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Ensuring Sales: A Theory of Inter-Firm Credit*

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Abstract: Delayed payment (trade credit) and prepayment are widely observed forms of inter-firm credit. We propose a simple theory to account for the prevalence of such credit. A downstream firm trades off inventory holding costs against lost sales. Lost final sales impose a negative externality on the upstream firm. The solution requires a subsidy limited by the value of inputs. Allowing the downstream firm to pay with a delay, an arrangement known as “trade credit,” is precisely such a solution. Further, solving a reverse externality accounts for the use of prepayment for inputs, even in the absence of any risk of default by the downstream firm. We clarify how input prices vary with such policies as well as when such instruments are more efficient than pure input price adjustments. Thus we account for inter-firm credit as an optimal instrument delivering a targeted inventory subsidy designed to prevent lost sales. The theory offers an explanation for the widespread use of net terms, and the fact that prepayment always carries a zero interest rate. Our results are also consistent with non-responsiveness of trade credit charges to fluctuations in the bank rate as well as market demand, and the fact that trade credit is negatively related to supplier profit level and inventory, but positively related to supplier profit margin.

KEYWORDS: *Trade credit, prepayment, input price variation, externality, inventory subsidy, the Burkart-Ellingsen critique*

JEL CLASSIFICATION: D2, L2

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1 INTRODUCTION

Delivery of inputs to a firm does not always coincide with payment by the firm. In some cases the supplier requires the buyer to prepay, and in other cases it allows the buyer to make a delayed payment. A short term delay in payments is a widely observed form of inter-firm credit. Delayed payments granted, usually called trade credit, account for about 15% of the assets of U.S. manufacturing firms. The importance of such credit is further underlined by its role in transmitting monetary policy shocks (see, for example, Brechling and Lipsey (1963), Meltzer (1960) and Nilsen (2002)).

Much of inter-firm credit is supplied at zero interest. Prepayment is always a pure cash advance, and therefore zero-interest credit. For trade credit, “net” terms are in widespread use. The most frequently used terms are “net 30,” requiring the buyer to repay within 30 days, again at zero interest cost to the buyer.¹ Trade credit has an implicit interest cost for the buyer only when the seller sets up a discount for early repayment,² and the buyer does not repay within the discount period. However, Ng, Smith and Smith (1999) report that in 15 out of 27 industry categories, discount terms are generally not offered, and pure net terms are used. Net terms are common across all but 4 of the other 12 categories. A recent study by Burkart, Ellingsen and Giannetti (2008) reports³ that only 20% of suppliers offer a discount and 50% of the most important suppliers do not offer them. In fact, even for discount terms, evidence suggests that many suppliers (72% in the Ng-et al study) allow some customers - especially long-term ones - to take the discount so long as payment is made within the net period, even after the discount period has elapsed.

The literature has often focused on a purely financial motivation for providing trade credit and considers the cost implicit in discount terms when repayment is not made within the discount period. This ignores pure net terms. Further, even in the case of discount terms, if the sole purpose were to impose a high interest rate, it would be far simpler to specify a standard loan contract at a high rate. But such terms are not observed anywhere in the developed world. Instead, every trade credit agreement includes

¹As discussed later, an indirect cost can arise through simultaneous input price adjustments. However, the important point is that there is an element of inventory subsidy in trade credit and prepayment that changes the marginal incentives.

²For example, a 2% discount for payment within 10 days may be added to a net 30 term.

³The Ng et al. (1999) study uses COMPUSTAT firms (i.e. relatively large publicly traded firms), while Burkart et al. (2008) use data from National Survey of Small Business Finances.

a zero interest period lasting 10-60 days.

Why would a supplier of inputs provide short term credit at zero interest to its customers? We propose a simple theory of trade credit as a subsidy mitigating a negative externality. A downstream firm trades off inventory holding costs against lost sales. Lost final sales impose a negative externality on an upstream firm, which supplies inputs. However, the upstream firm can induce the downstream firm to internalize the externality by allowing delayed payments, which delivers an inventory subsidy. Our model accounts for the use of delayed payment for inputs (trade credit) as well as prepayment for inputs, which mitigates a reverse externality. We compare these policies with those based on varying the price of the input, which can also deliver a similar subsidy, and clarify when either type of policy is optimal.

A common argument advanced in justifying trade credit is that the input suppliers are better informed/can better monitor their customers compared to banks. An important paper by Burkart and Ellingsen (2004) reviews this literature concisely and provides a cogent critique. They argue that the key advantage of trade credit is that it is advanced via illiquid inputs. The commitment value of such inputs ameliorates the vulnerability of liquid funds to misuse. As they point out, this corrects a weakness in the earlier monitoring literature since it fails to explain why trade credit is limited to the value of inputs. In the Burkart-Ellingsen theory the illiquidity of the trade credit instrument automatically ties the credit to the value of inputs. However, since the illiquidity of inputs is critical, the theory cannot explain the practice of prepayment which is similar to (reverse) trade credit, but is an advance of liquid cash rather than illiquid inputs.

One explanation for prepayment in the context of developing countries is that it is a response to default risk. However, in developed western economies, often large, well established firms prepay suppliers, which cannot be explained by appealing to default risk⁴. Therefore, to us the challenge is to simultaneously explain (a) both trade credit (input advances by the upstream firm) and prepayment (cash advances by the downstream firm), (b) why all such advances are limited by the value of inputs (we call this “the Burkart-Ellingsen critique”), and (c) why most trade credit is offered at zero interest. We also attempt to shed light on the choice between price cuts and trade credit and how these

⁴As Schwartz (1974) and Ferris (1981) note, prepayment is observed in construction, shipbuilding, aircraft and parts of the defense industries. Typically, in such cases, the probability of default or non-collection is zero. Prepayment clauses are also often used in North American oil and gas contracts.

might be combined.⁵

Burkart and Ellingsen (2004) show that net terms can arise if both up and downstream firms are credit constrained. Provision of trade credit ameliorates an underlying moral hazard problem and eases this constraint by allowing the upstream firm to borrow against accounts receivable. Since the division of accounts receivable into interest and principal does not matter, net terms arise.⁶

We adopt a different approach based on lost-sales externalities experienced by one firm because the other optimally decides to delay production. Since trade credit (and prepayment) arises as a *subsidy* that solves an externality, the interest charges are naturally zero for both credit constrained firms as well as large unconstrained upstream firms with significant market power.

In our model a downstream firm facing stochastic final demand can either produce immediately after each sale, or wait for one or more periods before producing. If the downstream firm finds it profitable to follow a waiting strategy, it might lose some sales, generating a negative externality for the upstream firm. We show that by subsidizing the downstream firm's inventory holding, delayed payment (trade credit) induces the downstream firm to internalize the externality. The theory extends to reverse trade credit, i.e. prepayment, which arises when the upstream firm wants to wait, generating a negative externality for the downstream firm.

Further, the bank rate represents the opportunity cost of inventory holding. Therefore waiting is optimal when the bank rate is relatively high. Since a high bank rate is one of the factors that can *cause* waiting, a bank loan (at this high rate) cannot solve the problem of negative externality. To dissuade the downstream firm from waiting, a loan must be provided at a *subsidized* rate - which lowers the cost of immediate production and makes waiting less attractive. Such a subsidy is naturally bounded above by the value of inputs. Thus our theory is not subject to the Burkart-Ellingsen critique mentioned above. To summarize, the solution to the problem of waiting is a loan at a subsidized rate covering at most the value of inputs - and trade credit is precisely such an instrument. This clarifies

⁵Indeed, as Burkart et al. (2008) note, "Existing theories fail to explain why suppliers provide trade credit to customers with bargaining power instead of offering (larger) price reductions." A contribution of our paper is to attempt to address precisely such concerns.

⁶As a referee has pointed out to us, the argument can be strengthened further: if loans cannot be secured against any trade credit interest, the optimal trade credit interest is zero.

the reason for providing credit at a zero interest rate which is lower than the bank rate. This is also consistent with the fact (reported by Ng et al. (1999)) that trade credit interest rate does not vary with either the bank rate or market demand fluctuations.

However, such a subsidy can also be delivered by varying the input price. Thus while our model can account for trade credit, another question remains: is there a reason to use trade credit in preference to price variation? To answer this, we assume that the input price is fully effective in the sense that price adjustment by an upstream firm does not induce (potential) rivals to react to the adjustment. We then show that whenever the cost of capital facing the up and downstream firms are different, price-based and delayed/advance-payment-based policies result in different outcomes. Specifically, suppose the downstream firm has an incentive to switch away from immediate production, and the upstream firm wants to offer an inventory subsidy to prevent the problem. We show that if the upstream firm faces a relatively lower cost of capital, trade credit Pareto dominates price discounts. The optimal solution is to advance full trade credit (defer the entire payment for a unit of input), and simultaneously *raise* the price to the highest level consistent with satisfying the downstream participation constraint. On the other hand, if the downstream firm faces a lower cost of capital, price discounts generate a higher surplus, and the optimal policy is to *lower* the input price to the level necessary to induce immediate production, and offer no trade credit. Further, if we relax the assumption of fully effective price adjustments, the scope of trade credit increases further to cover even some cases in which the upstream firm faces worse credit terms compared to the downstream firm.

Which policy—full trade credit or a pure price discount—is more likely to be optimal? We show that the scope for delivering an inventory subsidy arises when the upstream margin is higher compared to the downstream margin. Further, we argue that such an upstream firm is likely to face better credit terms. Our results then imply that trade credit is preferable to a price cut. In other words, trade credit is the optimal policy precisely in circumstances in which an inventory subsidy is most relevant. A similar argument applies to prepayment.

EVIDENCE AND SCOPE

Let us now comment on the applicability and limitations of our approach. A more detailed discussion of evidence on trade credit and pricing, trade credit terms and implica-

tions for inventory is deferred till section 7.

As noted above, our theory of trade credit best fits industries where competition is not severe so that the upstream firm's margin is not low. This could arise because the upstream firm provides an input that is not a standardized good, or a longstanding relationship between upstream and downstream firms imply a significant cost of switching suppliers. Indeed, as Petersen and Rajan (1997) report, trade credit provision is positively related to gross margin/sales. Next, trade credit is often part of a longstanding relationship.⁷ Further, Burkart et al. (2008) find that "suppliers of differentiated goods and services have larger accounts receivable than suppliers of standardized goods." They conclude that, overall, evidence lends most support to theories maintaining that input suppliers cannot be easily substituted. Our theory, which relies on a positive supplier margin, fits this category.

As for the nature of inputs, our theory clearly suits industries in which physical inputs are used, such as manufacturing industries. For inputs that are services rather than physical goods, our model applies so long as it takes time to produce the input service after the order is placed. In that case, exactly as for physical inputs, if the downstream firm waits for a customer to arrive before putting in an order for inputs, the sale is missed with positive probability.

This naturally raises the question whether the theory can be meaningfully applied to goods and services that are tailored to customer needs and therefore intrinsically produced to order. Clearly, the incentive for waiting to produce cannot serve as basis for a theory, since such waiting is now automatic. However, while the effect captured in our simple model is ruled out, effects that are very similar in spirit can arise even under production-to-order in a richer model. Suppose the downstream firm can produce the tailored good faster if it holds a unit of input in its inventory rather than orders and receives the unit from the upstream firm. "Immediate production" in this case means that the downstream firm buys a unit of input and waits for a customer, while "wait-and-see" translates to the situation in which the firm submits an order for input only when a buyer arrives. Suppose all buyers are happy to proceed with their order in the first case, while a fraction $1 - q$ find the waiting time too long in the second case and withdraw their or-

⁷For example, Uchida, Udell and Watanabe (2006) find, for Japan, that longer relationships are associated with more trade credit. They also report an average relationship length of 29 years between downstream firms and their main suppliers.

ders. Clearly, the same incentives to wait as in our model arise here - and trade credit arises similarly as a solution.

Our theory also gives rise to comparisons between cases in which trade credit or prepayment is used, suggesting that trade credit (prepayment) is more likely to occur when the upstream margin is high (low) relative to the downstream margin. This is consistent with the finding by Burkart et al. (2008) that suppliers are more likely to deny trade credit to more profitable firms, which are also less likely to be offered trade credit. This also seems consistent with the observation (see footnote 4) that prepayment is made in industries such as construction, shipbuilding and aircraft production, where the downstream mark-up is likely to be significant. However, we are unaware of any detailed empirical study of prepayment, and proper verification of such claims must await further investigation of empirical regularities.

