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# The Effects of Nonlocal Impact Ionization on the Speed of Avalanche Photodiodes

P. J. Hambleton, B. K. Ng, S. A. Plimmer, J. P. R. David, and G. J. Rees

**Abstract**—The nonlocal enhancement in the velocities of charge carriers to ionization is shown to outweigh the opposing effects of dead space, increasing the avalanche speed of short avalanche photodiodes (APDs) over the predictions of a conventional local model which ignores both of these effects. The trends in the measured gain–bandwidth product of two short InAlAs APDs reported in the literature support this result. Relatively large speed benefits are predicted to result from further small reductions in the lengths of short multiplication regions.

**Index Terms**—Avalanche photodiode (APD), bandwidth, dead space, frequency response,  $\text{In}_{0.52}\text{Al}_{0.48}\text{As}$ , nonequilibrium, nonlocal, velocity enhancement.

## I. INTRODUCTION

THE INTRINSIC gain of avalanche photodiodes (APDs) provides increased sensitivity in the detection of weak optical signals. However, the time for the avalanche to build up and decay is prolonged in devices in which both electron and hole ionization contribute significantly to the gain process. The extent to which this delay limits APD bandwidth has traditionally been interpreted using local models, in which electrons and holes are assumed to ionize with a probability that depends only on the local electric field strength and also to travel at their saturated drift velocities,  $v_n$  and  $v_p$ . Such models predict an almost direct scaling of 3-dB bandwidth with inverse device length. However, these assumptions are valid only in long ( $> \sim 1.0 \mu\text{m}$ ) multiplication regions [1], [2], where, on average, carriers reach equilibrium with the electric field long before they ionize.

High-speed APDs require much shorter multiplication regions ( $< \sim 0.2 \mu\text{m}$ ) where nonequilibrium effects in the transport lead to deviations from the assumptions of the local model. It is already well known that these assumptions are unable to account for the spatial distribution of ionization events in short devices which determines gain and excess noise. Indeed, a carrier must travel some distance in the field before acquiring sufficient energy to ionize, and this “dead space” is an essential ingredient of a correct interpretation of the behavior of short APDs [3]. It has been argued that the influence of the dead space can be expected to increase the avalanche delay for a given mean gain, reducing the bandwidth relative to local model predictions [4], [5]. This follows since the fraction of the multiplication region in which a

carrier can ionize is reduced by dead space. The attainment of a given mean gain then becomes more reliant on ionization feedback processes [2]. By contrast, recent Monte Carlo (MC) modeling of avalanche in short multiplication regions [1] has shown that nonequilibrium enhancement of the mean velocities of carriers to ionization leads to significantly faster avalanches than in a model which is spatially equivalent, but in which carriers travel always at their saturated drift velocities.

These two nonequilibrium effects exert opposing influences on APD bandwidth and their strengths vary in different ways with the electric field. Precisely how the bandwidth of short APDs scales with either gain or device length is therefore not clear, although an appreciation of such trends is needed for the design of high performance APDs. Models of the bandwidth must include both of these nonequilibrium effects so that measurements made on short APDs can be interpreted correctly. Currently, the only techniques capable of achieving this directly are relatively complex, such as MC [1] and Fokker Planck models [6].

Although unsuitable for predicting the speed of thin APDs, local ionization expressions of the type developed by Emmons [7] or Kuvás and Lee [8] can be used as a standard against which to judge the competition between nonlocal effects. Such models are used widely and are relatively simple to apply, although their use in the nonlocal regime requires justification. Kuvás and Lee showed that the frequency-dependent avalanche current response to pure electron injection at angular frequency  $\omega$  is proportional to

$$1/(1 + j\omega/(2\pi f_{3\text{dB}})) \quad (1a)$$

with a 3-dB bandwidth of

$$f_{3\text{dB}} = v/(2\pi MK_n L) \quad (1b)$$

where  $M$  is the mean dc gain,  $K_n$  is a correction factor for the displacement current, dependent on the ratio of hole to electron ionization coefficients,  $v = 2v_n v_p / (v_n + v_p)$  is the harmonic mean of the electron and hole saturated drift velocities, and  $L$  is the length of the multiplication region. The first order roll-off given by (1a) is also obtained in a nonlocal situation because of an overwhelming exponential time decay which is seen to dominate the mean current impulse response in the presence of soft dead space, velocity enhancement, and diffusion effects [1] for a wide range of device widths and multiplication values. Indeed, this demonstrates the wider validity of the work of Hayat and Saleh [4] who showed that this exponential behavior is expected for the case of constant carrier velocities and a hard dead space model in the long time limit. These effects of velocity enhancement and dead space can be included together in the effective value assigned to  $v$ , which then becomes a fitting parameter to determine the net of nonlocal effects.

