T. Meshii et al., Engineering Fracture Mechanics, Vol. 68, No. 8, pp. 975-986 (2001. 5).

Stress Intensity Factor for a Circumferential Crack in a Finite-Length Thin to Thick Walled Cylinder under an Arbitrary Biquadratic Stress Distribution on the Crack Surfaces

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Abstract

This paper presents the development of a practical method, by using prepared tabulated data, to calculate the mode I stress intensity factor (SIF) for an inner surface circumferential crack in a finite length cylinder. The crack surfaces are subjected to an axisymmetric stress with an arbitrary biquadratic radial distribution. The method was derived by applying the authors' weight function for the crack. This work is based on the thin shell theory and the Petroski-Achenbach method. Our method is valid over a wide range of mean radius to wall thickness ratio, $R_m/W \geq 1$, and for relatively short cracks with $a/W \leq 0.5$. The difference between the SIF obtained by our method for the geometry and that from finite element analysis is within 5%. The method we developed describes the effect that cylinder length gives on the SIF. This effect needs to be considered for cylinders shorter than non-dimensional cylinder length $\beta H \leq 5$.

Key words: Fracture mechanics; Stress intensity factor; Circumferential crack; Weight function; Cylinder

1. Introduction

The stress intensity factor (SIF) of an inner surface circumferential crack in a cylinder is one of the fundamental quantities for evaluating the structural integrity of cracked pressure vessels. Under an assumption of infinite lengths, Nied [1] obtained analytical SIF solutions to uniform and linear stress distribution on the crack surfaces, and Wu [2] and Fett [3] gave a weight function for evaluating the SIF of the crack loaded with an arbitrary stress distribution on its surfaces. However, the infinite length assumption is not valid for a good many actual pressure vessels that have finite lengths. For this reason, we developed a weight function for the crack that enables evaluation of the influence of cylinder length on the SIF. It is based on the theory of the cylindrical shell and compliance [4]. As a result of the SIF calculation using our weight function, we found that the SIF of the crack for short cylinders is larger than for long cylinders when the annular cross-section of a cylinder and the applied stress distribution are the same. This indicates that the design engineers may be erring on the dangerous side, if they apply a SIF solution for the crack obtained under the assumption of infinite cylinder length. Thus we think that it is useful to present our weight function which allows evaluation of the SIF including the effect of cylinder length in a ready-to-use form.

Since our weight function is based on the cylindrical shell theory, there is an applicable range for the mean radius to wall thickness ratio R_m/W that comes from the theory. A tentative guideline is given as $R_m/W \geq 5.5$ [5] (for non-dimensional crack length $a/W \leq 0.6$). This number is based on concrete numerical evaluation, though it may be argued that $R_m/W = 5.5$ is out of the application range according to the shell theory. There is the possibility of extending the range of application by adding more rigorous numerical studies and this will be useful if the weight function is applicable to more thick(er)-walled cylinders.

In this paper, the applicable application range of our weight function for the crack is examined rigorously via comparisons between the SIF obtained by our weight function and those obtained by Finite Element Analysis (FEA). Subsequently, a method that enables immediate SIF evaluation with prepared tabulated data for the crack under an arbitrary biquadratic stress distribution on the crack surfaces is presented. The method is applicable to structures within the new application range.

2. Weight function for a circumferential crack in a finite-length cylinder

We derived a closed form SIF equation of a circumferential crack in a finite-length cylinder corresponding to an axisymmetric bending problem as shown in Fig. 1(a) as a product of ψ_f, a geometrically defined parameter described in the Appendix and K_M, SIF of an infinite length edge-cracked beam under pure bending moment M_0 [5], [6].

$$K_{M0}(a)=\psi_f \cdot K_M = \psi_f \cdot \left[\frac{M_0}{Z} \sqrt{\pi a} \cdot F_M(a/W) \right] \tag{1}$$

Here, $Z = W^2/6$ is the section modulus and F_M is the infinite length edge-cracked beam's correction factor for finite width and under pure bending. The moment M_0 is related to the linear stress distribution $\sigma(x)$ by Eq. (2), whose inner and outer stress difference is $2\sigma_0$, and is given by Eq. (3), taking the curvature effect into account. M_0 as well as Z is a quantity per unit circumferential length.

$$\sigma(x) = \sigma_0 \cdot \left[1 + \frac{W}{6R_\mathrm{m}} - 2\frac{x}{W} \right] \tag{2}$$

$$\frac{M_0}{Z} = \sigma_0 \cdot \left[1 - \frac{1}{12}\left(\frac{W}{R_\mathrm{m}}\right)^2 \right] \Big/ \left[1 + \frac{1}{12}\left(\frac{W}{R_\mathrm{m}}\right)^2 \right] \tag{3}$$

The first step in the derivation of Eq. (1) was to replace the cylindrical shell by a beam on an elastic foundation and the cracked part by a rotary spring (whose compliance is equal to the compliance increment $\Delta\lambda$ due to the presence of a crack for an infinitely long beam under pure bending). Then through some further steps, we found that the approximate value of the desired SIF could be evaluated as the SIF of an edge cracked beam under pure bending ($\psi_\mathrm{f} \cdot M_0$). The validity of the equation was shown in our previous paper [5].

