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Fuzzy Output Tracking Control and Filtering for Nonlinear Discrete-Time Descriptor Systems under Unreliable Communication Links

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***Abstract*—In this paper, the problems of output tracking control and filtering are investigated for Takagi-Sugeno fuzzy-approximation-based nonlinear descriptor systems in the discrete-time domain. Specially, the unreliability of the communication links between the sensor and actuator/filter is taken into account, and the phenomenon of packet dropouts is characterized by a binary Markov chain with uncertain transition probabilities, which may reflect the reality more accurately than the existing description processes. A novel bounded real lemma (BRL), which ensures the stochastic admissibility with H_∞ performance for fuzzy discrete-time descriptor systems despite the uncertain Markov packet dropouts, is presented based on a fuzzy basis-dependent Lyapunov function. By resorting to the dual conditions of the obtained BRL, a solution for the designed fuzzy output tracking controller is given. A design method for the full-order fuzzy filter is also provided. Finally, two examples are finally adopted to show the applicability of the achieved design strategies.**

***Index Terms*—Descriptor systems, output tracking control, filtering, packet dropouts, Takagi-Sugeno fuzzy approximation.**

I. Introduction

The dynamical models for many well-known systems including robotic systems, power systems, and economical systems are semi-state, and can be naturally described by descriptor systems. Compared with the standard state-space systems, the structures of descriptor systems contain infinite dynamical modes, which can generate undesired impulse behaviors. Analysis and synthesis of various linear descriptor systems have been an active research topic in control community (see, e.g. [1], [2]). Meanwhile, to overcome the enormous difficulties in analyzing the highly nonlinear descriptor models, a lot of research attention has been devoted to Takagi-Sugeno (T-S) fuzzy descriptor systems (see e.g. [3]-[8]). In this case, the theory for linear descriptor models can be extended to a highly nonlinear descriptor system by resorting to the ability of the T-S fuzzy rules in approximating the nonlinearities [9]-[16].

In some practical cases, the communication links between the nonlinear plant and controller/filter are unreliable and the phenomenon of stochastic packet dropouts may happen. The control system subject to continuous packet dropouts will degrade the performance of the traditional controller, and even lead to instability. In general, two main random processes have been utilized to depict the packet dropout process: Bernoulli distribution and Markov chain. The former treats the packet dropout process as a temporally independent process, which does not cover the cases of temporal correlated channels with "memory" [17]. The latter considers the temporal correlation among packet dropouts, which is more general and may reflect more reality [18]. In [19], the Markov process was adopted to describe the quantity of the multiple packet losses. A new Markov model of bounded packet dropouts was proposed in [20], where the late-arrival packets can be represented with fewer nonzero parameters in the transition probability (TP) matrix. The similar model was used to study the estimation problem for discrete-time T-S fuzzy systems [21]. Recently, the successive packet dropouts characterized by the binary Markov chains have received increasing research attention [22], [23]. Compared with the Markov models adopted in [19]-[21], only two TPs are needed in this model. Besides, the upper bound on the successive packet dropouts is not required to be known in advance. It is noted that the rates of packet dropouts or TPs in aforementioned results are known precisely a priori. In practice, it is hard to get the accurate estimation of the rates/TPs due to the complex environment [24]-[29]. In view of this point, the authors in [30] considered the uncertain rates of the packet losses under Bernoulli distribution, and studied the distributed filtering for sensor networks. However, there has been very little literature on Markov packet dropouts with uncertain TPs.

Output tracking control and filtering are two important and fundamental problems in control filed, which have been extensively studied for various state-space fuzzy systems (see

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e.g. [31]-[37]). For fuzzy descriptor systems, unfortunately, a literature search has shown that very few results have been reported on their output tracking control problems, not to mention the case of the random packet dropouts. Although several filtering results have been reported (see e.g. [38]-[40]), the issue of the random packet dropouts existed between plant and filter is not completely addressed. The above situation motivates this study.

In this paper, we aim to investigate the output tracking control and filtering problems for fuzzy descriptor systems in the discrete-time domain. The communication links between the nonlinear plant and controller/filter are assumed to be unreliable, and suffers from random packet dropouts. The achieved contributions can be summarized as follows:

1) A binary Markov chain with uncertain TPs is proposed to characterize the random packet dropouts, which may reflect more reality than the existing description processes. Moreover, a dropout compensation mechanism is introduced to reflect the recent-arrival packet.

2) The dual systems for discrete-time descriptor systems with disturbance are established for the first time, where the corresponding dual conditions can avoid the matrix transformation and facilitate the synthesis of H_∞ controller.

3) A novel bounded real lemma (BRL) is presented for discrete-time fuzzy descriptor systems in the presence of uncertain Markov packet dropouts.

4) Based on the dual conditions of the BRL, a solution for the designed fuzzy output tracking controller is given by constructing suitable matrix structures. An appropriate fuzzy filter solution method is further developed to tackle the uncertain Markov packet dropouts.

The remainder of the paper is structured as follows. Preliminaries are provided in Section II. The main design procedure is presented in Section III. The verification of proposed output tracking and filtering methods is shown in Section IV. Concluding remarks are finally formulated in Section V.

Notations: $\mathbb{R}^{m\times n}$ and $\mathbb{R}^n$ denotes, respectively, the set of $m\times n$ matrices and the n -dimensional Euclidean space. Symbol "$*$" specifies the symmetric element of a symmetric matrix. Symbols "$\triangleq$" and "$=$" denote the equality signs concerning definitions and assignments, respectively. Superscript "T" denotes the transpose. $\text{sym}\{X\}$ represents $X+X^T$. $\lambda_{\min}\{A\}$ stands for the minimum eigenvalue of the matrix A. $\oplus$ denotes the terms that are not relevant to the result. $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation.

II. Problem Formulation and Preliminaries

A. System Description

Consider a discrete-time fuzzy descriptor system given by

$$\begin{cases} Ex(k+1)=\sum_{i=1}^{r}\xi_i\left(\nu(k)\right)\left[A_ix(k)+B_iu(k)+D_iw(k)\right] \\ y(k)=\sum_{i=1}^{r}\xi_i\left(\nu(k)\right)C_ix(k) \end{cases}, \quad (1)$$

where $x(k)\in\mathbb{R}^n$ is the state vector, $y(k)\in\mathbb{R}^p$ is the measured output vector, $u(k)\in\mathbb{R}^m$ denotes the control input, $w(t)\in\mathbb{R}^q$ is the external disturbance belonging to the $\mathcal{L}_2[0,\infty)$ space, $E\in\mathbb{R}^{n\times n}$ may be singular with $\text{rank}(E)=s\le n$, $A_i\in\mathbb{R}^{n\times n}$, $B_i\in\mathbb{R}^{n\times m}$, $C_i\in\mathbb{R}^{p\times n}$, $D_i\in\mathbb{R}^{n\times q}$, $\xi_i\left(\nu(k)\right)\ge 0$ denote the membership functions satisfying $\sum_{i=1}^{r}\xi_i\left(\nu(k)\right)=1$, $\nu(k)$ denotes the premise vector.

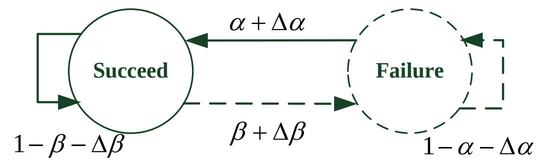

Fig. 1. Markov model for packet dropouts.

Under the unreliable communication environment like network, the transmitted measurement signal may be randomly lost due to the unreliable link. In this paper, the packet dropout is characterized by the variable $\theta(r_k)$ taking values in $\{0,1\}$, where $\{r_k, k\ge 0\}$ a discrete-time Markov process taking values in $\{1,2\}$, and is governed by a TP matrix

$$\Gamma\triangleq\{\pi_{\lambda\nu}+\Delta\pi_{\lambda\nu}\}=\begin{bmatrix}1-\alpha-\Delta\alpha & \alpha+\Delta\alpha\\ \beta+\Delta\beta & 1-\beta-\Delta\beta\end{bmatrix},\ \lambda,\nu=1,2, \quad (2)$$

where $\alpha\triangleq\text{Prob}\left(r_{k+1}=2\,|\,r_k=1\right)\in[0,1]$ denotes the recovery rate, $\beta\triangleq\text{Prob}\left(r_{k+1}=1\,|\,r_k=2\right)\in[0,1]$ denotes the failure rate, $\varepsilon_{1\min}<\Delta\alpha<\varepsilon_{1\max}$ and $\varepsilon_{2\min}<\Delta\beta<\varepsilon_{2\max}$ denote the uncertainties of the recovery rate and failure rate, respectively.

Remark 1: In this paper, the binary Markov chain with uncertain TPs is adopted to characterize the random packet dropouts (illustrated in Fig. 1.), which is more realistic than the previous processes with certain TPs [19]-[23]. If $\beta=1-\alpha$ and $\Delta\beta=-\Delta\alpha$, the corresponding TP matrix will change into $\Gamma=\begin{bmatrix}1-\alpha-\Delta\alpha & \alpha+\Delta\alpha\\ 1-\alpha-\Delta\alpha & \alpha+\Delta\alpha\end{bmatrix}$. In this case, the uncertain binary Markov process will reduce to a Bernoulli process with uncertain rate proposed in [30].

B. Fuzzy Output Tracking Control

Define $y_r(k)$ as the output reference signal to be tracked and $e(k)\triangleq y_r(k)-y(k)$ as the tracking error. To remove the steady-state tracking error, the integral term $\delta(k)\triangleq\sum_{l=0}^{k-1}e(l)$ is introduced. Define $\mathcal{X}(k)\triangleq\left[\delta^T(k)\quad x^T(k)\right]^T$, then the corresponding augmented system is given by

$$\begin{cases} \mathcal{E}\mathcal{X}(k+1)=\sum_{i=1}^{r}\xi_i\left(\nu(k)\right)\left[\mathcal{A}_i\mathcal{X}(k)+\mathcal{B}_iu(k)+\mathcal{D}_i\varpi(k)\right] \\ \mathcal{Z}(k)=\sum_{i=1}^{r}\xi_i\left(\nu(k)\right)\mathcal{C}_i\mathcal{X}(k) \end{cases}, \quad (3)$$

where $\mathcal{Z}(k)$ is the controlled output,

$\mathcal{E} \triangleq \begin{bmatrix} I_p & 0 \\ 0 & E \end{bmatrix}$, $\mathcal{A}_i \triangleq \begin{bmatrix} I_p & -C_i \\ 0 & A_i \end{bmatrix}$, $\mathcal{B}_i \triangleq \begin{bmatrix} 0 \\ B_i \end{bmatrix}$, $\mathcal{D}_i \triangleq \begin{bmatrix} I_p & 0 \\ 0 & D_i \end{bmatrix}$,

$\mathcal{C}_i \triangleq \begin{bmatrix} I_p & 0 \\ 0 & C_i \end{bmatrix}$, $\varpi(k) \triangleq \begin{bmatrix} y_r(k) \\ w(k) \end{bmatrix}$.

In this paper, based on the concept of parallel distributed compensation (PDC), the following state-feedback fuzzy output tracking controller (FOTC) is designed:

$$u(k) = \sum_{i=1}^{r} \xi_i\left(\nu(k)\right)\mathcal{K}_i \mathcal{X}(k), \tag{4}$$

where $\mathcal{K}_i \in \mathbb{R}^{m\times(n+p)}$ are the FOTC gain matrices to be solved.

Due to the unreliable link, the control signal $u(k)$ suffers from uncertain Markov packet dropouts. To compensate the packet dropout, it is assumed that the actuator keeps the current value unless the control signal is successfully received, that is

$$\hat{u}(k) = \begin{cases} u(k), & \text{if } \theta_k = 1 \\ \hat{u}(k-1), & \text{if } \theta_k = 0 \end{cases},$$

which can be further written as

$$\hat{u}(k) = \theta(r_k)u(k) + \bar{\theta}(r_k)\hat{u}(k-1), \tag{5}$$

where $\bar{\theta}(r_k) \triangleq (1-\theta(r_k))$.

Based on (3)-(5), the augmented system can be obtained as a fuzzy Markov jump descriptor system (FMJDS) described by

$$\begin{cases} \tilde{\mathcal{E}}\tilde{\mathcal{X}}(k+1) = \sum_{i=1}^{r} \xi_i\left(\nu(k)\right)\sum_{j=1}^{r} \xi_j\left(\nu(k)\right)\left[\tilde{\mathcal{A}}_{ij}(r_k)\tilde{\mathcal{X}}(k) \right.\\ \qquad\qquad \left. +\tilde{\mathcal{D}}_i\varpi(k)\right] \\ \mathcal{Z}(k) = \sum_{i=1}^{r} \xi_i\left(\nu(k)\right)\tilde{\mathcal{C}}_i\tilde{\mathcal{X}}(k) \end{cases}, \tag{6}$$

where

$\tilde{\mathcal{X}}(k) \triangleq \begin{bmatrix} \mathcal{X}(k) \\ \hat{u}(k-1) \end{bmatrix}$, $\tilde{\mathcal{E}} \triangleq \begin{bmatrix} \mathcal{E} & 0 \\ 0 & I_m \end{bmatrix}$, $\tilde{\mathcal{C}}_i \triangleq \begin{bmatrix} \mathcal{C}_i^T \\ 0_{m\times 2p} \end{bmatrix}^T$,

$\tilde{\mathcal{A}}_{ij}(r_k) \triangleq \begin{bmatrix} \mathcal{A}_i + \theta(r_k)\mathcal{B}_i\mathcal{K}_j & \bar{\theta}(r_k)\mathcal{B}_i \\ \theta(r_k)\mathcal{K}_j & \bar{\theta}(r_k)I_m \end{bmatrix}$, $\tilde{\mathcal{D}}_i \triangleq \begin{bmatrix} \mathcal{D}_i \\ 0 \end{bmatrix}$.