Finally, our theory suggests that trade credit is preferable to price cuts so long as the upstream firm faces credit terms that are better or at least not much worse than the downstream firm. If the former faces much worse terms relative to the latter *and* effective price cuts are available, our explanation for choosing trade credit over price cuts is less compelling. In such cases, other explanations—such as the joint credit expansion theory of Burkart and Ellingsen (2004)—are required.

RELATED LITERATURE

We discussed above the literature explaining trade credit as a solution to asymmetric information problems. Other papers have highlighted sales as motivation for trade credit.⁸ A further theory (Schwartz and Whitcomb (1979), Brennan, Maksimovic and Zechner (1988)) assumes that direct price discrimination is not possible, and explains trade credit as a device to enable this. If the seller does not have full information about its clients and must set up a menu of contracts, incentive compatibility requires the trade credit interest rate to be at least as high as the bank rate (see Brennan et al.). However, most trade credit

⁸An early contribution by Nadiri (1969) derives optimal trade credit by assuming that total sales is an increasing function of upstream credit provision. Further, Ferris (1981) suggests that firms facing a random input flow can reduce the transactions cost of holding liquid balances by using trade credit which allows payments at regular intervals. In this theory, trade credit and prepayment are identical. This is also vulnerable to the Burkart-Ellingsen critique, since any random costs (of production and/or transport) faced by the upstream firm should come under the scope of trade credit.

is provided at zero interest, which is lower than the bank rate. Further, this theory ties trade credit offers to characteristics of demand facing downstream firms, and cannot account for the observed invariance of the interest charge to market demand fluctuations. Our model accounts for these features, and also does not rely on ruling out price variation exogenously.⁹

Bougheas, Mateut and Mizen (2009) study a “storage-cost” model of trade credit. An upstream firm in a competitive industry faces a random demand for inputs from several downstream firms. If it carries some units in inventory, it does not miss cash sales. On the other hand, it has a storage cost of holding inventory, and if demand is low it wants to reduce storage cost by giving some units as trade credit to downstream firms. This is very different from our theory in which final demand is stochastic and trade credit arises as a solution to the downstream firm’s incentive to delay production. Further, the upstream firm in Bougheas et al. belongs to a competitive industry, while our model fits situations in which the upstream firm has market power. Despite the difference in focus, our model is consistent with the main evidence they uncover from data on UK firms: upstream inventory holding to sales ratio has a negative effect on accounts receivable to sales ratio (trade credit reduces the former and raises the latter).

The rest of the paper is organized as follows. Sections 2 and 3 set up the model and clarify the negative externality. Sections 4 to 6 explain how trade credit and price discounts can solve the problem, and compares the policies. Section 7 discusses evidence. Section 8 analyzes prepayment, section 9 discusses the relative likelihood of cases under which trade credit (prepayment) or price variation is optimal, and section 10 concludes. Proofs not in the body of the paper are collected in appendix A, and appendix B outlines a formal argument to show why the firm with the larger margin is likely to face a lower cost of credit.

⁹Vertical integration offers a different kind of solution to these problems, and could eliminate the externality pointed out here (as well as many of the problems identified in the literature). However, vertical integration involves greater governance costs as well as several types of internal organization costs (see Joskow (2005) for a succinct discussion) and is not necessarily feasible or optimal. We take it as given that the firms are managed separately. Further, banks that have longstanding relationships with clients might be able to vary credit terms over time to ameliorate the externality. We do not consider relationship lending here. Indeed, Burkart et al. (2008) note the empirical regularity that firms receiving trade credit secure financing from relatively uninformed banks: they borrow from numerous banks, and have shorter relationships with banks.

2 THE MODEL

An upstream firm produces an intermediate good which a downstream firm uses as input, and converts into a final consumption good. There are an infinite number of periods. Production of 1 unit by either firm takes exactly 1 period. The downstream firm has the capacity to hold 1 unit of final good in inventory. Timing is important since—given positive interest rates—a longer inventory holding period implies a higher (opportunity) cost. The structure of timing is as follows.

Period -1 is the set-up period for the upstream firm. At the start of period -1, the downstream firm can place an order for a unit of input. The upstream firm has the option of producing 1 unit in period -1¹⁰. Period 0, which follows, is the set-up period for the downstream firm. It can buy the unit of input from the upstream firm (if the latter has a unit available) and produce a unit of the final good, or choose to wait.

From period 1 onwards, the market is open. At the start of each period, a new customer (who buys a unit of the final good from the downstream firm) arrives with probability p , where $0 < p < 1$.

The arrival of a new customer in any period $t \geq 1$ leads to a successful sale if the downstream firm has a unit available in finished goods inventory. If the firm has no inventory (this can happen, for example, if the downstream firm chooses to wait in period 0 and a customer arrives at the start of period 1), and the customer fails to obtain a unit, he returns with probability q next period (where $0 < q < 1$), and if still not served, does not return.

The return probability q plays an important role. If $q = 0$, firms either produce immediately after each sale, or never produce. If $q > 0$, this gives firms a potential reason for waiting till a customer arrives to start production. As we clarify later, if either non-production or waiting to produce is optimal for a firm, a negative externality arises.

Let P denote the price of the final good and C denote the input price, which is also the (constant) marginal cost of the downstream firm. Let G denote the (constant) marginal cost of the upstream firm. P and G are given exogenously and satisfy $P > G$. The input

¹⁰Note that this period cannot be moved. In other words there is no sense in saying “the upstream firm should start production in a period later than -1.” Whenever the upstream firm produces the first unit, that is period -1.

price $C \in [G, P]$ is determined endogenously.

As noted in the introduction, our theory fits a scenario in which the input suppliers sell differentiated goods and often have long term relationships with downstream firms. Replacing a seller might then involve incurring additional costs. Therefore, the effective “outside option” for a downstream firm (the price at which it would decide to change suppliers) is the price charged by a rival firm plus possibly a mark-up to reflect the switching costs. Let \bar{C} denote this outside option cost.

Finally, let δ_u and δ_d denote the upstream and downstream discount factor respectively. If r_k is the rate of interest charged by banks to firm k , then $\delta_k = \frac{1}{(1+r_k)}$, where $0 < \delta_k \leq 1$, and $k \in \{u, d\}$. Later, in section 6, we consider reasons for and impact of any divergence between δ_u and δ_d .

PRODUCTION STRATEGIES

We consider two types of production strategies for each firm. A firm can produce immediately after a sale, or follow a waiting strategy. The details are as follows.

Downstream firm: The “immediate production” strategy is as follows.

- Place an order for a unit of input at the beginning of period -1. Subsequently, produce 1 unit in period 0.
- In any period $t > 0$, if a customer arrives at start of the period, sell and produce again. Otherwise carry over inventory to the next period.

A downstream firm following this strategy produces a unit initially and then immediately after every sale, ensuring that it always has a unit of final good in inventory. The “wait-and-see” strategy is as follows.

- Do not place an order for a unit of input at the beginning of period -1. Do nothing in period 0.
- In any period $t > 0$,
 - produce only if a new customer arrives at the start of period t and this customer is not served (i.e. produce in period $t > 0$ only when there is a strictly positive probability of a returning customer arriving at the start of period $t + 1$).
 - otherwise restart period 0 strategy.

This strategy says “produce if and only if (1) a customer arrives and (2) there is no unit in the inventory.” If these two conditions are met, production starts. Note that the customer who arrives is not served in this period and might return next period. The unit that is produced is therefore sold with a higher probability next period because the probability of at least one customer arriving next period is higher. Note also that waiting beyond this does not make any sense, as any unsatisfied customer today is, by construction, not around two periods from now.

Upstream firm: The immediate production strategy for the upstream firm is to produce 1 unit in period -1, and then produce 1 unit after every sale of a unit to the downstream firm. The wait-and-see strategy for the upstream firm is simply to produce only after an order has been received from the downstream firm. If the downstream firm follows the immediate production strategy, it places an order (and therefore the upstream firm produces) in period -1, and then after each sale of a final good.

Note that if both firms follow a wait-and-see strategy, production never gets started, and payoffs are zero for all.¹¹ Thus there are three non-trivial cases: the case in which both firms produce immediately, and two cases in which one of the two firms wait and the other produces immediately. In all these cases, the upstream firm produces in period -1. Thus for purposes of comparison across cases, we can ignore the cost of production in period -1 and simply suppose that the upstream firm starts life in period 0 with an endowment of 1 unit of output.

BENCHMARK EFFICIENCY

It is efficient to produce both the input and the final good immediately if the total surplus from immediate production is positive and also exceeds the surplus from following the wait-and-see strategy in producing either the input and/or the final good.

The parameters of the model are P, G, p, q, δ_u and δ_d where $\delta_k = 1/(1 + r_k)$, for $k \in \{u, d\}$. Let $r_m \equiv \max\{r_u, r_d\}$. As we show formally in appendix A.16, a *sufficient* condition for immediate production of both input and output to be efficient is that the expected rate of surplus from a sale exceeds the (highest) interest rate:

$$p \frac{(P - G)}{G} \geq r_m. \quad (2.1)$$

¹¹Even if a customer arrives at the start of period 1, ordering the input serves no purpose since it would take two periods to produce the input and then the final good, implying that the customer could not be served.

We assume this holds. Therefore the benchmark requires immediate production by both upstream and downstream firms.

Finally, a word on notation. The net present value payoff of a firm under any strategy is a function of the parameters $P, G, p, q, \delta_u, \delta_d$ as well as the input price C . For most parts, we can suppress these arguments without loss of clarity. However, in sections 5 and 6, we explicitly vary the input price, and then it is clearer to write payoffs with the argument C made explicit.

3 OPTIMAL STRATEGIES AND EXTERNALITY

3.1 DOWNSTREAM OPTIMUM

Recall from section 2 that the outside option input cost for the downstream firm is denoted by \bar{C} . Therefore if the upstream firm is simply selling inputs to the downstream firm it must charge a price $C \leq \min\{P, \bar{C}\}$. In this section we analyze the subgame after such an input price has been chosen. In other words, given any choice of C satisfying the above inequality, we analyze the optimal downstream strategy under different parameter configurations. Then in the next section we analyze the optimal upstream policy.

First, we need to consider the incentive of the downstream firm to participate in production at all. Clearly, if the rate of interest is high relative to the return from investment, the optimal choice is to not invest. Given any C , rate of return from investment for the downstream firm is given by $p(P - C)/C$. Since $p \in [0, 1]$, the best possible rate of return is $(P - C)/C$. The following result shows that if this rate is lower than the interest rate r_d (implying the condition $\delta_d > C/P$), the downstream firm optimally chooses not to invest.

Proposition 1 *If $\delta_d \leq C/P$, the optimal strategy of the downstream firm is not to invest.*

Next, let V_0^I be the value at time 0 under immediate production. In this case, the downstream firm incurs cost C initially to produce a unit. In each subsequent period, with probability p a sale is made and another unit is produced, implying that expected payoff in each period is $p(P - C)$. It follows that:

$$V_0^I = -C + \delta_d p (P - C) + \delta_d^2 p (P - C) + \dots = -C + \frac{\delta_d p (P - C)}{1 - \delta_d}. \quad (3.1)$$

Finally, let V_0^W be the payoff under the wait-and-see strategy. This can be derived as follows. Under this strategy, no production or sale happens in period 0 so that the payoff in that period is 0. The payoff starting period 1 depends on whether a customer arrives at the start of period 1 (which happens with probability p) or not. If not, the situation in period 1 is exactly as in period 0, and the value starting period 1 is simply V_0^W . If a customer does arrive at the start of period 1, let $V_1(1)$ denote the value starting period 1. We then have

$$V_0^W = 0 + \delta_d \left((1-p) V_0^W + p V_1(1) \right). \quad (3.2)$$

To calculate $V_1(1)$, note that once a customer arrives at the start of period 1, the downstream firm produces a unit in period 1 incurring cost C . Following this, at the beginning of period 2, there are 3 possible states: exactly 1 customer arrives; 2 customers (1 old and 1 new) arrive; and no one arrives. It is easy to calculate the payoff for each state, and the overall expected payoff EV_2 . $V_1(1)$ is then given by $(-C + \delta_d EV_2)$. Using this value in (3.2), we calculate the following expression for V_0^W . The detailed derivation is in the appendix.

Lemma 1 *The payoff of the downstream firm under the wait-and-see strategy is given by*

$$V_0^W = \frac{\delta_d p [\delta_d P (p + (1 - \delta_d)(1 - p)q) - C (1 - \delta_d(1 - p))]}{(1 - \delta_d)(1 - \delta_d + \delta_d p(2 - q))}. \quad (3.3)$$

Proposition 1 above shows the optimal choice for the case $\delta_d P \leq C$. The following result now derives the optimal strategy in the complementary case, shown in figure 1 (in section 4). Let p_ℓ^D denote the cutoff below which the downstream firm does not produce, and let p_{ic}^D denote the cutoff above which it follows the immediate production strategy.

