An analysis of the lifetime of OLSR networks

Jan-Maarten Verbree [a], Maurits de Graaf [b,*], Johann Hurink [a]

[a] The University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands
[b] Thales Nederland B.V., Division Land and Joint Systems, P.O. Box 88, 1270 AB Huizen, The Netherlands

ARTICLE INFO

ABSTRACT

The Optimized Link State Routing (OLSR) protocol is a well-known route discovery protocol for ad hoc networks. OLSR optimizes the flooding of link state information through the network using multipoint relays (MPRs). Only nodes selected as MPRs are responsible for forwarding control traffic. Many research papers aim to optimize the selection of MPRs with a specific purpose in mind: e.g., to minimize their number, to keep paths with high Quality of Service or to maximize the network lifetime (the time until the first node runs out of energy). In such analyzes often the effects of the network structure on the MPR selection are not taken into account. In this paper we show that the structure of the network can have a large impact on the MPR selection. In highly regular structures (such as grids) there is even no variation in the MPR sets that result from various MPR selection mechanisms. Furthermore, we study the influence of the network structure on the network lifetime problem in a setting where at regular intervals messages are broadcasted using MPRs. We introduce the 'maximum forcedness ratio', as a key parameter of the network to describe how much variation there is in the lifetime results of various MPR selection heuristics. Although we focus our attention to OLSR, a widely implemented protocol, on a more abstract level our results describe the structure of connected sets dominating the 2-hop neighborhood of a node.

© 2009 Elsevier B.V. All rights reserved.

1. Introduction

The Optimized Link State Routing (OLSR) protocol is a well-known and often implemented MANET (Mobile Ad Hoc Network) route discovery protocol. OLSR optimizes the flooding of link state information through the network by using multipoint relays (MPRs). Only nodes selected as MPRs are responsible for forwarding control traffic. Besides the mechanism for flooding *control* traffic like OLSR, there exist also flooding mechanisms for *data traffic*, like the Simplified Multicast Forwarding (SMF) protocol [6] that can work together with MPR selection algorithms (like the one in OLSR). An improved version of SMF is BMF [15]. Both mechanisms flood IP Multicast traffic over an OLSR network, where the MPRs identified by OLSR are used to optimize the flooding.

In this paper we present a graph related study investigating the impact of the network structure on the MPR selection. A main question in this context is how to measure the impact of the structure of the graph on the MPR selection. We show that for highly regular network structures (such as e.g. grids) MPR selection algorithms only can have a marginal influence on the choice of MPR nodes since many nodes are fixed as MPR nodes in the sense that every MPR selection algorithm has to choose these nodes. To demonstrate the influence of the fixed nodes, we consider the maximization of the network lifetime (the time at which the first communication fails due to depletion of battery-resources). We introduce a key parameter of networks (maximum forcedness ratio) and show how the network lifetime under different MPR selection algorithms

* Corresponding author. Tel.: +31 35 5248 149.
  E-mail addresses: maurits.degraaf@nl.thalesgroup.com (M. de Graaf), J.L.Hurink@utwente.nl (J. Hurink).

depends on this parameter. Although we focus our attention to OLSR, being a widely implemented protocol, on a more abstract level our results describe the structure of connected dominating sets covering the 2-hop neighborhood of a node.

This paper consists of two main parts. In Section 3 we provide structural results on MPR-sets that are independent of the selection algorithm that is used. In Sections 4 and 5 we apply this theory to grid graphs showing that for 'central nodes' in grid graphs all MPR selection algorithms yield the same sets. In Section 6 the network lifetime is addressed, showing that a new graph parameter, the 'maximum forcedness ratio' is strongly related to the degree in which the structure of the graph allows improvement of the network lifetime by a better MPR selection heuristic. For graphs with a maximum forcedness ratio close to 1, the concrete MPR selection heuristic has little impact on the resulting network lifetime.

2. Related work

2.1. MPR selection

The classical MPR selection problem is to find for a given node a set of MPRs of minimum size that covers the whole 2-hop neighborhood. Selecting the MPR-set of minimal cardinality has been proven to be NP-complete [14,17].

In practice, heuristic algorithms are used to select MPR-sets. To set up MPR-sets in a network, different MPR selection algorithms exist. In these algorithms each node (the selector nodes) independently chooses its MPR-set. These sets then act as relay nodes for messages sent by the selector node, and, thus can organize the broadcast communication in a network. The existing approaches mostly aim to optimize the selection of MPRs with a specific purpose in mind: e.g., to minimize their number (as was the objective in the original specification [5]), or to improve QoS (see [1]). In [12] other purposes are presented: to reduce the number of collisions, minimize the overlap between MPRs or maximize the global bandwidth.

The heuristics mentioned above have a structure that can be divided into three steps and use an incremental approach to compute an MPR-set. The first step always consists of selecting neighboring nodes as MPR that cover nodes in the 2-hop neighborhood that cannot be covered by other neighboring nodes. The second step extends this set in order to ensure that the complete 2-hop neighborhood is covered, and in the last step it is investigated if some of the current selected nodes can be dropped without violating the requested properties of an MPR-set. In [2] an interesting probabilistic analysis of the influence of the first step is given. The authors conclude that almost 75% of the relay nodes are selected by the first step of the heuristics. In this paper we show that for a specific class of graphs *all* MPRs are selected in the first step of the algorithm. In [8] MPR selection algorithms in a specific probabilistic setting are analyzed. In this setting the edges in the graph have a weight, which represents the probability of successful transmission over that edge. For this probabilistic edge model the MPR

selection heuristics are more complicated than the three step model.