C. Fuzzy Filtering

Let $z(k) \triangleq \sum_{i=1}^{r} \xi_i\left(\nu(k)\right)L_i x(k) \in \mathbb{R}^l$ be the signal to be estimated. Assuming that $u(t) \equiv 0$ in (1), and designing the following fuzzy filter:

$$\begin{cases} \mathcal{E}\mathcal{X}_f(k+1) = \sum_{i=1}^{r} \xi_i\left(\nu(k)\right)\left[\mathcal{A}_i^f \mathcal{X}_f(k) + \mathcal{B}_i^f y(k)\right] \\ \mathcal{Z}_f(k) = \sum_{i=1}^{r} \xi_i\left(\nu(k)\right)\mathcal{L}_i^f \mathcal{X}_f(k) \end{cases}, \tag{7}$$

where $\mathcal{X}_f(k) \in \mathbb{R}^n$ and $\mathcal{Z}_f(k) \in \mathbb{R}^l$ denote, respectively, the state and the estimated signal, $\mathcal{A}_i^f \in \mathbb{R}^{n\times n}$, $\mathcal{B}_i^f \in \mathbb{R}^{n\times p}$ and $\mathcal{L}_i^f \in \mathbb{R}^{l\times n}$ are the filter parameters to be determined.

To compensate the packet dropout, it is assumed that the filter keeps the recent value unless the measurement signal is successfully received, that is

$$\hat{y}(k) = \begin{cases} y(k), & \text{if } \theta_k = 1 \\ \hat{y}(k-1), & \text{if } \theta_k = 0 \end{cases},$$

which can be further written as

$$\hat{y}(k) = \theta(r_k)y(k) + \bar{\theta}(r_k)\hat{y}(k-1). \tag{8}$$

Let $\tilde{\mathcal{X}}(k) \triangleq \begin{bmatrix} x^T(k) & \mathcal{X}_f^T(k) & \hat{y}^T(k-1) \end{bmatrix}^T$ and $e(k) \triangleq z(k) - \mathcal{Z}_f(k)$. Based on (1), (7), and (8), the augmented filtering error system (FES) can be written as

$$\begin{cases} \tilde{\mathcal{E}}\tilde{\mathcal{X}}(k+1) = \sum_{i=1}^{r} \xi_i\left(\nu(k)\right)\sum_{j=1}^{r} \xi_j\left(z(k)\right)\left[\tilde{\mathcal{A}}_{ij}(r_k)\tilde{\mathcal{X}}(k) \right.\\ \qquad\qquad \left. +\tilde{\mathcal{D}}_i w(k)\right] \\ e(k) = \sum_{i=1}^{r} \xi_i\left(\nu(k)\right)\tilde{\mathcal{L}}_{ij}\tilde{\mathcal{X}}(k) \end{cases}, \tag{9}$$

where

$\tilde{\mathcal{E}} \triangleq \begin{bmatrix} E & 0 & 0 \\ 0 & \mathcal{E} & 0 \\ 0 & 0 & I_p \end{bmatrix}$, $\tilde{\mathcal{L}}_{ij} \triangleq \begin{bmatrix} L_i^T \\ -\mathcal{L}_j^{f,T} \\ 0 \end{bmatrix}^T$, $\tilde{\mathcal{D}}_i \triangleq \begin{bmatrix} D_i \\ 0 \\ 0 \end{bmatrix}$,

$\tilde{\mathcal{A}}_{ij}(r_k) \triangleq \begin{bmatrix} A_i & 0 & 0 \\ \theta(r_k)\mathcal{B}_j^f C_i & \mathcal{A}_j^f & \bar{\theta}(r_k)\mathcal{B}_j^f \\ \theta(r_k)C_i & 0 & \bar{\theta}(r_k)I_p \end{bmatrix}$.

Remark 2: When $E = I$, the fuzzy descriptor system (1) will reduce to a standard state-space fuzzy system. Thus, the obtained tracking control/filtering results can be further applied to regular T-S fuzzy systems.

During the development of the main results, the following definition and lemmas will be adopted.

Definition 1 [41]: The FMJDS (6) is referred to be

1) Regular if $\det\left(s\tilde{\mathcal{E}} - \tilde{\mathcal{A}}_{ij}(r_k)\right)$ are not identically zero, and causal if it is regular, and the term $\deg\left(\det\left(s\tilde{\mathcal{E}} - \tilde{\mathcal{A}}_{ij}(r_k)\right)\right)$ $=\text{rank}(\tilde{\mathcal{E}})$.

2) Stochastically stable if $\mathbb{E}\left\{\sum_{k=0}^{\infty}\|\mathcal{X}(k)\|^2 \mid \mathcal{X}(0), r_0\right\} < \infty$ holds for any $\mathcal{X}(0)$, and stochastically admissible if it is regular, causal and stochastically stable.

Lemma 1 [42]: Given $\mathcal{X}(k) \in \mathbb{R}^n$, for any symmetric matrix $\mathcal{P} = \mathcal{P}^T \in \mathbb{R}^{n\times n}$ and free matrix $\mathcal{Q} \in \mathbb{R}^{m\times n}$ such that $\text{rank}(\mathcal{Q}) < n$, the following inequalities are equivalent:

1) $\mathcal{X}^T(k)\mathcal{P}\mathcal{X}(k) < 0$, $\forall \mathcal{X}(k) \neq 0$, $\mathcal{Q}\mathcal{X}(k) = 0$,

2) $\exists \mathcal{R} \in \mathbb{R}^{n\times m}$ such that $\mathcal{P} + \mathcal{R}\mathcal{Q} + \mathcal{Q}^T\mathcal{R}^T < 0$.

Lemma 2: The problem of stochastic admissibility with H_∞ performance for system (6) is equivalent to the same problem for the following dual system:

$$\begin{cases} \tilde{\mathcal{E}}^T \tilde{\mathcal{X}}(k+1) = \sum_{i=1}^{r} \xi_i(\nu(k)) \sum_{j=1}^{r} \xi_j(\nu(k)) \Big[\tilde{\mathcal{A}}_{ij}^T(r_k) \tilde{\mathcal{X}}(k) \\ \qquad + \tilde{\mathcal{C}}_i^T \mathcal{Z}(k) \Big] \\ \varpi(k) = \sum_{i=1}^{r} \xi_i(\nu(k)) \tilde{\mathcal{D}}_i^T \tilde{\mathcal{X}}(k) \end{cases}. \quad (10)$$

Proof. Firstly, the regularity and causality between the pairs $\left(\tilde{\mathcal{E}}, \tilde{\mathcal{A}}_{ij}^{\lambda}\right)$ and $\left(\tilde{\mathcal{E}}^T, \tilde{\mathcal{A}}_{ij}^{\lambda,T}\right)$ are equivalent due to the fact that $\det\left(s\tilde{\mathcal{E}} - \tilde{\mathcal{A}}_{ij}^{\lambda}\right) = \det\left(s\tilde{\mathcal{E}}^T - \tilde{\mathcal{A}}_{ij}^{\lambda,T}\right)$ and $\deg\left(\det\left(s\tilde{\mathcal{E}} - \tilde{\mathcal{A}}_{ij}^{\lambda}\right)\right) = \deg\left(\det\left(s\tilde{\mathcal{E}}^T - \tilde{\mathcal{A}}_{ij}^{\lambda,T}\right)\right)$. Moreover, from [43], if the pairs $\left(\tilde{\mathcal{E}}, \tilde{\mathcal{A}}_{ij}^{\lambda}\right)$ are regular and causal, there exist two nonsingular matrices $\mathcal{S}$ and $\mathcal{T}$ such that $\tilde{\mathcal{E}} = \mathcal{S}\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\mathcal{T}$ and $\tilde{\mathcal{A}}_{ij}^{\lambda} = \mathcal{S}\begin{bmatrix} \bar{\tilde{\mathcal{A}}}_{ij}^{\lambda} & 0 \\ 0 & I \end{bmatrix}\mathcal{T}$. Thus, $\tilde{\mathcal{E}}^T = \mathcal{T}^T\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\mathcal{S}^T$ and $\tilde{\mathcal{A}}_{ij}^{\lambda,T} = \mathcal{T}^T\begin{bmatrix} \bar{\tilde{\mathcal{A}}}_{ij}^{\lambda,T} & 0 \\ 0 & I \end{bmatrix}\mathcal{S}^T$, which means the stochastic stability of the pairs $\left(\tilde{\mathcal{E}}, \tilde{\mathcal{A}}_{ij}^{\lambda}\right)$ is determined by $\bar{\tilde{\mathcal{A}}}_{ij}^{\lambda}$ and equivalent to the stochastic stability of $\bar{\tilde{\mathcal{A}}}_{ij}^{\lambda,T}$. Furthermore, the equality $\left\| \tilde{\mathcal{C}}_i \left(s\tilde{\mathcal{E}} - \tilde{\mathcal{A}}_{ij}^{\lambda}\right)^{-1} \tilde{\mathcal{D}}_i \right\|_{\infty} = \left\| \tilde{\mathcal{D}}_i^T \left(s\tilde{\mathcal{E}}^T - \tilde{\mathcal{A}}_{ij}^{\lambda,T}\right)^{-1} \tilde{\mathcal{C}}_i^T \right\|_{\infty}$ holds. So, the stochastic admissibility with H_∞ performance between systems (6) and (10) is equivalent. The proof is completed.

Remark 3: Up to date, only the dual conditions ensuring admissibility have been obtained for nominal discrete-time linear descriptor systems [1], which cannot be applied to the synthesis of H_∞ controller. Lemma 2 provides a dual system for discrete-time fuzzy descriptor system (1) where the stochastic admissibility with H_∞ performance is equivalent. The corresponding dual conditions can avoid the matrix transformation and facilitate the synthesis of H_∞ controller.

III. MAIN RESULTS

A. Admissibility Analysis

The following BRL is firstly presented, which ensures the stochastic admissibility conditions with noise attenuation performance for system (6).

Theorem 1: The FMJDS (6) is stochastically admissible with disturbance attenuation index γ, if there exist symmetric matrix variables $\tilde{\mathcal{P}}_i^{\lambda} > 0$, $\tilde{\mathcal{U}}_{ijs}^{\lambda}$, free matrix variables $\tilde{\mathcal{M}}_{js}^{\lambda}$, $\tilde{\mathcal{N}}_{js}^{\lambda}$, and $\tilde{\mathcal{G}}_{js}^{\lambda}$, $i, j, s \in \{1, 2, \cdots, r\}$, such that the following conditions are feasible for each $\lambda \in \{1, 2\}$:

$$\tilde{\Pi}_{iis}^{\lambda} < 0, \quad \varepsilon_{\lambda} = \{\varepsilon_{\lambda\min}, \varepsilon_{\lambda\max}\}, \quad (11)$$

$$\tilde{\Pi}_{ijs}^{\lambda} + \tilde{\Pi}_{jis}^{\lambda} < 0, \quad i < j, \quad \varepsilon_{\lambda} = \{\varepsilon_{\lambda\min}, \varepsilon_{\lambda\max}\}, \quad (12)$$

where

$$\tilde{\Pi}_{ijs}^{\lambda} \triangleq \begin{bmatrix} \tilde{\Xi}_{ijs}^{\lambda,11} & \tilde{\mathcal{M}}_{js}^{\lambda,T} - \tilde{\mathcal{A}}_{ij}^{\lambda,T}\tilde{\mathcal{N}}_{js}^{\lambda} & -\tilde{\mathcal{M}}_{js}^{\lambda,T}\tilde{\mathcal{D}}_i - \tilde{\mathcal{A}}_{ij}^{T}\tilde{\mathcal{G}}_{js}^{\lambda} \\ * & \tilde{\Xi}_{ijs}^{\lambda,22} & -\tilde{\mathcal{N}}_{js}^{\lambda,T}\tilde{\mathcal{D}}_i + \tilde{\mathcal{G}}_{js}^{\lambda} \\ * & * & -\gamma^2 I - \tilde{\mathcal{G}}_{js}^{\lambda,T}\tilde{\mathcal{D}}_i - \tilde{\mathcal{D}}_i^T\tilde{\mathcal{G}}_{js}^{\lambda} \end{bmatrix},$$

$$\tilde{\Xi}_{ijs}^{\lambda,11} \triangleq -\tilde{\mathcal{E}}^T\tilde{\mathcal{P}}_i^{\lambda}\tilde{\mathcal{E}} - \tilde{\mathcal{M}}_{js}^{\lambda,T}\tilde{\mathcal{A}}_{ij}^{\lambda} - \tilde{\mathcal{A}}_{ij}^{\lambda,T}\tilde{\mathcal{M}}_{js}^{\lambda} + \tilde{\mathcal{C}}_i^T\tilde{\mathcal{C}}_i,$$

$$\tilde{\Xi}_{ijs}^{\lambda,22} \triangleq \sum_{\nu\in\{1,2\}} \pi_{\lambda\nu}\tilde{\mathcal{P}}_s^{\nu} + \sum_{\nu\in\{1,2\},\nu\neq\lambda} \varepsilon_{\lambda}(\tilde{\mathcal{P}}_s^{\nu} - \tilde{\mathcal{P}}_s^{\lambda}) + \tilde{\mathcal{R}}^T\tilde{\mathcal{U}}_{ijs}\tilde{\mathcal{R}} + \tilde{\mathcal{N}}_{js}^{\lambda} + \tilde{\mathcal{N}}_{js}^{\lambda,T},$$

$\tilde{\mathcal{R}}$ is an arbitrary matrix satisfying $\tilde{\mathcal{R}}\tilde{\mathcal{E}} = 0$ and $\mathrm{rank}(\tilde{\mathcal{R}}) = n - s$.

