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Effect of Dead Space on Avalanche Speed

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Abstract—The effects of dead space (the minimum distance travelled by a carrier before acquiring enough energy to impact ionize) on the current impulse response and bandwidth of an avalanche multiplication process are obtained from a numerical model that maintains a constant carrier velocity but allows for a random distribution of impact ionization path lengths. The results show that the main mechanism responsible for the increase in response time with dead space is the increase in the number of carrier groups, which qualitatively describes the length of multiplication chains. When the dead space is negligible, the bandwidth follows the behavior predicted by Emmons but decreases as dead space increases.

Index Terms—Avalanche multiplication, avalanche photodiodes, frequency response, impact ionization.

I. INTRODUCTION

AVALANCHE photodiodes (APDs) are attractive options as photodetectors in optical communication systems because of the internal gain provided by avalanche multiplication. However, the randomness in position and time of the impact ionization events gives rise to noise and jitter. The excess noise factor, which describes the fluctuations in gain, has been studied by McIntyre [1] when the dead space, the minimum distance travelled by a carrier before acquiring enough energy to impact ionize, is neglected. However, in thin avalanching devices, where carriers are not in equilibrium with the local electric field, the dead space can represent a significant fraction of the avalanche width, resulting in lower excess noise than predicted by McIntyre's local analysis. More recently, there have been extensive studies of the excess noise characteristics in thin avalanching devices, both experimental [2]–[5] and theoretical [6]–[10].

The time response of avalanche multiplication has been investigated by Emmons [11] using a small signal, local analysis which neglects the effects of dead space. These simulations showed that for a given mean gain and ratio of impact ionization coefficients, the 3 dB bandwidth is inversely proportional to the avalanche width.

APDs designed to operate at high speed have thin multiplication regions (typically $<0.5 \mu\text{m}$) to reduce the carrier transit time. Using a similar local analysis [12], [13] concluded that a

gain-bandwidth product of 228 GHz should be achievable using an APD with a $0.1 \mu\text{m}$ bulk InAlAs multiplication layer [13]. However, at such small dimensions, the dead space becomes a significant fraction of the avalanche width.

The current impulse response of avalanche multiplication, the time-dependent current following injection of a carrier pair, has been studied, including the effects of dead space, using recurrence equations [14], [15]. Hayat and Saleh [14] and Bandyopadhyay *et al.* [15] found that for a given value of mean gain, the avalanche current decays more slowly as dead space increases. Using mean current impulse response and its standard deviation Hayat *et al.* have also investigated the effects of dead space on the bit-error-rate of optical communication systems [16]. Using a different recurrence technique, Hayat and Dong showed how to calculate the probability distribution of the avalanche duration of an APD [17]. Although the inclusion of dead space in their model is straightforward, these authors investigated only the local case.

While dead space can easily be incorporated into these recurrence techniques they do not readily provide insight into the mechanisms responsible for the increased response times which they predict. Furthermore, each specific recurrence technique is designed to evaluate the statistics of a specific parameter and further analyses are needed to obtain information on other parameters.

Monte Carlo models of semiconductor carrier transport automatically incorporate dead space effects and can provide realistic information on their consequences on temporal statistics. However these models are slow and also introduce other complexities, such as distributions in velocity and velocity overshoot of early ionizing carriers [18], making straightforward comparisons difficult for cases with and without dead space.

An alternative, more flexible approach for time response evaluation is therefore desirable to investigate the effects of dead space on APD speed. In this paper, we extend the basic framework of the random path length (RPL) model [7], which has been used successfully to predict mean gain and excess noise factor, to generate the temporal statistics of the multiplication process. The mean gain, probability distribution function (pdf) of multiplication, excess noise factor, current impulse response, and pdf of avalanche duration can be obtained simultaneously from the extended RPL model. Statistics from the RPL simulations are compared with those generated by recurrence equation techniques to assess the validity and accuracy of the extended RPL model. We also use the flexibility of the RPL model to explain the mechanism responsible for the increase in response time with dead space. Finally the reduction in device speed with dead space is calculated in terms of the 3 dB bandwidth.