2.2. Network lifetime problem

The network lifetime is an important parameter for battery-operated networks. Examples of such networks are personal area networks that are used in emergency situations. Such networks are deployed in regions where it is impractical to recharge/replace the battery of a node. This limited battery capacity of nodes participating in a MANET is a topic of a wide variety of literature on problems related to energy-efficiency. Many algorithms have been developed addressing the Network Lifetime Problem in general networks. From them, the following approaches are closely related to the topic of this paper:

(a) *Maximization of network lifetime for broadcast traffic.* Kang and Poovendran [9] present an algorithm that maximizes the static network lifetime. Low and Goh [11] consider the problem of maximizing the minimum residual energy that remains after a broadcast transmission from a source. Park and Sahni [13] present an alternative heuristic for determining a tree with maximum 'critical energy' (minimum residual energy). These references form a small collection of approaches in this area. Note, that all approaches above assume that the transmission originates from a single source and that none of the approaches provides a specific discussion of the impact on MPR selection.

(b) *Minimization of total energy consumption for broadcast traffic.* The problem of minimizing the total energy consumption for broadcast has been widely studied. The relation with the lifetime problem is that each broadcast reduces the sum of all battery capacities in the network with the total energy required for that broadcast. Cagalj et al. [3] and Liang [10] have proven independently that the minimum-energy broadcast problem with the objective of minimizing the total transmitted power is NP-hard. One of the first algorithms on broadcasting in wireless network with usage of the wireless multicast advantage is the Broadcast Incremental Power algorithm (BIP) [19], with its variants [20].

(c) *Extension of network lifetime by topology control.* The idea behind topology control is to reduce the number of connections in a network, to get a subnetwork with some given desired properties. This reduction can be realized by lowering the transmission power at certain nodes. The main issue is to find a topology with less connections and consequently less transmit power. The distributed algorithm XTC [18] is an algorithm that provides such a reduction. Calinescu et al. [4] study an approach where the lifetime of the network is maximized taking into account the energy cost to maintain the topology. Closest to the problem studied in this paper is [7], where adjustments are made to the MPR selection algorithm to increase the network lifetime.

3. The structure of MPR-sets: forced sets and fixed nodes

In this section we derive properties of multipoint relay (MPR) sets. A MPR-set of a node is defined as a subset of its neighbors which cover the complete 2-hop neighborhood of that node, i.e., if all vertices of an MPR-set of a vertex v forward a message received by v, the complete 2-hop neighborhood of v receives that message.

More formally, let $G = (V, E)$ be a connected graph (throughout this paper we assume bi-directional links), and let $N^k(u)$ denote the strict k-hop neighborhood of u, i.e., the set of nodes for which the shortest path to u has exactly k edges. A subset $M(u) \subset N^1(u)$ is called an MPR-set if $M(u)$ dominates $N^2(u)$, i.e., each node in $N^2(u)$ has a neighbor in $M(u)$. Furthermore, for a given MPR-set $M(u)$, we call nodes from this set an MPR of node u. Finally, we denote the set of all possible MPR-sets of u by $MPR(u)$.

To avoid circulating messages, MPRs only react on the first instance of a message. If this first instance is received by a neighboring node for which the given node is an MPR, the message is retransmitted, otherwise it is ignored. Further instances of the same message are ignored independently of the sender of this message. This is called 'duplicate message detection'. To be able to implement this process, every node maintains a duplicate set, in which all received messages are listed. This set is used to check if an incoming message already has been processed. Consequently, the following is possible: (1) a node receives a message from a node for which it is not an MPR and (2) later it receives the same message from a node for which it is an MPR. Both messages will not be retransmitted: in case (1) because the node was not an MPR, in case (2) because the message is in the duplicate set. However, it is still easy to see that broadcasting via MPR's in the above sense reaches all possible nodes in the network.

The existing MPR selection algorithms differ in the selection process of the sets $M(u)$, $u \in V$. In this section, we are interested in the subset of nodes of a neighborhood $N^1(u)$ which belong to every possible MPR-set of u. We denote this set by $F^1(u)$ and call it the forced set of node u. Note, that nodes in $F^1(u)$ are chosen as MPR for node u by every MPR selection algorithm. We also introduce the inverse notion of a forced MPR-set. For a given node $u \in V$, the set of nodes that force u to be MPR, is defined as $F^{-1}(u) = \{v | u \in F^1(v)\}$. Clearly, both definitions are related by: $v \in F^1(u) \iff u \in F^{-1}(v)$. In the remainder of this paper, when it is not of any interest to the situation at hand, we simply state that v is an MPR and omit the name of the selector node.

The following lemma gives a characterization of $F^{-1}(u)$ in terms of properties of the graph G.

Lemma 1. $v \in F^{-1}(u)$ if and only if there exists a node $v^* \in N(u)$ such that there is a unique 2-hop path from v to v^*, being the path $v - u - v^*$. In this case, node v^* belongs also to the set $F^{-1}(u)$.

Proof. ($\Leftarrow$) Suppose that there exists a node $v^* \in N(u)$ such that there is a unique 2-hop path from v to v^*, being the path $v - u - v^*$. Then any subset of $N^1(v)$ that dominates $N^2(v)$ must contain u in order to dominate v^*. So $v \in F^{-1}(u)$, and by symmetry also $v^* \in F^{-1}(u)$.