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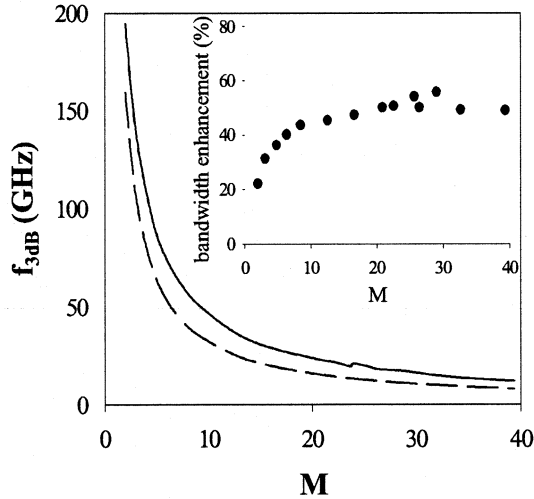


Fig. 1. The 3-dB bandwidth of mean avalanche current response for a 0.10- $\mu\text{m}$  multiplication region versus mean gain, predicted using the MC model (solid line) and the local model (dashed line). The inset shows the fractional increase of the MC model bandwidth over that of the local model.

This fitting parameter is expected to vary with device length and gain because of the field dependence of the competing processes of velocity enhancement and dead space. It is therefore instructive to examine trends in its expected behavior. To establish these trends, we compare predictions of the 3-dB bandwidth using a simple MC model, which accounts implicitly for both effects, with those obtained from a corresponding local model, for a range of device lengths and gains. We find that the effect of velocity enhancement increasingly dominates the opposing effect of dead space with increasing field so that, in fitting the 3-dB bandwidth using (1b) by adjusting  $v$ , this fitting parameter is expected to increase with decreasing multiplication region length. We find that the trends found for measurements in the literature of the bandwidth of two short  $\text{In}_{0.52}\text{Al}_{0.48}\text{As}$  APDs compare qualitatively with those predicted using our simple MC model.

## II. MODELING

Our simple MC model, previously described in [1], was used to compute the stochastic current induced following electron injection at one edge of the multiplication region. This current is given by Ramo's theorem [9] as  $i(t) = q \sum_i v_i / L$ , where  $q$  is the electronic charge and the sum is taken over the instantaneous velocities  $v_i$  of all carriers in the multiplication region at time  $t$ . An ensemble average of  $i(t)$  is taken over many equivalent trials to obtain the mean current impulse response from which the 3-dB bandwidth is obtained by Fourier transforming. For clarity, a model semiconductor was assumed in which the transport and ionization behavior of electrons and holes were treated as equivalent. The model also predicts a value of the saturated drift velocity of  $v_n (= v_p) = 6.65 \times 10^4 \text{ ms}^{-1}$  which is used in the corresponding local model, which assumes equal ionization coefficients for electrons and holes.

Fig. 1 compares predictions of the 3-dB bandwidth of mean avalanche current response from the MC model and from the local model for a 0.10- $\mu\text{m}$  multiplication region. It is evident that the MC model predicts the higher bandwidth for all values

