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Article:

Zheng, F C and Burr, A G orcid.org/0000-0001-6435-3962 (2005) Signal detection for orthogonal space-time block coding over time-selective fading channels: A PIC approach for the G(i) systems. *IEEE Transactions on Communications*. pp. 969-972. ISSN 0090-6778

<https://doi.org/10.1109/TCOMM.2005.849990>

Signal Detection for Orthogonal Space-Time Block Coding Over Time-Selective Fading Channels: A PIC Approach for the \mathcal{G}_i Systems

F. -C. Zheng and A. G. Burr

Abstract—One major assumption in all orthogonal space-time block coding (O-STBC) schemes is that the channel remains static over the entire length of the codeword. However, time selective fading channels do exist, and in such case the conventional O-STBC detectors can suffer from a large error floor in the high signal-to-noise ratio (SNR) cases. This paper addresses such an issue by introducing a parallel interference cancellation (PIC) based detector for the \mathcal{G}_i coded systems ($i = 3$ and 4).

Index Terms—Orthogonal space-time block coding (O-STBC), parallel interference cancellation (PIC), time selective fading channels, wireless communications.

I. INTRODUCTION

ORTHOGONAL space-time block coding (O-STBC) technology has attracted enormous interest due to its high diversity order and low decoding complexity [2], [3]. The low decoding complexity of O-STBC is directly due to the linear maximum-likelihood (ML) decoder at the receiver. The linear ML decoder, however, relies on the so-called “quasi-static channel” assumption: the channel remains static over the length of the entire codeword: $2T_s$ for the two transmit antenna (2-Tx) STBC (i.e., \mathcal{G}_2 – the Alamouti code [1]), $4T_s$ for the \mathcal{H}_i systems, and $8T_s$ for the \mathcal{G}_i systems ($i = 3$ for 3-Tx, $i = 4$ for 4-Tx, where T_s is the symbol period. For the definition of \mathcal{H}_i and \mathcal{G}_i , see [3]). While such an assumption is reasonable in most cases, time selective or fast fading channels do exist in practice, even for the 2-Tx case (see [4]–[6] and the references therein). In these scenarios, the channel state varies from symbol to symbol. Clearly, the 3- and 4-Tx O-STBC cases (especially the \mathcal{G}_i systems) are much more vulnerable to channel variation than the 2-Tx case due to the much longer STBC codeword.

The above channel variation will destroy the orthogonality of the channel matrix and therefore, cause inter element interference (IEI). The end effect of all this is an *irreducible error floor* in the bit error rate (BER) curves in the high signal-to-noise ratio (SNR) region. To suppress such an error floor in the 2-Tx case,

Paper approved by A. F. Naguib, the Editor for Wireless Communication of the IEEE Communications Society. Manuscript received August 3, 2003; revised November 22, 2004. This work was supported by the Engineering and Physical Sciences Research Council (EPSRC), U.K., under Grant GR/R94428/01. This paper was presented in part at the 59th IEEE Vehicular Technology Conference, Milan, Italy, May 2004.

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Digital Object Identifier 10.1109/TCOMM.2005.849990

an elegant decoder was presented in [4] and [5]. Since the decoder structure in [4] and [5] cannot be used for the 3- or 4-Tx case, a simple zero forcing (ZF) scheme for the \mathcal{G}_3 and \mathcal{G}_4 systems was proposed in [7] while a more effective version of the ZF scheme was derived in [8].

This paper proposes an alternative approach to the ZF detector in [7] and [8]. Based on the principle of parallel interference cancellation (PIC), the new detector (termed “PIC detector”) offers an even better performance than the ZF detector. The computational complexity of the PIC detector is higher than that of the conventional and the ZF detector, but is still very affordable. Only the \mathcal{G}_i coded systems are considered in this paper (the \mathcal{H}_i coded systems was addressed in [16] and [17]).

