Non-parametric models in the monitoring of engine performance and condition
Part 1: modelling of non-linear engine processes

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Abstract: This paper proposes the use of radial basis function (RBF) networks in the modelling of non-linear engine processes. A pertinent application of such a model is the reconstruction of cylinder pressure based upon the instantaneous angular velocity of the engine crankshaft. Distinction is made between parametric and non-parametric models and applications to which each is suited. The structure of an RBF model is presented and the use of this model in combustion pressure reconstruction is discussed. The paper concludes with a treatment of the practicalities associated with the implementation of an RBF model to typify a non-linear engine process.

Keywords: angular speed, cylinder pressure measurement, non-parametric engine model, radial basis function network, regressor selection, regularization

1 NON-LINEAR ENGINE MODELS

Many of the systems and processes operating within internal combustion engines are inherently non-linear; because of this, simple but accurate analytical models cannot be derived for them. A classic example of a non-linear engine process is the cyclic pressure variation within a combusting diesel engine cylinder. Owing to its practical importance, cylinder pressure estimation has been used as the process upon which to develop the non-linear modelling techniques illustrated in this paper. Combustion pressure waveform measurement and analysis play an important role in the improvement of performance, emissions control and condition monitoring in internal combustion engines. Conventional techniques are inapplicable for reliable, high-resolution cylinder pressure measurement on a routine or in-service basis. For these reasons an alternative approach to cylinder pressure estimation, based on easy-to-measure variables, is attractive. This paper introduces a family of non-parametric models, radial basis function (RBF) networks, which may be applied to the task of reconstructing cylinder pressure based on easy-to-obtain measurements of instantaneous crankshaft angular velocity and cylinder head vibration.

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At this stage it is worthwhile formally distinguishing between parametric and non-parametric models. Parametric models are differentiated from non-parametric models by the inference which may be drawn from their coefficients. In the case of the former, the aim is to arrive at values for a select few terms that are meaningful in the real world. It is assumed that a set of deterministic mathematical models of the system can be derived using physical laws. The system can then usually be written as a set of ordinary differential equations and algebraic equations:

$$y = f(q, \dot{q}, \ddot{q}, x, t, \boldsymbol{\Theta}) \tag{1}$$

where $\boldsymbol{q}$ represents the degrees of freedom, $\boldsymbol{y}$ the observed output variables, $\boldsymbol{x}$ the input variables and t the explicit time. Vector $\boldsymbol{\Theta}$ represents the unknown model parameters. Sufficient initial conditions must be supplied to arrive at values for q and its first and second derivatives. In order to determine the model parameters, $\boldsymbol{\Theta}$, it is necessary to substitute the measured time histories of all variables. This assumes that the measured variables are exact, which is certainly not the case for a real system where they will be polluted with noise and biased by instrumentation errors. Hopefully it is possible to have some idea of the error in each of the measurements, in which case it is possible to use this information to improve the prediction of the unknown model parameters.

In non-parametric modelling we have little *a priori* knowledge about the form of the true system. The system is still modelled (or it may be better to say typified) using an equation containing free parameters, but in a way in which the class of systems that the model can represent is very broad. The free parameters probably have no meaningful interpretation to real-world phenomena such as pipe diameters, expansion rates or specific heat capacity. Any meaning is purely coincidental. This does not imply that such parameters are useless and that attention should be paid only to system inputs and outputs, since their variation in a model that is updated some later time can reveal worthwhile information on changes to the system.

1.1 Parametric engine models

Considerable effort has been expended to reconstruct the pressure waveform indirectly and non-intrusively. Two previous approaches include:

(a) vibration measurement based reconstruction,
(b) crankshaft angular speed measurement reconstruction.