Proof. We first show the regularity and causality of the system. By pre-multiplying and post-multiplying (11) and (12) by $\begin{bmatrix} I & \tilde{\mathcal{A}}_{ij}^{\lambda,T} & 0 \\ 0 & I & 0 \\ 0 & I & I \end{bmatrix}$ and its transpose, respectively, we get

$$-\tilde{\mathcal{E}}^T\tilde{\mathcal{P}}_i^{\lambda}\tilde{\mathcal{E}} + \tilde{\mathcal{C}}_i^T\tilde{\mathcal{C}}_i + \tilde{\mathcal{A}}_{ij}^{\lambda,T} \sum_{\nu\in\{1,2\},\nu\neq\lambda} \varepsilon_{\lambda}(\tilde{\mathcal{P}}_s^{\nu} - \tilde{\mathcal{P}}_s^{\lambda})\tilde{\mathcal{A}}_{ij}^{\lambda} + \tilde{\mathcal{A}}_{ij}^{\lambda,T} \sum_{\nu\in\{1,2\}} \pi_{\lambda\nu}\tilde{\mathcal{P}}_s^{\nu}\tilde{\mathcal{A}}_{ij}^{\lambda} + \tilde{\mathcal{A}}_{ij}^{\lambda,T}\tilde{\mathcal{R}}^T\tilde{\mathcal{U}}_{ijs}\tilde{\mathcal{R}}\tilde{\mathcal{A}}_{ij}^{\lambda} < 0, \quad (13)$$

Since $\varepsilon_{\lambda} \le \pi_{\lambda\lambda}$, it follows that

$$\tilde{\mathcal{A}}_{ij}^{\lambda,T}\tilde{\mathcal{R}}^T\tilde{\mathcal{U}}_{ijs}\tilde{\mathcal{R}}\tilde{\mathcal{A}}_{ij}^{\lambda} - \tilde{\mathcal{E}}^T\tilde{\mathcal{P}}_i^{\lambda}\tilde{\mathcal{E}} < 0, \quad (14)$$

For singular matrix $\tilde{\mathcal{E}}$, there exist two nonsingular matrices $\mathcal{S}$ and $\mathcal{T}$ such that

$$\mathcal{S}\tilde{\mathcal{E}}\mathcal{T} = \begin{bmatrix} I_{p+s+m} & 0 \\ 0 & 0 \end{bmatrix}, \; \mathcal{S}\tilde{\mathcal{A}}_{ij}^{\lambda}\mathcal{T} = \begin{bmatrix} \tilde{\mathcal{A}}_{ij,11}^{\lambda} & \tilde{\mathcal{A}}_{ij,12}^{\lambda} \\ \tilde{\mathcal{A}}_{ij,21}^{\lambda} & \tilde{\mathcal{A}}_{ij,22}^{\lambda} \end{bmatrix},$$

$$\mathcal{S}^{-T}\tilde{\mathcal{P}}_i^{\lambda}\mathcal{S}^{-1} = \begin{bmatrix} \tilde{\mathcal{P}}_{i,11}^{\lambda} & \tilde{\mathcal{P}}_{i,12}^{\lambda} \\ \tilde{\mathcal{P}}_{i,21}^{\lambda} & \tilde{\mathcal{P}}_{i,22}^{\lambda} \end{bmatrix}, \; \tilde{\mathcal{R}}\mathcal{S}^{-1} = \begin{bmatrix} \tilde{\mathcal{R}}_{11} & \tilde{\mathcal{R}}_{12} \end{bmatrix}.$$

From $\tilde{\mathcal{R}}\tilde{\mathcal{E}} = 0$ and $\mathrm{rank}(\tilde{\mathcal{R}}) = n - s$, it can be concluded that $\tilde{\mathcal{R}}_{11} = 0$ and $\mathrm{rank}(\tilde{\mathcal{R}}_{12}) = n - s$, $\tilde{\mathcal{R}}_{12} \in \mathbb{R}^{(p+n+m)\times(n-s)}$.

By performing a congruence transformation to (14) by $\mathcal{T}^T$, we obtain

$$\begin{bmatrix} \tilde{\mathcal{A}}_{ij,11}^{\lambda} & \tilde{\mathcal{A}}_{ij,12}^{\lambda} \\ \tilde{\mathcal{A}}_{ij,21}^{\lambda} & \tilde{\mathcal{A}}_{ij,22}^{\lambda} \end{bmatrix}^T \begin{bmatrix} 0 & \tilde{\mathcal{R}}_{12} \end{bmatrix}^T \tilde{\mathcal{U}}_{ijs} \begin{bmatrix} 0 & \tilde{\mathcal{R}}_{12} \end{bmatrix} \begin{bmatrix} \tilde{\mathcal{A}}_{ij,11}^{\lambda} & \tilde{\mathcal{A}}_{ij,12}^{\lambda} \\ \tilde{\mathcal{A}}_{ij,21}^{\lambda} & \tilde{\mathcal{A}}_{ij,22}^{\lambda} \end{bmatrix} - \begin{bmatrix} I_{p+s+m} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{\mathcal{P}}_{i,11}^{\lambda} & \tilde{\mathcal{P}}_{i,12}^{\lambda} \\ \tilde{\mathcal{P}}_{i,21}^{\lambda} & \tilde{\mathcal{P}}_{i,22}^{\lambda} \end{bmatrix} \begin{bmatrix} I_{p+s+m} & 0 \\ 0 & 0 \end{bmatrix} < 0, \quad (15)$$

which can be simplified as

$$\begin{bmatrix} \oplus & \oplus \\ \oplus & \tilde{\mathcal{A}}_{ij,22}^{\lambda,T}\tilde{\mathcal{R}}_{12}^T\tilde{\mathcal{U}}_{ijs}\tilde{\mathcal{R}}_{12}\tilde{\mathcal{A}}_{ij,12}^{\lambda} \end{bmatrix} < 0, \quad (16)$$

The above inequality implies that $\tilde{\mathcal{A}}_{ij,12}^{\lambda}$ are nonsingular. Then, according to Definition 1, the FMJDS (6) is regular and causal.

Next, to prove the stochastic stability of the system, the following fuzzy stochastic Lyapunov function is constructed:

$$\mathcal{V}\left(\tilde{\mathcal{X}}_k, r_k\right)=\sum_{i=1}^{r}\xi_i\left(\nu(k)\right)\tilde{\mathcal{X}}^T(k)\tilde{\mathcal{E}}^T\tilde{\mathcal{P}}_i(r_k)\tilde{\mathcal{E}}\tilde{\mathcal{X}}(k), \quad (17)$$

Then, when $\varpi(k)\equiv 0$, one can obtain

$$\begin{aligned}
&\Delta\mathcal{V}\left(\tilde{\mathcal{X}}_k, r_k\right)\\
&=\mathbb{E}\left\{\mathcal{V}(\tilde{\mathcal{X}}_{k+1}, r_{k+1})\mid\tilde{\mathcal{X}}_k, r_k=\lambda\right\}-\mathcal{V}\left(\tilde{\mathcal{X}}_k, r_k\right)\\
&=\sum_{i=1}^{r}\xi_i\left(\nu(k+1)\right)\tilde{\mathcal{X}}^T(k+1)\tilde{\mathcal{E}}^T\sum_{\nu\in\{1,2\}}(\pi_{\lambda\nu}+\Delta\pi_{\lambda\nu})\tilde{\mathcal{P}}_i^{\nu}\tilde{\mathcal{E}}\tilde{\mathcal{X}}(k+1)\\
&-\sum_{i=1}^{r}\xi_i\left(\nu(k)\right)\tilde{\mathcal{X}}^T(k)\tilde{\mathcal{E}}^T\tilde{\mathcal{P}}_i^{\lambda}\tilde{\mathcal{E}}\tilde{\mathcal{X}}(k)\\
&=\sum_{i=1}^{r}\xi_i\left(\nu(k+1)\right)\tilde{\mathcal{X}}^T(k+1)\tilde{\mathcal{E}}^T\sum_{\nu\in\{1,2\}}\pi_{\lambda\nu}\tilde{\mathcal{P}}_i^{\nu}\tilde{\mathcal{E}}\tilde{\mathcal{X}}(k+1)\\
&+\sum_{i=1}^{r}\xi_i\left(\nu(k+1)\right)\tilde{\mathcal{X}}^T(k+1)\tilde{\mathcal{E}}^T\sum_{\nu\in\{1,2\},\nu\neq\lambda}\Delta\pi_{\lambda\nu}(\tilde{\mathcal{P}}_i^{\nu}-\tilde{\mathcal{P}}_i^{\lambda})\\
&\times\tilde{\mathcal{E}}\tilde{\mathcal{X}}(k+1)\\
&-\sum_{i=1}^{r}\xi_i\left(\nu(k)\right)\tilde{\mathcal{X}}^T(k)\tilde{\mathcal{E}}^T\tilde{\mathcal{P}}_i^{\lambda}\tilde{\mathcal{E}}\tilde{\mathcal{X}}(k)\\
&=\sum_{i=1}^{r}\xi_i\left(\nu(k)\right)\sum_{s=1}^{r}\xi_s\left(\nu(k+1)\right)\zeta^T(k)\begin{bmatrix}-\tilde{\mathcal{E}}^T\tilde{\mathcal{P}}_i^{\lambda}\tilde{\mathcal{E}} & 0\\ * & \mathbb{P}\end{bmatrix}\zeta(k)
\end{aligned}, \quad (18)$$

where

$$\zeta(k)\triangleq\begin{bmatrix}\tilde{\mathcal{X}}^T(k) & \tilde{\mathcal{X}}^T(k+1)\tilde{\mathcal{E}}^T\end{bmatrix}^T,$$

$$\mathbb{P}\triangleq\sum_{\nu\in\{1,2\}}\pi_{\lambda\nu}\tilde{\mathcal{P}}_s^{\nu}+\sum_{\nu\in\{1,2\},\nu\neq\lambda}\Delta\pi_{\lambda\nu}(\tilde{\mathcal{P}}_s^{\nu}-\tilde{\mathcal{P}}_s^{\lambda}).$$

Moreover, it is straightforward that the following equation holds:

$$\sum_{i=1}^{r}\xi_i\left(\nu(k)\right)\sum_{j=1}^{r}\xi_j\left(\nu(k)\right)\begin{bmatrix}-\tilde{\mathcal{A}}_{ij}^{\lambda} & I\end{bmatrix}\zeta(k)=0. \quad (19)$$

By Lemma 1, $\Delta\mathcal{V}\left(\tilde{\mathcal{X}}_k, r_k\right)<0$ is equivalent to

$$\begin{aligned}
&\begin{bmatrix}-\tilde{\mathcal{E}}^T\tilde{\mathcal{P}}_i^{\lambda}\tilde{\mathcal{E}} & 0\\ * & \mathbb{P}\end{bmatrix}+\begin{bmatrix}\tilde{\mathcal{M}}_{js}^{\lambda,T}\\ \tilde{\mathcal{N}}_{js}^{\lambda,T}\end{bmatrix}\begin{bmatrix}-\tilde{\mathcal{A}}_{ij}^{\lambda} & I\end{bmatrix}\\
&+\begin{bmatrix}-\tilde{\mathcal{A}}_{ij}^{\lambda} & I\end{bmatrix}^T\begin{bmatrix}\tilde{\mathcal{M}}_{js}^{\lambda,T}\\ \tilde{\mathcal{N}}_{js}^{\lambda,T}\end{bmatrix}^T\\
&=\hat{\tilde{\Pi}}_{ijs}^{\lambda}=\frac{\varepsilon_{\lambda\max}-\Delta\pi_{\lambda\nu}}{\varepsilon_{\lambda\max}-\varepsilon_{\lambda\min}}\tilde{\Pi}_{iis}^{\lambda}\left(\varepsilon_{\lambda}=\varepsilon_{\lambda\min}\right)\\
&+\frac{\Delta\pi_{\lambda\nu}-\varepsilon_{\lambda\min}}{\varepsilon_{\lambda\max}-\varepsilon_{\lambda\min}}\tilde{\Pi}_{iis}^{\lambda}\left(\varepsilon_{\lambda}=\varepsilon_{\lambda\max}\right)<0
\end{aligned}, \quad (20)$$

where

$$\hat{\tilde{\Pi}}_{ijs}^{\lambda}\triangleq\begin{bmatrix}-\tilde{\mathcal{E}}^T\tilde{\mathcal{P}}_i^{\lambda}\tilde{\mathcal{E}} & \tilde{\mathcal{M}}_{js}^{\lambda,T}-\tilde{\mathcal{A}}_{ij}^{\lambda,T}\tilde{\mathcal{N}}_{js}^{\lambda}\\ * & \mathbb{P}+\tilde{\mathcal{N}}_{js}^{\lambda}+\tilde{\mathcal{N}}_{js}^{\lambda,T}\end{bmatrix}.$$

From $\tilde{\mathcal{R}}\tilde{\mathcal{E}}=0$, for any symmetric matrices $\tilde{\mathcal{U}}_{ijs}^{\lambda}$, the following equation holds:

$$\begin{aligned}
0=&\sum_{i=1}^{r}\xi_i\left(\nu(k)\right)\sum_{j=1}^{r}\xi_j\left(\nu(k)\right)\sum_{s=1}^{r}\xi_s\left(\nu(k+1)\right)\\
&\times\tilde{\mathcal{X}}^T(k+1)\tilde{\mathcal{E}}^T\tilde{\mathcal{R}}^T\tilde{\mathcal{U}}_{ijs}^{\lambda}\tilde{\mathcal{R}}\tilde{\mathcal{E}}\tilde{\mathcal{X}}(k+1),
\end{aligned} \quad (21)$$

Then, in the light of the conditions (20) and (21), we obtain

$$\begin{aligned}
&\Delta\mathcal{V}\left(\tilde{\mathcal{X}}_k, r_k\right)\\
&=\sum_{i=1}^{r}\xi_i\left(\nu(k)\right)\sum_{j=1}^{r}\xi_j\left(\nu(k)\right)\sum_{s=1}^{r}\xi_s\left(\mathrm{z}(k+1)\right)\zeta^T(k)\hat{\tilde{\Pi}}_{ijs}^{\lambda}\zeta(k)\\
&=\sum_{i=1}^{r}\xi_i^2\left(\nu(k)\right)\sum_{s=1}^{r}\xi_s\left(\nu(k+1)\right)\zeta^T(t)\hat{\tilde{\Pi}}_{iis}^{\lambda}\zeta(t),\\
&+\sum_{i<j}^{r}\xi_i\left(\nu(k)\right)\xi_j\left(\nu(k)\right)\sum_{s=1}^{r}\xi_s\left(\nu(k+1)\right)\zeta^T(k)(\hat{\tilde{\Pi}}_{ijs}^{\lambda}+\hat{\tilde{\Pi}}_{jis}^{\lambda})\zeta(k)\\
&<0
\end{aligned} \quad (22)$$

which implies

$$\mathbb{E}\left\{\mathcal{V}(\tilde{\mathcal{X}}_{k+1}, r_{k+1})\mid\tilde{\mathcal{X}}_k,\lambda\right\}-\mathcal{V}\left(\tilde{\mathcal{X}}_k, r_k\right)<-\rho\tilde{\mathcal{X}}^T(k)\tilde{\mathcal{X}}(k), \quad (23)$$

where $\rho\triangleq\lambda_{\min}\left\{-\hat{\tilde{\Pi}}_{ijs}^{\lambda}, i, j, s\in\{1,2,\cdots,r\},\lambda\in\{1,2\}\right\}$. Then, one can readily get $\sum_{k=0}^{\infty}\mathbb{E}\left\{\tilde{\mathcal{X}}^T(k)\tilde{\mathcal{X}}(k)\right\}<\frac{1}{\rho}\mathbb{E}\left\{\mathcal{V}\left(\tilde{\mathcal{X}}(0), r_0\right)\right\}$. By Definition 1, the FMJDS (6) is stochastically admissible.