Proposition 2 *For $\delta_d P > C$, there exist cutoffs p_ℓ^D and p_{ic}^D where $0 \leq p_\ell^D < p_{ic}^D < 1$ such that (a) for any $p \geq p_{ic}^D$, the optimal choice is immediate production, (b) for $p_\ell^D < p < p_{ic}^D$, wait-and-see is optimal, and (c) for $p \leq p_\ell^D$ no investment is optimal. p_ℓ^D and p_{ic}^D are given by:*

$$p_\ell^D = \max \left\{ 0, \frac{(1 - \delta_d)(C - \delta_d q P)}{\delta_d((P - C) - (1 - \delta_d) q P)} \right\}, \quad (3.4)$$

$$p_{ic}^D = \frac{(1 - \delta_d)C}{\delta_d((P - C) - q(\delta_d P - C))}. \quad (3.5)$$

Note that the parameter q is crucial in distinguishing waiting from non-investment. Clearly, if $q = 0$, $p_\ell^D = p_{ic}^D$, and the choice is simply between immediate production and non-investment since the probability of a sale cannot now be increased by delaying production till a customer arrives.

3.2 UPSTREAM PAYOFFS AND EXTERNALITY

Just as $\delta_d P > C$ is necessary for downstream investment to take place (proposition 1), a necessary condition for upstream investment to be viable is $\delta_u C > G$. We assume this holds. We also assume, for the analysis in this section, that the upstream firm always produces immediately after a sale. This is true if G is low relative to C .¹² The case in which upstream firm waits is analyzed in section 8.

Let U_0^I and U_0^W denote the payoffs of the upstream firm when the downstream firm chooses the strategy of immediate production and wait-and-see, respectively.

Lemma 2

$$U_0^I = (C - G) + \frac{\delta_u p (C - G)}{1 - \delta_u}, \quad (3.6)$$

$$U_0^W = \frac{\delta_u p}{(1 - \delta_u) + \delta_u p (2 - q)} U_0^I. \quad (3.7)$$

We have assumed that $\delta_u C > G$, which implies $C - G > 0$, and therefore $U_0^I > 0$. Note that the coefficient of U_0^I on the right hand side of equation (3.7) is strictly lower than 1. Therefore $U_0^I > U_0^W > 0$, i.e. the upstream firm would want the downstream firm to invest and follow the immediate production strategy for all $p > 0$. Thus the downstream firm generates a negative externality if it either decides to wait and see ($p_\ell^D \leq p < p_{ic}^D$) or not to invest at all ($p < p_\ell^D$).

¹²The upstream payoff from waiting is derived formally in Lemma 3 in section 8. The result shows that immediate production is better for the upstream firm than waiting whenever C is high and/or G is low. By waiting, the upstream firm saves interest cost on G , but since waiting forces the downstream firm to wait as well, some sales are lost with positive probability. Clearly, therefore, immediate production is better if C is high relative to G .

4 DELAYED PAYMENT

Trade credit is an offer to delay the payment for a fraction $\tau \in [0, 1]$ of the cost of inputs till the next order¹³ - which occurs when a customer arrives next and a sale takes place. In the analysis below we fix the input price C and work out the optimal trade credit given this price. Later, in section 6, we analyze the overall optimal policy by allowing both price variation and trade credit.

4.1 OPTIMAL TRADE CREDIT

Without any trade credit, the payoff of the downstream firm from immediate production is given by equation (3.1): $V_0^I = -C + \frac{\delta_d p (P-C)}{1-\delta_d}$. Suppose trade credit of τC , where $\tau \in [0, 1]$, is offered, and assume that the trade credit offer is such that the downstream firm chooses to produce immediately. Then the downstream firm pays $(1 - \tau) C$ to the upstream firm on delivery of each unit of the input good it purchases. Thus the first period payoff is $-(1 - \tau) C$. In each subsequent period, if there is a sale made by the downstream firm, it receives the price P , repays τC , and incurs a new cost of production of $(1 - \tau) C$. Thus in each period after period 1, the payoff is the same as before, and given by $p (P - C)$. Thus the payoff under trade credit, denoted by V_0^T , is given by

$$V_0^T = -(1 - \tau) C + \frac{\delta_d p (P - C)}{1 - \delta_d} = V_0^I + \tau C. \quad (4.1)$$

This proves the following result, which is very useful for later calculations.

Proposition 3 *Suppose the input price is C , and for each unit of input sold, trade credit is offered for a fraction $\tau \in [0, 1]$ of the input cost. Suppose also that τ is such that the downstream incentive to produce immediately is satisfied. Then the trade credit scheme can be represented by a total transfer of τC from the upstream to the downstream firm.*

A trade credit offer is feasible if it satisfies downstream incentive to produce immediately, as well as the upstream participation constraint. Using equation (4.1), the downstream incentive constraint is given by $V_0^T = V_0^I + \tau C \geq \max\{0, V_0^W\}$. Next, let U_0^T denote the upstream payoff under trade credit. We need $U_0^T \geq \max\{0, U_0^W\}$. Using the proposition

¹³We implicitly assume that the trade credit interest rate is zero. Later, in section 6.3 we show that this is without loss of generality.

above, $U_0^T = U_0^I - \tau C$. Further, from (3.7), $U_0^W > 0$. Thus the upstream participation constraint simplifies to $U_0^I - \tau C \geq U_0^W$.

Without trade credit, the downstream firm invests and follows the immediate production strategy for $p \geq p_{ic}^D$, and for $p < p_{ic}^D$ there is a loss of efficiency. The following result shows that the optimal trade credit offer τ^*C restores efficiency for $p < p_{ic}^D$ whenever the upstream participation-in-trade-credit constraint holds. Full efficiency is achieved if $G = 0$.

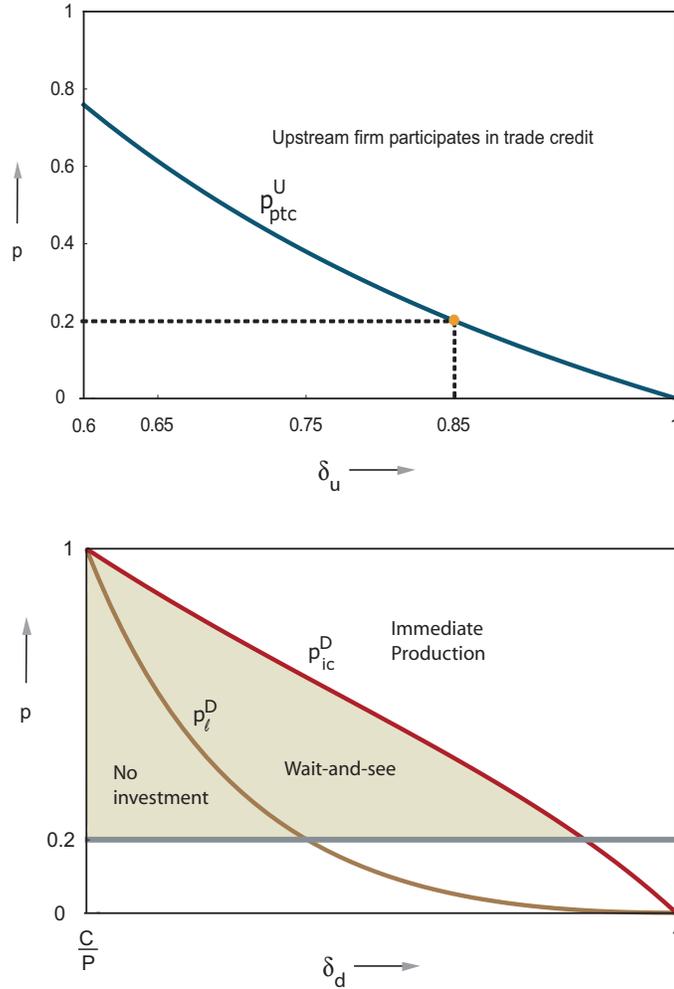


Figure 1: The upstream participation-in-trade-credit constraint is satisfied for $p \geq p_{ptc}^U$. Here, for $\delta_u = 0.85$, $p_{ptc}^U = 0.2$. For all values of δ_d for which $p_{ic}^D > 0.2$ (the shaded region), trade credit can restore first best on $[0.2, p_{ic}^D]$. The figure is drawn here for $C/P = 0.6$, $q = 0.6$ and $G = 0.1$.

Proposition 4 *The optimal trade credit fraction $\tau^* \in [0, 1]$ is given by*

$$\tau^* = \frac{1}{C} \left(\max\{0, V_0^W, V_0^I\} - V_0^I \right).$$

Further, there exists a cutoff $p_{ptc}^U \geq 0$ which is such that the upstream participation-in-trade-credit constraint holds whenever $p \geq p_{ptc}^U$, implying that efficiency is restored for any p such that $p_{ptc}^U \leq p < p_{ic}^D$. The cutoff p_{ptc}^U is increasing in G and decreasing in δ_u , and goes to 0 as either $G \rightarrow 0$ or $\delta_u \rightarrow 1$.

The result shows that if either G is low or δ_u is high, the interval $[p_{ptc}^U, p_{ic}^D]$ is well defined and trade credit restores efficiency on this interval.¹⁴ The efficiency gain under trade credit is illustrated in figure 1.

5 DISCOUNTED INPUT PRICE

Let us now show that a price-based policy can also solve the externality problem. The intuition is straightforward: a price cut encourages immediate production by reducing the cost of obtaining inputs, thereby subsidizing inventory costs.

For this section and the next, it is convenient to change notation slightly and write payoffs with the argument C made explicit.

Recall that any contract between the up and downstream firms must give the latter a payoff that is at least as high as that resulting from either not investing or investing and buying inputs at a cost \bar{C} (the input price charged by a rival supplier plus a mark-up reflecting switching costs). Suppose the upstream firm offers a (discounted) price C_D to a downstream firm if the latter produces immediately after a sale and a price \bar{C} if it pursues a wait-and-see strategy.¹⁵ Since $V_0^I(C_D)$ is decreasing in C_D , there exists $C_D \leq \min\{P, \bar{C}\}$ such that immediate production under a price C_D is more attractive to the downstream firm compared to any other strategy.

¹⁴Note also that the parameter q has an ambiguous effect on trade credit. On the one hand, an increase in q increases the incentive to wait (it raises V_0^W), but it also lowers the loss from waiting (raises U_0^W), lowering the incentive to provide trade credit.

¹⁵If $\bar{C} \leq P$, offering any $C < \bar{C}$ in this case only encourages the downstream firm not to produce immediately. If $\bar{C} > P$, then any price in the interval $[P, \bar{C}]$ has the same incentive: the downstream firm would prefer not to produce at these prices, and C_D simply needs to ensure participation in production.

The optimal choice of C_D , denoted by C_D^* , is given by $V_0^I(C_D^*) = \max\{0, V_0^W(\bar{C}), V_0^I(\bar{C})\}$, i.e.

$$-C_D^* + \frac{\delta_d p(P - C_D^*)}{1 - \delta_d} = \max\{0, V_0^W(\bar{C}), V_0^I(\bar{C})\}. \quad (5.1)$$

Finally, we need to check that the upstream participation constraint holds. The result below shows when this is achieved.¹⁶

Proposition 5 *For any $C \leq \bar{C}$ such that $\max\{V_0^W(\bar{C}), 0\} > V_0^I(C)$, there exists $C_D^* < C$ such that $V_0^I(C_D^*) = \max\{V_0^W(\bar{C}), 0\}$. Further, for $\delta_d \geq \delta_u$, there exists a cutoff $p_{pdp}^U \in (0, 1)$ which is such that the upstream participation-in-discounted-price constraint holds for $p \geq p_{pdp}^U$ implying that efficiency is restored for $p \in [p_{pdp}^U, p_{ic}^D]$. The cutoff p_{pdp}^U is decreasing in G and decreasing in δ_d , and goes to 0 as either $G \rightarrow 0$ or $\delta_d \rightarrow 1$.*

6 TRADE CREDIT VERSUS PRICE VARIATION

As discussed in the introduction, our model fits a scenario of imperfect price competition in the upstream sector with firms enjoying a degree of market power. Evidence suggests that input suppliers sell differentiated products, and downstream firms are often in longstanding relationships with particular suppliers, implying that substituting input suppliers could be costly. The outside option \bar{C} , as defined in section 2, is the outside input price plus a mark-up reflecting such switching costs.