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II. RPL MODEL

In the RPL technique [7], the random ionization path length of an electron x_e is described by its pdf $h_e(x_e)$, and the model uses the Monte Carlo technique [19] to generate various avalanche statistics. In the ‘‘hard dead space model,’’ $h_e(x_e)$ is given by

$$h_e(x_e) = \begin{cases} 0, & x_e \leq d_e \\ \alpha^* \exp[-\alpha^*(x_e - d_e)], & x_e > d_e \end{cases} \quad (1)$$

where d_e and α^* are the dead space and ‘‘activated’’ ionization coefficient for electrons. d_e is taken as

$$d_e = \frac{\varepsilon_{the}}{E} \quad (2)$$

where ε_{the} is the electron ionization threshold energy, and E is the applied electric field. α^* can be conveniently deduced from the local electron ionization coefficient α for the appropriate uniform electric field by equating mean ionization path lengths in the local and nonlocal cases [20]

$$\frac{1}{\alpha} = d_e + \frac{1}{\alpha^*}. \quad (3)$$

By substituting uniformly distributed random numbers, r , between 0 and 1 into the survival probability out to distance x_e

$$S_e(x_e) = 1 - \int_0^{x_e} h_e(x) dx. \quad (4)$$

x_e can be generated from

$$x_e = d_e - \frac{\ln(r)}{\alpha^*}. \quad (5)$$

By replacing $h_e(x)$, d_e , α^* , α , ε_{the} , $S_e(x)$, and x_e with $h_h(x)$, d_h , β^* , β , ε_{thh} , $S_h(x)$, and x_h , similar expressions for hole impact ionization path length can be obtained.

Each trial in an RPL simulation terminates when all carriers have left the multiplication region of width w . Each injected carrier gives rise to a multiplication, labeled M , which is a random variable, owing to the stochastic nature of the impact ionization process. The value of M for each trial is recorded to build up reliable avalanche statistics by performing an ensemble average over many trials. Spatially determined statistics such as mean gain $\langle M \rangle$ and excess noise factor F can be calculated using

$$\langle M \rangle = \frac{1}{n} \sum_{i=1}^n M_i \quad (6)$$

and

$$F = \frac{1}{n \langle M \rangle^2} \sum_{i=1}^n (M_i^2) \quad (7)$$

where M_i is the multiplication resulting from trial i , and n is the total number of RPL trials in the simulation. The pdf of M for a given mean gain depends on w and the dead space. This is reflected in the variation in F , which is a measure of the spread in the pdf.

Mean gain and excess noise factors for a 0.2 μm APD were calculated from the RPL simulations and are shown in Fig. 1. The ionization coefficients and threshold energies used in these and all subsequent simulations in this work are for GaAs and are taken from [21]. The RPL results agree with those given by the

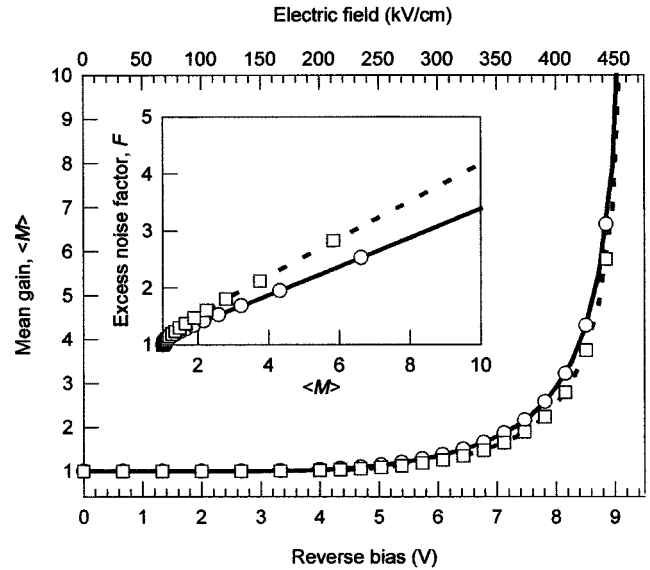


Fig. 1. Mean gain and excess noise factor for a 0.2 μm thick APD using threshold energies and ionization coefficients for GaAs. The results generated by RPL and by the recurrence technique for electron injection (\circ , solid line) and hole injection (\square , dashed line) are indistinguishable.