Finally, we shall establish the H_∞ performance. In this case, the inequality (20) can be further written as

$$\begin{aligned}
&\begin{bmatrix}-\tilde{\mathcal{E}}^T\tilde{\mathcal{P}}_i^{\lambda}\tilde{\mathcal{E}} & 0 & 0\\ * & \mathbb{P} & 0\\ * & * & 0\end{bmatrix}+\begin{bmatrix}\tilde{\mathcal{M}}_{js}^{\lambda,T}\\ \tilde{\mathcal{N}}_{js}^{\lambda,T}\\ \tilde{\mathcal{G}}_{js}^{\lambda,T}\end{bmatrix}\begin{bmatrix}-\tilde{\mathcal{A}}_{ij}^{\lambda} & I & -\tilde{\mathcal{D}}_i\end{bmatrix}\\
&+\begin{bmatrix}-\tilde{\mathcal{A}}_{ij}^{\lambda} & I & -\tilde{\mathcal{D}}_i\end{bmatrix}^T\begin{bmatrix}\tilde{\mathcal{M}}_{js}^{\lambda,T}\\ \tilde{\mathcal{N}}_{js}^{\lambda,T}\\ \tilde{\mathcal{G}}_{js}^{\lambda,T}\end{bmatrix}^T<0
\end{aligned}. \quad (24)$$

Under zero initial condition, $\mathcal{V}\left(\tilde{\mathcal{X}}_k, r_k\right)|_{k=0}=0$, and in the light of condition (11) and (12), we have

$$\begin{aligned}
&\mathbb{E}\left\{\sum_{k=0}^{\infty}\left[\mathcal{Z}^T(k)\mathcal{Z}(k)-\gamma^2\varpi^T(k)\varpi(k)+\Delta\mathcal{V}\left(\tilde{\mathcal{X}}_k, r_k\right)\right]\right\}\\
&=\sum_{k=0}^{\infty}\bar{\zeta}^T(k)\tilde{\Pi}_{ijs}^{\lambda}\bar{\zeta}(k)<0
\end{aligned}, \quad (25)$$

where $\bar{\zeta}(k)=\begin{bmatrix}\zeta^T(k) & \varpi^T(k)\end{bmatrix}^T$.

Based on the initial condition and $\mathcal{V}\left(\tilde{\mathcal{X}}_k, r_k\right)\geq 0$, we readily obtain $\mathbb{E}\left\{\sum_{k=0}^{\infty}\|\mathcal{Z}(k)\|^2\right\}<\gamma^2\sum_{k=0}^{\infty}\|\varpi(k)\|^2$. This completes the proof.

Remark 4: Theorem 1 presents a BRL to ensure the stochastic admissibility with H_∞ performance for FMJDS (6) despite the uncertain Markov packet dropouts. To reduce the design conservatism, the FBDLF (16) is introduced instead of the common Lyapunov function. Moreover, Lemma 1 is adopted to avoid the crossing terms between the system and Lyapunov matrices, and facilitate the controller/ filter synthesis.

B. FOTC Design

Define $\mathcal{I}_1 \triangleq \begin{bmatrix} I_{n+p} & 0_{(n+p)\times m}\end{bmatrix}^T$ and $\mathcal{I}_2 \triangleq \begin{bmatrix} 0_{m\times(n+p)} & I_{m\times m}\end{bmatrix}^T$, based on the obtained BRL, the following theorem specifies the FOTC (4) to achieve H_∞ tracking performance for plant (1) despite the Markov packet dropouts.

Theorem 2: For specified ε, $\mathcal{J}_1$, and $\mathcal{J}_2$, the FMJDS (6) is stochastically admissible with disturbance attenuation index γ, if there exist symmetric matrix variables $\mathcal{P}_{i,11}^\lambda \in \mathbb{R}^{(n+p)\times(n+p)}$, $\mathcal{P}_{i,22}^\lambda \in \mathbb{R}^{m\times m}$, and $\tilde{\mathcal{U}}_{ijs}$, free matrix variables $\mathcal{P}_{i,12}^\lambda \in \mathbb{R}^{(n+p)\times m}$, $\mathcal{M}_{js,21}^\lambda \in \mathbb{R}^{m\times(n+p)}$, $\mathcal{M}_{js,22}^\lambda \in \mathbb{R}^{m\times m}$, $\mathcal{N}_{js,21}^\lambda \in \mathbb{R}^{m\times(n+p)}$, $\mathcal{N}_{js,22}^\lambda \in \mathbb{R}^{m\times m}$, $\mathcal{G}_{js,21}^\lambda \in \mathbb{R}^{m\times 2p}$, $\mathcal{M} \in \mathbb{R}^{(n+p)\times(n+p)}$ and $\bar{\mathcal{K}}_i \in \mathbb{R}^{m\times(n+p)}$, $i,j,s \in \{1,2,\cdots,r\}$, such that for each $\lambda \in \{1,2\}$:

$$\begin{bmatrix} \mathcal{P}_{i,11}^\lambda & \mathcal{P}_{i,12}^\lambda \\ * & \mathcal{P}_{i,22}^\lambda \end{bmatrix} > 0, \tag{26}$$

$$\Pi_{iis}^\lambda < 0, \quad \varepsilon_\lambda = \{\varepsilon_{\lambda\min}, \varepsilon_{\lambda\max}\}, \tag{27}$$

$$\Pi_{ijs}^\lambda + \Pi_{jis}^\lambda < 0, \quad i<j, \quad \varepsilon_\lambda = \{\varepsilon_{\lambda\min}, \varepsilon_{\lambda\max}\}, \tag{28}$$

where

$$\Pi_{ijs}^\lambda \triangleq \begin{bmatrix} \Xi_{ijs}^{\lambda,11} & \Xi_{ijs}^{\lambda,12} & \Xi_{ijs}^{\lambda,13} \\ * & \Xi_{ijs}^{\lambda,22} & \Xi_{ijs}^{\lambda,23} \\ * & * & -\gamma^2 I - \mathrm{sym}\{\mathcal{C}_i \mathcal{M}\mathcal{J}_2\} \end{bmatrix},$$

$$\begin{aligned}\Xi_{ijs}^{\lambda,11} \triangleq & -\mathcal{I}_1\mathcal{E}\mathcal{P}_{i,11}^\lambda\mathcal{E}^T\mathcal{I}_1^T - \mathcal{I}_2\mathcal{P}_{i,22}^\lambda\mathcal{I}_2^T - \mathrm{sym}\{\mathcal{I}_1\theta(\lambda)\mathcal{B}_i\bar{\mathcal{K}}_j\mathcal{I}_1^T \\ & + \mathcal{I}_1\mathcal{E}\mathcal{P}_{i,12}^\lambda\mathcal{I}_2^T + \mathcal{I}_1\bar{\theta}(\lambda)\mathcal{B}_i\mathcal{M}_{js,21}^\lambda\mathcal{I}_1^T + \mathcal{I}_1\theta(\lambda)\mathcal{B}_i\bar{\mathcal{K}}_j \mathrm{J}_1\mathcal{I}_2^T \\ & + \mathcal{I}_1\mathcal{A}_i\mathcal{M}\mathcal{I}_1^T + \mathcal{I}_2\theta(\lambda)\bar{\mathcal{K}}_j\mathcal{I}_1^T + \mathcal{I}_1\bar{\theta}(\lambda)\mathcal{B}_i\mathcal{M}_{js,22}^\lambda\mathcal{I}_2^T \\ & + \mathcal{I}_2\theta(\lambda)\bar{\mathcal{K}}_j \mathrm{J}_1\mathcal{I}_2^T + \mathcal{I}_2\bar{\theta}(\lambda)\mathcal{M}_{js,21}^\lambda\mathcal{I}_1^T + \mathcal{I}_1\mathcal{A}_i\mathcal{M}\mathrm{J}_1\mathcal{I}_2^T \\ & + \mathcal{I}_2\bar{\theta}(\lambda)\mathcal{M}_{js,22}^\lambda\mathcal{I}_2^T\} + \mathcal{I}_1\mathcal{D}_i\mathcal{D}_i^T\mathcal{I}_1^T\end{aligned},$$

$$\begin{aligned}\Xi_{ijs}^{\lambda,12} \triangleq & -\mathcal{I}_1\varepsilon\mathcal{A}_i\mathcal{M}\mathcal{I}_1^T - \mathcal{I}_2\varepsilon\theta(\lambda)\bar{\mathcal{K}}_j\mathcal{I}_1^T - \mathcal{I}_2\varepsilon\theta(\lambda)\bar{\mathcal{K}}_j\mathrm{J}_1\mathcal{I}_2^T \\ & + \mathcal{I}_1\mathcal{M}_{js,21}^{\lambda,T}\mathcal{I}_2^T - \mathcal{I}_2\bar{\theta}(\lambda)\mathcal{N}_{js,21}^\lambda\mathcal{I}_1^T - \mathcal{I}_1\varepsilon\theta(\lambda)\mathcal{B}_i\bar{\mathcal{K}}_j\mathcal{I}_1^T \\ & - \mathcal{I}_1\bar{\theta}(\lambda)\mathcal{B}_i\mathcal{N}_{js,21}^\lambda\mathcal{I}_1^T + \mathcal{I}_2\mathcal{M}_{js,22}^\lambda\mathcal{I}_2^T - \mathcal{I}_1\varepsilon\mathcal{A}_i\mathcal{M}\mathrm{J}_1\mathcal{I}_2^T, \\ & + \mathcal{I}_1\mathcal{M}\mathcal{I}_1^T - \mathcal{I}_2\bar{\theta}(\lambda)\mathcal{N}_{js,22}^\lambda\mathcal{I}_2^T - \mathcal{I}_1\varepsilon\theta(\lambda)\mathcal{B}_i\bar{\mathcal{K}}_j\mathrm{J}_1\mathcal{I}_2^T \\ & - \mathcal{I}_1\bar{\theta}(\lambda)\mathcal{B}_i\mathcal{N}_{js,22}^\lambda\mathcal{I}_2^T + \mathcal{I}_2\mathcal{J}_1^T\mathcal{M}^T\mathcal{I}_1^T\end{aligned}$$

$$\begin{aligned}\Xi_{ijs}^{\lambda,13} \triangleq & -\mathcal{I}_1\theta(\lambda)\mathcal{B}_i\bar{\mathcal{K}}_j\mathcal{J}_2 - \mathcal{I}_2\mathcal{J}_1^T\mathcal{M}^T\mathcal{C}_i^T - \mathcal{I}_2\theta(\lambda)\bar{\mathcal{K}}_j\mathcal{J}_2 \\ & - \mathcal{I}_1\mathcal{A}_i\mathcal{M}\mathcal{J}_2 - \mathcal{I}_2\bar{\theta}(\lambda)\mathcal{G}_{js,21}^\lambda - \mathcal{I}_1\bar{\theta}(\lambda)\mathcal{B}_i\mathcal{G}_{js,21}^\lambda \\ & - \mathcal{I}_1\mathcal{M}^T\mathcal{C}_i^T\end{aligned},$$

$$\begin{aligned}\Xi_{ijs}^{\lambda,22} \triangleq & \mathcal{I}_1\sum_{v\in\{1,2\}}\pi_{\lambda v}\mathcal{P}_{s,11}^v\mathcal{I}_1^T + \mathcal{I}_1\sum_{v\in\{1,2\},v\neq\lambda}\varepsilon_\lambda(\mathcal{P}_{s,11}^v - \mathcal{P}_{s,11}^\lambda)\mathcal{I}_1^T \\ & + \mathcal{I}_2\sum_{v\in\{1,2\}}\pi_{\lambda v}\mathcal{P}_{s,22}^v\mathcal{I}_2^T + \mathcal{I}_2\sum_{v\in\{1,2\},v\neq\lambda}\varepsilon_\lambda(\mathcal{P}_{s,22}^v - \mathcal{P}_{s,22}^\lambda)\mathcal{I}_2^T \\ & + \mathcal{I}_1\mathcal{R}_{11}^T\tilde{\mathcal{U}}_{ijs}\mathcal{R}_{11}\mathcal{I}_1^T + \mathcal{I}_2\mathcal{R}_{12}^T\tilde{\mathcal{U}}_{ijs}\mathcal{R}_{12}\mathcal{I}_2^T + \mathrm{sym}\{\mathcal{I}_1\varepsilon\mathcal{M}\mathcal{I}_1^T \\ & + \mathcal{I}_1\sum_{v\in\{1,2\}}\pi_{\lambda v}\mathcal{P}_{s,12}^v\mathcal{I}_2^T + \mathcal{I}_1\sum_{v\in\{1,2\},v\neq\lambda}\varepsilon_\lambda(\mathcal{P}_{s,12}^v - \mathcal{P}_{s,12}^\lambda)\mathcal{I}_2^T \\ & + \mathcal{I}_1\mathcal{R}_{11}^T\tilde{\mathcal{U}}_{ijs}\mathcal{R}_{12}\mathcal{I}_2^T + + \mathcal{I}_1\varepsilon\mathcal{M}\mathcal{J}_1\mathcal{I}_2^T + \mathcal{I}_2\mathcal{N}_{js,21}^\lambda\mathcal{I}_1^T \\ & + \mathcal{I}_2\mathcal{N}_{js,22}^\lambda\mathcal{I}_2^T\}\end{aligned},$$