($\Rightarrow$) Let $v \in F^{-1}(u)$ and suppose that every node $v^* \in N^2(v)$ can be reached via a 2-hop path $v - w - v^*$ with $w \neq u$. In this case $N^1(v) \setminus u$ is a possible MPR-set of v, which contradicts the fact that u is forced to be MPR. Consequently, there exists a $v^* \in N^2(v)$ for which the only 2-hop path between v and v^* is $v - u - v^*$. Since we have bi-directional links, $v^* \in N^2(v)$ means that also $v \in N^2(v^*)$. If we combine this with the fact that u is the only node that connects to both v and v^*, u has to be MPR for v^* in each possible MPR selection. Thus, $v^* \in F^{-1}(u)$. $\square$

An extreme case occurs, when all neighbors of a node u force u to be MPR, i.e., $F^{-1}(u) = N(u)$. In such a case we call u a *fixed MPR*. An example of a fixed MPR is e.g. the center node in a star topology. In Fig. 1 and Table 1 we present an example graph with its forced and fixed nodes. In this example there is only one fixed node, namely node u_4.

Fixed nodes have an important impact on the network lifetime. The following proposition states that fixed nodes provide an upper bound to the network lifetime independent of which nodes initiate the broadcasts. In the setting of this paper, we assume that different broadcasts do not interfere in time (a new broadcasts does not start before the previous is finished). Therefore, the network lifetime can be expressed in the number of messages that can been broadcasted until the first node runs out of energy.

To formulate the proposition, we introduce the notion of the Network Lifetime $NLT(G)$ of a graph G. This value denotes the maximum number of messages which can be broadcasted within the network represented by G, if MPR flooding is used. Note, that in this definition the nodes which broadcast the messages may be chosen in such a way that a maximal lifetime is achieved.

Proposition 2. Let $G = (V, E)$ be a connected graph with a set $S \neq \emptyset$ of fixed MPRs. Furthermore, let the initial battery capacity of a node u be denoted by $E(u)$ and the battery cost per transmission of a message in node $u \in V$ be $C(u)$. Then,
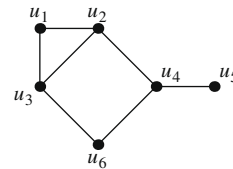

Fig. 1. An example network with forced and fixed nodes.

Table 1
$F^1(u)$ and $F^{-1}(u)$ for the example network in Fig. 1.

Node u	$F^1(u)$	$F^{-1}(u)$
u_1	$\{u_2, u_3\}$	$\emptyset$
u_2	$\{u_4\}$	$\{u_1, u_4\}$
u_3	$\emptyset$	$\{u_6\}$
u_4	$\{u_2\}$	$\{u_2, u_5, u_6\}$
u_5	$\{u_4\}$	$\emptyset$
u_6	$\{u_3, u_4\}$	$\emptyset$

$$NLT(G) \leqslant \min_{u \in S} \left\lfloor \frac{E(u)}{C(u)} \right\rfloor. \tag{1}$$

Proof. Let u be a fixed MPR in a connected network, so u is MPR for every neighbor. We claim that u transmits each broadcast message exactly once. To see this, we distinguish two cases. If u is the source, it obviously transmits the message once. If u is not the source, the message reaches node u via one of its neighbors. Since u is fixed MPR, the first message that arrives at u is being relayed and the (possibly) next duplicate messages are ignored. So, every broadcast message reduces the battery of a node $u \in S$ exactly once with $C(u)$. This immediately gives the bound stated in the proposition. $\square$

Note, that we need the fact that u is MPR for all its neighbors to ensure that it relays all messages it receives. If node u would not be MPR for some neighbor v and the first message that u receives comes from v, then u would not relay this message due to the duplicate message detection property.

The previous proposition provides an upper bound to the network lifetime. In general, the lifetime of the network may be even smaller, if a non-fixed MPR, say w, exists with a low ratio $\frac{E(w)}{C(w)}$. However, if we assume that this is not the case, the given bound is tight as we show below.

Corollary 3. Let $G = (V, E)$ be a connected graph with a set $S \neq \emptyset$ of fixed MPRs. If

$$\min_{w \in V} \left\lfloor \frac{E(w)}{C(w)} \right\rfloor \geqslant \min_{u \in S} \left\lfloor \frac{E(u)}{C(u)} \right\rfloor, \tag{2}$$

then $NLT(G) = \min_{u \in S} \left\lfloor \frac{E(u)}{C(u)} \right\rfloor$.

Proof. By the duplicate message detection property, a single message will reduce the battery capacity of a node w by at most $C(w)$. Therefore node w cannot run out of battery before $\frac{E(w)}{C(w)}$ broadcasts. Thus, the inequality (2) guarantees that the bound in Proposition 2 is tight. $\square$

4. The structure of MPR-sets in grid graphs

In this section, we apply the results of the previous section to grid structures. Grid structures are characterized by their regular structure and turn out to have restricted possibilities to vary their MPR-sets. Even though grid graphs in their pure form hardly occur in practical settings, a lot of real networks have grid-like structures or sub-structures and, as a consequence, the results derived for pure grid graphs occur in some 'weaker' sense also in these networks.

Formally, we denote by $G_{m \times n}(r)$ a graph with $m \cdot n$ nodes on grid points of a $m \times n$ grid. We assume the horizontal and vertical distance between neighboring grid points to be 1. Furthermore, two nodes u and v are connected by an edge in $G_{m \times n}(r)$ if and only if $d(u, v) \leqslant r$, where $d(u, v)$ denotes the Euclidean distance between the nodes u and v. Note, that n and m determine the vertex set and r determines the edge set of the graph $G_{m \times n}(r)$. As examples (see Fig. 2) we consider the graphs for $r = 1$ and $r = \sqrt{2}$. $G_{m \times n}(1)$ has only horizontal and vertical edges and $G_{m \times n}(\sqrt{2})$ has also edges between diagonal neighboring grid points. Note, that $G_{m \times n}(1)$ is what normally is considered as a grid graph.