On the other hand, our weight function to evaluate the SIF corresponding to a problem shown in Fig. 1(b) was derived by applying the Petroski-Achenbach method to the SIF solution K_{M0} given by Eq. (1) and taking the crack opening displacement corresponding to Eq. (1) $v(x; a)$ as

$$v(x;a) = \frac{8}{E'}\sqrt{\frac{a-x}{2\pi}} \cdot \left[K_{M0}(a) - C_3 \cdot (a-x) \right] \tag{4}$$

where x = 0 shows the mouth point. Here E' = E (Young's modulus) for plane stress and E'=E/(1-ν^2) for plane strain, with ν being Poisson's ratio. Coefficient C_3 was obtained from Eq. (4) as follows by using crack mouth opening displacement (CMOD), $\delta(a)$: v at x = 0.

$$C_3(a) = \frac{1}{a}\left(K_{M0}(a) - \frac{E'}{8}\sqrt{\frac{2\pi}{a}} \cdot \delta(a) \right) \tag{5}$$

Taking into account that CMOD of an edge-cracked strip under pure bending M_0, δ_beam, is given by a closed form equation [7] and that $\delta(a) = \psi_f \cdot \delta_{beam}(a)$ is analogous to Eq. (1), all the unknowns in Eq. (4) and (5) were determined.

By substituting $v(x; a)$ into Eq. (6), the desired weight function $w(x; a)$ was obtained. By applying this $w(x; a)$ to Eq. (7), the SIF of the crack under an arbitrary stress distribution $\sigma(x)$ on the crack surface (Fig.1(b)) can be evaluated.

$$w(x;a) \equiv \frac{E'}{2K_{M0}(a)} \cdot \frac{\partial v(x;a)}{\partial a} \tag{6}$$

$$K(a) = \int_0^a \frac{R_i + x}{R_i + a} \sigma(x) \cdot w(x;a) dx \tag{7}$$

Here, R_i is the inner radius of the cylinder. The formal definition of w is as follows [4]:

$$\begin{aligned} & w(x;a) \cdot [\sqrt{\pi} a^2 W \sqrt{a-x} \cdot \psi_f \cdot F_M] \\ & = \sqrt{2x} \cdot F_M \cdot (Wx \cdot \psi_f + 2a(a-x) \cdot \frac{\partial \psi_f}{\partial \xi}) \\ & + (a-x) \times \{2a \cdot \psi_f \cdot (\sqrt{2x} \cdot \frac{\partial F_M}{\partial \xi} + (a-x) \cdot \frac{\partial V}{\partial \xi}) + V \cdot [W(2a+x) \cdot \psi_f + 2a(a-x) \cdot \frac{\partial \psi_f}{\partial \xi}]\} \end{aligned} \tag{8}$$

where $\xi = a/W$ and V in Eq. (8) is a non-dimensional function of ξ that defines δ_{beam} in Eq. (9) [7].

$$\delta_{beam}(a) = \frac{4M_0}{ZE'} \cdot a \cdot V(\xi) \tag{9}$$

The actual formulation of F_M, ψ_f and V used for numerical studies in the following sections is given in the Appendix.

3. Application range of the weight function for circumferential crack in finite length cylinders

3.1 *Effect of mean radius to wall thickness ratio R_m/W on the SIF*

Since our weight function is based on the cylindrical shell theory, there is an applicable range for the mean radius to wall thickness ratio R_m/W that comes out of the theory. A tentative guideline is given as

$R_m/W \geq 5.5$ [6] (for non-dimensional crack length $a/W \leq 0.6$) as a result of numerical studies, although $R_m/W = 5.5$ may be too small to apply the shell theory. It may be possible to extend the range of application by adding more rigorous numerical studies.

First, we studied the practical application limit of our weight function from the standpoint of R_m/W. A circumferentially cracked long cylinder under uniform tension σ_m was chosen as a benchmark problem for this purpose, because an analytical solution, K_{anal}, has been obtained under an infinite cylinder length assumption by Nied [1]. SIF evaluated by our weight function, K_{Meshii}, was compared with K_{anal} for the crack with a/W = 0.1, 0.2, 0.3, 0.4, 0.5 in a cylinder of R_m/W = 0.61, 0.75, 0.93, 1.17, 4.50, 9.50 (inner/outer radius ratio R_i/R_o = 0.1, 0.2, 0.3, 0.4, 0.8, 0.9), and summarized in Fig. 2. In all cases we used σ_m = 9.8 MPa, wall thickness W = 10 mm, cylinder length H=150 mm, Young's modulus E=206 GPa and Poisson's ratio $\nu = 0.3$.

It can be seen from Fig. 2 that $R_m/W \geq 0.93$ is necessary in order to make the difference between K_{Meshii} and K_{anal} be 5% or less for the evaluated case. Therefore, we chose R_m/W =1 as a tentative candidate of the application limit and proceeded to further studies. The range apparently exceeds the one expected from the cylindrical shell theory.

Next we investigated the case where the length of the cylinder is finite. In this case, the solutions from our method are compared with those from FEA as there are no analytical solutions. We first looked at the accuracy of our FEA solutions before examining the effect of the cylinder length on the SIF. A circumferentially cracked long cylinder (H = 150 mm) under uniform tension σ_m = 9.8 MPa was chosen again as a benchmark problem. K_{FEA}: SIF evaluated from FEA results was compared with K_{anal}: Nied's analytical solution [1] (obtained with the assumption of infinite cylinder length) for cracks with a/W = 0.1, 0.2, 0.3, 0.4, 0.5 in a cylinder of R_m/W =9.5, 4.5, 2, 1 (R_i/R_o = 0.9, 0.8, 0.6, 0.33). E and ν are identical to those in the previous cases. SIF by FEA was evaluated by applying displacement correlation technique (DCT) to the displacements of the crack tip singular elements. The final SIF solution K_{FEA} is the converged value obtained through the process of singular element refinement. K_{anal} for R_i/R_o = 0.33 was interpolated from the values for R_i/R_o = 0.3 and 0.4.