$\Xi_{ijs}^{\lambda,23} \triangleq -\mathcal{I}_1\varepsilon\mathcal{M}^T\mathcal{C}_i^T - \mathcal{I}_2\varepsilon\mathcal{J}_1^T\mathcal{M}^T\mathcal{C}_i^T + \mathcal{I}_1\mathcal{M}\mathcal{J}_2 + \mathcal{I}_2\mathcal{G}_{js,21}^\lambda$,

$\tilde{\mathcal{R}} \triangleq [\mathcal{R}_{11} \quad \mathcal{R}_{12}]$ is an arbitrary matrix satisfying $\tilde{\mathcal{R}}\tilde{\mathcal{E}} = 0$ and $\mathrm{rank}(\tilde{\mathcal{R}}) = n - s$. Moreover, the gain matrices of the FOTC (4) are computed by

$$\mathcal{K}_i = \mathcal{M}^{-1}\bar{\mathcal{K}}_i. \tag{29}$$

Proof. Based on Lemma 2, the dual conditions of Theorem 1 can be written as

$$\hat{\Pi}_{iis}^\lambda < 0, \ \varepsilon_\lambda = \{\varepsilon_{\lambda\min}, \varepsilon_{\lambda\max}\}, \tag{30}$$

$$\hat{\Pi}_{ijs}^\lambda + \hat{\Pi}_{jis}^\lambda < 0, \ i<j, \ \varepsilon_\lambda = \{\varepsilon_{\lambda\min}, \varepsilon_{\lambda\max}\}, \tag{31}$$

where

$$\hat{\Pi}_{ijs}^\lambda \triangleq \begin{bmatrix} \hat{\Xi}_{ijs}^{\lambda,11} & \tilde{\mathcal{M}}_{js}^{\lambda,T} - \tilde{\mathcal{A}}_{ij}^\lambda\tilde{\mathcal{N}}_{js}^\lambda & -\tilde{\mathcal{M}}_{js}^{\lambda,T}\tilde{\mathcal{C}}_i^T - \tilde{\mathcal{A}}_{ij}^\lambda\tilde{\mathcal{G}}_{js}^\lambda \\ * & \hat{\Xi}_{ijs}^{\lambda,22} & -\tilde{\mathcal{N}}_{js}^{\lambda,T}\tilde{\mathcal{C}}_{ii}^T + \tilde{\mathcal{G}}_{js}^\lambda \\ * & * & -\gamma^2 I - \tilde{\mathcal{G}}_{js}^{\lambda,T}\tilde{\mathcal{C}}_i^T - \tilde{\mathcal{C}}_i\tilde{\mathcal{G}}_{js}^\lambda \end{bmatrix},$$

$\hat{\Xi}_{ijs}^{\lambda,11} \triangleq -\tilde{\mathcal{E}}\tilde{\mathcal{P}}_i^\lambda\tilde{\mathcal{E}}^T - \tilde{\mathcal{M}}_{js}^{\lambda,T}\tilde{\mathcal{A}}_{ij}^{\lambda,T} - \tilde{\mathcal{A}}_{ij}^\lambda\tilde{\mathcal{M}}_{js}^\lambda + \tilde{\mathcal{D}}_i\tilde{\mathcal{D}}_i^T$,

$$\begin{aligned}\hat{\Xi}_{ijs}^{\lambda,22} \triangleq & \sum_{v\in\{1,2\}}\pi_{\lambda v}\tilde{\mathcal{P}}_s^v + \sum_{v\in\{1,2\},v\neq\lambda}\varepsilon_\lambda(\tilde{\mathcal{P}}_s^v - \tilde{\mathcal{P}}_s^\lambda) + \tilde{\mathcal{R}}^T\tilde{\mathcal{U}}_{ijs}\tilde{\mathcal{R}} \\ & + \tilde{\mathcal{N}}_{js}^\lambda + \tilde{\mathcal{N}}_{js}^{\lambda,T}\end{aligned},$$

$\tilde{\mathcal{R}}$ is an arbitrary matrix satisfying $\tilde{\mathcal{R}}\tilde{\mathcal{E}}^T = 0$ and $\mathrm{rank}(\tilde{\mathcal{R}}) = n - s$.

Denote the Lyapunov matrices $\tilde{\mathcal{P}}_i^\lambda$ as $\tilde{\mathcal{P}}_i^\lambda \triangleq \begin{bmatrix} \mathcal{P}_{i,11}^\lambda & \mathcal{P}_{i,12}^\lambda \\ * & \mathcal{P}_{i,22}^\lambda \end{bmatrix}$, and their positivity is ensured by condition (26). To obtain the suitable stabilization conditions, the structures of matrix variables $\tilde{\mathcal{M}}_{js}^\lambda$, $\tilde{\mathcal{N}}_{js}^\lambda$ and $\tilde{\mathcal{G}}_{js}^\lambda$ are further defined as

$$\tilde{\mathcal{M}}_{js}^\lambda \triangleq \begin{bmatrix} \mathcal{M} & \mathcal{M}\mathcal{J}_1 \\ \mathcal{M}_{js,21}^\lambda & \mathcal{M}_{js,22}^\lambda \end{bmatrix}, \ \tilde{\mathcal{N}}_{js}^\lambda \triangleq \begin{bmatrix} \varepsilon\mathcal{M} & \varepsilon\mathcal{M}\mathcal{J}_1 \\ \mathcal{N}_{js,21}^\lambda & \mathcal{N}_{js,22}^\lambda \end{bmatrix},$$

$$\tilde{\mathcal{G}}_{js}^\lambda \triangleq \begin{bmatrix} \mathcal{M}\mathcal{J}_2 \\ \mathcal{G}_{js,21}^\lambda \end{bmatrix}, \tag{32}$$

where $\mathcal{M}$ is nonsingular, ε, $\mathcal{J}_1 \in \mathbb{R}^{(n+p)\times m}$, $\mathcal{J}_2 \in \mathbb{R}^{(n+p)\times 2p}$ are tuning parameters.

In this case, we have

$$\tilde{\mathcal{A}}_{ij}^\lambda\tilde{\mathcal{M}}_{js}^\lambda \triangleq \begin{bmatrix} \mathcal{A}_i\mathcal{M} + \theta(\lambda)\mathcal{B}_i\mathcal{M}\mathcal{K}_i + \bar{\theta}(\lambda)\mathcal{B}_i\mathcal{M}_{js,21}^\lambda & \mathcal{A}_i\mathcal{M}\mathcal{J}_1 + \theta(\lambda)\mathcal{B}_i\mathcal{M}\mathcal{K}_i\mathcal{J}_1 + \bar{\theta}(\lambda)\mathcal{B}_i\mathcal{M}_{js,22}^\lambda \\ \theta(\lambda)\mathcal{M}\mathcal{K}_i + \bar{\theta}(\lambda)\mathcal{M}_{js,21}^\lambda & \theta(\lambda)\mathcal{M}\mathcal{K}_i\mathcal{J}_1 + \bar{\theta}(\lambda)\mathcal{M}_{js,22}^\lambda \end{bmatrix},$$

$$\tilde{\mathcal{A}}_{ij}^\lambda\tilde{\mathcal{N}}_{js}^\lambda \triangleq \begin{bmatrix} \varepsilon\mathcal{A}_i\mathcal{M} + \theta(\lambda)\varepsilon\mathcal{B}_i\mathcal{M}\mathcal{K}_i + \bar{\theta}(\lambda)\mathcal{B}_i\mathcal{N}_{js,21}^\lambda & \varepsilon\mathcal{A}_i\mathcal{M}\mathcal{J}_1 + \theta(\lambda)\varepsilon\mathcal{B}_i\mathcal{M}\mathcal{K}_i\mathcal{J}_1 + \bar{\theta}(\lambda)\mathcal{B}_i\mathcal{N}_{js,22}^\lambda \\ \theta(\lambda)\varepsilon\mathcal{M}\mathcal{K}_i + \bar{\theta}(\lambda)\mathcal{N}_{js,21}^\lambda & \theta(\lambda)\varepsilon\mathcal{M}\mathcal{K}_i\mathcal{J}_1 + \bar{\theta}(\lambda)\mathcal{N}_{js,22}^\lambda \end{bmatrix},$$

$$\tilde{\mathcal{A}}_{ij}^{\lambda}\tilde{\mathcal{G}}_{js}^{\lambda} \triangleq \begin{bmatrix} \mathcal{A}_i\mathcal{M}\mathcal{J}_2 + \theta(\lambda)\mathcal{B}_i\mathcal{M}\mathcal{K}_i\mathcal{J}_2 + \bar{\theta}(\lambda)\mathcal{B}_i\mathcal{G}_{js,21}^{\lambda} \\ \theta(\lambda)\mathcal{M}\mathcal{K}_i\mathcal{J} + \bar{\theta}(\lambda)\mathcal{G}_{js,21}^{\lambda} \end{bmatrix}, \quad (33)$$

By substituting the expressions of $\tilde{\mathcal{M}}_{js}^{\lambda}$, $\tilde{\mathcal{N}}_{js}^{\lambda}$ and $\tilde{\mathcal{G}}_{js}^{\lambda}$ into (30) and (31), and defining $\bar{\mathcal{K}}_i \triangleq \mathcal{M}\mathcal{K}_i$, we obtain the conditions (27) and (28). The proof is completed.

Remark 5: Theorem 1 provides a computation method for the designed FOTC (4). The tuning parameters ε, $\mathcal{J}_1$ and $\mathcal{J}_2$ need to be given in advance. A simple choice for $\mathcal{J}_1$ and $\mathcal{J}_2$ is $J_1 = \begin{bmatrix} I_m & 0_{m\times(n+p-m)} \end{bmatrix}^T$ and $J_2 = \begin{bmatrix} I_{2p} & 0_{2p\times(n-p)} \end{bmatrix}^T$.

Remark 6: To obtain the gain matrices of the FOTC (4), a regular method is to make an equivalent matrix transformation for conditions (11) and (12). As a consequence, the dimensions of the matrix inequalities will be enlarged and more computational burden will be consumed. To overcome this issue, the dual conditions (30) and (31) are presented based on Lemma 2, which can avoid the matrix transformation and facilitate the controller synthesis. It is worth mentioning that the designed FOTC in (4) is off-line, so the computation cost in Theorems 2 does not affect its practical application.

C. Fuzzy Filter Design

Define $\mathcal{I}_1 \triangleq \begin{bmatrix} I_n & 0_{n\times(n+p)} \end{bmatrix}^T$, $\mathcal{I}_2 \triangleq \begin{bmatrix} 0_n & I_n & 0_{n\times p} \end{bmatrix}^T$, and $\mathcal{I}_3 \triangleq \begin{bmatrix} 0_{p\times 2n} & I_{p\times p} \end{bmatrix}^T$. Based on the obtained BRL in Theorem 1, the following theorem specifies the fuzzy filter (7) to achieve estimation performance for plant (1) despite the Markov packet dropouts.