The grid structure of a graph induces two important properties, which are the basic elements for the proof of the theorem presented below:

- Translation property.

  If u, v are grid points and $\vec{a}$ is a vector such that $u + \vec{a}$ is a grid point, then $v + \vec{a}$ is also a grid point.

- Symmetry property.

  If v is a grid point, then the point obtained by mirroring v through another grid point is again a grid point.

Note, that for the two mentioned properties we assume that the grid structure is large enough; i.e. that the translated or mirrored point is still within the grid. Also for the following results, we are not interested in the specific issues at the border of the graph, but we concentrate on central nodes. A node $v \in G_{m \times n}(r)$ is called a *j-hop central node*, with $j \in \{1, 2\}$, if v is located in the $(m - 2j\lfloor r \rfloor) \times (n - 2j\lfloor r \rfloor)$ subgrid that is created by removing $j\lfloor r \rfloor$ grid rows from the upper and lower side of the grid and $j\lfloor r \rfloor$ grid columns from the left and right side of the grid. Nodes in $G_{m \times n}(r)$ that are not *j-hop central nodes*, are called *border nodes*. Note that the *j-hop neighborhood of a *j-hop central node v is point symmetrical in $v, j = 1, 2$.

Using the above terminology, we now apply the results of the previous section to grid graphs. The following theo-
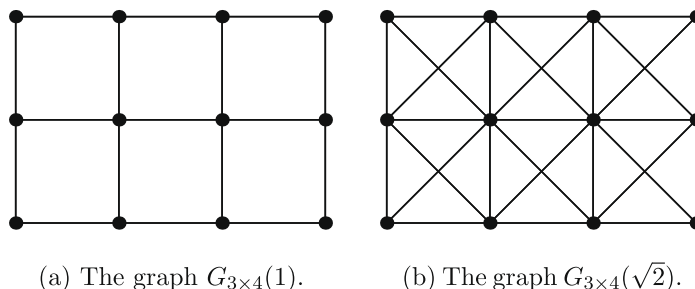

(a) The graph $G_{3 \times 4}(1)$.   (b) The graph $G_{3 \times 4}(\sqrt{2})$.

Fig. 2. Two graphs on grids.

rem states the rather surprising fact that, for a 2-hop central node u in a grid graph we have $F^{-1}(u) = F^1(u)$ and that $F^1(u)$ is an MPR-set of u. As can be seen from Fig. 1, this is in general not true.

To keep the paper concise, we only present a sketch of the proof of Theorem 4. A full proof can be found in [16].

Theorem 4. *For every 2-hop central node $u \in G_{m \times n}(r)$ with $r \geqslant 1$, the set $F^{-1}(u)$ is an MPR-set of u, i.e. $F^{-1}(u) \in MPR(u)$.*

Proof. (*Sketch*). Let u be a 2-hop central node in $G_{m \times n}(r)$ with $r \geqslant 1$.

(1) Then $v \in F^{-1}(u)$ if and only if the mirror image $\bar{v}^u$ (obtained by point mirroring v in u) is the only node in $N(u)$ with a unique 2-hop path to v.

(2) Using this, it follows from the translation property that $v \in F^{-1}(u)$ if and only if $u \in F^{-1}(v)$, hence $F^{-1}(u) = F^1(u)$.

(3) Next a geometrical characterization of $F^{-1}(u)$ is given: $v \in N(u)$ is in $F^{-1}(u)$ if and only if v is an extreme point of the convex hull $C(u)$ of $N(u)$; i.e. the smallest convex space in which all nodes in $N(u)$ are located (a point $x \in C$ is called an *extreme point* if it is not an interior point of any line segment in C).

(4) Let u be a 2-hop central node in $G_{m \times n}(r)$ with $r \geqslant 1$. Then for every node $w \in N^2(u)$ it holds that $w \in C_2(u)$. Here, $C_2(u)$ is the convex hull multiplied by a factor of 2: $C_2(u) = \{2x \| x \in C(u)\}$.

(5) The proof is completed by showing: all grid points in $C_2(u)$ are the grid points defined by $\bigcup_{f \in F^{-1}(u)} N(f)$. This can be proven by a simple geometric argument. □

5. MPR selection algorithms

The analysis in the previous section forms the fundament for our main result on MPR selection algorithms for grid graphs. Before presenting the theorem we discuss the structure of MPR selection algorithms. MPR selection algorithms are localized algorithms, where each node $u \in V$ selects an MPR-set $M(u)$, independently from the other nodes. Most MPR selection algorithms use the following structure to calculate an MPR-set $M(u)$ for node u:

(1) Start with an empty MPR-set of node u, and add nodes of $N(u)$ that are the only neighbor of a node in $N^2(u)$. So, after this step $M(u) = F^1(u)$.

(2) While there are still uncovered nodes in $N^2(u)$, select the nodes from $N(u)$ that cover at least one uncovered node and provide the highest revenue (the definition of revenue depends on the MPR selection algorithm).

(3) Optimize the MPR-set by attempting to remove a node from $M(u)$ and checking if $N^2(u)$ is still dominated. If this is the case, the node is removed from $M(u)$. Nodes are removed in the order 'lowest revenue first'.

For every MPR selection algorithm, the MPR-set of node u contains at least the forced set of node u. Consequently, Step 1 of the three-step algorithmic procedure has to be part of each MPR selection algorithm. A procedure to optimize the selected MPR-set, as in Step 3, can also be expected to be part of an MPR selection algorithm. However, there is some variation possible in the order the nodes are considered for discarding. Between Step 1, and Step 3, there has to be a Step 2, that selects non-forced MPRs according to some optimization criteria. These criteria vary per MPR selection algorithm and, therefore, this step characterizes the MPR selection algorithm. For example, instead of adding nodes with maximum coverage, one can add nodes from $N(u)$ in the order 'most energy first'.