recurrence equations formulated by Hayat *et al.* [6], confirming that for calculations of spatial multiplication statistics, RPL is equivalent to the recurrence technique, which has been used to successfully model photomultiplication and excess noise measurements [3], [5].

The RPL technique can also be extended to generate temporal statistics by assuming constant velocities for electrons and holes. Knowledge of the position and velocity of each carrier in the avalanche region enables calculation of the instantaneous current from Ramo’s theorem [22]. The avalanche duration, the mean current impulse response, and its variance are then easily generated in a single simulation unlike in the case of solving the recurrence equations which need to be solved separately for each quantity, adding a factor of 3–4 times to the computation time. The RPL simulation can be 50–100 times faster than even a simple Monte Carlo simulation [18] depending on $\langle M \rangle$ and w and is adequate for determining the spatial ionization properties that are important in this case.

III. RANDOM RESPONSE TIME AND CURRENT IMPULSE RESPONSE

For comparison, the RPL techniques and the recurrence equations described in [14] and [17] are used to calculate the mean impulse response and the pdf of avalanche duration. In this work, dead space is included in the method proposed in [17]. In all temporal calculations, a constant velocity of 10^5 ms^{-1} is assumed for both electrons and holes, although different velocities for electrons and holes can be readily incorporated in the RPL model.

The RPL and the recurrence technique simulations give indistinguishable results for current impulse response, as shown in Fig. 2 for a 0.2 μm APD with $\beta = \alpha$ at $\langle M \rangle = 20$. The results for $d/w = 0$ and $d/w = 0.3$ are also compared in Fig. 2. Since calculations using ionization coefficients and threshold energies

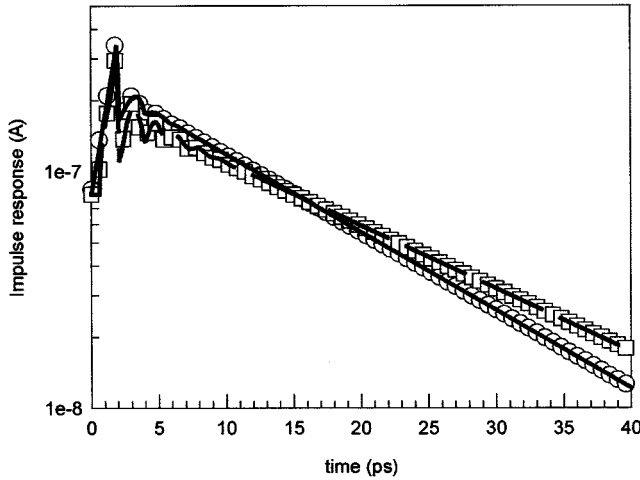


Fig. 2. Current impulse response of a $0.2 \mu\text{m}$ -thick APD with a mean gain of 20 without dead space (solid line) for a field of 426 kV/cm and including dead space (dotted line) for a field of 506 kV/cm . RPL results (\circ for $d/w = 0$ and \square for $d/w = 0.3$) and those of the recurrence technique (lines) are indistinguishable.