To compare a policy of price variation with trade credit, we need to clarify two aspects related to the outside option. Suppose both the incumbent and rival suppliers offer the same trade credit terms, and suppose $\bar{C} < P$. In this case, a downstream firm would switch to a rival supplier if the incumbent upstream firm charges a price $C > \bar{C}$, so that the input price is “flexible up to \bar{C} .” On the other hand, if either there are no rival firms or if the adjustment costs are high so that $\bar{C} \geq P$, the outside option places no constraint on the upstream firm’s input price C , i.e. it can be adjusted all the way up to P if necessary. In this case, we say that the price policy is “**fully flexible**”.

Second, we have so far assumed \bar{C} is fixed exogenously. However, it might itself respond to price changes by the upstream firm. We say that a policy of price cuts by an upstream

¹⁶As the next section shows, a price discount is in fact optimal only when $\delta_d \geq \delta_u$ - and so we report results for this case only. In fact, for $\delta_d < \delta_u$, it is even more difficult to satisfy the upstream participation constraint.

firm is subject to “competitive pressure” if a price reduction by the firm leads to a fall in the price charged by rival suppliers, reducing the downstream outside option. A price policy is defined to be “**fully effective**” if it is not subject to competitive pressure. In other words, a fully effective price policy allows the upstream firm to reduce prices without affecting \bar{C} .

To compare trade credit with price variation, we first assume the following. We relax this later and show how the conclusions change.

Assumption 1 *The upstream price policy is fully effective and fully flexible.*

We now show that trade credit and price variation lead to different outcomes when the costs of outside funds facing the upstream and downstream firms are different. Specifically, whenever the upstream firm faces a lower cost of funds, trade credit is optimal, while a price discount is optimal if the downstream firm faces better terms. The intuition is as follows.

A dollar of trade credit represents a transfer of the interest costs of \$1 from upstream to downstream. But a reduction of \$1 in the input price is like a loan for which neither interest costs nor the principal amount are paid back. Therefore if the upstream firm is indifferent between \$1 of trade credit and \$ x of price cut, it must be that $x < 1$. It follows that for the downstream firm, trade credit reduces current interest costs on a higher amount (1 rather than x), effectively injecting more immediate cash, while a price cut requires less to be paid back (0 rather than 1) after a sale occurs in the future. If a downstream firm is cash-strapped and more impatient than the upstream firm, it prefers to receive trade credit, while a downstream firm with relatively greater patience prefers a price cut.

There are several reasons why the upstream and downstream firms might face different credit terms from outside lenders such as banks. We outline a formal argument in **appendix B** along the following lines. An upstream firm has a natural advantage over a bank in observing sales whenever sales are followed immediately by an input order.¹⁷

¹⁷Under trade credit, the downstream firm places an order immediately after each sale, making sales observable to the input supplier. Thus the input supplier is the natural candidate to offer trade credit. In other words, the upstream firm’s advantage over a bank in observing sales derives simply from the fact that it is the input supplier.

Suppose outside lenders such as banks cannot observe each sale, and therefore lend over a fixed duration. Also suppose the upstream firm has greater market power and earns a higher mark-up per unit compared to the downstream firm (i.e. $C - G$ high, $P - C$ low). Repayment of any loan from a bank depends on achieving a certain number of sales for each firm. Given the mark-ups, this number is clearly greater for the downstream firm. When p falls, the ability of the downstream firm to make a timely repayment is affected more than that of the upstream firm, implying that lenders face a greater risk in lending to the former and charge a higher rate.

6.1 OPTIMAL TRADE CREDIT AND PRICE POLICY

To clarify the choice between the policies, let us first compare the total surplus under trade credit and price discounting for any given input price C . Suppose at this price the downstream firm does not have an incentive to produce immediately, but it is possible to restore the incentive with either a discounted price $C_D < C$ or trade credit τ . The total surplus under trade credit is $S^T(C) \equiv V_0^I(C) + \tau C + U_0^I(C) - \tau C = V_0^I(C) + U_0^I(C)$. The total surplus under a price discount is $S^D(C_D) \equiv V_0^I(C_D) + U_0^I(C_D)$. It follows that

$$S^T(C) - S^D(C_D) = p \left(\frac{\delta_u - \delta_d}{(1 - \delta_u)(1 - \delta_d)} \right) (C - C_D). \quad (6.1)$$

Thus $S^T(C) \gtrless S^D(C_D)$ as $\delta_u \gtrless \delta_d$.

Next, we assume that the upstream firm sets the input price subject to satisfying the downstream participation constraint and characterize the overall optimal policy. Note that either policy is optimal if $\delta_u = \delta_d$.

Proposition 6 *Suppose assumption 1 holds, and suppose $\max\{0, V_0^W(\bar{C})\} > V_0^I(C)$. The optimal policy is given by the following:*

- If $\delta_u > \delta_d$, the optimal policy is $\tau = 1$ and $C = C^*$, where

$$\frac{\delta_d p (P - C^*)}{1 - \delta_d} = \max\{0, V_0^W(\bar{C})\}. \quad (6.2)$$

- If $\delta_u < \delta_d$, the optimal policy is $\tau = 0$ and $C = C_D^*$ where C_D^* is given by equation (5.1).

Thus if the upstream firm faces better credit terms ($\delta_u > \delta_d$), the optimal policy is to offer full trade credit (i.e. advance the input and defer the entire payment) and have the highest possible input price that satisfies the downstream participation constraint. On the other hand, if $\delta_u < \delta_d$, the optimal policy is purely price-based, and requires lowering the input price to the level that just makes it incentive compatible for the downstream firm to pursue the immediate production strategy.

6.2 RELAXING THE ASSUMPTION OF FULLY EFFECTIVE AND FULLY FLEXIBLE PRICING

Let us now relax assumption 1 and show how this affects the result above.

Relaxing fully effective pricing If the upstream firm is subject to competitive pressure so that a price cut leads to a fall in the outside option price of the downstream firm (so that pricing is not fully effective), the efficacy of the price instrument is reduced. The effectiveness of trade credit, on the other hand, does not depend upon the behavior of competing firms. A firm receiving trade credit has the incentive to produce immediately irrespective of offers by competing input sellers. This leads to the following result which shows that trade credit can be optimal even for some cases in which the upstream firm faces worse credit terms compared to the downstream firm (i.e. for a range of values of δ_u below δ_d).

Corollary 1 *If \bar{C} is an increasing function of C (so that pricing is not fully effective), there exists $\delta_* < \delta_d$ such that for any $\delta_u > \delta_*$ the optimal policy is to set $\tau = 1$ and set C to satisfy the downstream participation constraint given by (6.2).*

Relaxing fully flexible pricing Further, if $\bar{C} < P$ so that the input price is only flexible up to \bar{C} , we get the following result. As discussed in section 7, this provides an explanation for the fact that larger clients are offered trade credit as well as a lower price.

Corollary 2 *If $\bar{C} < P$, trade credit must be offered along with a price less than or equal to \bar{C} .*

6.3 TRADE CREDIT INTEREST RATE

We have assumed a zero trade credit interest rate throughout. Intuitively, this is without loss of generality because in our model trade credit serves as a subsidy. Adding an interest

rate simply reduces the subsidy. But if it makes sense to offer the subsidy in the first place, adding interest serves no purpose. We verify this formally below. Suppose a rate of interest r_t is charged for delayed payment. The following result shows that we can set $r_t = 0$ in all cases without loss of generality. The proof is relegated to the appendix.

Corollary 3 *Whenever it is optimal for the upstream firm to offer full trade credit ($\tau = 1$), its payoff is independent of r_t and in other cases it is optimal to set $r_t = 0$.*

7 RELATING TO EVIDENCE

Trade Credit and Firm Margin, Profit and Sales: Petersen and Rajan (1997) find that trade credit provision increases in the supplier’s margin. At the same time, firms that experience a decline in sales, as well as firms that have lower net income (lower cash flow) offer more trade credit. Further, Burkart et al. (2008) find that suppliers are more likely to deny trade credit to more profitable firms, which are also less likely to be offered trade credit. While these might appear anomalous (Petersen and Rajan do say their finding is surprising), these are precisely the predictions of our model.

As the margin $C - G$ rises, the upstream firm has more to lose from lost sales, and therefore has more of an incentive to provide an inventory subsidy. On the other hand, the parameter p captures the volatility of final good sales (and therefore also of input sales). When p is high, the level of sales is high and the downstream firm has no incentive to wait, eliminating the need for trade credit. This explains why trade credit provision increases in the supplier’s margin and at the same time firms suffering sales declines and lower cash-flow offer more trade credit. Further, considering the firms that receive trade credit, as the downstream firm’s margin ($P - C$) rises, and/or sales are high (high p), it has less incentive to wait, reducing the need for trade credit.

Trade Credit and Input Pricing: Our theory suggests that along with trade credit, price is raised to the highest level consistent with downstream participation constraint. However, corollary 2 in section 6.2 shows that if the outside option $\bar{C} < P$, prices are not fully flexible, and trade credit must be combined with a price of at most \bar{C} .

In other words, given that the *competition environment* is a confounding factor, testing this aspect of the theory is difficult. Burkart et al. (2008) report that larger firms with many

suppliers are offered more trade credit and lower prices. As discussed above, our theory is not inconsistent with this evidence.¹⁸

Trade Credit Terms and Relation with the Rate of Interest: As explained above, our model is consistent with a trade credit interest rate (of zero) which is below a positive bank rate. Further, in the model the duration of trade credit is naturally bounded by purchase of input at one end and sale of final good at the other, implying that the model is consistent with the observation that trade credit is usually short term credit.

Next, the *decision to offer trade credit* depends crucially on the bank interest rates as well as market demand. A high bank rate and/or low p create incentives to move away from immediate production, generating a scope for trade credit. But *when offered*, the zero rate charged does not respond to bank rate changes, or changes in market demand. This is consistent with the observed insensitivity of the trade credit interest rate to changes in the bank rate.

The relation between bank rates and trade credit is summarized in Figure 1. Suppose the bank rate increases so that r_u and r_d rise - i.e. δ_u and δ_d fall. This raises the incentive to wait (or even to not invest), creating greater scope for trade credit. On the other hand, this reduces upstream incentive to provide trade credit, and large enough rise in the rate would violate the upstream participation constraint. A similar effect arises from lowering p . Indeed, if the economic situation is such that a large rise in the bank rate is also accompanied by a fall in p , the same effect (scope for trade credit higher, but harder to satisfy the upstream participation constraint) is strengthened.

Inventory Investment: In their paper on inventories, Blinder and Maccini (1991) point out that “the question of why inventory investment seems to be insensitive to changes in real interest rates remains open, important, and troublesome.” In our model, trade credit

¹⁸However, if generally firms receiving trade credit (irrespective of the competition environment) have lower cost of input, this would not be a good explanation. An alternative explanation in the case of firms with multiple suppliers – which is not captured by our simple model, but not inconsistent with it – is as follows. With many suppliers, a sale is not automatically followed by an order, which means any given supplier may not be sure when a sale takes place - i.e. when the usefulness of its trade credit expires (and then potentially starts to indirectly subsidize the purchase of the next unit of input from another supplier). In this case, to encourage early repayment, a price discount incentive may be given. This could result in a lower cost of inputs for relatively credit unconstrained large firms. Indeed, Burkart et al. (2008) themselves propose repayment incentive as an explanation of discounts to large firms.

ensures that for a large range of parameter values the downstream firm adopts immediate production, implying unit inventory per period (the firm either produces 1 unit or carries 1 unit) which is insensitive to the interest rate. Maccini, Moore and Schaller (2004) offer an explanation to the inventory insensitivity puzzle based on the claim that firms care about the long-run level of the interest rate and not about short run rate fluctuations. Our theory complements their work by identifying an additional reason to suggest that inventory investment by firms might not respond to transitory shocks, but does respond to large and persistent changes in the real interest rate.

8 PREPAYMENT

We now proceed with the assumption that the downstream firm chooses to produce immediately ($(P - C)$ high) and analyze the upstream firm's incentive to delay production, making a case for prepayment.¹⁹

8.1 OPTIMAL STRATEGY OF THE UPSTREAM FIRM

Let U_0^{UI} and U_0^{UW} denote the payoffs of the upstream firm when the upstream firm chooses the strategy of immediate production and wait-and-see, respectively.

Lemma 3

$$U_0^{UI} = C - G + \frac{\delta_u p (C - G)}{1 - \delta_u},$$

$$U_0^{UW} = C + \frac{\delta_u p (\delta_u C - G)}{(1 - \delta_u) (1 + \delta_u p (1 - \delta_u q (1 - p)))}.$$

We now derive the optimal strategy of the upstream firm. Recall from section 3.2 that when the upstream firm simply sells units of input to the downstream firm at a price C , the former chooses not to produce at all if $\delta_u C \leq G$. The next result derives the optimal choice for the complementary case.