Based on the above considerations, we present our main theorem of the study on MPR selection in graphs $G_{m \times n}(r)$. It states that the MPR-set for a 2-hop central node in a grid graph is equal to its forced set.

Theorem 5. *For every 2-hop central node $u \in G_{m \times n}(r)$ with $r \geqslant 1$, the set $F^1(u)$ is selected as MPR-set of node u if a three step MPR selection algorithm is used.*

Proof. Since $F^1(u) = F^{-1}(u)$ (see the proof of Theorem 4) and since $F^{-1}(u)$ dominates $N^2(u)$ (Theorem 4), after Step 1 the set $M(u)$ dominates $N^2(u)$. Thus, Step 2 of the general three step MPR selection algorithm is never processed. (Step 2 of the algorithm is only processed if there are uncovered nodes in $N^2(u)$). So there are no nodes added to the MPR-set and therefore $F^1(u)$ is selected as MPR-set of u. Step 3 cannot remove any of the nodes in the MPR-set as they are all forced to be MPR. □

This theorem implies that in the case of graphs of type $G_{m \times n}(r)$, MPR selection algorithms do not influence the lifetime of 2-hop central nodes. Since the above results mainly are a consequence of the regular structure (translation, symmetry) of the graphs, we may expect similar results for all graphs with regular (sub) structures.

6. Network lifetime simulations

In this section we describe simulation results for MPR flooding. First, we describe how the theoretical results derived for grid graphs are supported by simulations. Afterwards, we complement the analysis on grid graphs by concentrating on random graphs. For the simulations we use three MPR selection algorithms: MinCar, MaxWill and MaxWillMinForced. These algorithms are described in Table 2. (For the 'revenue' we refer back to Section 5.)

6.1. Grid graphs

We performed simulations to verify the results for grid graphs presented in this paper. The simulations we present here consider grid graphs $G_{m \times m}(r)$ with $m = 10, \ldots, 15$ and $r = 1, 2, 3$ on the torus. In [16] more simulations are described, including simulations on grid graphs on the plane. As before, we assume the broadcasts do not interfere in time, so that the network lifetime can be expressed as the number of broadcasts. Each node starts with an amount

Table 2
The MPR selection algorithms considered in this paper.

MinCar	Revenue is defined as 'the number of uncovered nodes from $N^2(u)$ that are covered'. This leads to the well-known algorithm minimizing the cardinality of $MPR(u)$.
MaxWill	Revenue is defined as 'willingness'. OLSR has eight values available for the willingness (from 0 ("will never") to 7 ("will always")). This MPR selection algorithm selects first the nodes with the highest willingness. When willingness indicates the remaining energy of the node, this algorithm attempts to maximize the minimum energy of the network by saving energy of nodes with a low remaining energy. In this paper we assume 'willingness' to be equal to the residual energy, to avoid rounding effects.
MaxWillMinForced	Revenue is defined as $s(v) = E(v)(1 - f(v))$. The term $E(v)$ denotes the residual energy of a node v. This additional element aims to further improve the network lifetime. It provides a look-ahead on the residual energy in the future, as it describes the expectation that a node will consume much energy as it is forced to be MPR by many nodes.

Table 3
The network lifetime realized by MPR flooding with the different MPR selection algorithms for the torus variant of the grid graphs. The results are presented as a 3-tuple (x_1, x_2, x_3) where x_1 denotes the network lifetime according to algorithm MinCar, x_2 according to MaxWill and x_3 according to MaxWillMinForced.

Range	Grid size					
	10×10	11×11	12×12	13×13	14×14	15×15
1	(20,20,20)	(20,20,20)	(20,20,20)	(20,20,20)	(20,20,20)	(20,20,20)
2	(80,80,80)	(36,36,36)	(76,76,76)	(50,50,50)	(68,68,68)	(39,39,39)
3	(77,87,87)	(49,45,45)	(45,36,36)	(43,43,43)	(49,49,49)	(43,43,43)

of energy sufficient for 20 broadcasts. For each algorithm the same, randomly chosen, sendpattern is used. For the MPR selection algorithms: MinCar, MaxWill and MaxWillMinForced the network lifetime is presented in Table 3.

The results of the simulations in Table 3 and the additional simulations in [16] show that: (1) on the torus, for grid graphs $G_{m \times m}(r)$ with r 'small' with respect to $m(r < m/4)$ (corresponding to the 2-hop central node hypothesis in Theorem 4), the three considered MPR selection algorithms give exactly the same lifetimes. (2) When

$r = 1$ for both torus and plane all algorithms yield exactly the same lifetime. (3) On the plane (see [16]), for grid graphs $G_{m \times m}(r)$, with $r > 1$ some border effects occur, resulting in different network lifetimes for the three algorithms.