II. MODEL FOR THE \mathcal{G}_4 SYSTEM

Consider a typical \mathcal{G}_4 encoded 4-Tx O-STBC system with 4 transmit (4-Tx) and 1 receive (1-Rx) antennas. A group of four complex symbols, s_1, s_2, s_3 , and s_4 , are passed through a \mathcal{G}_4 encoder before transmitted over $8T_s$. The encoder output is therefore, a 8×4 matrix $\mathbf{C} = [c_{ij}]$, where c_{ij} is $\pm s_k$, or $\pm s_k^*$ (conjugate of s_k), and is transmitted by Tx j at time i . Also, by letting the channel gain from Tx j to the Rx at time i be $h_j(i)$, the received signal at time i is

$$r_i = \sum_{j=1}^4 h_j(i)c_{ij} + n_i \quad (1)$$

where n_i is a complex additive white Gaussian noise (AWGN) with zero mean and a variance of σ_n^2 (therefore, $0.5\sigma_n^2$ per dimension). Also, $h_j(i)$ is subject to Rayleigh fading but is normalized, i.e., $\sigma_h^2 = 1$, or $\text{Re}[h_j(i)], \text{Im}[h_j(i)] \sim N(0, 0.5)$.

The “quasi-static channel” assumption in all O-STBC schemes requires that $h_j(i) = \text{constant}$ over the entire codeword length. This, in the case of \mathcal{G}_4 and \mathcal{G}_3 systems, means that the channel remains static over $8T_s$. It has been shown in [7], [8], [14] that even under normal vehicle speeds, the assumption of “quasi-static channel” may not hold for the 4-Tx STBC (e.g., \mathcal{G}_4 and \mathcal{G}_3) systems. As such, this paper assumes that the channel is static over T_s only and from one T_s to the next, it is time variant. Clearly, this is a much more general and realistic model with the quasi-static channel now becoming a special case. Perfect channel state information (CSI) is assumed in this paper. For the estimation of CSI, see [6] and [9]–[12].

III. CONVENTIONAL O-STBC DETECTOR FOR THE \mathcal{G}_4 SYSTEM

For symbol group $\mathbf{s} = [s_1, s_2, s_3, s_4]^T$, the code matrix for \mathcal{G}_4 is [2], [3]

$$\mathbf{C} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2 & s_1 & -s_4 & s_3 \\ -s_3 & s_4 & s_1 & -s_2 \\ -s_4 & -s_3 & s_2 & s_1 \\ s_1^* & s_2^* & s_3^* & s_4^* \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3^* & s_4^* & s_1^* & -s_2^* \\ -s_4^* & -s_3^* & s_2^* & s_1^* \end{bmatrix}. \quad (2)$$

From (1), the “manipulated received signal vector” $\mathbf{r} = [r_1, \dots, r_4, r_5^*, \dots, r_8^*]^T$ can then be written as

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n} \quad (3)$$

where the channel matrix

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_{12}(1) & \mathbf{h}_{34}(1) \\ \mathbf{h}'_{34}(3) & -\mathbf{h}'_{12}(3) \\ \mathbf{h}_{12}^*(5) & \mathbf{h}_{34}^*(5) \\ -\mathbf{h}_{34}^*(7) & \mathbf{h}_{12}^*(7) \end{bmatrix} \quad (4)$$

$$\mathbf{h}_{ij}(l) = \begin{bmatrix} h_i(l) & h_j(l) \\ h_j(l+1) & -h_i(l+1) \end{bmatrix}, \quad \mathbf{h}'_{ij}(l) = \begin{bmatrix} h_i(l) & -h_j(l) \\ h_j(l+1) & h_i(l+1) \end{bmatrix}, \quad l = 1, 3, 5, 7, \text{ and} \\ \mathbf{n} = [n_1, \dots, n_4, n_5^*, \dots, n_8^*]^T.$$

At the receiver end, the conventional O-STBC detector assumes that the channel is time invariant over the entire $8T_s$ period. Regardless of any channel variation, the following expression is effectively employed:

$$\mathbf{r} = \hat{\mathbf{H}}\mathbf{s} + \mathbf{n} \quad (5)$$

where $\hat{\mathbf{H}}$ is the estimated channel matrix for the “quasi-static channel”:

$$\hat{\mathbf{H}} = \begin{bmatrix} \hat{\mathbf{h}}_{12} & \hat{\mathbf{h}}_{34} \\ \hat{\mathbf{h}}'_{34} & -\hat{\mathbf{h}}'_{12} \\ \hat{\mathbf{h}}_{12}^* & \hat{\mathbf{h}}_{34}^* \\ -\hat{\mathbf{h}}_{34}^* & \hat{\mathbf{h}}_{12}^* \end{bmatrix} \quad (6)$$

$$\text{with } \hat{\mathbf{h}}_{ij} = \begin{bmatrix} \hat{h}_i & \hat{h}_j \\ \hat{h}_j & -\hat{h}_i \end{bmatrix}, \quad \hat{\mathbf{h}}'_{ij} = \begin{bmatrix} \hat{h}_i & -\hat{h}_j \\ \hat{h}_j & \hat{h}_i \end{bmatrix}, \text{ and } \hat{h}_j = h_j(1).$$