The vibration measurement based approach involves recording vibrations at a position either on the engine block or on the cylinder head. An appropriately identified transfer function, usually in the form of a parametric model, is used to compute the pressure waveform from the filtered vibration acceleration signal [**1–4**]. It has been established that the monitored vibration signal is highly linearly correlated with the pressure waveform, particularly over the section of the waveform that corresponds to the combustion. Owing to the dispersion of the vibration signal as it propagates through the engine block, the transfer function has traditionally been estimated by cepstral analysis of both the vibration signals and the pressure signals in order to obtain robust reconstruction results. Although this method has been employed successfully in the detection of certain types of fault, distortion exists in the critical cylinder compression and combustion sections of the reconstructed pressure waveform. Therefore, these transfer function based procedures cannot model accurately the characteristic features associated with the onset of combustion.

An approach based on the instantaneous angular speed of the crankshaft is considered to be more successful and is therefore more widely used. The measurement system is non-intrusive and is relatively convenient to set up. Moreover, the reconstruction process makes use of the identified engine model and provides insight into the physical mechanisms responsible for crankshaft speed fluctuations. Unfortunately, the major difficulties with this method lie in the reconstruction algorithms and model simplification. Previous attempts at overcoming these difficulties have been made through two approaches:

(a) improvement of the model based algorithm,
(b) introduction of pattern recognition techniques.

In the model based algorithm, an inverse equation is derived algebraically or using an iterative procedure. By solving the engine momentum equation, the instantaneous pressure waveform can then be computed using the measured instantaneous angular speed as an input [**5–12**]. In common with many inverse problems, the solution suffers from instability at certain points owing to near singularity of the inverse matrix. This occurs at TDC and BDC during the engine cycle when the effective torque radius is near to zero. The result is an unstable reconstruction over the crucial central section of the pressure waveform. Generally speaking, the model based approach has the disadvantage of being inextricably tied to the simplifying assumptions needed to construct the model. Model errors and inadequate assumptions lead to considerable deviations. In fact, a comprehensive comparison of the reconstructed pressure waveforms with empirical measurements has not been found in any of the previously mentioned work.

1.2 Pattern recognition

A novel concept in pressure waveform reconstruction has been the use of pattern recognition techniques. In this approach, a knowledge base of signature patterns is collected. Each pattern corresponds to a known operating parameter of the engine system. A measured speed signature is then compared with the patterns in the knowledge base using pattern recognition techniques. The magnitude of the cylinder pressure is then obtained from the magnitude of similar patterns taken from each cylinder. Since no assumptions are made about the engine model involved in the pattern recognition based method, this approach has the advantage of typifying the behaviour of the real, physical system. However, only a limited size of knowledge base can be formed and it cannot reflect the continuous nature of possible engine operating regimes. Therefore, interpolation techniques have been introduced to overcome this difficulty. In Brown [**13**], pattern recognition was employed in association with linear interpolation in order to predict the peak pressure of the cylinder waveform. The overall standard deviation of the prediction errors is less than 6 per cent. However, because the technique is still a prototype, the overall waveform of the cylinder pressure cannot be given. Hence many important pressure parameters cannot be analysed.

1.3 Non-parametric models

Following on from these novel pattern recognition concepts, this paper investigates waveform reconstruction using a radial basis function network. This form of non-parametric model combines features of both pattern recognition and interpolation. Moreover, the trained network is, in effect, a series of locally valid models with non-linear interpolation between the sub-models. In operation, the most relevant local models are activated and the results are combined to form an overall prediction. Through use of an appropriate training regime, this is likely to ensure a parsimonious representation of the model. This procedure allows accurate predictions to be produced from a relatively small amount of stored data, using a compact model with relatively few free parameters.