The difference between K_{FEA} and K_{anal} was 1% or less (max. 0.86%) for cases shown in Fig. 3, thus the validity of K_{FEA} was confirmed. An identical procedure will be used to evaluate K_{FEA} in the following paragraph.

3. 2 *Effect of the cylinder length*

In the above study we chose H/W= 15 intuitively, for cylinders with various R_m/W, to satisfy the long cylinder assumption from the practical point of view. However, if we start to work on giving a SIF solution given in a table, it will be useful to know the specific cylinder length that satisfies the infinite length assumption. Labbens et al. [8] pointed out that the infinite length assumption is satisfied for thin cylinders with length of $\beta H \geq 5$ (β is a quantity which is used in replacing the cylindrical shell by a beam on an elastic foundation and its definition is given in the Appendix). We studied the cylinder length that satisfies the infinite length assumption, including thick-walled cylinders. In this study, we also checked to see if the tentatively set application range $R_m/W \geq 1$ for our weight function is valid for finite length cylinders.

SIF changes due to varying H/W = 4, 10 and 15 for cracks of a/W = 0.1, 0.3 and 0.5 in cylinders of R_m/W = 1, 2, 4.5 and 9.5, respectively, were evaluated for three stress distributions on the crack surfaces; i) uniform stress distribution σ_m = 9.8 MPa, ii) linear stress distribution whose inner and outer stress difference $2\sigma_0$=19.6 MPa, iii) quadratic stress distribution $\sigma(x) = 98\,(1 - x/W)^2$ MPa. Studies on cases of $(R_m/W, H/W)$ =(9.5, 12), (4.5, 8), (2, 6) were added to those above, because we thought $\beta H = 5$ to be a key value. We also added the case of $(R_m/W, H/W)$ = (1, 2) as a data for $\beta H < 5$.

For each case, K_{FEA}: SIF obtained from FEA+DCT and K_{Fett}: SIF evaluated from Fett's weight function were compared with K_{Meshii}: SIF by our weight function (Eq. (8)). This is shown in Figs. 4-6. In these figures, we chose K_{Meshii} as a reference value instead of K_{FEA} because we thought continuity to be the preferable characteristics for the value. K_{Fett}, also a candidate for the reference value in this respect, was excluded because it does not properly evaluate the effect of cylinder length on the SIF.

Note that we also evaluated the SIF by Wu's weight function [2] but it was not included in the figures, because $R_m/W \leq 2$ is out of their application range. Nied's analytical solutions (under infinite cylinder length assumption) K_{Nied} for uniform stress distribution [1] was also excluded from the figures, after we confirmed the difference between K_{Nied} and K_{Fett} as below 1% for the cases we studied.

From Figs. 4-6, it is very clear that cylinder length affects the SIF. If we focus our attention on K_{FEA}, we see that the difference between K_{Meshii} and K_{FEA} is as small as 5%, or less, for the examined cases, which suggests that K_{Meshii} can account for the cylinder length accurately enough for practical use. We emphasize that this is also true for thick-walled cylinders up to $R_m/W = 1$. As we checked the validity of K_{Meshii}, the application range of the infinite cylinder length assumption can now be read from the difference between K_{Fett} and K_{Meshii}. From Figs. 4-6, a guideline $\beta H \geq 5$ (which Labbens et al. [8] proposed for thin-walled cylinders) seems to be valid for thick-walled cylinders up to $R_m/W = 1$, though parameter β has no more meaning than a formal expression for thick-walled cylinders.

Note that there exists a minimum cylinder length for the difference between K_{FEA} and K_{Meshii} in order to satisfy the value of 5% or less. If we select $\beta H \geq 2.5$ as this minimum cylinder length, we can see from Figs. 4-6 that the desired accuracy can be expected for thick to thin cylinders including $R_m/W = 1$. In this case, better accuracy is expected for thin-walled cylinders. In summary, from a practical standpoint our weight function is applicable for $R_m/W \geq 1$, $0.1 \leq a/W \leq 0.5$ and $\beta H \geq 2.5$.

4. SIF for an inner surface circumferential crack in a finite-length cylinder

Now that the application range of our weight function has become clear from the studies above, we will present our weight function in more practical form for engineers. We will derive a method which enables immediate SIF evaluation for the crack in cylinders satisfying the applicable range. That is, we will give a dimensionless geometric function K_j in tables, which gives SIF $K(a)$ for a circumferential crack subjected to crack face loading $\sigma(x)$ defined by the following equation.

$$\sigma(x)=\sum_{j=0}^{4}\sigma_j\left(\frac{x}{W}\right)^j;\quad K(a)=\sqrt{W}\sum_{j=0}^{4}\sigma_j K_j \tag{10}$$

Here, $K_j(j$=0~4), $k_j(j$=0~5) was defined as follows.

$$K_j=\frac{R_\mathrm{i}k_j+Wk_{j+1}}{\sqrt{W}(R_\mathrm{i}+a)};\ k_j=\frac{1}{W^j}\int_0^a x^j w(x;a)dx \tag{11}$$

K_j is a non-dimensional function of dimensionless parameters $\xi = a/W$, βW (eventually R_m/W) and βH. K_j for various ξ, βW and βH are shown in Tables 1-6.