Theorem 3: For specified ε, $\mathcal{J}_1$, and $\mathcal{J}_2$, the FES (6) is stochastically admissible with disturbance attenuation index γ, if there exist symmetric matrix variables $\mathcal{P}_{i,11}^{\lambda} \in \mathbb{R}^{n\times n}$, $\mathcal{P}_{i,22}^{\lambda} \in \mathbb{R}^{n\times n}$, $\mathcal{P}_{i,33}^{\lambda} \in \mathbb{R}^{p\times p}$, and $\tilde{\mathcal{U}}_{ijs}$, free matrix variables $\mathcal{P}_{i,12}^{\lambda} \in \mathbb{R}^{n\times n}$, $\mathcal{P}_{i,13}^{\lambda} \in \mathbb{R}^{n\times p}$, $\mathcal{P}_{i,23}^{\lambda} \in \mathbb{R}^{n\times p}$, $\mathcal{M}_{js,11}^{\lambda} \in \mathbb{R}^{n\times n}$, $\mathcal{M}_{js,13}^{\lambda} \in \mathbb{R}^{n\times p}$, $\mathcal{M}_{js,21}^{\lambda} \in \mathbb{R}^{n\times n}$, $\mathcal{M}_{js,23}^{\lambda} \in \mathbb{R}^{n\times p}$, $\mathcal{M}_{js,33}^{\lambda} \in \mathbb{R}^{p\times p}$, $\mathcal{N}_{js,11}^{\lambda} \in \mathbb{R}^{n\times n}$, $\mathcal{N}_{js,13}^{\lambda} \in \mathbb{R}^{n\times p}$, $\mathcal{N}_{js,21}^{\lambda} \in \mathbb{R}^{n\times n}$, $\mathcal{N}_{js,23}^{\lambda} \in \mathbb{R}^{n\times p}$, $\mathcal{N}_{js,33}^{\lambda} \in \mathbb{R}^{p\times p}$, $\mathcal{G}_{js,11}^{\lambda} \in \mathbb{R}^{q\times n}$, $\mathcal{G}_{js,13}^{\lambda} \in \mathbb{R}^{q\times p}$, $\mathcal{M} \in \mathbb{R}^{n\times n}$, $\bar{\mathcal{A}}_j^f \in \mathbb{R}^{n\times n}$, $\bar{\mathcal{B}}_j^f \in \mathbb{R}^{n\times p}$, and $\bar{\mathcal{L}}_j^f \in \mathbb{R}^{l\times n}$, $i,j,s \in \{1,2,\cdots,r\}$, such that for each $\lambda \in \{1,2\}$:

$$\begin{bmatrix} \mathcal{P}_{i,11}^{\lambda} & \mathcal{P}_{i,12}^{\lambda} & \mathcal{P}_{i,13}^{\lambda} \\ * & \mathcal{P}_{i,22}^{\lambda} & \mathcal{P}_{i,23}^{\lambda} \\ * & * & \mathcal{P}_{i,33}^{\lambda} \end{bmatrix} > 0, \quad (34)$$

$$\Pi_{iis}^{\lambda} < 0, \quad \varepsilon_{\lambda} = \{\varepsilon_{\lambda\min}, \varepsilon_{\lambda\max}\}, \quad (35)$$

$$\Pi_{ijs}^{\lambda} + \Pi_{jis}^{\lambda} < 0, \quad i < j, \quad \varepsilon_{\lambda} = \{\varepsilon_{\lambda\min}, \varepsilon_{\lambda\max}\}, \quad (36)$$

where

$$\Pi_{ijs}^{\lambda} \triangleq \begin{bmatrix} \Xi_{ijs}^{\lambda,11} & \Xi_{ijs}^{\lambda,12} & \Xi_{ijs}^{\lambda,13} & \mathcal{I}_1 L_i^T - \mathcal{I}_2\mathcal{L}_j^{f,T} \\ * & \Xi_{ijs}^{\lambda,22} & \Xi_{ijs}^{\lambda,23} & 0 \\ * & * & \Xi_{ijs}^{\lambda,33} & 0 \\ * & * & * & -I_l \end{bmatrix},$$

$$\begin{aligned} \Xi_{ijs}^{\lambda,11} \triangleq & -\mathcal{I}_1 E^T\mathcal{P}_{i,11}^{\lambda} E\mathcal{I}_1^T - \mathcal{I}_2\mathcal{E}^T\mathcal{P}_{i,22}^{\lambda}\mathcal{E}\mathcal{I}_2^T - \mathcal{I}_3 E^T\mathcal{P}_{i,33}^{\lambda} E\mathcal{I}_3^T \\ & -\mathrm{sym}\{\mathcal{I}_1 E^T\mathcal{P}_{i,12}^{\lambda}\mathcal{E}\mathcal{I}_2^T + \mathcal{I}_1 E^T\mathcal{P}_{i,13}^{\lambda}\mathcal{I}_3^T + \mathcal{I}_2\mathcal{E}^T\mathcal{P}_{i,23}^{\lambda}\mathcal{I}_3^T \\ & +\mathcal{I}_1\mathcal{M}_{js,11}^{\lambda} A_i\mathcal{I}_1^T + \mathcal{I}_1\theta(\lambda)\bar{\mathcal{B}}_j^f C_i\mathcal{I}_1^T + \mathcal{I}_2\mathcal{M}_{js,21}^{\lambda} A_i\mathcal{I}_1^T \\ & +\mathcal{I}_1\theta(\lambda)\mathcal{M}_{js,13}^{\lambda} C_i\mathcal{I}_1^T + \mathcal{I}_1\bar{\mathcal{A}}_j^f\mathcal{I}_2^T + \mathcal{I}_1\bar{\theta}(\lambda)\bar{\mathcal{B}}_j^f\mathcal{I}_3^T \\ & +\mathcal{I}_1\bar{\theta}(\lambda)\mathcal{M}_{js,13}^{\lambda}\mathcal{I}_3^T + \mathcal{I}_2\theta(\lambda)\bar{\mathcal{B}}_j^f C_i\mathcal{I}_1^T + \mathcal{I}_2\bar{\mathcal{A}}_j^f\mathcal{I}_2^T \\ & +\mathcal{I}_3\mathcal{M}_{js,31}^{\lambda} A_i\mathcal{I}_1^T + \mathcal{I}_3\theta(\lambda)\bar{\mathcal{B}}_j^f C_i\mathcal{I}_1^T + \mathcal{I}_3\mathcal{J}_1\bar{\mathcal{A}}_j^f\mathcal{I}_2^T \\ & +\mathcal{I}_2\theta(\lambda)\mathcal{M}_{js,23}^{\lambda} C_i\mathcal{I}_1^T + \mathcal{I}_2\bar{\theta}(\lambda)\mathcal{M}_{js,23}^{\lambda}\mathcal{I}_3^T \\ & +\mathcal{I}_3\theta(\lambda)\mathcal{M}_{js,33}^{\lambda} C_i\mathcal{I}_1^T + \mathcal{I}_3\bar{\theta}(\lambda)\mathcal{M}_{js,33}^{\lambda}\mathcal{I}_3^T \\ & +\mathcal{I}_2\bar{\theta}(\lambda)\bar{\mathcal{B}}_j^f\mathcal{I}_3^T + \mathcal{I}_3\bar{\theta}(\lambda)\mathcal{J}_1\bar{\mathcal{B}}_j^f\mathcal{I}_3^T\} \end{aligned},$$

$$\begin{aligned} \Xi_{ijs}^{\lambda,12} \triangleq & \mathcal{I}_3\mathcal{M}_{js,33}^{\lambda}\mathcal{I}_3^T - \mathcal{I}_1 A_i^T\mathcal{N}_{js,11}^{\lambda,T}\mathcal{I}_1^T - \mathcal{I}_1\theta(\lambda)\varepsilon C_i^T\bar{\mathcal{B}}_j^{f,T}\mathcal{I}_1^T \\ & -\mathcal{I}_1\theta(\lambda)C_i^T\mathcal{N}_{js,13}^{\lambda,T}\mathcal{I}_1^T - \mathcal{I}_1 A_i^T\mathcal{N}_{js,21}^{\lambda,T}\mathcal{I}_2^T - \mathcal{I}_2\varepsilon\bar{\mathcal{A}}_j^{f,T}\mathcal{I}_1^T \\ & -\mathcal{I}_1\theta(\lambda)\varepsilon C_i^T\bar{\mathcal{B}}_j^{f,T}\mathcal{I}_2^T - \mathcal{I}_1 A_i^T\mathcal{N}_{js,31}^{\lambda,T}\mathcal{I}_3^T - \mathcal{I}_2\varepsilon\bar{\mathcal{A}}_j^{f,T}\mathcal{I}_2^T \\ & -\mathcal{I}_2\varepsilon\bar{\mathcal{A}}_j^{f,T}\mathcal{J}_1^T\mathcal{I}_3^T - \mathcal{I}_3\bar{\theta}(\lambda)\varepsilon\bar{\mathcal{B}}_j^{f,T}\mathcal{I}_1^T + \mathcal{I}_1\mathcal{M}_{js,13}^{\lambda}\mathcal{I}_3^T \\ & -\mathcal{I}_1\theta(\lambda)C_i^T\mathcal{N}_{js,23}^{\lambda,T}\mathcal{I}_2^T + \mathcal{I}_2\mathcal{M}_{js,21}^{\lambda}\mathcal{I}_1^T + \mathcal{I}_3\mathcal{M}_{js,31}^{\lambda}\mathcal{I}_1^T \\ & -\mathcal{I}_1\theta(\lambda)\varepsilon C_i^T\bar{\mathcal{B}}_j^{f,T}\mathcal{I}_3^T + \mathcal{I}_1\mathcal{M}_{js,11}^{\lambda}\mathcal{I}_1^T + \mathcal{I}_3\mathcal{J}_1\mathcal{M}\mathcal{I}_2^T \\ & +\mathcal{I}_1\mathcal{M}\mathcal{I}_2^T - \mathcal{I}_1\theta(\lambda)C_i^T\mathcal{N}_{js,33}^{\lambda,T}\mathcal{I}_3^T + \mathcal{I}_2\mathcal{M}_{js,23}^{\lambda}\mathcal{I}_3^T \\ & +\mathcal{I}_2\mathcal{M}\mathcal{I}_2^T - \mathcal{I}_3\bar{\theta}(\lambda)\mathcal{N}_{js,13}^{\lambda,T}\mathcal{I}_1^T - \mathcal{I}_3\bar{\theta}(\lambda)\varepsilon\bar{\mathcal{B}}_j^{f,T}\mathcal{I}_2^T \\ & -\mathcal{I}_3\bar{\theta}(\lambda)\varepsilon\bar{\mathcal{B}}_j^{f,T}\mathcal{J}_1^T\mathcal{I}_3^T - \mathcal{I}_3\bar{\theta}(\lambda)\mathcal{N}_{js,23}^{\lambda,T}\mathcal{I}_2^T \\ & -\mathcal{I}_3\bar{\theta}(\lambda)\mathcal{N}_{js,23}^{\lambda,T}\mathcal{I}_3^T \end{aligned},$$

$$\begin{aligned} \Xi_{ijs}^{\lambda,13} \triangleq & -\mathcal{I}_1\mathcal{M}_{js,11}^{\lambda} D_i\mathcal{I}_1^T - \mathcal{I}_2\mathcal{M}_{js,21}^{\lambda} D_i\mathcal{I}_1^T - \mathcal{I}_1\theta(\lambda)C_i^T\mathcal{G}_{js,13}^{\lambda,T}\mathcal{I}_1^T \\ & -\mathcal{I}_3\mathcal{M}_{js,31}^{\lambda} D_i\mathcal{I}_1^T - \mathcal{I}_1\theta(\lambda)C_i^T\bar{\mathcal{B}}_j^{f,T}\mathcal{I}_1^T - \mathcal{I}_3\bar{\theta}(\lambda)\mathcal{G}_{js,13}^{\lambda,T}\mathcal{I}_1^T, \\ & -\mathcal{I}_3\bar{\theta}(\lambda)\bar{\mathcal{B}}_j^{f,T}\mathcal{J}_2^T\mathcal{I}_1^T - \mathcal{I}_2\bar{\mathcal{A}}_j^{f,T}\mathcal{J}_2^T\mathcal{I}_1^T - \mathcal{I}_1 A_i^T\mathcal{G}_{js,11}^{\lambda,T}\mathcal{I}_1^T \end{aligned}$$

$$\begin{aligned} \Xi_{ijs}^{\lambda,22} \triangleq & \mathcal{I}_1\mathcal{R}_{11}^T\tilde{\mathcal{U}}_{ijs}\mathcal{R}_{11}\mathcal{I}_1^T + \mathcal{I}_2\mathcal{R}_{12}^T\tilde{\mathcal{U}}_{ijs}\mathcal{R}_{12}\mathcal{I}_2^T + \mathcal{I}_3\mathcal{R}_{13}^T\tilde{\mathcal{U}}_{ijs}\mathcal{R}_{13}\mathcal{I}_3^T \\ & \mathcal{I}_1\sum_{\nu\in\{1,2\}}\pi_{\lambda\nu}\mathcal{P}_{s,11}^{\nu}\mathcal{I}_1^T + \mathcal{I}_1\sum_{\nu\in\{1,2\},\nu\neq\lambda}\varepsilon_{\lambda}(\mathcal{P}_{s,11}^{\nu} - \mathcal{P}_{s,11}^{\lambda})\mathcal{I}_1^T \\ & +\mathcal{I}_2\sum_{\nu\in\{1,2\}}\pi_{\lambda\nu}\mathcal{P}_{s,22}^{\nu}\mathcal{I}_2^T + \mathcal{I}_2\sum_{\nu\in\{1,2\},\nu\neq\lambda}\varepsilon_{\lambda}(\mathcal{P}_{s,22}^{\nu} - \mathcal{P}_{s,22}^{\lambda})\mathcal{I}_2^T \\ & +\mathcal{I}_3\sum_{\nu\in\{1,2\}}\pi_{\lambda\nu}\mathcal{P}_{s,33}^{\nu}\mathcal{I}_3^T + \mathcal{I}_3\sum_{\nu\in\{1,2\},\nu\neq\lambda}\varepsilon_{\lambda}(\mathcal{P}_{s,33}^{\nu} - \mathcal{P}_{s,33}^{\lambda})\mathcal{I}_3^T \\ & +\mathrm{sym}\{\mathcal{I}_1\mathcal{N}_{js,13}^{\lambda}\mathcal{I}_3^T + \mathcal{I}_1\mathcal{N}_{js,11}^{\lambda}\mathcal{I}_1^T + \mathcal{I}_2\mathcal{N}_{js,21}^{\lambda}\mathcal{I}_1^T \\ & +\mathcal{I}_2\mathcal{N}_{js,23}^{\lambda}\mathcal{I}_3^T + \mathcal{I}_1\sum_{\nu\in\{1,2\}}\pi_{\lambda\nu}\mathcal{P}_{s,13}^{\nu}\mathcal{I}_3^T + \mathcal{I}_2\sum_{\nu\in\{1,2\}}\pi_{\lambda\nu}\mathcal{P}_{s,23}^{\nu}\mathcal{I}_3^T \\ & +\mathcal{I}_1\sum_{\nu\in\{1,2\},\nu\neq\lambda}\varepsilon_{\lambda}(\mathcal{P}_{s,12}^{\nu} - \mathcal{P}_{s,12}^{\lambda})\mathcal{I}_2^T + \mathcal{I}_1\varepsilon\mathcal{M}\mathcal{I}_2^T + \mathcal{I}_2\varepsilon\mathcal{M}\mathcal{I}_2^T \\ & +\mathcal{I}_3\varepsilon\mathcal{J}_1\mathcal{M}\mathcal{I}_2^T + \mathcal{I}_1\mathcal{R}_{11}^T\tilde{\mathcal{U}}_{ijs}\mathcal{R}_{13}\mathcal{I}_3^T + \mathcal{I}_2\mathcal{R}_{12}^T\tilde{\mathcal{U}}_{ijs}\mathcal{R}_{13}\mathcal{I}_3^T \\ & +\mathcal{I}_1\sum_{\nu\in\{1,2\},\nu\neq\lambda}\varepsilon_{\lambda}(\mathcal{P}_{s,13}^{\nu} - \mathcal{P}_{s,13}^{\lambda})\mathcal{I}_3^T + \mathcal{I}_1\mathcal{R}_{11}^T\tilde{\mathcal{U}}_{ijs}\mathcal{R}_{12}\mathcal{I}_2^T \end{aligned},$$