Let p_{ic}^U denote the cutoff above which the upstream incentive to follow the immediate production strategy is satisfied.

¹⁹This paper provides an explanation for prepayment in developed market economies with little or no chance of default. Including a default risk (perhaps because of poor enforcement of law) would obviously increase the scope for prepayment, as this now also serves as insurance against default.

Proposition 7 For any given C, G and δ_u satisfying $\delta_u C > G$, there exists a unique cutoff $p_{ic}^U \in (0, 1)$ such that for $p \geq p_{ic}^U$, the optimal choice is immediate production, and for $p < p_{ic}^U$, wait-and-see is optimal.

8.2 DOWNSTREAM OPTIMUM

Let V_0^{UI} and V_0^{UW} denote the payoffs of the downstream firm when the upstream firm chooses the strategy of immediate production and wait-and-see, respectively. V_0^{UI} is the same as V_0^I given by (3.1). We derive V_0^{UW} below.

Lemma 4
$$V_0^{UW} = -C + \frac{\delta_d p (P - \delta_d C)}{(1 - \delta_d) (1 + \delta_d p (1 - \delta_d q (1 - p)))}$$
.

The following characterizes the downstream optimum.

Proposition 8 Whenever the downstream firm prefers immediate production to waiting itself, it also prefers the upstream sector to produce immediately rather than wait and see. Formally, $V_0^{UI} > V_0^{UW}$ for $p \geq p_{ic}^D$, where p_{ic}^D is given by equation (3.5).

8.3 PREPAYMENT

Suppose prepayment is offered for a fraction $\theta \in [0, 1]$ of the upstream price at every instance of placing an order for inputs, and θ is such that the upstream incentive to produce immediately is restored. Then, as in the case of trade credit, the prepayment scheme can be represented by a total transfer of θC from the downstream firm to the upstream firm. Since the argument is exactly similar to proposition 3, we omit the proof here.

Let p_{pp}^D denote the cutoff above which the downstream participation-in-prepayment constraint holds. Recall that p_{ic}^U is the cutoff below which the upstream firm chooses the wait-and-see strategy (creating a scope for prepayment) and p_{ic}^D is the cutoff above which the downstream firm chooses immediate production. The following result characterizes the outcome under prepayment.

Proposition 9 Under condition 2.1, for any $G > 0$ there exists $\Delta > 0$ such that for $C - G \leq \Delta$, we have $p_{pp}^D < p_{ic}^U$, and prepayment restores full efficiency on the non-empty interval $[\max\{p_{pp}^D, p_{ic}^D\}, p_{ic}^U]$.

8.4 PREPAYMENT VERSUS INPUT PRICE MARK-UP

As under trade credit, we can compare the total surplus under prepayment ($S^{PP}(C)$) to that under a price mark-up ($S^M(C_M)$). Suppose the upstream firm does not have an incentive to produce immediately, but it is possible to restore the incentive with either a price mark-up $C_M > C$ or prepayment θ . Similar to equation (6.1), $S^M(C_M) - S^{PP}(C) = p \left(\frac{\delta_u - \delta_d}{(1 - \delta_u)(1 - \delta_d)} \right) (C_M - C)$. Thus $S^{PP}(C) \geq S^M(C_M)$ as $\delta_d \geq \delta_u$. Finally, it is easy to show that if $\delta_u > \delta_d$, the optimal policy is a pure price mark-up that is sufficient to induce immediate production, and if $\delta_u < \delta_d$, the optimal policy is to prepay the full input price, and then set the lowest possible price consistent with upstream participation. Further, analogously to the case of trade credit before, if prices are not fully flexible, the scope for prepayment expands. We omit the details.

9 COMPARING ACROSS POLICIES: TRADE CREDIT, PREPAYMENT, PRICE VARIATION

First, note that the scope for trade credit or a price discount depends on both downstream incentive to move away from immediate production, and upstream incentive to provide a remedial subsidy. It can be checked easily that as the downstream margin ($P - C$) rises, $V_0^I - V_0^W$ rises. Therefore the incentive to move away from immediate production declines. Further, as the upstream margin ($C - G$) rises, $U_0^I - U_0^W$ rises, raising the incentive to provide a remedial subsidy if needed. Similar reasoning can be applied to the case of prepayment/price mark-up, the scope for which rises and falls in the down and upstream margins respectively. The following statement summarizes these facts:

The scope for trade credit and/or a discounted input price increases in $C - G$, and decreases in $P - C$, while the scope for prepayment and/or an input price mark-up decreases in $C - G$, and increases in $P - C$.

If the upstream margin is high relative to the downstream margin, a scope for trade credit and/or a price discount arises. But which policy is optimal? As discussed in section 6 (and appendix B), in this case the upstream firm is likely to face easier credit terms, which implies that trade credit, rather than a price discount, is the optimal policy instrument. Similarly, if $(P - C)$ is high relative to $(C - G)$, a scope for prepayment and/or a price

mark-up arises. And, as section 8.4 shows, this is also a case in which the downstream firm is likely to face easier credit terms, which makes prepayment, rather than a price mark-up, the optimal instrument. The table below summarizes our results about the relative likelihood of the direction of a subsidizing policy and the optimal instrument to implement the policy.

	$\delta_u > \delta_d$	$\delta_u < \delta_d$
$(P - C)$ low, $(C - G)$ high	Trade credit	Price discount
$(P - C)$ high, $(C - G)$ low	Price mark-up	Prepayment

Further, as discussed before, if price cuts are not fully effective (flexible), trade credit (prepayment) is optimal even if the upstream firm faces worse (better) terms.

10 CONCLUSION

The literature on trade credit has often emphasized that a discount term represents a high implicit interest rate. We ask a complementary question: why is so much trade credit supplied on net terms, charging zero interest? Indeed, even credit including a discount period is not a simple high cost loan - it includes an interest free period. Further, prepayment, which is reverse trade credit, is simply a cash advance provided at an exactly zero interest rate.

In our theory, a downstream firm cuts inventories if the opportunity cost of holding inventories exceeds the expected loss through foregone sales. But lost sales create an externality for the upstream firm. Given that a relatively high bank rate is one of the factors that generate the incentive to save on inventories, bank loans cannot solve the problem. The solution must be a loan at a *subsidized rate* and trade credit delivers precisely such a subsidy. Further, the upstream firm has an incentive to provide such credit because it benefits from increased sales when the downstream firm internalizes the externality. Similarly, prepayment arises as a response to a reverse externality originating from upstream cost saving. Importantly, an inventory subsidy is limited by the value of inputs. This implies that our theory is consistent with the fact that the extent of trade credit (and prepayment) is determined by the value of inputs, and is therefore immune to the Burkart-Ellingsen critique.

The theory clarifies why trade credit and prepayment are provided at zero interest, which

is necessarily lower than the bank rate, and explains the invariance of trade credit terms to fluctuations in the bank rate as well as market demand. Further, in our theory trade credit is a complement - not substitute - of bank lending. The literature often implies that such complementarity arises from credit rationing. Our theory, in contrast, predicts complementarity even in the absence of such constraints. Finally, note that while trade credit is a zero-interest loan, it is not necessarily cheap since the input price could be raised simultaneously.

However, a price discount could also deliver the subsidy implicit in trade credit. Therefore the use of trade credit requires further justification. We show that trade credit Pareto dominates price cuts when the upstream firm faces better credit terms relative to the downstream firm. This is likely to be the case when the former earns a higher margin compared to the latter. But the scope for trade credit or price discounting arises precisely when the upstream firm earns a relatively higher margin. Thus the circumstance in which the upstream firm would like to induce the downstream firm to adopt the immediate production strategy is also naturally the case in which trade credit is preferred to price variation. A similar analysis applies to prepayment. Finally, if price cuts are not fully effective because they are matched by rival firms, trade credit is also optimal for a range of cases in which the upstream firm faces worse credit terms compared to the downstream firm.

In sum, our model shows that it is plausible to take a view of inter-firm credit as an optimal instrument delivering a targeted subsidy which covers at most the input costs and adjusts inventory incentives to prevent lost sales.

11 APPENDIX A: PROOFS

A.1 PROOF OF LEMMA 1

We know from (3.2) that $V_0^W = 0 + \delta_d ((1 - p) V_0^W + p V_1(1))$ where $V_1(1)$ is the value starting period 1 if a customer arrives at the start of period 1. To calculate $V_1(1)$, recall that once a customer arrives at the start of period 1, there are 3 possible states at the beginning of period 2: **State 1:** Exactly one customer arrives. Probability of this event is $(p + q - 2p q)$. The firm sells and gets P , and then restarts period 0 strategy. Thus payoff is $P + V_0^W$.

State 2: 2 customers (1 old and 1 new) arrive. Probability of this event is $p q$. The firm sells to the returning customer and gets P , and then onwards gets $V_1(1)$. Thus payoff is $P + V_1(1)$.

State 3: No one arrives. Probability of this event is $(1 - p)(1 - q)$. Let the payoff in this state be denoted by $V_2(0)$. This is given by:

$$V_2(0) = 0 + \delta_d \left(p (P + V_0^W) + (1 - p) V_2(0) \right). \quad (\text{A.1})$$

From the above,

$$V_1(1) = -C + \delta_d \left[(p + q - 2p q)(P + V_0^W) + p q(P + V_1(1)) + (1 - p)(1 - q)V_2(0) \right].$$

Using this as well as equations (3.2) and (A.1), we can solve for V_0^W and obtain the stated value.||

A.2 PROOF OF PROPOSITION 1

From equation (3.3), we can write V_0^W as $V_0^W = f(\delta_d)g(\delta_d)$, where $f(\delta_d) = \frac{\delta_d P}{1 - \delta_d}$, and

$$g(\delta_d) = \frac{P(p + (1 - \delta_d) q (1 - p)) - C/\delta_d + C (1 - p)}{D},$$

where $D = (1 - \delta_d)/\delta_d + p(2 - q)$.

$g(C/P) = -\frac{C(P-C)(1-p)(1-q)}{(P-C)+p(2-q)C} \leq 0$, with strict inequality for $p < 1$. Thus at $\delta_d = C/P$, $V_0^W \leq 0$, with strict inequality for $p < 1$. Further, at $\delta_d = C/P$, $V_0^I = -C(1 - p) \leq 0$.

Now, $f(\cdot) > 0$, $f'(\cdot) > 0$. Further,

$$g'(\cdot) = \frac{1}{\delta_d^2 D^2} \left(p(1 - \delta_d^2 q(2 - q)(1 - p))P + (1 - \delta_d)^2 q(1 - p)P + p(1 - q)C \right) > 0,$$

where the last step follows from the fact that the maximized value of $q(2 - q)$ is 1, and therefore $1 - \delta_d^2 q(2 - q)(1 - p) > 0$. Thus $\frac{\partial V_0^W}{\partial \delta_d} > 0$ whenever $g(\delta_d) > 0$. But since $f(\delta_d) > 0$ always, whenever $g(\delta_d) > 0$ it is also true that $V_0^W > 0$.

Therefore, if $g(\hat{\delta}_d) > 0$ for any $\hat{\delta}_d > 0$, then $g(\cdot)$ is positive and increasing for all $\delta_d > \hat{\delta}_d$.

Since for $\delta_d = C/P$, both V_0^I and V_0^W are non-positive, from the above it follows that for all $\delta_d < C/P$ they are negative. Thus for any investment to take place, a necessary condition is $\delta_d > C/P$.||

A.3 PROOF OF PROPOSITION 2

From equation (3.3), $V_0^W \geq 0$ implies

$$p \geq \frac{(1 - \delta_d)(C - \delta_d q P)}{\delta_d((P - C) - (1 - \delta_d)qP)}, \quad (\text{A.2})$$

where strict equality implies $V_0^W = 0$. From equation (3.1), $V_0^I \geq 0$ implies

$$p \geq \frac{(1 - \delta_d)C}{\delta_d(P - C)}, \quad (\text{A.3})$$

where, similarly, strict equality implies $V_0^I = 0$. It can be easily checked that for any $\delta_d \in (C/P, 1)$, the expression on the right hand side of (A.3) strictly exceeds that of (A.2).

Thus the participation constraint for investment is given by $p \geq p_\ell^D$ where p_ℓ^D is the maximum of 0 and the expression on the right hand side of (A.2).

Next, the optimal choice is immediate production if $V_0^I \geq V_0^W$. Solving from equations (3.1) and (3.3), this is the case if $p \geq p_{ic}^D$ where p_{ic}^D is as in equation (3.5).