Note that the non-dimensional cylinder length βH can be written as $\beta H = [3(1-\nu^2)]^{1/4}(H/W)/(R_\mathrm{m}/W)^{1/2}$ according to the definition of β in the Appendix and that it is a function of R_m/W. As K_j (j = 0~4) for βH = 5 coincides with those for βH = 10 to two significant figures as in Tables 1-6, $\beta H \geq 5$ as an infinite cylinder length criteria was reconfirmed for various R_m/W.

When we examine K_j for a specific geometry, we can see that $K_j > K_{j+1}$ for j = 0~3. In this case, K_j (j = 2~4) corresponding to the higher order stress term σ_j (j = 2~4) was less than approximately one-tenth of K_0. This suggests the fact that the error in SIF due to stress linearization (popular for design engineers) is about 10%.

5. Conclusions

This paper presents a method to calculate the mode I SIF with prepared tabulated data for an inner surface circumferential crack in a finite-length cylinder. The cracks are subjected to axisymmetric stress with an arbitrary biquadratic radial distribution. The method can take into account the effect of various geometric parameters (including cylinder length) on the SIF. The method, derived by applying the authors' weight function for the crack, was expected to be restricted according to the theory of cylindrical shell. However, difference in SIF obtained from our method and FEA for a geometry satisfying $R_m/W \geq 1$, $0.1 \leq a/W \leq 0.5$, $\beta H \geq 2.5$ was small as 5% or less. This fact suggests that our SIF method is applicable to wider range of geometry than expected. Moreover, validity of an infinite cylinder length criteria $\beta H \geq 5$ (which was proposed for thin-walled cylinders) for thick-walled cylinders up to $R_m/W = 1$ was shown, though parameter β has no more meaning than a formal expression for thick-walled cylinders.

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Appendix

β which appears in the text of this paper is a parameter defined in replacing the problem of a cylinder with axisymmetric bending loads on its edges (Fig. A1 left) with the problem of a beam on an elastic foundation with bending loads on its ends (Fig. A1 right). β is defined as follows, by formally writing the flexural rigidity of the prismatic beam as $D = EW^3/12(1-\nu^2)$ [10],

$$\beta^4 = \frac{EW}{4R_\mathrm{m}^2 D} = \frac{3(1-\nu^2)}{(R_\mathrm{m}W)^2} \tag{A1}$$

where R_m : mean radius, W : wall thickness, E : Young's modulus and ν : Poisson's ratio. Note that $1/\beta$ has the dimension of length.

Actual ψ_f [5], F_M, V [7] and $\Delta\lambda$ [9] (used to calculate ψ_f) used in numerical studies are as follows.:

$$\psi_\mathrm{f} = \frac{\sinh \beta H + \sin \beta H}{\sinh \beta H + \sin \beta H + \beta D \cdot \Delta\lambda(\cosh \beta H + \cos \beta H - 2)} \tag{A2}$$

$$F_M(\xi) = \sqrt{\frac{2}{\pi\xi}\tan\frac{\pi\xi}{2}} \cdot \frac{0.923 + 0.199\{1 - \sin(\pi\xi/2)\}^4}{\cos(\pi\xi/2)} \tag{A3}$$

$$V(\xi) = 0.8 - 1.7\xi + 2.4\xi^2 + \frac{0.66}{(1-\xi)^2} \tag{A4}$$

$$\Delta\lambda(\xi) = \frac{\pi(1.1215)^2}{2E'} \cdot \frac{\xi^2\left[1 + \xi(1-\xi)(0.44 + 0.25\xi)\right]}{(1-\xi)^2(1+2\xi)^2} \cdot \left(\frac{6}{W}\right)^2 \tag{A5}$$

List of tables and figures

Fig. 5 Effect of cylinder length on SIF under linear stress distribution ($\nu = 0.3$)

Fig. 6 Effect of cylinder length on SIF under quadratic stress distribution ($\nu = 0.3$)

Fig. A1 Replacement of axisymmetric bending problem of a cylinder by a beam on an elastic foundation

Table 1 Non-dimensional stress intensity factor for circumferential crack in a finite length cylinder

(R_m/W =10, ν = 0.3)

βH	ξ	K_0	K_1	K_2	K_3	K_4
10	0.1	0.658414	0.039620	0.003027	0.000253	0.000022
	0.2	0.996956	0.116991	0.017668	0.002931	0.000510
	0.3	1.358750	0.230173	0.051177	0.012598	0.003267
	0.4	1.777830	0.384610	0.111557	0.036146	0.012391
	0.5	2.240890	0.582462	0.206808	0.082710	0.035146
5	0.1	0.658547	0.039625	0.003028	0.000253	0.000022
	0.2	0.997654	0.117047	0.017674	0.002932	0.000510
	0.3	1.360640	0.230394	0.051214	0.012606	0.003268
	0.4	1.781770	0.385215	0.111692	0.036182	0.012401
	0.5	2.247970	0.583792	0.207174	0.082829	0.035189
4	0.1	0.659352	0.039657	0.003030	0.000253	0.000022
	0.2	1.001880	0.117382	0.017712	0.002937	0.000511
	0.3	1.372140	0.231741	0.051442	0.012651	0.003278
	0.4	1.806000	0.388931	0.112520	0.036399	0.012464
	0.5	2.291910	0.592050	0.209448	0.083568	0.035453
3	0.1	0.662462	0.039782	0.003037	0.000253	0.000022
	0.2	1.018500	0.118697	0.017861	0.002957	0.000514
	0.3	1.418490	0.237167	0.052359	0.012833	0.003317
	0.4	1.907100	0.404438	0.115979	0.037307	0.012725
	0.5	2.483590	0.628071	0.219368	0.086796	0.036606