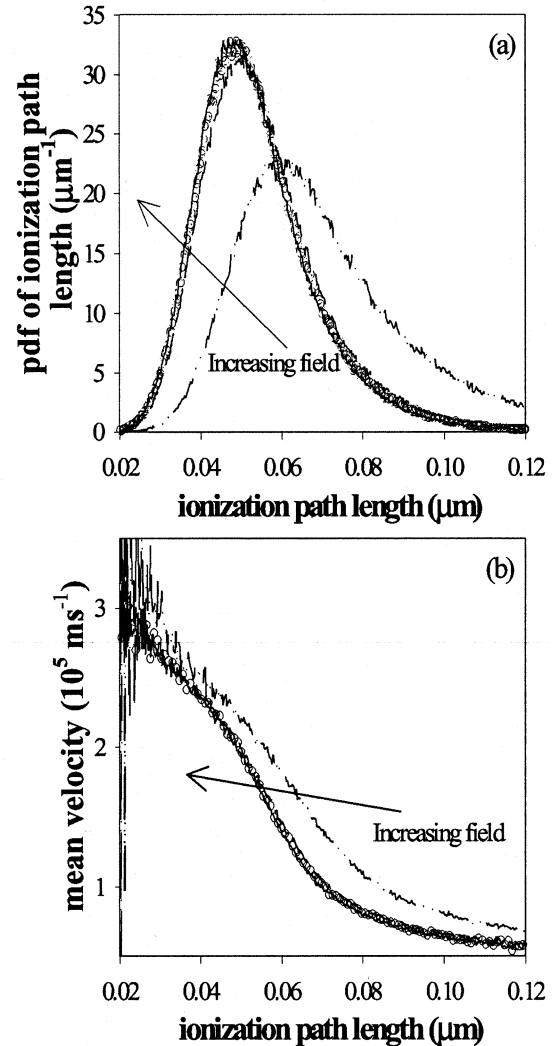


Fig. 2. (a) Pdfs of ionization path length and (b) mean velocity of carriers to ionization as a function of ionization path length predicted by the MC model for fields  $E = 6.469 \times 10^7 \text{ Vm}^{-1}$  (dashed double dotted line),  $E = 7.550 \times 10^7 \text{ Vm}^{-1}$  (short dash line),  $E = 7.690 \times 10^7 \text{ Vm}^{-1}$  (open circles), and  $E = 7.750 \times 10^7 \text{ Vm}^{-1}$  (solid line) giving mean gain,  $M = 3$ ,  $M = 13$ ,  $M = 24$ , and  $M = 39$  in a 0.1- $\mu\text{m}$  multiplication region. The ionization path length pdfs and the mean velocities for each ionization path length converge with increasing field. The MC noise in the velocity at small ionization path lengths is a result of the relatively small number of ionization events and relatively large velocity fluctuations.

of gain. The fractional increase in the bandwidth is plotted separately in the inset. Since this is always positive, the enhancement in ionizing carrier velocities must consistently overcompensate for the effect of dead space. Moreover, the former clearly increases faster with field than the latter at low gain, as follows from the initial rise in the curve. For  $M > 10$ , however, the bandwidth enhancement saturates at around 50%. This is to be expected, since the competing effects on the bandwidth depend primarily on electric field, whereas the gain increases without limit close to breakdown. Thus, for appreciable values of gain, small increments in field produce large changes in gain but have a negligible influence on the effects of velocity enhancement and dead space on bandwidth.

This argument is supported by Fig. 2(a), which depicts ionization path length probability density functions (pdfs), and

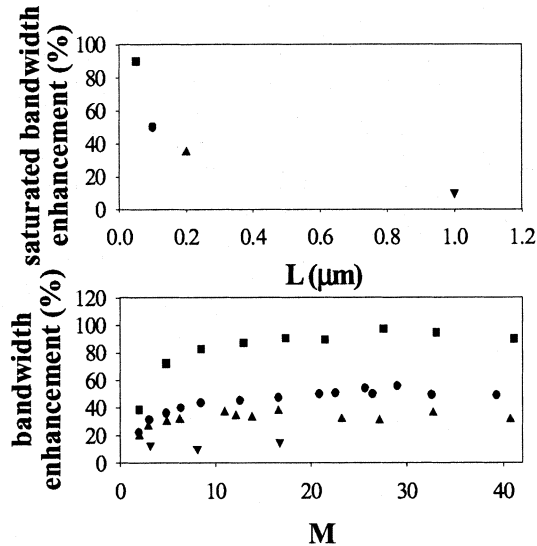


Fig. 3. Relative enhancement in the 3-dB bandwidth versus mean gain for the MC model over that predicted by the local model for multiplication regions of length,  $L = 0.05 \mu\text{m}$  (square),  $L = 0.10 \mu\text{m}$  (circle),  $L = 0.20 \mu\text{m}$  (triangle up), and  $L = 1.00 \mu\text{m}$  (triangle down). The top panel shows the saturated values of the enhancements shown in the lower panel versus multiplication region length.

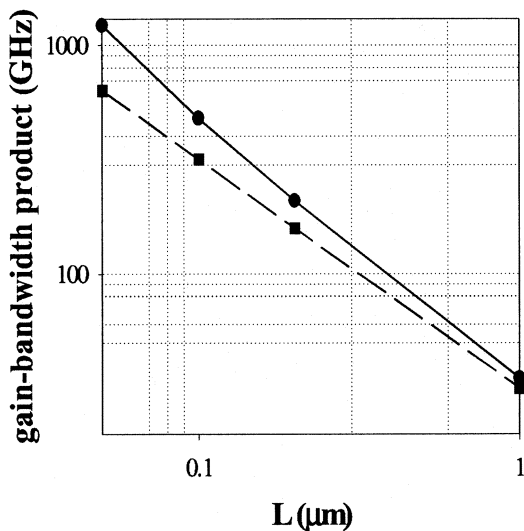


Fig. 4. Gain–bandwidth product at saturation versus multiplication region length predicted both by the MC model (circles) and the local model (squares). The lines are a guide to the eye.