It can be proved that the “linear maximum-likelihood (ML)” \mathcal{G}_4 detector in [3] is equivalent to the following two-step procedure [7], [8]:

[Step C1] Apply linear transform $\Psi = \hat{\mathbf{H}}^H$ to the received signal \mathbf{r} :

$$\mathbf{r}' = \Psi\mathbf{r} = \Psi\hat{\mathbf{H}}\mathbf{s} + \mathbf{v}. \quad (7)$$

[Step C2] Carry out the “linear ML” detection:

$$\hat{s}_i = \arg \left\{ \min_{s_m \in \mathcal{S}} |[\mathbf{r}']_i - \lambda_i s_m|^2 \right\}, \quad i = 1, 2, 3, 4. \quad (8)$$

Here, $\mathbf{v} = \Psi\mathbf{n}$, \mathcal{S} is the symbol alphabet, $[\mathbf{r}']_i$ the i th element of \mathbf{r}' , and $\lambda_i = 2 \sum_{j=1}^4 |\hat{h}_j|^2$ (i.e., the i th element of diagonal $\Psi\hat{\mathbf{H}}$).

In reality, however, the true physical process in Step C1 is

$$\mathbf{r}' = \Psi\mathbf{r} = \Psi\mathbf{H}\mathbf{s} + \mathbf{v}. \quad (9)$$

Note that in general $\Psi\mathbf{H}$ in (9) is nondiagonal

$$\Phi = \Psi\mathbf{H} = [\phi_{ij}]_{4 \times 4}. \quad (10)$$

The conventional 2-step linear detection procedure is truly ML if and only if the channel is truly static over $8T_s$, in which case $\mathbf{H} = \hat{\mathbf{H}}$ and thus $\phi_{ij} = 0$ for $i \neq j$, and $\phi_{ii} = \lambda_i$.

For a time selective fading channel, however, $\mathbf{H} \neq \hat{\mathbf{H}}$ and thus $\phi_{ij} \neq 0$ for $i \neq j$. Physically, this leads to inter element interference (IEI). The value of the nondiagonal ϕ_{ij} depends upon the time-selectivity of the channel. When using the above conventional detector (thus the \mathcal{G}_4 decoder in [3]) for a time selective fading channel, these nonzero ϕ_{ij} 's are effectively ignored, resulting in extra detection errors in addition to those caused by the AWGN. These extra errors will form an irreducible error floor in the BER curves in the high SNR region.

IV. PIC DETECTOR FOR THE \mathcal{G}_4 SYSTEM

Although $\phi_{ij} \neq 0$ (for $i \neq j$) in (10), it is also true that normally $|\phi_{ij}| \ll |\phi_{ii}|$. This is because under normal vehicle speeds or Doppler spread, the channel variation over $8T_s$ still tends to be relatively small. As an example, let us consider the following popular AR(1) model for time-selective channels [4]–[11]:

$$h_j(i+m) = \alpha_m h_j(i) + w_j(i+m) \quad (11)$$

where $w_j(i)$ is another i.i.d. complex Gaussian random variable having zero mean and variance σ_w^2 and being statistically independent of $h_j(i)$. Also, $\alpha_m = E[h_j(i)h_j^*(i+m)] = J_0(2\pi m f_d T_s)$, where f_d is the Doppler frequency and $J_0(\cdot)$ is the 0th order Bessel function of the first kind. If the fading paths originate sufficiently far away from the receiver, α_m can be assumed to be the same for all the transmitter antennas. To illustrate the dominance of ϕ_{ii} , similarly to [14], we introduce the following “nondiagonal index”:

$$\mu = E \left[\frac{\|\Phi_{\text{ND}}\|_{\text{F}}^2}{\|\text{diag}(\Phi)\|_{\text{F}}^2} \right] \quad (12)$$

where $\|\mathbf{A}\|_{\text{F}}$ denotes the Frobenius norm of \mathbf{A} , and matrix Φ_{ND} comprises the nondiagonal elements of Φ : $\Phi_{\text{ND}} = \Phi - \text{diag}(\Phi)$. Clearly, the above index μ reflects the ratio of the squared magnitude of the nondiagonal elements to the squared magnitude of the diagonal elements in Φ , and its value is dependent upon the Doppler spread of the channel. The relation between the nondiagonal index μ and $f_d T_s$ is demonstrated in Fig. 1, which is obtained using 40 000 \mathbf{H} and $\hat{\mathbf{H}}$ realizations. It is easy to see that for the normal range of Doppler spreads we always have $\mu \ll 1$.

Based on the above observation, we can now apply the principle of parallel interference cancellation (PIC) to (9). As is well

known, PIC (although suboptimum) is an effective yet simple approach in multiuser detection of CDMA (there is a rich literature on PIC for CDMA. See e.g., [15] and all the references therein).

A. Algorithm

[Step P1] *Initialization*: Set iteration number $k = 0$, and obtain $s_i^{(0)}$ from the conventional O-STBC decoder via (7) and (8): $s_i^{(0)} = \hat{s}_i$.

[Step P2] *Iteration*: For iteration number $k = 1, 2, \dots, I$,

$$\mathbf{r}'^{(k)} = \mathbf{r}' - \Phi_{\text{NDS}}^{(k-1)}. \quad (13)$$

The symbol detection for the current iteration can then be achieved via a simple least square approach

$$s_i^{(k)} = \arg \left\{ \min_{s_m \in \mathcal{S}} |[\mathbf{r}'^{(k)}]_i - \phi_{ii} s_m|^2 \right\}, \quad i = 1, 2, 3, 4 \quad (14)$$

where \mathcal{S} is the symbol alphabet, and $[\mathbf{r}'^{(k)}]_i$ is the i th element of $\mathbf{r}'^{(k)}$. Also, $\mathbf{s}^{(k)} = [s_1^{(k)}, s_2^{(k)}, s_3^{(k)}, s_4^{(k)}]^T$.

As in a CDMA PIC detector, the above procedure comprises two components: tentative symbol estimation (TSE) and tentative interference subtraction (TIS). With the iterations progressing, the TSE will contain fewer and fewer errors, making the TIS more and more accurate. Once the IEI related errors have been eliminated, (14) becomes a linear ML procedure. This explains why the PIC detector potentially offers a much better performance than the corresponding ZF detector. As to the number of iterations, our simulations have shown that $I = 3$ iterations normally deliver most of the gain.

B. Algorithm Discussion

1) *Quasi-Static Channels and Algorithm Evolution*: When the channel is indeed static over the entire $8T_s$ period (i.e., $\mathbf{H} = \hat{\mathbf{H}}$), the Initialization step (i.e., the conventional O-STBC decoder) will give the optimum solution. In such a special situation, as $\Phi_{\text{ND}} = \mathbf{0}$ and $\phi_{ii} = \lambda_i$, the iterations in (13) and (14) will not alter the already optimum initial solution. To this extent, the PIC detection algorithm represents an evolution of the conventional O-STBC detector.

2) *Algorithm Complexity*: Compared with the conventional \mathcal{G}_4 decoder, the main computation increase for the PIC detector is from the calculation of matrix Φ and the extra iterations in (13) and (14). It is easy to show that the total increase involves $[32 + M + (3 + M)I]$ complex number (CN) multiplications and $[28 + M + 3I]$ CN additions per symbol, where M is the modulation level and I is the number of iterations. This means, for $M = 16$ and $I = 3$, 105 CN multiplications and 53 CN additions per symbol. On the other hand, the complexity increase of the ZF scheme in [8] is 34 CN multiplications and 25 CN additions (regardless of M). Therefore, the computational complexity of the PIC detector is higher than that of both the conventional and the ZF decoders [7], [8] (the surprisingly lower complexity of the ZF schemes is due to the special matrix algorithms in [7] and [8], which only involves the inversion of 2×2 matrices). As I is normally small (≤ 3), however, the extra computation in the PIC detector is still moderate (compared with the full ML search over all 4 symbols, whose complexity is $O(M^4)$), and can well be justified by the enormous performance improvement.