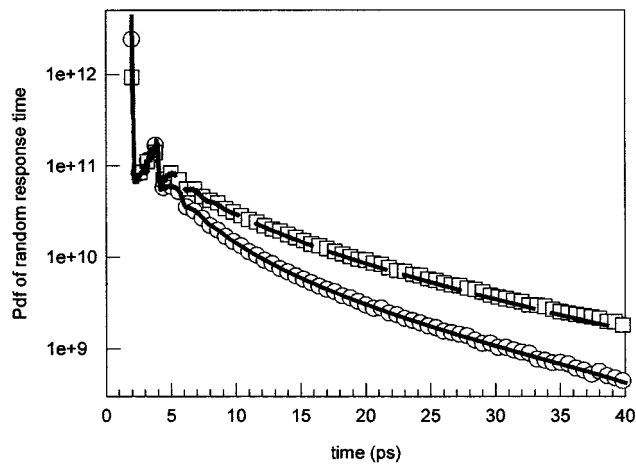


Fig. 3. Pdf's of random response time for a $0.2 \mu\text{m}$ thick APD with a mean gain of 20, ignoring dead space (solid line) for a field of 426 kV/cm and including dead space (dotted line) for a field of 506 kV/cm . RPL results (\circ for $d/w = 0$ and \square for $d/w = 0.3$) and the recurrence technique results (lines) are indistinguishable.

for GaAs in [21] suggest $d_e/w = 0.295$ and $d_h/w = 0.315$ in a $0.2 \mu\text{m}$ APD with $\langle M \rangle = 20$, the value of $d/w = 0.3$ chosen here is not unreasonable. A fixed ratio of β/α is used to allow clear comparisons between results with and without dead space. Since α and β of most semiconductors become similar at high fields, the use of $\beta = \alpha$ is appropriate.

The current impulse response shows a slower decay when the effects of dead space are included, consistent with the earlier observations of Hayat *et al.* [14] and Bandyopadhyay *et al.* [15]. Pdf's of avalanche duration are compared in Fig. 3, including and excluding the effects of dead space. The results are again indistinguishable for the RPL and the recurrence techniques. A slower decay is again observed when dead space is included.

The excellent agreement between the RPL and the recurrence techniques of current impulse response and pdf of avalanche duration results for cases with and without dead space confirms that these techniques are equivalent for calculating both spatial

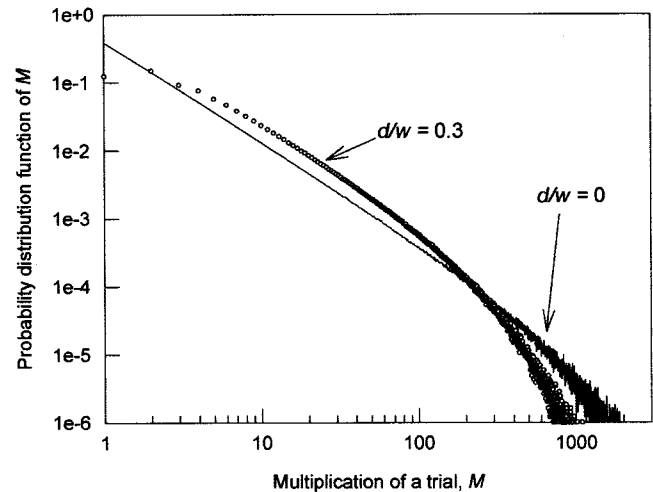


Fig. 4. Pdf's of M for $d/w = 0$ (line) and $d/w = 0.3$ (symbols) for $\langle M \rangle = 20$.

and temporal statistics under the constant carrier velocity assumption.

IV. DISCUSSION

Figs. 2 and 3 show that dead space increases the response time of an APD. To investigate the origins of this increase, we consider in detail the processes contributing to the gain measured from an APD, which corresponds to the mean $\langle M \rangle$ of M taken over its pdf.

Pdf's of M , both including and excluding the effects of dead space, for a mean gain of 20 are compared in Fig. 4. The number of trials (typically 5 million trials per simulation) for the simulations was chosen to give negligible noise in the data for $M > 100$. The probability of an injected carrier impact ionizing increases as the dead space becomes more significant. More importantly the dead space changes the shape of the pdf by reducing the number of trials resulting in large M and generally narrowing its pdf. One might therefore reasonably expect that a larger M would correspond to a longer avalanche duration, so that a narrower multiplication pdf with the same $\langle M \rangle$ should correspond to a faster response. However, this contradicts with our numerical predictions of current impulse response and avalanche duration in Figs. 2 and 3.

