 27, 253-267.

Meshii, T. and Watanabe, K., 1999. Maximum stress intensity factor for a circumferential crack in a finite length thin-walled cylinder under transient radial temperature distribution. Engineering Fracture Mechanics. 63, 23-38.

Meshii, T. and Watanabe, K., 2001a. Analytical approach to crack arrest tendency under cyclic thermal stress for an inner-surface circumferential crack in a finite-length cylinder. Transactions of the ASME, Journal of Pressure Vessel Technology. 123, 220-225.

Meshii, T. and Watanabe, K., 2001b. Stress intensity factor for a circumferential crack in a finite-length thin to thick-walled cylinder under an arbitrary biqudratic stress distribution on the

T. Meshii, et al., Nuclear Engineering and Design, Vol. 236, No. 10, pp. 1081-1085 (2006. 5).

crack surfaces. Engineering Fracture Mechanics. 69, 975-986.

Abramowitz, M. and Stegun, I. A., 1972. Handbook of Mathematical Functions, 9th printing, Dover Publications, New York.

T. Meshii, et al., Nuclear Engineering and Design, Vol. 236, No. 10, pp. 1081-1085 (2006. 5).

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Table 2 Comparison of upper limit of SIF range $\Delta K_{\mathrm{cylmax}}$ under thermal striping with that by our simplified method $2K_{\max}$ (Eq. (12)) for long cylinder of $\beta H = 10$

Fig. 1 Circumferentially cracked cylinder under thermal striping

Fig. 2 Comparison of $\Delta\sigma_{\mathrm{max.app}}$ and $\Delta\sigma_{\max}$ for wide range of r_{m}/W

Table 1 Comparison of maximum inner-outer wall stress difference under thermal striping for large heat flux by Eq. (19) & (13)

r_m/W	x_1	(a) $\Delta\sigma_{max}$ / $\Delta\sigma_0$	F_o for (a)	Ω for (a)	(b) $\Delta\sigma_{max, app}$ / $\Delta\sigma_0$	(b)/(a)
10	14.621	1.143	0.00171	130.5	1.261	1.103
6	8.350	1.140	0.00516	43.3	1.254	1.100
1	0.625	1.106	0.732	0.311	1.168	1.056

Table 2 Comparison of upper limit of SIF range ΔK_{cylmax} under thermal striping with that by our simplified method $2K_{max}$ (Eq. (12)) for long cylinder of βH =10

r_m/W	a/W	0.1	0.2	0.3	0.4	0.5
10	(a) $\Delta K_{cylmax}/\Delta K_{\Delta T}$	0.246	0.292	0.313	0.327	0.329
	Ω for (a)	198	140	122	116	111
	(b) $2K_{max}/\Delta K_{\Delta T}$	0.299	0.359	0.385	0.397	0.398
	(b)/(a)	1.215	1.229	1.230	1.214	1.210
6	(a) $\Delta K_{cylmax}/\Delta K_{\Delta T}$	0.245	0.285	0.296	0.297	0.286
	Ω for (a)	64.5	45.8	41.0	37.9	36.0
	(b) $2K_{max}/\Delta K_{\Delta T}$	0.297	0.349	0.365	0.362	0.341
	(b)/(a)	1.212	1.225	1.233	1.219	1.192
1	(a) $\Delta K_{cylmax}/\Delta K_{\Delta T}$	0.234	0.231	0.198	0.163	0.127
	Ω for (a)	0.367	0.293	0.255	0.243	0.227
	(b) $2K_{max}/\Delta K_{\Delta T}$	0.273	0.282	0.250	0.206	0.156
	(b)/(a)	1.167	1.221	1.263	1.264	1.228

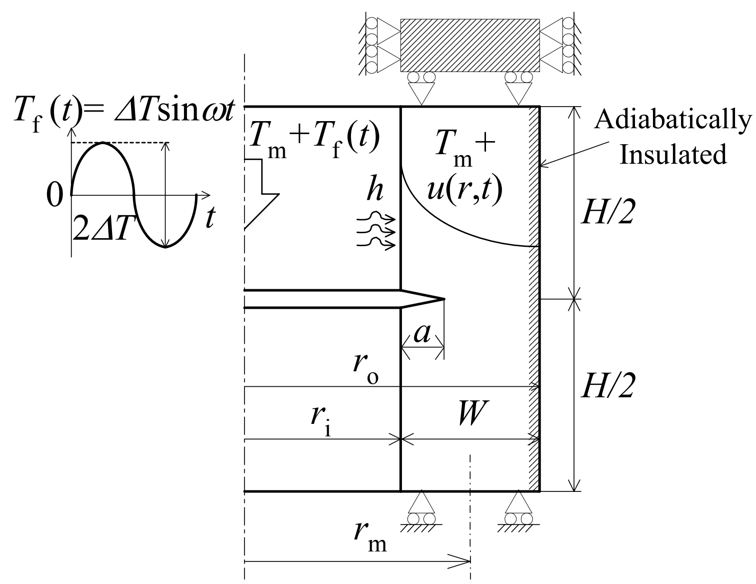

Fig. 1 Circumferentially cracked cylinder under thermal striping

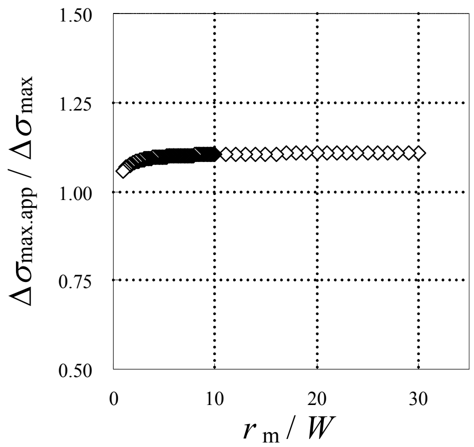

Fig. 2 Comparison of $\Delta\sigma_{max.app}$ and $\Delta\sigma_{max}$ for wide range of r_m/W