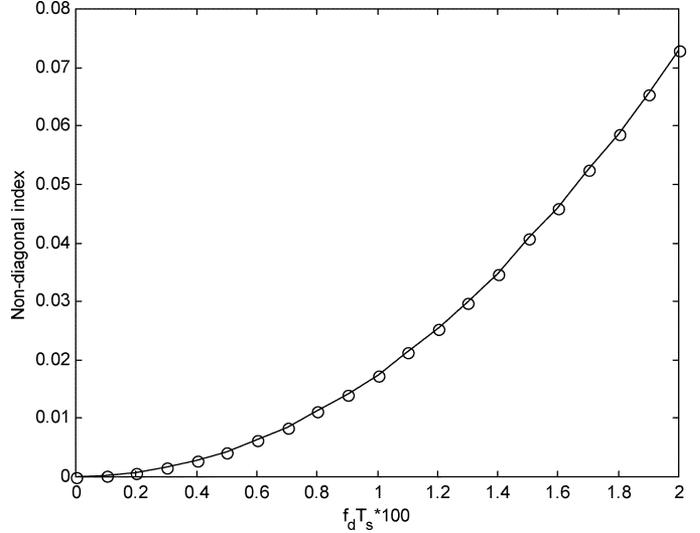


Fig. 1. Value of nondiagonal index μ with respect to $f_d T_s$.

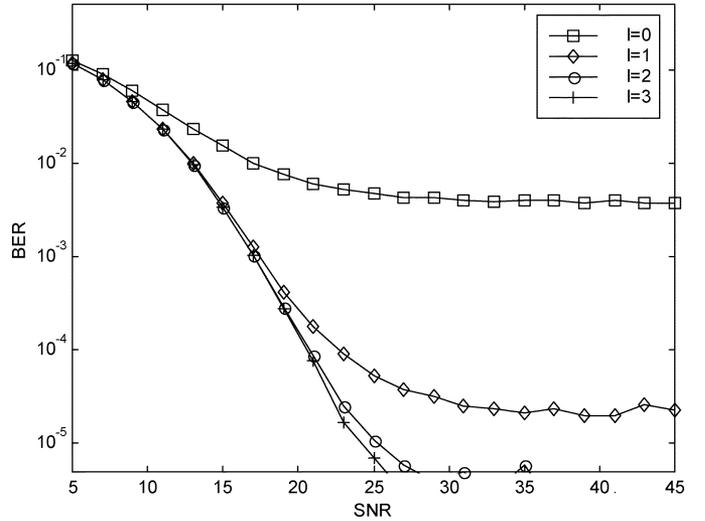


Fig. 2. BER performance of the PIC \mathcal{G}_4 detector with respect to the number of iterations (I).

3) *\mathcal{G}_3 Encoded Systems*: By setting $h_4(i) = 0$ in (1) and all the other related equations, the above PIC detector can directly be used in the \mathcal{G}_3 encoded systems.

V. SIMULATIONS

The \mathcal{G}_4 system under 16-QAM (Gray encoded) modulation and the time selective fading channel in (11) are employed. The signal to noise ratio (SNR) at the receiver is defined as $SNR = (M_T \sigma_h^2 E_s) / \sigma_n^2 = M_T E_s / \sigma_n^2$ (since $\sigma_h^2 = 1$), where E_s is the Tx power at each antenna, and $M_T = 4$ (for the \mathcal{G}_4 system). Also, the UMTS symbol rates are considered: $T_s = SF / (3.84 \times 10^6)$ seconds with $f = 2$ GHz, where SF is the spreading factor (in UMTS) but in our simulations is simply a parameter for T_s adjustment.