Table 2 Non-dimensional stress intensity factor for circumferential crack in a finite length (R_m/W =8, ν = 0.3)

βH	ξ	K_0	K_1	K_2	K_3	K_4
10	0.1	0.656131	0.039532	0.003022	0.000253	0.000022
	0.2	0.986629	0.116195	0.017578	0.002919	0.000509
	0.3	1.332410	0.227166	0.050675	0.012500	0.003244
	0.4	1.724800	0.376676	0.109812	0.035692	0.012261
	0.5	2.148590	0.565535	0.202209	0.081226	0.034619
5	0.1	0.656279	0.039537	0.003023	0.000253	0.000022
	0.2	0.987394	0.116256	0.017585	0.002920	0.000509
	0.3	1.334430	0.227403	0.050715	0.012508	0.003247
	0.4	1.728910	0.377309	0.109953	0.035729	0.012272
	0.5	2.155750	0.566883	0.202581	0.081347	0.034662
4	0.1	0.657174	0.039573	0.003025	0.000253	0.000022
	0.2	0.992031	0.116623	0.017627	0.002926	0.000510
	0.3	1.346790	0.228851	0.050960	0.012556	0.003258
	0.4	1.754250	0.381202	0.110822	0.035958	0.012338
	0.5	2.200250	0.575265	0.204892	0.082100	0.034932
3	0.1	0.660632	0.039712	0.003033	0.000253	0.000022
	0.2	1.010280	0.118070	0.017792	0.002948	0.000513
	0.3	1.396780	0.234711	0.051952	0.012753	0.003300
	0.4	1.860680	0.397558	0.114474	0.036916	0.012613
	0.5	2.396080	0.612150	0.215064	0.085412	0.036116

Table 3 Non-dimensional stress intensity factor for circumferential crack in a finite length cylinder

(R_m/W =6, ν = 0.3)

βH	ξ	K_0	K_1	K_2	K_3	K_4
10	0.1	0.652631	0.039397	0.003015	0.000252	0.000022
	0.2	0.971336	0.115018	0.017447	0.002902	0.000506
	0.3	1.294590	0.222854	0.049957	0.012359	0.003215
	0.4	1.651180	0.365675	0.107393	0.035063	0.012081
	0.5	2.025170	0.542916	0.196067	0.079246	0.033916
5	0.1	0.652799	0.039403	0.003015	0.000252	0.000022
	0.2	0.972189	0.115086	0.017455	0.002903	0.000506
	0.3	1.296780	0.223111	0.050001	0.012367	0.003217
	0.4	1.655470	0.366336	0.107541	0.035102	0.012092
	0.5	2.032300	0.544265	0.196440	0.079367	0.033959
4	0.1	0.653825	0.039445	0.003018	0.000252	0.000022
	0.2	0.977390	0.115499	0.017502	0.002909	0.000507
	0.3	1.310230	0.224692	0.050268	0.012421	0.003228
	0.4	1.682050	0.370434	0.108457	0.035343	0.012162
	0.5	2.077020	0.552721	0.198777	0.080129	0.034232
3	0.1	0.657768	0.039603	0.003027	0.000253	0.000022
	0.2	0.997819	0.117121	0.017686	0.002934	0.000511
	0.3	1.364730	0.231097	0.051353	0.012636	0.003275
	0.4	1.794210	0.387723	0.112324	0.036360	0.012455
	0.5	2.275030	0.590167	0.209125	0.083503	0.035440

Table 4 Non-dimensional stress intensity factor for circumferential crack in a finite length cylinder

(R_m/W =4, ν = 0.3)

βH	ξ	K_0	K_1	K_2	K_3	K_4
10	0.1	0.646295	0.039154	0.003001	0.000251	0.000022
	0.2	0.945175	0.113011	0.017223	0.002873	0.000502
	0.3	1.232960	0.215839	0.048789	0.012130	0.003166
	0.4	1.537080	0.348638	0.103648	0.034089	0.011803
	0.5	1.843660	0.509664	0.187037	0.076334	0.032881
5	0.1	0.646499	0.039162	0.003002	0.000251	0.000022
	0.2	0.946165	0.113090	0.017232	0.002874	0.000502
	0.3	1.235380	0.216125	0.048838	0.012139	0.003168
	0.4	1.541540	0.349331	0.103804	0.034130	0.011815
	0.5	1.850630	0.510991	0.187406	0.076454	0.032924
4	0.1	0.647735	0.039212	0.003005	0.000251	0.000022
	0.2	0.952217	0.113572	0.017287	0.002881	0.000503
	0.3	1.250290	0.217886	0.049137	0.012199	0.003181
	0.4	1.569340	0.353645	0.104772	0.034386	0.011888
	0.5	1.894460	0.519349	0.189726	0.077213	0.033196
3	0.1	0.652487	0.039403	0.003016	0.000252	0.000022
	0.2	0.976075	0.115473	0.017503	0.002910	0.000507
	0.3	1.311220	0.225083	0.050360	0.012442	0.003233
	0.4	1.688160	0.372084	0.108912	0.035476	0.012203
	0.5	2.091590	0.556934	0.200161	0.080625	0.034420