$$+\mathcal{I}_2\sum_{v\in\{1,2\},v\neq\lambda}\varepsilon_\lambda(\mathcal{P}^v_{s,23}-\mathcal{P}^\lambda_{s,23})\mathcal{I}_3^T+\mathcal{I}_1\sum_{v\in\{1,2\}}\pi_{\lambda v}\mathcal{P}^v_{s,12}\mathcal{I}_2^T$$

$$+\mathcal{I}_3\mathcal{N}^\lambda_{js,31}\mathcal{I}_1^T+\mathcal{I}_3\mathcal{N}^\lambda_{js,33}\mathcal{I}_3^T\Big\}$$

$$\Xi^{\lambda,23}_{ijs}\triangleq-\mathcal{I}_1\mathcal{N}^\lambda_{js,11}D_i\mathcal{I}_1^T-\mathcal{I}_2\mathcal{N}^\lambda_{js,21}D_i\mathcal{I}_1^T-\mathcal{I}_3\mathcal{N}^\lambda_{js,31}D_i\mathcal{I}_1^T+\mathcal{I}_1\mathcal{G}^{\lambda,T}_{js,11}\mathcal{I}_1^T+\mathcal{I}_2\mathcal{M}^T\mathcal{J}_2^T\mathcal{I}_1^T+\mathcal{I}_3\mathcal{G}^{\lambda,T}_{js,13}\mathcal{I}_1^T,$$

$$\Xi^{\lambda,33}_{ijs}\triangleq-\gamma^2I-\mathrm{sym}\left\{\mathcal{G}^\lambda_{js,11}D_i\right\},$$

$\tilde{\mathcal{R}}=\begin{bmatrix}\mathcal{R}_{11} & \mathcal{R}_{12} & \mathcal{R}_{13}\end{bmatrix}$ is any arbitrary matrix satisfying $\tilde{\mathcal{R}}\tilde{\mathcal{E}}=0$ and $\mathrm{rank}\,(\tilde{\mathcal{R}})=n-s$.

Moreover, the parameters of the fuzzy filter (7) are computed by

$$\mathcal{A}^f_j=\mathcal{M}^{-1}\bar{\mathcal{A}}^f_j,\ \mathcal{B}^f_j=\mathcal{M}^{-1}\bar{\mathcal{B}}^f_j,\text{ and }\mathcal{L}^f_j=\bar{\mathcal{L}}^f_j. \quad (37)$$

Proof. Denote the Lyapunov matrices $\tilde{\mathcal{P}}^\lambda_i$ as $\tilde{\mathcal{P}}^\lambda_i\triangleq\begin{bmatrix}\mathcal{P}^\lambda_{i,11} & \mathcal{P}^\lambda_{i,12} & \mathcal{P}^\lambda_{i,13}\\ * & \mathcal{P}^\lambda_{i,22} & \mathcal{P}^\lambda_{i,23}\\ * & * & \mathcal{P}^\lambda_{i,33}\end{bmatrix}$, and their positivity is ensured by condition (34). The structures of matrix variables $\tilde{\mathcal{M}}^\lambda_{js}$, $\tilde{\mathcal{N}}^\lambda_{js}$ and $\tilde{\mathcal{G}}^\lambda_{js}$ are specified as

$$\tilde{\mathcal{M}}^\lambda_{js}\triangleq\begin{bmatrix}\mathcal{M}^\lambda_{js,11} & \mathcal{M} & \mathcal{M}^\lambda_{js,13}\\ \mathcal{M}^\lambda_{js,21} & \mathcal{M} & \mathcal{M}^\lambda_{js,23}\\ \mathcal{M}^\lambda_{js,31} & \mathcal{J}_1\mathcal{M} & \mathcal{M}^\lambda_{js,33}\end{bmatrix}^T,$$

$$\tilde{\mathcal{N}}^\lambda_{js}\triangleq\begin{bmatrix}\mathcal{N}^\lambda_{js,11} & \varepsilon\mathcal{M} & \mathcal{N}^\lambda_{js,13}\\ \mathcal{N}^\lambda_{js,21} & \varepsilon\mathcal{M} & \mathcal{N}^\lambda_{js,23}\\ \mathcal{N}^\lambda_{js,31} & \varepsilon\mathcal{J}_1\mathcal{M} & \mathcal{N}^\lambda_{js,33}\end{bmatrix}^T,$$

$$\tilde{\mathcal{G}}^\lambda_{js}\triangleq\begin{bmatrix}\mathcal{G}^\lambda_{js,11} & \mathcal{J}_2\mathcal{M} & \mathcal{G}^\lambda_{js,13}\end{bmatrix}^T, \quad (38)$$

where $\mathcal{M}$ is nonsingular, ε, $\mathcal{J}_1\in\mathbb{R}^{p\times n}$, $\mathcal{J}_2\in\mathbb{R}^{q\times n}$ are tuning parameters.

Thus, we have

$$\tilde{\mathcal{M}}^{\lambda,T}_{js}\tilde{\mathcal{A}}^\lambda_{ij}=\left[\begin{matrix}\mathcal{M}^\lambda_{js,11}A_i+\theta(\lambda)\mathcal{M}\mathcal{B}^f_jC_i+\theta(\lambda)\mathcal{M}^\lambda_{js,13}C_i\\ \mathcal{M}^\lambda_{js,21}A_i+\theta(\lambda)\mathcal{M}\mathcal{B}^f_jC_i+\theta(\lambda)\mathcal{M}^\lambda_{js,23}C_i\\ \mathcal{M}^\lambda_{js,31}A_i+\theta(\lambda)\mathcal{J}_1\mathcal{M}\mathcal{B}^f_jC_i+\theta(\lambda)\mathcal{M}^\lambda_{js,33}C_i\end{matrix}\right.$$
$$\left.\begin{matrix}\mathcal{M}\mathcal{A}^f_j & \bar{\theta}(\lambda)\mathcal{M}\mathcal{B}^f_j+\bar{\theta}(\lambda)\mathcal{M}^\lambda_{js,13}\\ \mathcal{M}\mathcal{A}^f_j & \bar{\theta}(\lambda)\mathcal{M}\mathcal{B}^f_j+\bar{\theta}(\lambda)\mathcal{M}^\lambda_{js,23}\\ \mathcal{J}_1\mathcal{M}\mathcal{A}^f_j & \bar{\theta}(\lambda)\mathcal{J}_1\mathcal{M}\mathcal{B}^f_j+\bar{\theta}(\lambda)\mathcal{M}^\lambda_{js,33}\end{matrix}\right],$$

$$\tilde{\mathcal{N}}^{\lambda,T}_{js}\tilde{\mathcal{A}}^\lambda_{ij}=\left[\begin{matrix}\mathcal{N}^\lambda_{js,11}A_i+\theta(\lambda)\varepsilon\mathcal{M}\mathcal{B}^f_jC_i+\theta(\lambda)\mathcal{N}^\lambda_{js,13}C_i\\ \mathcal{N}^\lambda_{js,21}A_i+\theta(\lambda)\varepsilon\mathcal{M}\mathcal{B}^f_jC_i+\theta(\lambda)\mathcal{N}^\lambda_{js,23}C_i\\ \mathcal{N}^\lambda_{js,31}A_i+\theta(\lambda)\varepsilon\mathcal{J}_1\mathcal{M}\mathcal{B}^f_jC_i+\theta(\lambda)\mathcal{N}^\lambda_{js,33}C_i\end{matrix}\right.$$
$$\left.\begin{matrix}\varepsilon\mathcal{M}\mathcal{A}^f_j & \bar{\theta}(\lambda)\varepsilon\mathcal{M}\mathcal{B}^f_j+\bar{\theta}(\lambda)\mathcal{N}^\lambda_{js,13}\\ \varepsilon\mathcal{M}\mathcal{A}^f_j & \bar{\theta}(\lambda)\varepsilon\mathcal{M}\mathcal{B}^f_j+\bar{\theta}(\lambda)\mathcal{N}^\lambda_{js,23}\\ \varepsilon\mathcal{J}_1\mathcal{M}\mathcal{A}^f_j & \bar{\theta}(\lambda)\varepsilon\mathcal{J}_1\mathcal{M}\mathcal{B}^f_j+\bar{\theta}(\lambda)\mathcal{N}^\lambda_{js,33}\end{matrix}\right],$$

$$\tilde{\mathcal{G}}^{\lambda,T}_{js}\tilde{\mathcal{A}}^\lambda_{ij}=\left[\mathcal{G}^\lambda_{js,11}A_i+\theta(\lambda)\mathcal{J}_2\mathcal{M}\mathcal{B}^f_jC_i+\theta(\lambda)\mathcal{G}^\lambda_{js,13}C_i\quad \mathcal{J}_2\mathcal{M}\mathcal{A}^f_j\quad \bar{\theta}(\lambda)\mathcal{J}_2\mathcal{M}\mathcal{B}^f_j+\bar{\theta}(\lambda)\mathcal{G}^\lambda_{js,13}\right], \quad (39)$$

Replace $\tilde{\mathcal{R}}$ and $\tilde{\mathcal{C}}_i$ by $\begin{bmatrix}\mathcal{R}_{11} & \mathcal{R}_{12} & \mathcal{R}_{13}\end{bmatrix}$ and $\tilde{\mathcal{L}}_{ij}$, respectively. By substituting the expressions of system matrices and matrix variables into (11) and (12), and define $\bar{\mathcal{A}}^f_j\triangleq\mathcal{M}\mathcal{A}^f_j$, $\bar{\mathcal{B}}^f_j\triangleq\mathcal{M}\mathcal{B}^f_j$, $\bar{\mathcal{L}}^f_j\triangleq\mathcal{L}^f_j$, we get the conditions (35) and (36). The proof is completed.

Remark 7: Similar to Theorem 2, the tuning parameters ε, $\mathcal{J}_1$ and $\mathcal{J}_2$ need to be set in advance. A simple choice for $\mathcal{J}_1$ and $\mathcal{J}_2$ is $J_1=\begin{bmatrix}I_p & 0_{p\times(n-p)}\end{bmatrix}$ and $J_2=\begin{bmatrix}I_q & 0_{q\times(n-q)}\end{bmatrix}$.

IV. Simulation Examples

Example 1 (fuzzy output tracking control). The inverted pendulum controlled by a dc motor via a gear train is adopted, and the physical system is illustrated in Figs. 2 and 3. The plant's parameters $\mathrm{g}=9.8\,\mathrm{m/s^2}$, $\mathrm{K_m}=0.1\,\mathrm{Nm/A}$, M=1kg, L=1 m, $\mathrm{R}_a=1\,\Omega$, N=10, $\mathrm{K_b}=0.1\ \mathrm{Vs/rad}$. Define the state variables $x_1(t)\triangleq\theta_\mathrm{p}(t)$, $x_2(t)\triangleq\dot{\theta}_\mathrm{p}(t)$, $x_3(t)\triangleq\mathrm{I_a}(t)$, the physical system and can be described by the following nonlinear equation:

$$\begin{cases}\dot{x}_1(t)=x_2(t)\\ \dot{x}_2(t)=9.8\sin x_1(t)+x_3(t)\\ \dot{x}_3(t)=-2x_2(t)-2x_3(t)+2u(t)+w(t)\end{cases}, \quad (40)$$

Let $\mathrm{T}=0.1$ be the fixed step of discretization and $x_4(k)=9.8\mathrm{T}\sin x_1(k)$. By applying the Euler's discretization approach [44], the nonlinear equation (40) can be transformed into the following nonlinear discrete-time descriptor system:

$$\begin{cases}x_1(k+1)=x_1(k)+\mathrm{T}x_2(k)\\ x_2(k+1)=x_2(k)+\mathrm{T}x_3(k)+x_4(k)\\ x_3(k+1)=(1-2\mathrm{T})x_3(k)+\mathrm{T}\left(-2x_2(k)+2u(k)+w(k)\right)\\ 0=\mathrm{T}9.8\sin x_1(k)-x_4(k)\end{cases}. \quad (41)$$

Then, under $x_1(k)\in\begin{bmatrix}-\pi & \pi\end{bmatrix}$, the plant (41) can be described by the following fuzzy descriptor system [45]:

$$\begin{cases}Ex(k+1)=\sum_{i=1}^{2}\xi_i\left(x_1(k)\right)\left[A_ix(k)+B_iu(k)+D_iw(k)\right]\\ y(k)=x_1(k)\end{cases}, \quad (42)$$

where $x(k)=\begin{bmatrix}x_1^T(k) & x_2^T(k) & x_3^T(k) & x_4^T(k)\end{bmatrix}^T$,

$$E=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&0\end{bmatrix},\ B_1=B_2=\begin{bmatrix}0\\0\\0.2\\0\end{bmatrix},\ D_1=D_2=\begin{bmatrix}0\\0\\0.1\\0\end{bmatrix},$$

$$A_1=\begin{bmatrix}1&0.1&0&0\\0&1&0.1&1\\0&-0.2&0.8&0\\0.98&0&0&-1\end{bmatrix},\ A_2=\begin{bmatrix}1&0.1&0&0\\0&1&0.1&1\\0&-0.2&0.8&0\\0&0&0&-1\end{bmatrix},$$

and the normalized fuzzy weighting functions are depicted in Fig. 4.