Five vehicle speeds are simulated: $V = 0$ (quasi-static channels), 70, 100, 130, and 160 km/h (leading to different α_m values) for SF = 128 (or equivalently $V = 0, 35, 50, 65$, and 80 km/h for SF = 256). These correspond to the $f_d T_s$ values of 0, 0.0043, 0.0062, 0.0080, and 0.0099 (see Fig. 1 for the corresponding values of the nondiagonal index μ). For

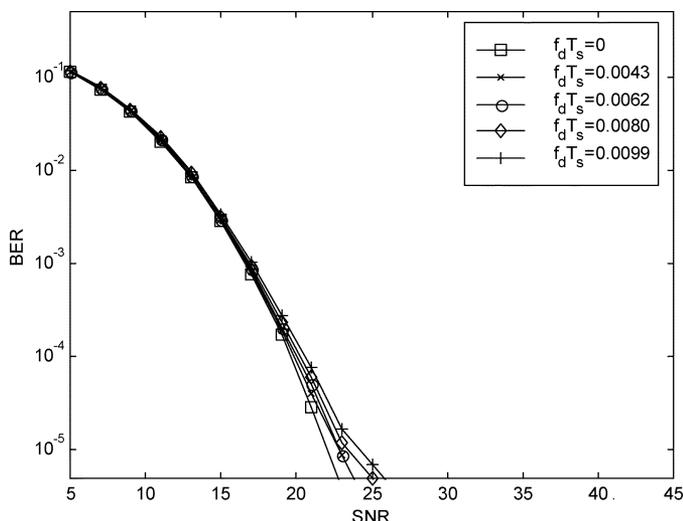


Fig. 3. BER performance of the PIC \mathcal{G}_4 detector for $I = 3$.

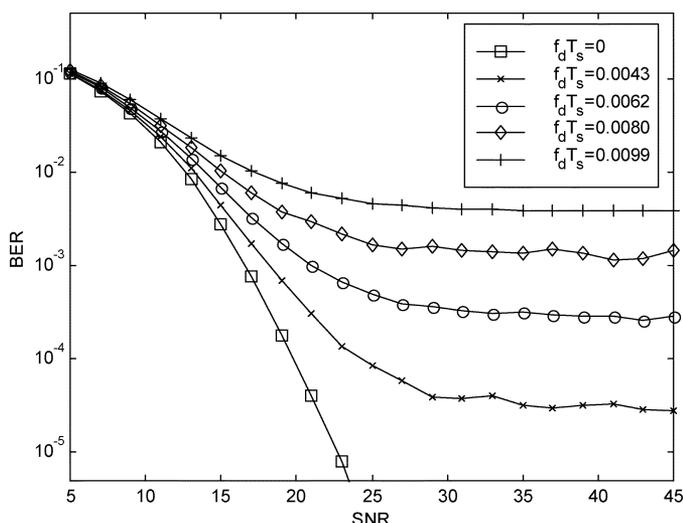


Fig. 4. BER performance of the conventional \mathcal{G}_4 detector (i.e., $I = 0$).

the case of $f_d T_s = 0.0099$, the BER details of the PIC for $I = 0$ (i.e., the conventional decoder), 1, 2, and 3 are shown in Fig. 2. Clearly, two or three iterations deliver most of the performance gain even for such a case of relatively high speed. For all the other speeds, only the BER results for $I = 3$ are plotted in Fig. 3. For comparison, the results of the conventional \mathcal{G}_4 detector are shown in Fig. 4. It is easy to see that the PIC detector exhibits no error floor while the conventional \mathcal{G}_4 detector does. Most importantly, the BER degradation of the PIC caused by channel variation is very small indeed within the considered $f_d T_s$ range. The penalty, however, is a relatively higher computational complexity.

Our simulations (not shown here) have also indicated that the above PIC detector works well right up to around 240 km/h (150 miles/h) for SF = 128 (i.e., $f_d T_s = 0.0148$). For the even higher speeds (or $f_d T_s$ values), however, it may incur more than 3 iterations or even a residual error floor. This obviously is not

an issue for the \mathcal{G}_i ZF schemes in [7] and [8]. All the above observations also apply to the \mathcal{G}_3 coded 3-Tx systems.

VI. CONCLUSIONS

This paper has presented a PIC based detector structure for the \mathcal{G}_i O-STBC systems over time-selective fading channels. While the conventional \mathcal{G}_i detectors under such conditions tend to suffer from a considerable irreducible error floor in the high SNR cases, the PIC detector shows no error floor at all for the normal speed range. The PIC's relatively higher computational cost can be justified by its enormous performance gain.

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