Finally, we check that $p_\ell^D < p_{ic}^D < 1$. $p_{ic}^D < 1$ is true if $\frac{(1 - \delta_d)C}{\delta_d((P - C) - q(\delta_d P - C))} < 1$, which simplifies to $\delta_d q < 1$, which is true.

Next, since $(P - C) - q(\delta_d P - C) > (P - C) - (\delta_d P - C) = (1 - \delta_d)P > 0$, it is clear that $p_{ic}^D > 0$. Thus $p_\ell^D < p_{ic}^D$ is true if

$$\frac{(1 - \delta_d)(C - \delta_d q P)}{\delta_d((P - C) - (1 - \delta_d)qP)} < \frac{(1 - \delta_d)C}{\delta_d((P - C) - q(\delta_d P - C))},$$

which implies $(\delta_d P - C)(P(1 - \delta_d q) + C) > 0$, which is true, since $\delta_d > C/P$.

Thus for $p \geq p_{ic}^D$, immediate production is optimal, for $p \in (p_{\ell}^D, p_{ic}^D)$, the payoff from waiting is higher than immediate production ($V_0^W > V_0^I$), and satisfies the participation constraint ($V_0^W > 0$). Finally, for $p \leq p_{\ell}^D$, the payoff from investment is negative, and not investing is optimal. ||

A.4 PROOF OF LEMMA 2

If the downstream firm adopts the immediate production strategy, in each period a sale is made with probability p . This also means that in each period the upstream firm makes a sale to the downstream firm with probability p . In this case, the upstream payoff is as follows. As noted at the end of section 2, without loss of generality we can ignore the cost in period -1 (in effect assuming that the upstream firm starts with a unit of endowment in period 0). In period 0, the upstream firm sells this endowment and then produces another unit, making the period payoff $C - G$. Similarly, in any period $t > 0$, a unit is produced if a sale is made to the downstream firm (which happens if the downstream firm sells a unit) so that the expected payoff is $p(C - G)$ in each such period. Thus $U_0^I = (C - G) + \delta_u p(C - G) + \dots$, which explains equation (3.6).

If the downstream firm adopts the wait-and-see strategy, the payoff of the upstream firm can be calculated as follows. In what follows, the term “customer” means a customer for the downstream firm, i.e. a final customer.

Let $U_1(1)$ be the value starting period 1 if a customer arrives at the start of period 1, and $U_1(0)$ be the value starting period 1 if no one arrives at the start of period 1. In the latter case, the situation is exactly as at the start of period 0 and thus $U_1(0) = U_0^W$. Then

$$U_0^W = 0 + \delta_u \left((1 - p) U_0^W + p U_1(1) \right). \quad (\text{A.4})$$

To calculate $U_1(1)$, note that once a customer arrives at the start of period 1, the upstream firm sells a unit and earns $C - G$ in period 1. Following this, at the beginning of period 2, there are 3 possible states (exactly as in the proof of Lemma 1). **State 1:** Exactly 1 customer arrives (probability $p + q - 2p q$). The downstream firm sells and restarts period 0 strategy. Thus upstream payoff is U_0^W . **State 2:** 2 customers (1 old and 1 new) arrive (probability $p q$). The downstream firm sells to the returning customer and restarts pro-

duction. Thus upstream firm is in exactly the same situation as at the start of period 1 if a customer arrives, and the payoff is $U_1(1)$. **State 3:** No customer arrives (probability $(1-p)(1-q)$). The payoff in this state (denoted by $U_2(0)$) is derived as follows. If a customer arrives tomorrow, the downstream firms sells the unit previously produced, and restarts period 0 strategy. The upstream payoff is U_0^W . If no one arrives tomorrow, the upstream firm gets $U_2(0)$. Thus

$$U_2(0) = 0 + \delta_u \left(p U_0^W + (1-p) U_2(0) \right). \quad (\text{A.5})$$

From the above,

$$U_1(1) = (C - G) + \delta_u \left[(p + q - 2p q) U_0^W + p q U_1(1) + (1-p)(1-q) U_2(0) \right]. \quad (\text{A.6})$$

From equations (A.4), (A.5) and (A.6), we can solve for U_0^W , and obtain:

$$U_0^W = \frac{(1 - \delta_u + \delta_u p) \delta_u p (C - G)}{(1 - \delta_u)(1 - \delta_u + \delta_u p (2 - q))} = \frac{\delta_u p}{1 - \delta_u + \delta_u p (2 - q)} U_0^I,$$

where the second step uses equation (3.6).||

A.5 PROOF OF PROPOSITION 4

The optimal trade credit offer must be such that the downstream incentive constraint binds, i.e. $V_0^I + \tau^* C = \max\{0, V_0^W\}$.

From (A.2), $V_0^W \geq 0$ as $p \geq p_\ell^D$, where p_ℓ^D is given by equation (3.4). Thus for $p < p_\ell^D$, τ^* is given by $V_0^I + \tau^* C = 0$, and for $p \in [p_\ell^D, p_{ic}^D)$, τ^* is given by $V_0^I + \tau^* C = V_0^W$, where p_{ic}^D is given by equation (3.5). Finally, from equations (3.1) and (3.3), $V_0^I \geq V_0^W$ if $p \geq p_{ic}^D$. Thus for $p \geq p_{ic}^D$, $\tau^* = 0$. Thus $\tau^* = \frac{1}{C} (\max\{0, V_0^W, V_0^I\} - V_0^I)$, which is as stated.

Let us check that $\tau^* \in [0, 1]$ in all cases. For $p \geq p_{ic}^D$, $\tau^* = 0$. For $p < p_\ell^D$, $V_0^I < V_0^W < 0$. Thus $\tau^* = -V_0^I/C > 0$. Also, from (3.1), $-V_0^I \leq C$ (since $C \leq P$). Thus $\tau^* \in [0, 1]$.

Next, for $p \in [p_\ell^D, p_{ic}^D)$, $V_0^W > 0$ and $V_0^W > V_0^I$. Thus $\tau^* = (V_0^W - V_0^I)/C > 0$. Also, from equations (3.1) and (3.3), for $p \in [p_\ell^D, p_{ic}^D)$,

$$\frac{V_0^W - V_0^I}{C} = 1 - \frac{\delta_d p}{1 - \delta_d} \frac{\delta_d p (1 - q) (P - C) + (1 - \delta_d)(1 - \delta_d q (1 - p)) P}{C (1 - \delta_d + \delta_d p (2 - q))} < 1.$$

Thus in this case $0 < \tau^* < 1$.

Next, let us check that τ^* satisfies all the participation and incentive compatibility constraints. By construction the payoff under τ^* satisfies the downstream incentive constraint.

Finally, the upstream participation constraint must hold. The upstream payoff under optimal trade credit is given by $U_0^I - \tau^*C$. Since $\tau^* \leq 1$, it suffices to show that $U_0^I - C > U_0^W$. Using equations (3.6) and (3.7), and solving for p , the upstream participation-in-trade-credit constraint is given by

$$p \geq p_{\text{ptc}}^U \equiv \left(\frac{1 - \delta_u}{\delta_u} \right) \frac{G(2 - q) + \sqrt{G(4C(1 - q) + Gq^2)}}{2(1 - q)(C - G)}.$$

Clearly, this is only possible if $p_{\text{ptc}}^U \leq 1$, which is true if either G is low or δ_u is not very low. It is also clear that if either $G \rightarrow 0$ or $\delta_u \rightarrow 1$, $p_{\text{ptc}}^U \rightarrow 0$ in which limiting case full efficiency is attained.

For $p \geq p_{\text{ic}}^D$ the immediate production strategy is chosen optimally even without trade credit. The optimal trade credit satisfies the participation constraint of the upstream firm, and ensures that the downstream firm follows the immediate production strategy for any p such that $p_{\text{ptc}}^U \leq p < p_{\text{ic}}^D$. \parallel

A.6 PROOF OF PROPOSITION 5

The discussion in section 5 proves the claim in the first sentence. Let us now check the upstream participation constraint. This always holds if $\max\{V_0^W(\bar{C}), 0\} = 0$, as in this case the downstream firm does not produce at all - generating a zero payoff for the upstream firm, while a price of C_D^* generates a positive payoff.

Next, let $\max\{V_0^W(\bar{C}), 0\} = V_0^W(\bar{C})$. In this case the discounted price C_D^* is such that $V_0^I(C_D^*) = V_0^W(\bar{C})$. Let $\mu(x) \equiv \frac{xp}{1 - x + xp(2 - q)}$. Using equations (3.1) and (3.3), and solving, we get $C_D^* = \mu(\delta_d)(\bar{C} + (1 - q\delta_d)P)$.

Now, the upstream participation constraint is given by $U_0^W(\bar{C}) \leq U_0^I(C_D^*)$. Using equations (3.6) and (3.7), this simplifies to

$$\mu(\delta_u)(\bar{C} - G) \leq C_D^* - G = \mu(\delta_d)(\bar{C} + (1 - q\delta_d)P) - G, \quad (\text{A.7})$$

where the second step uses the expression for C_D^* from above. Solving, we get

$$\delta_u \leq \delta_u^{\max} \equiv \frac{p\delta_d(\bar{C} + P(1 - q\delta_d)) - G(1 - \delta_d + p(2 - q)\delta_d)}{p\bar{C} - G(1 - p + pq) + (1 - p(2 - q))\delta_d(pP + G(1 - p(1 - q))) - pPq\delta_d}.$$

Note that the upstream participation constraint holds for any $\delta_d \geq \delta_u$ if and only if $\delta_u^{\max} \geq \delta_d$. Solving for p from $\delta_u^{\max} = \delta_d$ from above, and discarding the solution exceeding 1, this is true whenever

$$p \geq p_{\text{pdp}}^U = \frac{(1 - \delta_d)G}{\delta_d((1 - \delta_d q)P - (1 - q)G)}.$$

p_{pdp}^U is positive and it can be directly verified that it is below 1. Further, it is easy to see that $p_{\text{pdp}}^U \rightarrow 0$ if either $G \rightarrow 0$ or $\delta_d \rightarrow 1$.

A.7 PROOF OF PROPOSITION 6

Recall from section 2 that the outside option for the downstream firm is given by \bar{C} . Therefore its outside option payoff is $V_{\text{out}} \equiv \max\{0, V_0^W(\bar{C}), V_0^I(\bar{C})\}$. Now suppose a price C is offered to the downstream firm if it follows immediate production, and a price C' is offered if it follows wait-and-see. Clearly, trade credit must satisfy $V_0^I(C) + \tau^*C = \max\{0, V_0^W(C'), V_0^I(C), V_{\text{out}}\}$.

Suppose $\max\{0, V_0^W(C'), V_0^I(C), V_{\text{out}}\} = V_0^W(C')$. Since V_0^W is decreasing in the input price, it follows that $C' \leq \bar{C}$ (otherwise the outside option is simply $V_0^W(\bar{C})$). In this case $\tau^*C = V_0^W(C') - V_0^I(C)$. Therefore $U_0^T(C) = U_0^I(C) + V_0^I(C) - V_0^W(C')$, which increases in C' , and therefore the optimal value of C' is simply \bar{C} . It follows that $\max\{0, V_0^W(C'), V_0^I(C), V_{\text{out}}\} = \max\{0, V_0^W(\bar{C}), V_0^I(C)\}$. If the maximum is $V_0^I(C)$, trade credit is zero. Therefore only two cases need to be considered.

Case 1: $\max\{0, V_0^W(\bar{C}), V_0^I(C)\} = 0$. In this case $\tau^*C = -V_0^I(C)$. Therefore

$$\begin{aligned} U_0^T(C) &= U_0^I(C) - \tau^*C = U_0^I(C) + V_0^I(C) \\ &= \frac{\delta_d p P}{1 - \delta_d} + pC \frac{(\delta_u - \delta_d)}{(1 - \delta_u)(1 - \delta_d)} - \left(\frac{1 - \delta_u(1 - p)}{1 - \delta_u} \right) G. \end{aligned} \quad (\text{A.8})$$

If $\delta_u > \delta_d$, the optimal policy is therefore $C = P$. In this case $\tau^*P = -V_0^I(P)$ implies $\tau^* = 1$. Next, if $\delta_u < \delta_d$, the optimal policy is to lower the price to $C = C_D^*$ and put $\tau^* = 0$, where C_D^* is as in section 5. Finally, any $C \in [0, P]$ with $\tau^*C = -V_0^I(C)$ is optimal if $\delta_u = \delta_d$.