To understand this apparent contradiction, we must recognize that trials resulting in the same M can comprise multiplication chains of different lengths involving different numbers of "carrier groups." Carrier groups are defined as follows. Injected electrons are in the first electron group, as are new electrons descended from the injected electron, and all such electrons will leave the multiplication region together. New holes created by the first group of electrons and further holes created by these new holes form the first hole group, which may generate a second electron group if any of themselves ionizes, and so on. Carrier groups are tracked in Fig. 5 for a trial with $M = 5$ and 4 groups of carriers. The flexibility of the RPL model allows recording of the pdf of the number of carrier groups for each M , which is not readily available in the recurrence equation techniques.

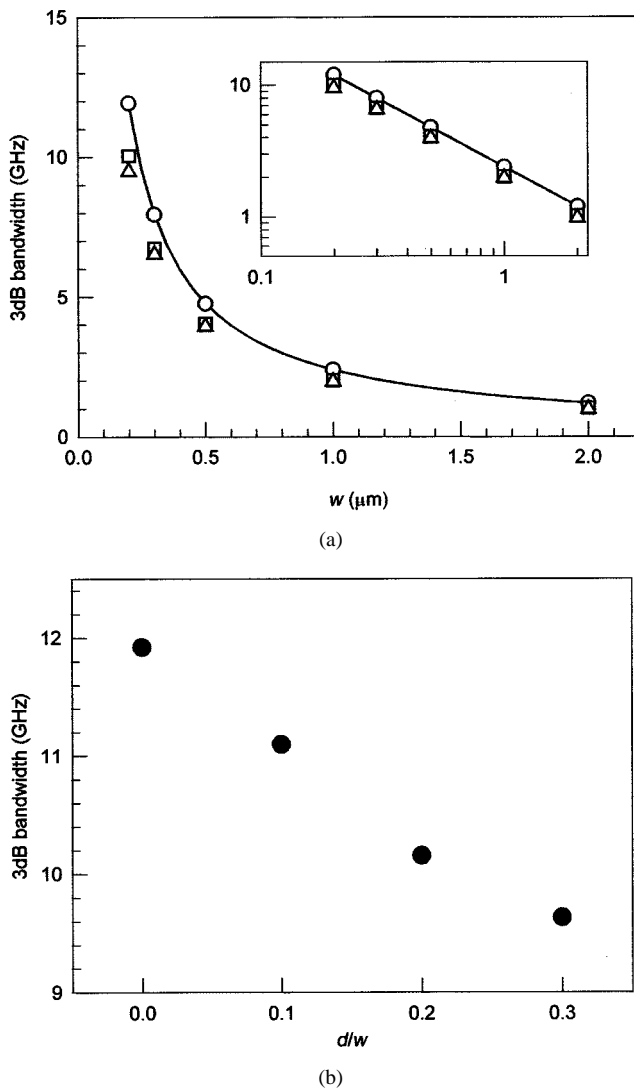


Fig. 7. (a) Bandwidth deduced by Fourier transforming the current impulse response generated by RPL. All results correspond to a mean gain of 20. Results for $d/w = 0$ (\circ) agree with Emmons' local analysis of frequency response (line). Bandwidth reduction is apparent when d/w is increased to 0.2 (\square) and 0.3 (\triangle). The same results are plotted on a logarithmic scale in the inset. (b) Bandwidth versus d/w for $w = 0.2 \mu\text{m}$ and $\langle M \rangle = 20$.

V. CONCLUSIONS

The RPL technique extended to model the temporal response of avalanche multiplication has been shown to be numerically equivalent to various recurrence techniques, while being more flexible and capable of providing more information.

The mechanism responsible for the increased avalanche multiplication response time with dead space has been investigated using this RPL model. It is found that dead space increases the number of carrier groups and hence, the length of the multiplication chains as impact ionization events initiated by feedback carriers become more important. This degrades the improvement in bandwidth expected by reducing the multiplication layer thickness.

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