It is assumed that $\alpha=0.9$, $\beta=0.1$, $0.01<\Delta\alpha<0.02$,

$-0.02<\Delta\beta<-0.01$, by solving Theorem 2 with $\varepsilon=1.2$, $\mathcal{J}_1=[1\quad 0_{1\times4}]^T$, $\mathcal{J}_2=[I_2\quad 0_{2\times3}]^T$, $\mathcal{R}_{11}=[0\quad 0\quad 0\quad 1]$ and $\mathcal{R}_{12}=0$, and $\gamma=25$, we obtain the gain matrix variables of the FOTC as

$$\mathcal{K}_1=[3.3013\quad -68.1172\quad -25.9659\quad -3.9975\quad -12.2256],$$
$$\mathcal{K}_2=[3.3539\quad -68.4705\quad -26.2373\quad -4.0436\quad -12.3473]. \tag{43}$$

The reference output tracking command is set as

$$y_r(k)=\begin{cases}0.5, & 10\le k\le 20\\ 0, & \text{otherwise}\end{cases}, \tag{44}$$

Under the zero initial condition and the external disturbance $w(k)=8e^{-0.15k}\sin(2k)$, the responses of output signal and the referenced signal are depicted in Fig. 5, the response of FOTC is shown in Fig. 6, and the situation of data packet dropouts is given in Fig. 7. The simulation results verify that, under the designed FOTC (4), the measurement output can effectively track the referenced output signal despite the uncertain Markov packet dropouts.

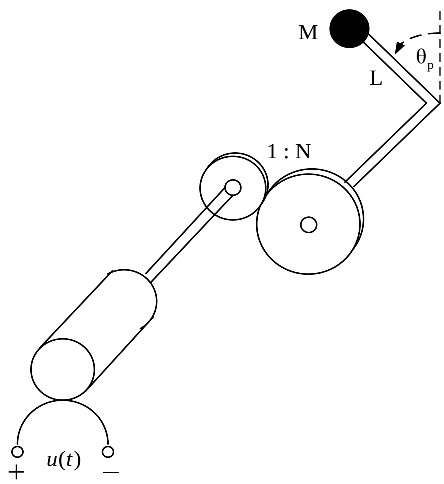

Fig. 2. The schemetic diagram of controlled inverted pendulum.

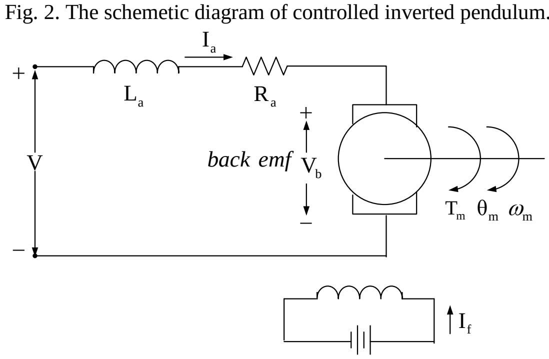

Fig. 3. The schemetic diagram of armature-controlled dc motor.

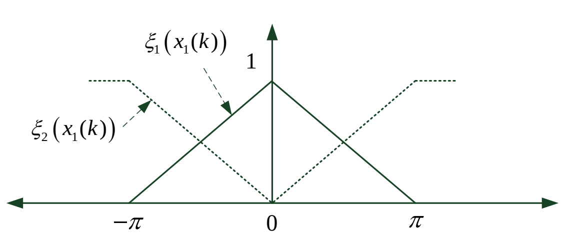

Fig. 4. Fuzzy weighting functions.

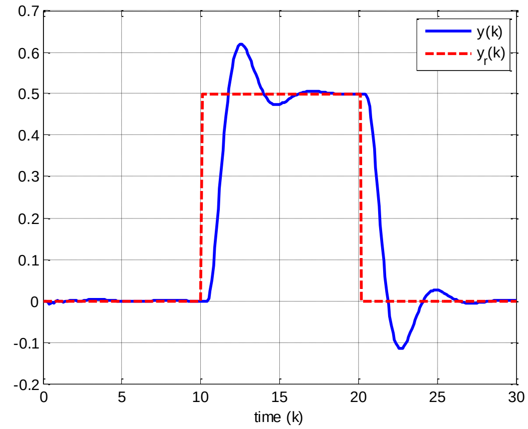

Fig. 5. Output tracking responses.

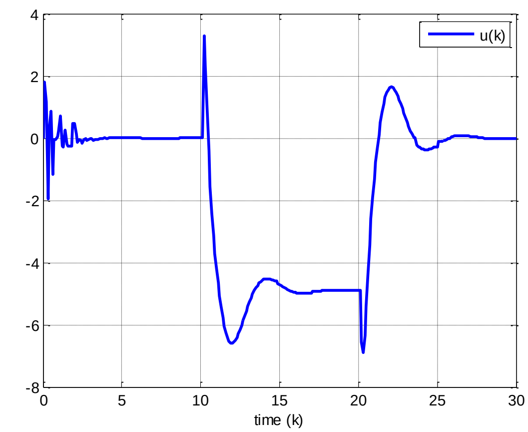

Fig. 6. Response of the FOTC.

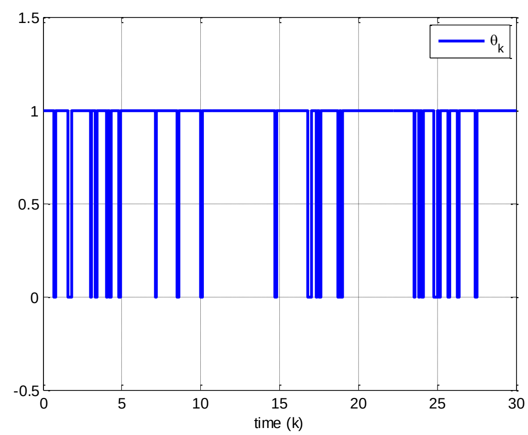

Fig. 7. Data packet dropouts.

Example 2 (fuzzy filtering). A nonlinear tunnel diode circuit is considered, and the diagram is provided in Fig. 8. The circuit's parameters are chosen as $\mathrm{C}=20\ \mathrm{mF}$, $\mathrm{L}=1000\ \mathrm{mH}$, $\mathrm{R}=10\ \Omega$, and $\mathrm{i_D}=0.002\mathrm{V_D}(t)+0.01\mathrm{V_D^3}(t)$. Defining the state variables $x_1(t)=\mathrm{V_C}(t)$ and $x_2(t)=\mathrm{i_L}(t)$, the nonlinear circuit can be described by

$$\begin{cases}\dot{x}_1(t)=-0.1x_1(t)-0.5x_1^3(t)+50x_2(t)\\ \dot{x}_2(t)=-x_1(t)-10x_2(t)+w(t)\end{cases}. \tag{45}$$

Let $\mathrm{T}=0.1$ be the fixed step of discretization and

$x_3(k) = -0.5\mathrm{T}x_1^3(k)$. By applying the Euler's discretization approach [44], the nonlinear system (45) can be transformed into the following discrete-time model:

$$\begin{cases} x_1(k+1) = x_1(k) + \mathrm{T}\left(-0.1x_1(k) + 50x_2(k)\right) + x_3(k) \\ x_2(k+1) = x_2(k) + \mathrm{T}\left(-x_1(k) - 10x_2(k) + w(k)\right) \\ \qquad 0 = \mathrm{T}0.5x_1^3(k) + x_3(k) \end{cases} . \quad (46)$$

Under $x_1(k) \in [-3 \quad 3]$, the plant (46) can be further described by

$$\begin{cases} Ex(k+1) = \sum_{i=1}^{2} \xi_i\left(x_1(k)\right)\left[A_i x(k) + D_i w(k)\right] \\ \qquad y(k) = z(k) = x_1(k) \end{cases}, \quad (47)$$

where the normalized fuzzy weighting functions are depicted in Fig. 9, and

$x(k) = \left[x_1^T(k) \quad x_2^T(k) \quad x_3^T(k)\right]^T$, $D_1 = D_2 = [0 \quad 0.1 \quad 0]^T$,

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ A_1 = \begin{bmatrix} 0.99 & 5 & 1 \\ -0.1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ A_2 = \begin{bmatrix} 0.99 & 5 & 1 \\ -0.1 & 0 & 0 \\ 0.45 & 0 & 1 \end{bmatrix}.$$

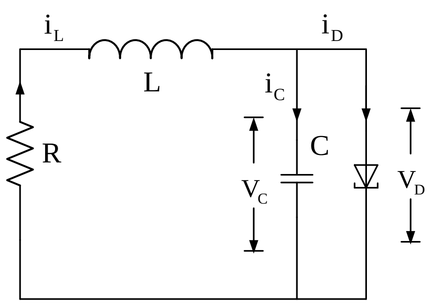

Fig. 8. Tunnel diode circuit.

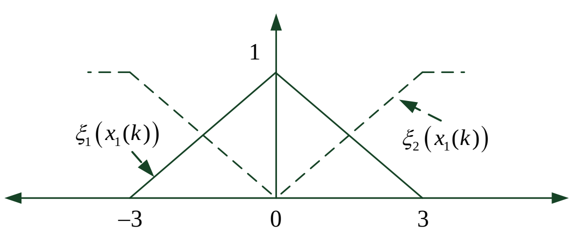

Fig. 9. Fuzzy weighting functions.

It is assumed that $\alpha = 0.4$, $\beta = 0.7$, $-0.1 < \Delta\alpha < 0.2$, $-0.2 < \Delta\beta < 0.1$, by solving Theorem 3 with $\varepsilon = 1.2$, $\mathcal{J}_1 = \mathcal{J}_2 = [1 \quad 0_{1\times 2}]$, $\mathcal{R}_{11} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $\mathcal{R}_{12} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\mathcal{R}_{13} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, and $\gamma = 1$, we obtain the filter parameters as

$$\mathcal{A}_1^f = \begin{bmatrix} -0.1513 & 1.6571 & 0.2832 \\ -0.0169 & 0.3787 & 0.0358 \\ 0.0231 & 0.1272 & 1.1990 \end{bmatrix},\ \mathcal{B}_1^f = \begin{bmatrix} -1.3087 \\ 0.0040 \\ 0.0104 \end{bmatrix},$$

$$\mathcal{A}_2^f = \begin{bmatrix} -0.1463 & 1.6131 & 0.1750 \\ -0.0193 & 0.3657 & 0.0555 \\ 0.0283 & 0.1219 & 0.8909 \end{bmatrix},\ \mathcal{B}_2^f = \begin{bmatrix} -1.2595 \\ -0.0042 \\ -0.0796 \end{bmatrix},$$

$$\mathcal{L}_1^f = [0.0109 \quad 0.0265 \quad -0.0047],$$
$$\mathcal{L}_2^f = [0.0135 \quad 0.0105 \quad -0.0890]. \quad (48)$$

With initial condition $x(0) = [2 \quad 1 \quad -0.4]^T$ and fuzzy filter (7), the filtering error under the external disturbance $w(k) = 8e^{-0.15k}\sin(2k)$ is demonstrated in Fig. 10, and the situation about data packet dropouts is given in Fig. 11. The simulation results verify that the system state can be estimated successfully by the designed fuzzy filter despite the uncertain Markov packet dropouts.

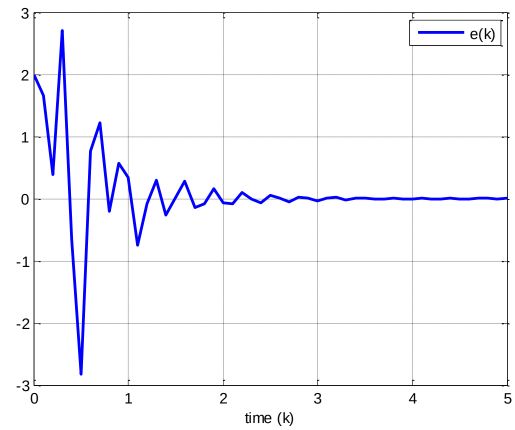

Fig. 10. Response of filtering error.

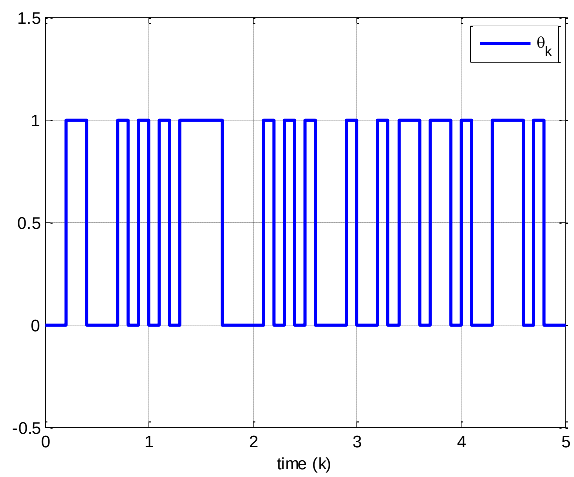

Fig. 11. Data packet dropouts.

V. Conclusion

This paper has investigated the problems of FOTC and filter design for discrete-time fuzzy descriptor systems, which are limited by unreliable communication links. The phenomenon of data packet dropouts has been characterized by a binary Markov chain with uncertain transition probabilities. A novel BRL has been given to guarantee the stochastic admissibility with noise attenuation performance. Based on the dual conditions of the BRL, a FOTC design approach has been provided by choosing the suitable structures of the specified variable matrices. A computational method for the designed fuzzy filter is further provided. Finally, two examples have been adopted to demonstrate the effectiveness of the developed design strategies. In future work, we shall consider the problem of limited communication resources by introducing event-triggered transformation mechanism. The problem of tracking control based on output measurement only will also be studied by designing a suitable static output feedback fuzzy controller.

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