Fig. 2(b), which depicts the dependence of mean velocity to ionization on ionization path length, for a range of electric fields corresponding to increasing values of multiplication in a  $0.1\text{-}\mu\text{m}$  multiplication region. As the field increases, both sets of curves shift to shorter distances so that ionization path lengths and the time to ionization are reduced, on average. However, the incremental changes in pdf and mean velocity to ionization become negligible with increasing  $M$ , leading to saturation of the bandwidth enhancement, as shown in the inset of Fig. 1.

Similar calculations of the bandwidth enhancement versus gain were carried out for a range of multiplication region lengths. The results are shown in Fig. 3 (bottom panel). For

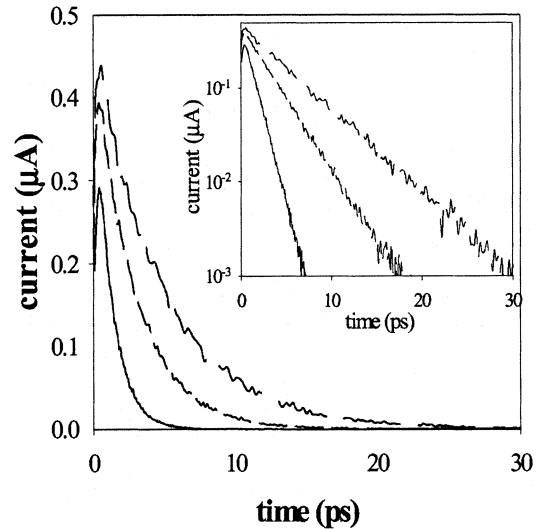


Fig. 5. Mean current impulse response predicted by the MC model in which electron and hole transport are no longer treated as equivalent following electron injection into a  $0.1\text{-}\mu\text{m}$  multiplication region for mean gain,  $M = 3$  (solid line),  $M = 8$  (short dashed line), and  $M = 15$  (long dashed line). The inset shows the same curves with current plotted on a log scale.

each length, the enhancement in bandwidth first increases and then saturates with gain with a value which increases with decreasing multiplication region length, as summarized in the top panel. The results again show that the effect of velocity enhancement increases faster with field than the delaying effect of the dead space. The practical implications of this are summarized in Fig. 4 in terms of the gain–saturated–bandwidth product predicted by the MC model and the local model.

The applicability of (1a) in the nonlocal regime follows from the fact that, despite strong influences, both of enhanced carrier velocities to ionization and of dead space effects, the mean current impulse response nonetheless is dominated by exponential decay. This is especially true at high values of gain where measurements of the gain–bandwidth product are most usefully determined. This was demonstrated in [1] using our MC model for the case where the transport of electrons and holes was treated in an equivalent manner. Exponential decay was also demonstrated analytically in [4] for the case of a hard dead space model and constant carrier velocities. Fig. 5 demonstrates a more general example of the dominance of exponential decay using our MC model for a  $0.10\text{-}\mu\text{m}$  multiplication region where electron and hole transport was modeled according to parameters given in [10] for GaAs and no equivalence is assumed.

### III. COMPARISON WITH EXPERIMENT

Lenox *et al.* [11] measured the frequency response of a pair of similar, separate absorption, charge and multiplication APDs with  $\text{In}_{0.52}\text{Al}_{0.48}\text{As}$  multiplication regions of length  $L = 0.40 \mu\text{m}$  and  $L = 0.20 \mu\text{m}$ . We have fitted their measured values of gain–bandwidth product  $Mf_{3\text{ dB}}$  to (1) to obtain an effective value of mean carrier velocity  $v_{\text{eff}}$  for each device.  $K_n$  was calculated using the expression of Kuvás and Lee [8], valid for large values of gain and conveniently independent of carrier velocities, using values for the ionization coefficients ratio taken from the work of Watanabe *et al.* [12]. The factor

TABLE I  
DETAILS OF FITTING TO THE EXPERIMENTAL RESULTS IN [11]

L ( $\mu\text{m}$ )	$M \times f_{3\text{dB}}$ (GHz)	$K_n$	$v_n$ ( $10^4 \text{ ms}^{-1}$ )	$v_{\text{eff}}$ ( $10^4 \text{ ms}^{-1}$ )	Dead space slowing factor, s	$v_{\text{eff}}^*$ ( $10^4 \text{ ms}^{-1}$ )
0.40	130	0.24	4.5	7.9	0.9	8.8
0.20	290	0.27	4.5	9.7	0.8	12.1

$K_n$  results from a calculation in [8] which assumes local ionization and depends on the ratio of ionization coefficients. We extend its use here to the nonlocal regime in order to compare predictions of the nonlocal and local models. Because of the dependence of ionization coefficients on electric field, the local value we use here for  $K_n$  leads to a conservatively low estimate of  $v_{\text{eff}}$ .