These observations coincide with the theoretical considerations in Theorem 4. Results (1) and (2) directly follow, where result (3) can be explained from the fact that not all nodes are 2-hop central nodes, and therefore the algorithms have some freedom in choice.
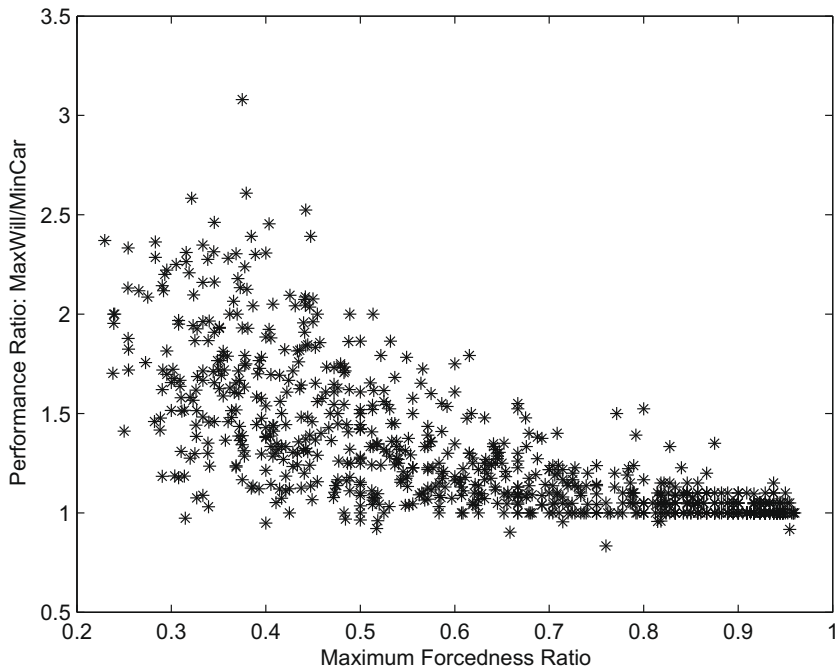

Fig. 3. The effect of the Maximum Forcedness Ratio on the performance differences using MPR flooding with MPRs selected by the MaxWill algorithm and the MinCar algorithm.

6.2. Random graphs

In Section 3 we have shown that fixed nodes provide an upper bound to the network lifetime, independent of the MPR selection algorithms. We therefore are interested in the impact of 'almost' fixed nodes on the network lifetime performances of different MPR selection algorithms. To that order, we introduce the *Forcedness Ratio* to define 'how fixed' a node is. For a node u the Forcedness Ratio $f(u)$ is defined as the fraction of nodes in $N^1(u)$ that is forced to choose u as MPR,

$$f(u) = \frac{|F^{-1}(u)|}{|N^1(u)|}.$$

Obviously, we have $0 \leqslant f(u) \leqslant 1$ and $f(u) = 1$ if and only if node u is fixed. If $f(u)$ is close to 1, node u is forced to be MPR by many of its neighbors. We define $MFR := \max_{u \in V} f(u)$. Evidently, $MFR = 1$ if there is a fixed node in the network. Consequently, when MFR is close to 1, we expect the MPR selection algorithms too have little influence on the network lifetime.

To analyze the relation between the MFR and the network lifetime realized by different MPR selection algo-

rithms, we generated a set of networks with a wide range of MFRs. Based on preliminary simulations, we choose to place 150 nodes in a square field of 1000×1000 units, while selecting for each simulation a transmission range from the set $[200, 250, 300, 350]$ and assigning this range to all nodes. For every transmission range we created 200 networks, resulting in 800 networks in total.

In each of these networks we initiated broadcast messages according to the same sendpattern. The messages are broadcasted by MPR flooding, where the MPRs are selected by the different algorithms. We discuss them separately.

6.2.1. MaxWill versus MinCar
The results of the simulations are presented in Fig. 3. The Performance Ratio in this graph is defined as the network lifetime using MaxWill divided by the network lifetime using MinCar.

Analyzing the graph, we see that when the Maximum Forcedness Ratio approaches 1 the difference in network lifetimes become smaller. This can also be concluded from Table 4 in which the mean and the standard deviation of the Performance Ratio is listed per intervals of

Table 4
The mean and standard deviation of the Performance Ratio concerning MaxWill and MinCar per interval of the Maximum Forcedness Ratios.

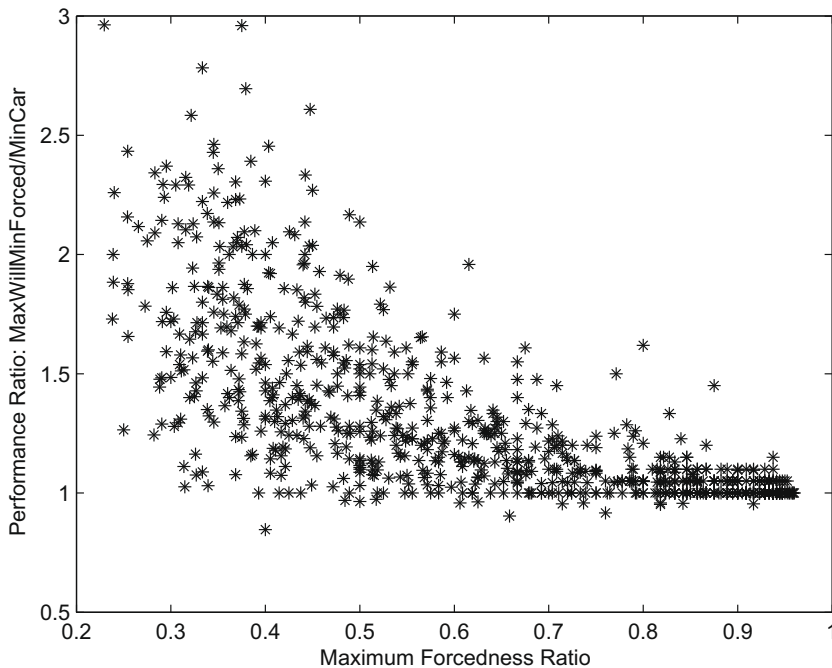|	Maximum Forcedness Ratio intervals							
	$[0.2, 0.3)$	$[0.3, 0.4)$	$[0.4, 0.5)$	$[0.5, 0.6)$	$[0.6, 0.7)$	$[0.7, 0.8)$	$[0.8, 0.9)$	$[0.9, 1)$
Mean	1.89	1.71	1.52	1.29	1.19	1.10	1.05	1.02
SD	0.32	0.39	0.33	0.22	0.15	0.11	0.06	0.03

Fig. 4. The effect of the Maximum Forcedness Ratio on the performance differences using MPR flooding with MPRs selected by the MaxWillMinForced algorithm and the MinCar algorithm.