Case 2: $\max\{0, V_0^W(\bar{C}), V_0^I(C)\} = V_0^W(\bar{C})$. In this case

$$U_0^T(C) = \frac{\delta_d p P}{1 - \delta_d} + p C \frac{(\delta_u - \delta_d)}{(1 - \delta_u)(1 - \delta_d)} - \left(\frac{1 - \delta_u(1 - p)}{1 - \delta_u} \right) G - V_0^W(\bar{C}). \quad (\text{A.9})$$

But maximizing this with respect to C is the same as maximizing the expression for $U_0^T(C)$ given by equation (A.8). Therefore, as before, if $\delta_u < \delta_d$ the optimal policy is $\tau^* = 0$ and $C = C_D^*$.

If $\delta_u > \delta_d$, the optimal policy is to have the highest possible input price that satisfies the downstream participation constraint. Now, suppose $\tau < 1$. Then it is possible to raise τ further, and also raise C to keep the downstream firm indifferent. This increases the upstream payoff. Therefore the optimal policy sets $\tau^* = 1$, and the associated input price satisfying the downstream participation constraint (as given in the statement of the proposition).||

A.8 PROOF OF COROLLARY 1

From equations (A.8) and (A.9), the upstream payoff from offering trade credit τ and charging price C is

$$U_0^T(C) = \frac{\delta_d p P}{1 - \delta_d} + p C \frac{(\delta_u - \delta_d)}{(1 - \delta_u)(1 - \delta_d)} - \left(\frac{1 - \delta_u(1 - p)}{1 - \delta_u} \right) G - \phi,$$

where $\phi \equiv \max\{0, V_0^W(\bar{C})\}$. It follows that

$$\frac{\partial}{\partial C} U_0^T(C) = p \frac{(\delta_u - \delta_d)}{(1 - \delta_u)(1 - \delta_d)} - \frac{\partial \phi}{\partial C}.$$

Since \bar{C} is an increasing function of C , ϕ is decreasing weakly in C . Further, as C falls, \bar{C} falls, and below a certain level of \bar{C} , $V_0^W(\bar{C})$ is necessarily positive. In this case ϕ decreases strictly in C . It follows that if $\delta_u - \delta_d$ is not large (whatever the sign of the term), the second term (which is positive) can dominate, and in that case C should be as high as possible. It follows that the optimal policy in this case is as in the first part of proposition 6: offer full trade credit and the highest price satisfying downstream participation constraint. This proves that there exists $\delta_* < \delta_d$ such that for all $\delta_u > \delta_*$, the stated policy is optimal. ||

A.9 PROOF OF COROLLARY 2

In offering trade credit, the upstream firm must satisfy the downstream participation constraint given by $V_0^I(C) + \tau C = \phi$, where $\phi \equiv \max\{0, V_0^W(\bar{C})\}$. Given any $\tau \in (0, 1)$, \hat{C} solves the equation above where $\hat{C} = \frac{\delta_d p P - (1 - \delta_d) \phi}{(1 - \tau)(1 - \delta_d) + p\delta_d}$. Suppose the outside option $\bar{C} < P$. Then the optimal input price is given by $C^* = \min[\bar{C}, \hat{C}]$. If $\phi = 0$ and $\tau = 1$, $\hat{C} = P$, but $C^* = \bar{C} < P$. And if $\phi = V_0^W(\bar{C})$, C^* could be even lower than \bar{C} .||

A.10 PROOF OF COROLLARY 3

The payoff of the downstream firm under trade credit is as in section 4, except in each period after the initial period an extra repayment of $r_t \tau C$ must be made. The payoff is thus given by

$$V_0^T(C) = -(1 - \tau)C + \frac{\delta_d p(P - C - r_t \tau C)}{1 - \delta_d} = V_0^I(C) + \tau C \left(1 - \frac{\delta_d p r_t}{1 - \delta_d}\right).$$

In offering a trade credit and price policy, the upstream firm must satisfy the downstream participation constraint given by $V_0^I(C) + \tau C \left(1 - \frac{\delta_d p r_t}{1 - \delta_d}\right) = \phi$, where $\phi \equiv \max\{0, V_0^W(\bar{C})\}$. Given any $\tau \in (0, 1)$, C^* solves the equation above where

$$C^* = \frac{\delta_d p P - (1 - \delta_d) \phi}{(1 - \tau)(1 - \delta_d) + p\delta_d(1 + r_t \tau)}.$$

The upstream payoff is then given by $U_0^T(C^*) = (C^* - G) + \frac{\delta_u p(C^* - G)}{1 - \delta_u} - \tau C^* \left(1 - \frac{\delta_u p r_t}{1 - \delta_u}\right)$. Substituting the value of C^* from above and differentiating with respect to τ ,

$$\frac{\partial U_0^T(C^*)}{\partial \tau} = \frac{p(1 + r_t)(\delta_u - \delta_d)(\delta_d p P - (1 - \delta_d)\phi)}{(1 - \delta_u)((1 - \tau)(1 - \delta_d) + \delta_d p(1 + r_t \tau))^2}.$$

For any given $r_t \geq 0$, this is strictly positive if $\delta_u > \delta_d$. Therefore, for $\delta_u > \delta_d$, $\tau^* = 1$. Using this, $U_0^T(C^*) = \frac{\delta_u(\delta_d p P - (1 - \delta_d)\phi) - \delta_d G(1 - (1 - p)\delta_u)}{(1 - \delta_u)\delta_d}$, which is independent of r_t . We can therefore set $r_t = 0$ without loss of generality.

For $\delta_u < \delta_d$, if $\tau^* = 0$, r_t is obviously irrelevant. Further, if for some $\delta_u < \delta_d$, $\tau \in (0, 1)$, we have

$$\frac{\partial U_0^T(C^*)}{\partial r_t} = \frac{p(1 - \tau)\tau(\delta_u - \delta_d)(\delta_d p P - (1 - \delta_d)\phi)}{(1 - \delta_u)((1 - \tau)(1 - \delta_d) + \delta_d p(1 + r_t \tau))^2}.$$

Clearly, the upstream payoff in this case is strictly decreasing in r_t , and therefore it is optimal to have $r_t = 0$.||

A.11 PROOF OF LEMMA 3

U_0^{UI} is the same as U_0^I given by (3.6). Next, let us calculate U_0^{UW} . The upstream firm produces once the downstream firm places an order after selling a unit. In period 0, the downstream firm spends C and starts production. Thus the upstream firm earns C in period 0. In period 1 there are two possible states - either a customer arrives or no customer arrives. Let $U_1(0)$ be the value from period 1 onwards in the second state. If a customer arrives, the downstream firm makes a sale, and the upstream firm produces in period 1. Thus the payoff in period 1 is $-G$, and let the value from period 2 onwards be denoted by $U_2(1)$. Thus

$$U_0^{UW} = C + \delta_u (p (-G + \delta_u U_2(1)) + (1 - p) U_1(0)). \quad (\text{A.10})$$

$U_1(0)$ can be calculated as follows:

$$U_1(0) = 0 + \delta_u p (-G + \delta_u U_2(1)) + \delta_u (1 - p) U_1(0). \quad (\text{A.11})$$

Let us now calculate $U_2(1)$. Since the upstream firm produces in period 1, it sells a unit at the start of period 2. Subsequently, there are two possible states exactly as in period 1. Denoting the probability that no customer arrives at start of period 3 by ψ , we have

$$U_2(1) = C + (1 - \psi) \delta_u (-G + \delta_u U_2(1)) + \psi \delta_u U_1(0). \quad (\text{A.12})$$

Now, since the upstream firm produces in period 1, the downstream firm can only produce in period 2. Any customer arriving at the start of period 2 cannot be served. In period 3, no customer arrives if there is neither any returning customer from period 2, nor any new customer arrives²⁰ $\psi = (1 - p)(1 - pq)$. Substituting the value of ψ in (A.12), and using equations (A.10) to (A.12), we get the required expression for U_0^{UW} .||

²⁰The probability that there is a returning customer is the probability that a customer arrives (p) times the probability that he returns (q). Thus the probability of no returning customer is $(1 - pq)$. The probability of no new customer arriving in period 3 is $(1 - p)$.

A.12 PROOF OF PROPOSITION 7

Let $\Delta U \equiv U_0^{UW} - U_0^{UI}$. From lemma 3, we have $\frac{\partial U_0^{UI}}{\partial p} = \frac{\delta_u(C - G)}{1 - \delta_u}$, and $\frac{\partial U_0^{UW}}{\partial p} = \left[\frac{\delta_u(\delta_u C - G)}{1 - \delta_u} \right] \left[\frac{1 - \delta_u^2 p^2 q}{1 + \delta_u p(1 - \delta_u q) + \delta_u^2 p^2 q} \right]$. The first term in square brackets is lower than $\frac{\partial U_0^{UI}}{\partial p}$, and the second term in square brackets is less than 1. It follows that $\frac{\partial \Delta U}{\partial p} < 0$.

Next, $\lim_{p \rightarrow 1} \Delta U = -\frac{\delta_u C - G}{1 - \delta_u^2} < 0$, and $\lim_{p \rightarrow 0} \Delta U = G > 0$. Finally, ΔU is continuous in p . The facts above imply that there exists a unique $p \in (0, 1)$ (denoted by p_{ic}^U) that satisfies $\Delta U = 0$, and $\Delta U \geq 0$ as $p \leq p_{ic}^U$.

A.13 PROOF OF LEMMA 4

In period 0, the downstream firm spends C and starts production. In period 1 there are two possible states - either a customer arrives or no customer arrives. Let Y_1 be the value in the first state and Y_0 be the value in the second state. Then

$$V_0^{UW} = -C + \delta_d p Y_1 + \delta_d (1 - p) Y_0. \quad (\text{A.13})$$

Now,

$$Y_0 = 0 + \delta_d (p Y_1 + (1 - p) Y_0). \quad (\text{A.14})$$

Let us calculate Y_1 next. In period 1, if a customer arrives, the downstream firm sells the output produced in period 0, and gets P . Following a sale, the downstream firm places an order for a unit of input, and produces a unit in period 2. Any customer arriving in period 2 does not get served, but might return next period. At the start of period 3, the downstream firm has 1 unit of output to sell.

In period 3, no customer arrives (see footnote 20) with probability is $(1 - p)(1 - p q)$, and the payoff is Y_0 . If at least one customer arrives, the situation is exactly like the state in period 1 in which a customer arrives and the subsequent payoff is Y_1 .²¹ It follows that

$$Y_1 = P - \delta_d C + \delta_d^2 ((1 - (1 - p)(1 - p q)) Y_1 + (1 - p)(1 - p q) Y_0).$$

²¹Any customer who does not get served in period 3 is lost completely since the downstream firm can only sell the next unit of output in period 5. Thus whether 1 or 2 customers arrive in period 3 makes no difference.

Using this as well as equations (A.13) and (A.14), we obtain the stated expression for V_0^{UW} .||

A.14 PROOF OF PROPOSITION 8

The proof proceeds through the following lemma.

Lemma 5 $V_0^{UI} - V_0^{UW}$ is strictly increasing in p .

Proof: Since $V_0^{UI} = V_0^I$, using the values from equations (3.1) and lemma 4, and simplifying, we get

$$\Delta V \equiv V_0^{UI} - V_0^{UW} = \frac{\delta_d p}{1 - \delta_d} \left[\frac{(P - C)}{1 + (1/f(p))} - \frac{C(1 - \delta_d)}{1 + f(p)} \right],$$

where $f(p) = p \delta_d (1 - \delta_d q) + p^2 \delta_d^2 q$. Note that $f'(p) > 0$. Thus the first term inside the square brackets on the right hand side of the above equation is strictly increasing in p , and the second term is strictly decreasing in p . Thus the term inside the square bracket is strictly increasing in p . The coefficient of this term is also strictly increasing in p . This proves the result.||

For p close to 0, $f(p)$ is close to 0, and $\Delta V < 0$. As $p \rightarrow 1$, $f(p) \rightarrow \delta_d$. Thus $\lim_{p \rightarrow 1} \Delta V = \frac{\delta_d}{1 - \delta_d^2} (\delta_d P - C) > 0$ where the inequality follows from the fact that for the downstream firm to invest at all we must have $\delta_d P > C$ (proposition 1). Combined with the lemma above, and the fact that ΔV is continuous in p , we conclude that there exists a unique $0 < p_c < 1$ such that $\Delta V = 0$ at $p = p_c$. Using the values of V_0^{UI} and V_0^{UW} ,

$$p_c = \frac{1 - \delta_d q}{2 \delta_d q} \left(\sqrt{1 + \frac{4q(1 - \delta_d)C}{(1 - \delta_d q)^2 (P - C)}} - 1 \right).$$

From lemma 5, for $p > p_c$, $V_0^{UI} > V_0^{UW}$.