Table 5 Non-dimensional stress intensity factor for circumferential crack in a finite length cylinder

(R_m/W =2, ν = 0.3)

βH	ξ	K_0	K_1	K_2	K_3	K_4
10	0.1	0.629187	0.038504	0.002965	0.000249	0.000022
	0.2	0.882734	0.108241	0.016691	0.002803	0.000492
	0.3	1.099730	0.200687	0.046268	0.011635	0.003059
	0.4	1.312210	0.315015	0.096250	0.032164	0.011252
	0.5	1.515970	0.449442	0.170647	0.071039	0.030998
5	0.1	0.629462	0.038516	0.002966	0.000249	0.000022
	0.2	0.883961	0.108340	0.016703	0.002804	0.000492
	0.3	1.102430	0.201010	0.046323	0.011646	0.003061
	0.4	1.316630	0.315717	0.096410	0.032206	0.011264
	0.5	1.522090	0.450640	0.170985	0.071151	0.031038
4	0.1	0.631123	0.038583	0.002970	0.000249	0.000022
	0.2	0.891442	0.108943	0.016772	0.002814	0.000493
	0.3	1.119010	0.203004	0.046665	0.011714	0.003076
	0.4	1.344140	0.320083	0.097402	0.032470	0.011341
	0.5	1.560590	0.458184	0.173110	0.071852	0.031291
3	0.1	0.637582	0.038844	0.002985	0.000250	0.000022
	0.2	0.921525	0.111369	0.017050	0.002851	0.000499
	0.3	1.188810	0.211392	0.048103	0.012002	0.003139
	0.4	1.466050	0.339433	0.101799	0.033637	0.011679
	0.5	1.740490	0.493430	0.183040	0.075129	0.032473

Table 6 Non-dimensional stress intensity factor for circumferential crack in a finite length cylinder

(R_m/W =1, ν = 0.3)

βH	ξ	K_0	K_1	K_2	K_3	K_4
10	0.1	0.592321	0.037127	0.002890	0.000244	0.000021
	0.2	0.778424	0.100306	0.015809	0.002687	0.000475
	0.3	0.914655	0.179523	0.042732	0.010939	0.002909
	0.4	1.042680	0.274248	0.087213	0.029799	0.010573
	0.5	1.168000	0.384456	0.152780	0.065228	0.028920
5	0.1	0.592670	0.037141	0.002890	0.000244	0.000021
	0.2	0.779786	0.100420	0.015822	0.002689	0.000475
	0.3	0.917238	0.179849	0.042790	0.010950	0.002911
	0.4	1.046360	0.274869	0.087358	0.029839	0.010585
	0.5	1.172490	0.385400	0.153055	0.065320	0.028953
4	0.1	0.594786	0.037228	0.002895	0.000244	0.000021
	0.2	0.788120	0.101116	0.015904	0.002700	0.000477
	0.3	0.933245	0.181870	0.043145	0.011023	0.002927
	0.4	1.069460	0.278763	0.088270	0.030085	0.010657
	0.5	1.200970	0.391381	0.154800	0.065908	0.029169
3	0.1	0.603064	0.037570	0.002915	0.000246	0.000022
	0.2	0.822199	0.103966	0.016237	0.002745	0.000484
	0.3	1.002390	0.190601	0.044681	0.011334	0.002996
	0.4	1.174990	0.296553	0.092439	0.031213	0.010988
	0.5	1.338010	0.420166	0.163199	0.068739	0.030204

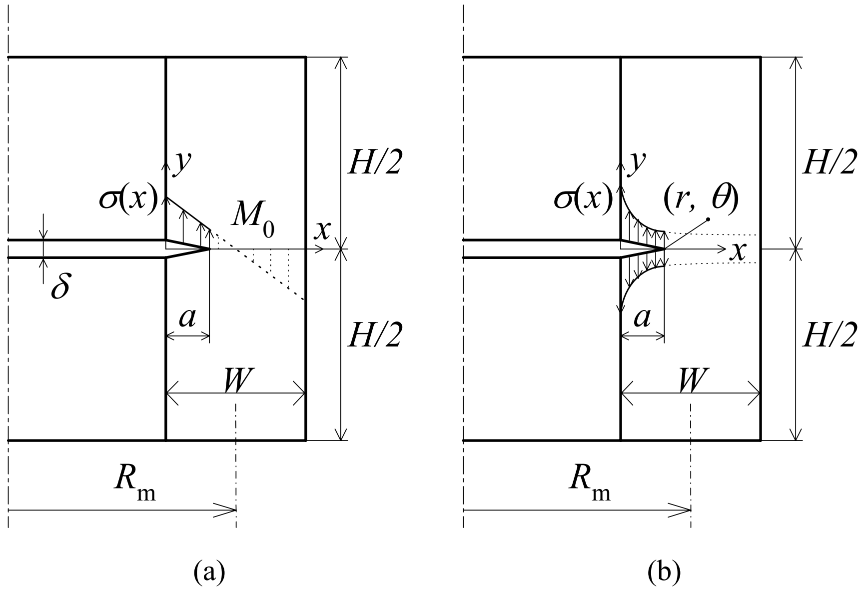

Fig. 1 A cylinder with a circumferential crack under arbitrary stress distribution