Table 5
The mean and standard deviation of the Performance Ratio concerning MaxWillMinForced and MinCar per interval of the Maximum Forcedness Ratios.

	Maximum Forcedness Ratio intervals							
	$[0.2, 0.3)$	$[0.3, 0.4)$	$[0.4, 0.5)$	$[0.5, 0.6)$	$[0.6, 0.7)$	$[0.7, 0.8)$	$[0.8, 0.9)$	$[0.9, 1)$
Mean	1.91	1.74	1.51	1.29	1.18	1.10	1.05	1.01
SD	0.41	0.40	0.34	0.22	0.17	0.11	0.07	0.03

the Maximum Forcedness Ratio. The results support the expected effect that the 'more fixed' the 'most fixed' node is, the smaller the performance differences are. If the Maximum Forcedness Ratio is smaller, also bigger performance differences occur. The graph also shows that in almost every simulation the network lifetime of MPR flooding using MaxWill is larger than the network lifetime of MPR flooding using MinCar.

6.2.2. MaxWillMinForced versus MinCar

The relation between the performances of MPR flooding using MaxWillMinForced and MinCar to select MPRs are shown in Fig. 4 and Table 5. In the graph the Performance Ratio is defined as the network lifetime using the MaxWillMinForced MPR selection algorithm divided by the network lifetime using the MinCar algorithm. There are some similarities between this graph and the corresponding table and the ones discussed in the previous section, which is not surprising, as the MaxWillMinForced algorithm is based on MaxWill and only adds a sort of look-ahead for the energy consumption. So, the performances vary less if the Maximum Forcedness Ratio approaches one and the MaxWillMinForced algorithm is almost in every situation better than the MinCar.

6.2.3. MaxWillMinForced versus MaxWill

The performance comparison between the MaxWillMinForced and MaxWill MPR selection algorithms does not yield a clear winner. The absence of a winner is underlined by the mean of the Performance Ratio, which equals 1.0023. When we compare MaxWillMinForced with MaxWill we see that for 424 of the 800 simulations the selection algorithms lead to exactly the same network lifetime. MaxWillMinForced beats MaxWill in 203 simulations, but on the other hand, MaxWill beats MaxWillMinForced in 173 simulations.

7. Conclusions

We presented an analysis of MPR flooding by looking separately at MPRs, MPR flooding and MPR selection. By this, we are able to point out the effects of the specific elements of MPR flooding. Our conclusions are that for general graphs fixed nodes provide an upper bound to the network lifetime, independent of the MPR selection algorithm.

For grid graphs all MPR selection algorithms provide the same MPR-set for 2-hop central nodes, namely the set of forced nodes that is selected in the first step of a three step MPR selection algorithm. Since this result is a consequence of the regular structure (translation, symmetry) of the graphs, we may expect similar results for all graphs with regular (sub) structures.

For random graphs, the Maximum Forcedness Ratio parameter, that we introduce in this paper, seems a good descriptor of the degree in which MPR selection algorithms yield different network lifetimes. In random graphs with an MFR close to 1 there is less difference between MPR selection algorithms than in graphs with MFR close to 0. Therefore, it is not worthwhile to investigate new MPR selection algorithms if the networks under consideration have an MFR close to 1. More generally, all computational studies on MPR selection should take into account the MFR to ensure that no wrong conclusions are drawn from the achieved results.

As a byproduct of the simulations we got some insight in the effectiveness of some MPR selection algorithms for random networks. MPR flooding with MPR-sets selected by MaxWill or MaxWillMinForced leads to a significantly longer network lifetime compared to MPR flooding with the MPR-sets selected by MinCar. However, the comparison between MaxWill and MaxWillMinForced does not yield a clear winner, even if the networks have a low MFR. Combined with the fact that MaxWillMinForced is more difficult to implement, MaxWill therefore seems a good choice to implement in OLSR networks where lifetime of the network is important.

While in this paper we focus on mechanisms for homogeneous broadcast traffic, further research is needed to verify if similar conclusions hold for heterogeneous unicast traffic.

References

[1] H. Badis, K. Al Agha, An efficient QOLSR extension protocol for QoS in ad hoc networks, in: Proceedings of IEEE Vehicular Technology Conference, 2004, pp. 2650–2653.

[2] A. Busson, N. Mitton, E. Fleury, Analysis of the multi-point relay selection in OLSR and implications, IFIP International Federation for Information Processing Challenges in Ad Hoc Networking, 197/2006, Springer, Boston, 2006, pp. 387–396.

[3] M. Cagalj, J. Hubaux, C. Enz, Minimum-energy broadcast in all-wireless networks: Np-completeness and distribution issues, in: Proceedings of the Annual International Conference on Mobile Computing and Networking, MOBICOM, 2002, pp. 172–182.

[4] G. Calinescu, S. Kapoor, A. Olshevsky, A. Zelikovsky, Network lifetime and power assignment in ad hoc wireless networks, in: Proceedings of ESA 2003, Lecture Notes in Computer Science, vol. 2832, 2003, pp. 114–126.