The results are recorded in Table I, which also shows the measured saturated drift velocity for electrons  $v_n$  [13], [14]. We are not aware of any corresponding measurements of the saturated hole velocity,  $v_p$ , and so we have assumed  $v_p = v_n$ . The enhancement of the gain–bandwidth product over the local model predictions amounts to  $\sim(v_{\text{eff}} - v_n)/v_n = 76\%$  for the  $L = 0.4 \mu\text{m}$  device and  $116\%$  for the  $L = 0.2 \mu\text{m}$  device. While the apparent enhanced speed of the longer device might be ascribed to an underestimate of the mean saturated drift velocity, the increased enhancement in speed for the shorter device seems to provide compelling evidence for the effects which we predict. We have also tried to make a realistic estimate of the mean enhanced velocity to ionization by factoring out the opposing bandwidth reduction resulting from the effects of dead space. We estimated this latter speed reduction factor  $s$  by comparing local model predictions with those of the model used in [1] in which the spatial distribution of ionization events was as predicted by the MC model but in which all carriers travelled with their saturated drift velocities. These estimates were obtained as  $s = 0.9$  for the larger device and  $s = 0.8$  for the shorter device. We deduce the enhanced mean velocity to ionization as  $v_{\text{eff}}^* = v_{\text{eff}}/s$ , giving an overall velocity enhancement of  $(v_{\text{eff}}^* - v_n)/v_n = 95\%$  and  $169\%$  respectively for the long and short devices.

#### IV. DISCUSSION

The fact that the enhancement in bandwidth predicted by the MC model over the value predicted by the local model saturates at high gain shows that the gain–bandwidth product is independent of gain, even in the presence of nonlocal effects. This is consistent with published measurements of gain–bandwidth curves for short devices [11]. It follows that, for any given device length, a single effective velocity  $v_{\text{eff}}$  can be used to assess the outcome of the competition between nonlocal effects on bandwidth for all but the lowest gains. The low gain frequency response, where the gain–bandwidth product is predicted to increase with gain, is in any case generally dominated by RC and transit time limitations [15].

The trend in the values of  $v_{\text{eff}}$  obtained from fitting to the results in [11] is consistent with the modeled trends shown in Fig. 3. These estimates of  $v_{\text{eff}}$  are conservatively low since the possibility of ionization outside the multiplication region, par-

ticularly in the adjacent charge and  $n$ -cladding layers, has been neglected. This would lead to a larger value of  $L$  in (1), resulting in an even larger enhancement in the effective value of  $v_{\text{eff}}$  over the local model prediction.

The enhancement of the fitted values of  $v_{\text{eff}}$  over the saturated electron drift velocity provides evidence of enhancement in the mean velocities to ionization at high fields. The further increase in the fitted value of  $v_{\text{eff}}$  as the device length is reduced provides compelling evidence of this effect, which appears to outweigh the opposing effects of dead space on the bandwidth of short APDs.

The accelerated increase in bandwidth for short devices predicted in Fig. 3 promises large speed benefits from further small reductions in the lengths of short devices.

#### V. CONCLUSION

A significant net enhancement in the bandwidth of short ( $< 0.2 \mu\text{m}$ ) multiplication regions is predicted over local model predictions owing to the dominance of a nonlocal enhancement in the mean velocities of carriers to ionization over the nonlocal speed reduction due to dead space effects. This net bandwidth enhancement is expected to increase as the length of the multiplication region is reduced. Experimental evidence from the literature is offered to support this prediction and carriers in a  $0.2 \mu\text{m}$   $\text{In}_{0.52}\text{Al}_{0.48}\text{As}$  multiplication region are estimated to travel, on average,  $40\%$  faster than those traveling in a  $0.4\text{-}\mu\text{m}$  region. Furthermore, the average carrier velocity in each device appears to be considerably higher than the electron saturated drift velocity reported for this material. This net enhancement in bandwidth is predicted to accelerate with decreasing multiplication region length so that large speed benefits are expected from further small reductions in the lengths of short devices. The frequency response of short APDs can be fitted by the expression deduced from the local model provided that an enhanced carrier velocity is used, which is independent of gain but increases with decreasing multiplication region length.

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