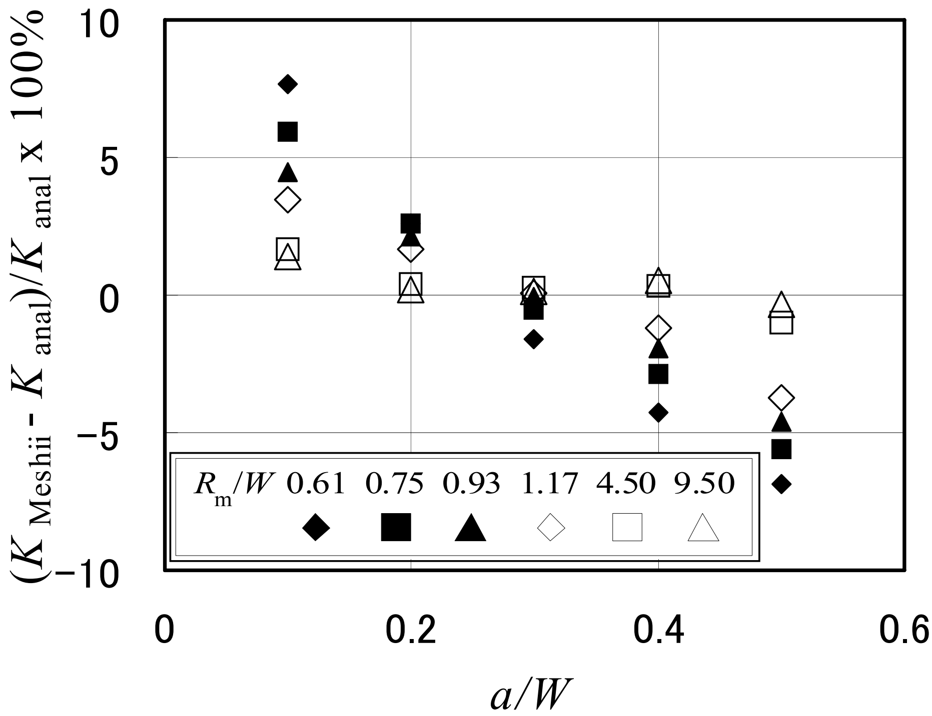

Fig. 2 Effect of R_m/W on SIF under uniform stress distribution ($H/W = 15$, $\nu = 0.3$)

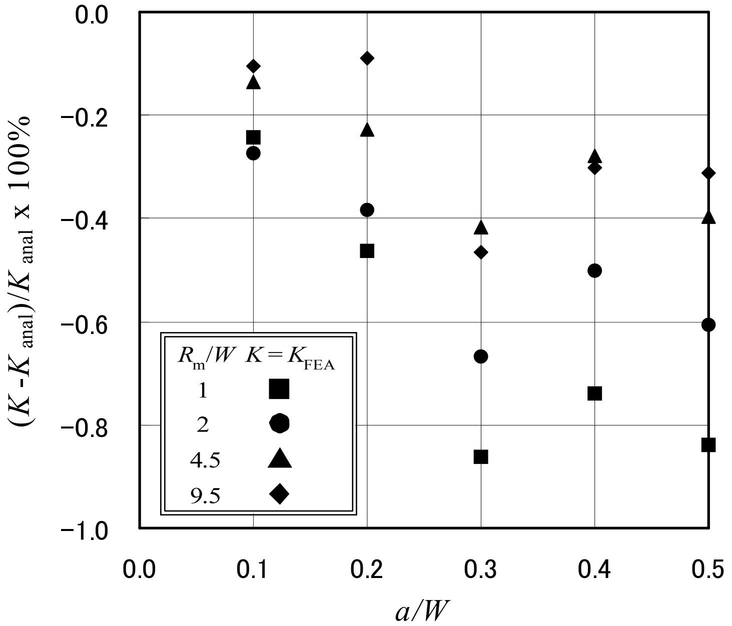

Fig. 3 Comparison of SIF of circumferential crack in a long cylinder under uniform stress (H/W =15, ν= 0.3)