[5] T. Clausen, P. (editors) Jacquet, C. Adjih, A. Laouiti, P. Minet, P. Muhlethaler, A. Qayyum, L. Viennot, Optimized link state routing protocol (OLSR). RFC 3626, Network Working Group, 2003.

[6] J. Dean, W. Chao, J.P. Macker, Simplified multicast forwarding in mobile ad hoc networks. in: Proceedings of the IEEE Military Communications Conference, 2004, pp. 744–750.

[7] N. Ghanem, S. Boumerdassi, E. Renault, New energy saving mechanisms for mobile ad-hoc networks using OLSR, in: PE-WASUN '05: Proceedings of the 2nd ACM International Workshop on Performance Evaluation of Wireless Ad Hoc, Sensor, and Ubiquitous Networks, 2005, pp. 273–274.

[8] F. Ingelrest, D. Simplot-Ryl, Maximizing the probability of delivery of multipoint relay broadcast protocol in wireless ad hoc networks with a realistic physical layer, in: Proceedings of the International Conference on Mobile Ad-Hoc and sensor networks, vol. 2006, 2006, p. 143.

[9] I. Kang, R. Poovendran, Maximizing network lifetime of broadcasting over wireless stationary ad hoc networks, Mobile Networks and Applications 10 (6) (2005) 879–896.

[10] W. Liang, Constructing minimum-energy broadcast trees in wireless ad hoc networks, in: Proceedings of the International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc), 2002, pp. 112–122.

[11] C.P. Low, L.W. Goh, On the construction of energy-efficient maximum residual battery capacity broadcast trees in static ad hoc wireless networks, Computer Communications 29 (1) (2005) 93–102.

[12] B. Mans, N. Shreshta, Performance evaluation of approximation algorithms for multipoint relay selection, in: Proceedings of the Third Annual Mediterranean Ad Hoc Networking Workshop, 2004.

[13] J. Park, S. Sahni, Maximum lifetime broadcasting in wireless networks, in: 3rd ACS/IEEE International Conference on Computer Systems and Applications 2005, 2005, pp. 1–8.

[14] A. Qayyum, L. Viennot, A. Laouiti, Multipoint relaying for flooding broadcast messages in mobile wireless networks, in: HICSS '02: Proceedings of the 35th Annual Hawaii International Conference on System Sciences (HICSS'02), vol. 9, 2002, p. 298.

[15] E. Tromp, Basic Multicast Forwarding in Mobile Ad Hoc Networks. <http://sourceforge.net/projects/olsr-bmf>, 2007.

[16] J. Verbree, Network Lifetime in Wireless Networks, Master Thesis, University of Twente, 2007. <http://eprints.eemcs.utwente.nl/15856>.

[17] L. Viennot, Complexity Results on Election of Multipoint Relays in Wireless Networks, Technical Report RR-3584, INRIA, 1998.

[18] R. Wattenhofer, A. Zollinger, Xtc: a practical topology control algorithm for ad-hoc networks, in: Proceedings – International Parallel and Distributed Processing Symposium, IPDPS 2004, vol. 18, 2004, pp. 2969–2976 (Abstracts and CD-ROM).

[19] J.E. Wieselthier, G.D. Nguyen, A. Ephremides, On the construction of energy-efficient broadcast multicast trees in wireless networks, Proceedings – IEEE INFOCOM 2 (2000) 585–594.

[20] J.E. Wieselthier, G.D. Nguyen, A. Ephremides, Resource management in energy-limited, bandwidth-limited, transceiver-limited wireless networks for session-based multicasting, Computer Networks 39 (2) (2002) 113–131.

Jan-Maarten Verbree (1982) received a master degree at the University of Twente in 2007. He came in touch with research on wireless networks during his internship at the Distributed Computing Group at the ETH in Zurich in 2006. After his internship he decided to work more on wireless network for his final thesis. He accomplished his final thesis, titled 'Network Lifetime in Wireless Networks', at the 'Discrete Mathematics and Mathematical Programming' group at the department of Applied Mathematics at the University of Twente. Although the research found place at the University of Twente, the main questions in the thesis found there origin at Thales Netherlands, Land & Joint Systems. Since january 2008 he works as a software engineer on various projects. His current job is at Neelen & Schuurmans, a watermanagement consultancy company in Utrecht.

Maurits de Graaf (1966) received a PhD degree at the University of Amsterdam in 1994 for the thesis Graphs and curves on surfaces. From 1994 to 1999 he was with KPN Research, working on traffic management methods for ATM networks. Since 1999 he is with the innovation department of Thales Netherlands, Land & Joint systems where he is involved in various collaborative projects with other companies and institutions. Topics addressed are: IP, IPv6, VoIP, IEEE 802.11, IEEE 802.1q, and wireless routing algorithms. Since 2006 he is involved in the Casimir project (funded by the National Science foundation in the Netherlands), where he is working part-time at the University of Twente with the department of Stochastic Operations Research on the development of algorithms to increase the lifetime of wireless ad-hoc communication networks.

Johann Hurink received a PhD degree at the University of Osnabrück (Germany) in 1992 for a thesis on a scheduling problem occurring in the area of public transport. From 1992 until 1998 he has been an assistant professor at the same university working on local search methods and complex scheduling problems. This work resulted in a Habilitation thesis at the University of Osnabrück in 1999. From 1998 until 2005 he has been an assistant professor, from 2005 until 2009 an associate professor, and since 2009 a full professor in the group 'Discrete Mathematics and Mathematical Programming' at the department of Applied Mathematics at the University of Twente. Current work includes the application of optimization techniques and scheduling models to problems from logistics, health care, and telecommunication.