From proposition 2, $V_0^I > V_0^W$ for $p > p_{ic}^D$. Using the values of p_c and p_{ic}^D , it is easy (but laborious) to see that

$$(p_{ic}^D)^2 - (p_c)^2 = \left(\frac{2q(1 - \delta_d)C}{(1 - \delta_d q)((P - C) - q(\delta_d P - C))} \right)^2 \left(\frac{q(\delta_d P - C)}{(P - C)} \right) > 0,$$

where the last step follows from the fact that the relevant case for downstream production to take place at all is $\delta_d P > C$. Since p_{ic}^D and p_c are both positive, the above shows that $p_{ic}^D > p_c$. It follows that for $p \geq p_{ic}^D$, $V_0^{UI} > V_0^{UW}$.||

A.15 PROOF OF PROPOSITION 9

Prepayment is made if the upstream firm has an incentive to wait without it, and both $U_0^{PP} \geq U_0^{UW}$ as well as $V_0^{PP} \geq V_0^{UW}$. Since a prepayment scheme can be represented by a total transfer of θC , $U_0^{PP} = U_0^{UI} + \theta C$, and $V_0^{PP} = V_0^{UI} - \theta C$.

Whenever $U_0^{UW} > U_0^{UI}$, let θ^* be such that upstream constraint binds $U_0^{UI} + \theta^* C = U_0^{UW}$. The downstream sector then participates in prepayment if $V_0^{UI} - \theta^* C \geq V_0^{UW}$. Let us check that $0 < \theta^* < 1$. For $U_0^{UW} > U_0^{UI}$, clearly $\theta^* > 0$. Further, from lemma 3, $U_0^{UW} < C + \delta_u p(\delta_u C - G)/(1 - \delta_u)$. Using the expression for U_0^{UI} from the same lemma, it follows that $U_0^{UW} - U_0^{UI} < G - \delta_u p C/(1 - \delta_u) < G$. Thus $\theta^* < G/C < 1$.

Let us now establish a few facts that we then use to prove the result. **Fact 1:** as $C \downarrow G/\delta_u$, $p_{ic}^U \uparrow 1$, and thus $p_{ic}^U = 1$ for all $C \leq G/\delta_u$. **Fact 2:** from section A.12, we have $\frac{\partial(U_0^{UW} - U_0^{UI})}{\partial p} < 0$.

Next, p_{pp}^D solves $V_0^{UI} - V_0^{UW} = \theta^* C$ where $\theta^* C = \max\{0, U_0^{UW} - U_0^{UI}\}$. As $p \rightarrow 1$, $U_0^{UW} - U_0^{UI} \rightarrow \frac{\delta_u C - G}{1 - \delta_u^2}$. Therefore, we have **Fact 3:** for $C \leq G/\delta_u$, as $p \rightarrow 1$, $\theta^* C \rightarrow 0$.

We now proceed with the proof. First, let us show that p_{pp}^D exists, is unique, and, for $C \leq G/\delta_u$, is bounded away from 1. Consider the equation $V_0^{UI} - V_0^{UW} = \theta^* C$. From section A.14, at $p = p_c$, the left hand side is 0. The right hand side is positive for $p < p_{ic}^U$. But, from fact 1, $p_{ic}^U = 1$ for $C \leq G/\delta_u$. Since $p_c < 1$, the right hand side is positive at $p = p_c$.

At $p = 1$, for $C \leq G/\delta_u$, the right hand side is 0 (fact 3), while the left hand side is positive and at its largest value, which follows from lemma 5 in section A.14. Finally, using the same lemma as well as fact 2, $V_0^{UI} - V_0^{UW} - \theta^* C$ is strictly increasing in p and this is also continuous in p . These imply that $p_{pp}^D \in (p_c, 1)$ exists, is unique, and is bounded away from 1. Therefore, using fact 1, we have the following:

$$\text{For } C \leq G/\delta_u, p_{pp}^D < p_{ic}^U. \quad (*)$$

Finally, it is easy to see from equation (3.5) that for $\delta_d P > C$, $p_{ic}^D < 1$. Now, condition (2.1) holds by assumption, and since $r_m = (1 - \delta_m)/\delta_m \geq (1 - \delta_d)/\delta_d$, we have $p(P - G) \geq (1 - \delta_d)/\delta_d$. Since the relevant case is $p < 1$ (for $p = 1$ no scope for prepayment arises), we have $(P - G) > (1 - \delta_d)/\delta_d$, which in turn implies $\delta_d P > G$. Therefore for C close

to G , we do have $\delta_d P > C$, and therefore $p_{ic}^D < 1$. Combining this with fact 1, we can conclude that:

$$\text{There is some } \widehat{C} \in (G, G/\delta_u] \text{ such that for } C \leq \widehat{C}, p_{ic}^D < p_{ic}^U. \quad (**)$$

From (*) and (**), for $C \leq \widehat{C} \leq G/\delta_u$, we have $\max\{p_{ic}^D, p_{pp}^D\} < p_{ic}^U$. In other words, there is $\Delta \leq \frac{1-\delta_u}{\delta_u}G$ such that for $C - G \leq \Delta$, the interval specified in the statement of the proposition is non-empty. On this interval the downstream firm produces immediately, and the upstream firm is induced also to produce immediately through prepayment. Therefore prepayment restores efficiency on this interval. ||

A.16 PROOF OF SUFFICIENCY OF CONDITION (2.1)

First, note that the total surplus from immediate production of both input and final good is always greater than that from immediate production of input but a wait-and-see strategy for producing the final good. This is because once the input is produced (and the cost G expended), there is no gain from waiting one or more periods to produce the final good. There are no further costs of producing the final good, and therefore no interest cost is saved by waiting. On the other hand, by waiting some final good sales are missed.

Therefore we simply need to compare the surplus from immediate production of both input and final good (denoted by S_0^I) to the surplus from a strategy of waiting to produce input and immediate production of final good (denoted by S_0^{UW}).

These surplus values can be derived simply by setting $C = G$ and adding the corresponding values for upstream and downstream firms: $S_0^I = V_0^I(G) + U_0^I(G)$, and $S_0^{UW} = V_0^{UW}(G) + U_0^{UW}(G)$. Let these be evaluated with respect to some interest rate r and discount factor $\delta = 1/(1+r)$. We get

$$\begin{aligned} S_0^I &= -G + \frac{\delta p(P-G)}{1-\delta}, \\ S_0^{UW} &= \frac{\delta p(P-G)}{(1-\delta)(1+\delta p(1-\delta q(1-p)))}. \end{aligned}$$

Now, immediate production of both input and final good is efficient if $S_0^I \geq S_0^{UW}$. This is true if

$$\frac{P-G}{G} \geq \frac{(1-\delta)(Z+1)}{p\delta(Z+2)},$$

where $Z \equiv p\delta(1 - (1 - p)\delta)$. Note that $(1 - \delta)/\delta = r$. Further, since $Z \leq 1$, $(Z + 1)/(Z + 2) \leq 2/3 < 1$. Therefore a sufficient condition to ensure $S_0^I \geq S_0^{UW}$ is $\frac{P-G}{G} \geq \frac{r}{p}$. Since it must be that the appropriate interest rate $r \leq \max\{r_u, r_d\} \equiv r_m$, it is sufficient to have $\frac{P-G}{G} \geq \frac{r_m}{p}$ which is the stated condition. ||

12 APPENDIX B

We argued in section 6 that if the upstream firm earns a margin $(C - G)$ that is high (low) relative to the downstream margin $(P - C)$, the upstream (downstream) firm is also likely to face easier credit terms. In this section we sketch an argument to show how this might arise.

We assume that a bank knows the investment environment (i.e. knows the values of p, q, P, C, G) but cannot observe each act of sale of input or output or provision of inter-firm credit. The bank lends over a certain fixed term that encompasses several production and sales opportunities. Let n denote the number of potential sales periods inside the lending period - i.e. if both firms produced immediately and if a customer arrived in each period there would be n sales over the term of the bank loan.

For simplicity assume that both firms have an incentive to produce immediately. If this is not the case, the algebra below would be more complicated, but it would not change the nature of the calculations.

The number of sales k in n periods follows a Binomial distribution with success probability p . Let $f(k) \equiv \binom{n}{k} p^k (1 - p)^{n-k}$. Suppose the competitive rate of interest is r . For a loan of L the competitive bank must earn an expected return of $(1 + r)L$. Therefore the break-even condition is

$$\sum_{k=0}^{K^*} f(k)k(P - C) + \left(1 - \sum_{k=0}^{K^*} f(k)\right) L(1 + r_d) = L(1 + r), \quad (\text{B.1})$$

where r_d is the rate charged by the bank to the downstream firm and where K^* is such that $K^*(P - C) \leq L(1 + r_d) \leq (K^* + 1)(P - C)$.

The left hand side of (B.1) can be written as $\sum_{k=0}^n f(k) \min\{k(P - C), L(1 + r_d)\}$. Since for some values of $k \geq 0$, $k(P - C) < L(1 + r_d)$, it must be that $r_d > r$. Further, as $(P - C)$

falls, $\min(k(P - C), L(1 + r_d))$ falls weakly for all k and strictly for some k . It follows that the left hand side of (B.1) falls as $(P - C)$ falls. Thus to satisfy (B.1), r_d must rise.

A similar calculation determines r_u (in condition (B.1), replace $(P - C)$ by $(C - G)$ and r_d by r_u). Since r_d decreases in $(P - C)$, it is also immediate that $r_d \geq r_u$ as $(C - G) \geq (P - C)$.

REFERENCES

- Blinder, Alan S. and Louis J. Maccini**, "Taking Stock: A Critical Assessment of Recent Research on Inventories," *Journal of Economic Perspectives*, 1991, 5, 73–96. 22
- Bougheas, Spiros, Simona Mateut, and Paul Mizen**, "Corporate trade credit and inventories: New evidence of a trade-off from accounts payable and receivable," *Journal of Banking and Finance*, 2009, 33, 300–307. 7
- Brechling, Frank P.R. and Richard G. Lipsey**, "Trade Credit and Monetary Policy," *Economic Journal*, 1963, 73, 618–641. 1
- Brennan, Michael J., Vojislav Maksimovic, and Josef Zechner**, "Vendor Financing," *Journal of Finance*, 1988, 43(5), 1127–1141. 6
- Burkart, Mike and Tore Ellingsen**, "In-Kind Finance: A Theory of Trade Credit," *American Economic Review*, 2004, 94(3), 569–590. 2, 3, 6
- , —, and Mariassunta Giannetti**, "What You Sell Is What You Lend? Explaining Trade Credit Contracts," *Review of Financial Studies*, 2008, Forthcoming. 1, 3, 5, 6, 7, 21, 22
- Ferris, J. Stephen**, "A Transactions Theory of Trade Credit Use," *Quarterly Journal of Economics*, 1981, 94(2), 243–270. 2, 6
- Joskow, Paul**, "Vertical Integration," in C. Menard and M. M. Shirley, eds., *Handbook of New Institutional Economics*, Springer, 2005, chapter 13, pp. 319–348. 7
- Maccini, Louis J., Bartholomew Moore, and Huntley Schaller**, "The Interest Rate, Learning, and Inventory Investment," *American Economic Review*, 2004, 94(5), 1303–1327. 23
- Meltzer, Allan H.**, "Mercantile Credit, Monetary Policy, and Size of Firms," *Review of Economics and Statistics*, 1960, 42(4), 429–437. 1
- Nadiri, Ishaq M.**, "The Determinants of Trade Credit in the U.S. Total Manufacturing Sector," *Econometrica*, 1969, 37(3), 408–423. 6
- Ng, Chee K., Janet Kiholm Smith, and Richard L. Smith**, "Evidence on the Determinants of Credit Terms in Interfirm Trade," *Journal of Finance*, 1999, 54(3), 1109–1129. 1, 4

- Nilsen, Jeffrey H.**, "Trade Credit and the Bank Lending Channel," *Journal of Money, Credit and Banking*, 2002, 34(1), 226–253. 1
- Petersen, Mitchell and Raghuram Rajan**, "Trade Credit: Theories and Evidence," *Review of Financial Studies*, 1997, 10(3), 661–691. 5, 21
- Schwartz, Robert A.**, "An Economic Model of Trade Credit," *Journal of Financial and Quantitative Analysis*, 1974, 9 (4), 643–657. 2
- Schwartz, Robert A. and David K. Whitcomb**, "The Trade Credit Decision," in James L. Bicksler, ed., *Handbook of Financial Economics*, North-Holland, 1979, pp. 257–273. 6
- Uchida, Hirofumi, Gregory F. Udell, and Wako Watanabe**, "Are Trade Creditors Relationship Lenders?," 2006. working paper. 5