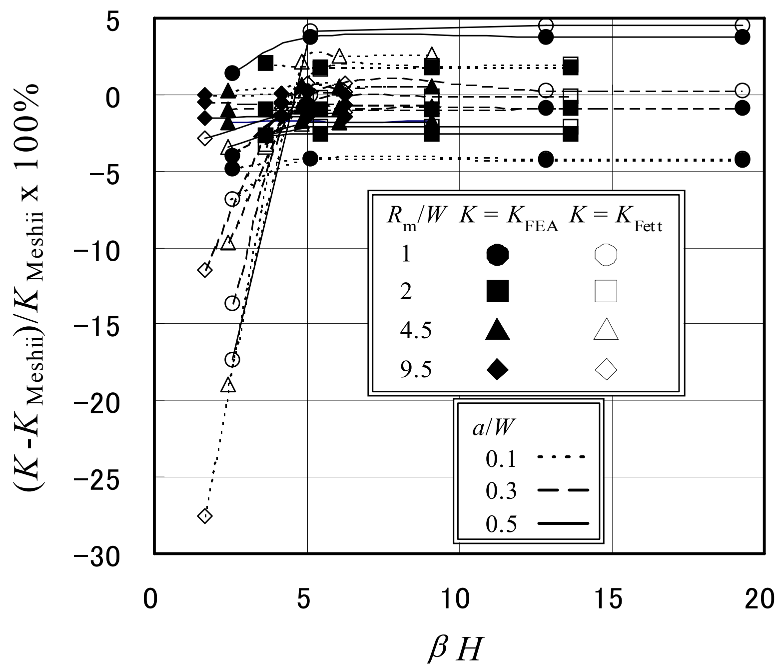

Fig. 4 Effect of cylinder length on SIF under uniform stress distribution (ν = 0.3)

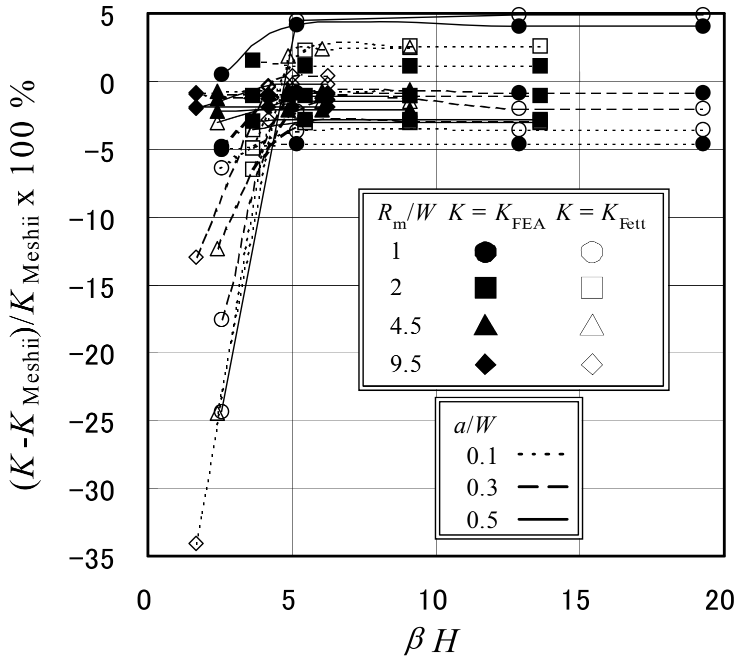

Fig. 5 Effect of cylinder length on SIF under linear stress distribution ($\nu = 0.3$)

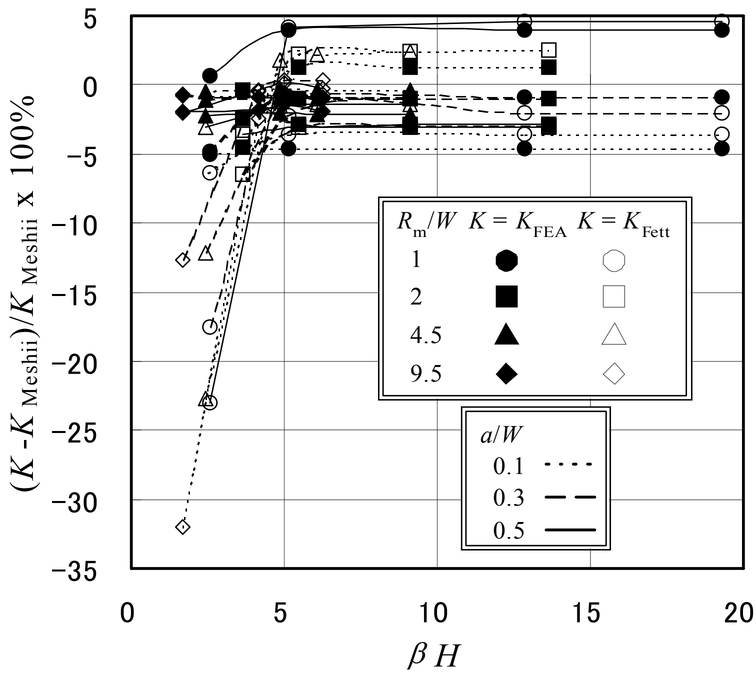

Fig. 6 Effect of cylinder length on SIF under quadratic stress distribution ($\nu = 0.3$)

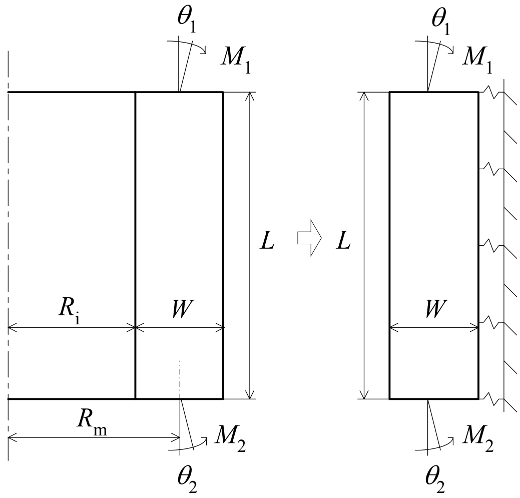

Fig. A1 Replacement of axisymmetric bending problem of a cylinder by a beam on an elastic foundation