2 CYLINDER PRESSURE AND CRANKSHAFT SPEED

Conventional parametric models of the cylinder pressure in engines have been derived from balance equations for the angular momentum of reciprocating components. When the engine is operating in a steady state condition and the crankshaft system is modelled as rigid, the engine crankshaft motion is described by the torque balance equation [**5**, **8**, **10**, **11**]:

$$J(\theta)\ddot{\theta} = M_g - M_m - M_f - M_l \tag{2}$$

Here the crank angle is a function of time $\theta = \theta(t)$, $\ddot{\theta} = \mathrm{d}^2\theta/\mathrm{d}t^2$, and $J(\theta)$ is the time-varying engine inertia. This non-linear term is necessary to describe engine operation at high speeds. M_g represents the gas torque generated by the combustion pressure, $P(\theta)$. The effective torque radius, $R(\theta)$, derived using the engine geometry, is introduced to enable the transformation from piston force to crank torque as

$$R(\theta) = r\left\{\sin(\theta) + \frac{r}{l}\frac{\sin\theta\cos\theta}{\sqrt{[1-(r^2/l^2)\sin^2\theta]}}\right\} \tag{3}$$

If the piston area is A, the gas torque may then be calculated from

$$M_g(\theta) = AR(\theta)P(\theta) \tag{4}$$

M_m is the torque due to the effective reciprocating mass and the piston motion. This torque does not contribute net kinetic energy to the system but creates significant fluctuation in the resultant torque at the combustion frequency; it is determined by the engine geometry and speed. For the non-linear engine model [**10**, **11**], this torque is expressed as a function of the instantaneous crankshaft speed and angular momentum:

$$M_m = \frac{1}{2}\frac{\mathrm{d}J(\theta)}{\mathrm{d}\theta}\dot{\theta}^2 \tag{5}$$

M_f is the torque due to friction and the pumping action of the engine, and M_l is the external load torque. In the general case, this external load torque cannot be calculated as a dependent variable using terms related to the instantaneous angular velocity, $\dot{\theta}$.

The four components of torque, M_g, M_m, M_f and M_l, can either be measured or estimated and then substituted in equation (2). This is then solved to give the combustion pressure as a function of θ, $\dot{\theta}$, $\ddot{\theta}$, the load torque $M_l(\theta)$ and the friction torque $M_f(\theta)$:

$$P_g(\theta) = \frac{1}{AR(\theta)}\left[J(\theta)\ddot{\theta} + \frac{1}{2}\frac{\mathrm{d}J(\theta)}{\mathrm{d}\theta}\dot{\theta}^2 + M_l(\theta) + M_f(\theta)\right] \tag{6}$$

From this equation, the gas pressure, P_g, may be rewritten as a function of three independent variables:

$$P_g(\theta) = \Psi(\ddot{\theta}, \dot{\theta}, \theta, M_f + M_l) \tag{7}$$

Owing to the ease of pressure, torque and speed measurement and the low cost of the hardware involved, crank speed based cylinder pressure reconstruction has been explored by many researchers [**5**, **8–11**, etc.]. However, the results obtained by researchers in solving equation (6) using measured data have involved a number of assumptions because of the non-linearity in the angular momentum term $J(\theta)$ at high speed, the stochastic nature of the combustion process and the extra measurement required in determination of the external mean load. A non-parametric model circumvents many of the difficulties inherent in the model based approach to cylinder pressure reconstruction. RBF networks have been shown to possess universal function approximation capabilities and can be used to establish a mapping between the instantaneous crankshaft angular velocity and the cylinder pressure.

In the work reported in references [**10**], [**11**] and [**14**], the friction and pump torques were estimated using the engine speed and geometry. Ribbens [**15**] argued that crankshaft acceleration measurements contain information on both the average and time-varying engine torque applied to the crankshaft. Owing to the following relationship between crankshaft acceleration and torque,

$$\ddot{\theta} = \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = \frac{\mathrm{d}\dot{\theta}}{\mathrm{d}\theta}\frac{\mathrm{d}\theta}{\mathrm{d}t} = \dot{\theta}\frac{\mathrm{d}\dot{\theta}}{\mathrm{d}\theta} \tag{8}$$

previous work suggests that the torque measurement can be avoided in neural network modelling. Hence, the neural network development can be based on the assumption that the cylinder pressure may be expressed as a function of the instantaneous angular velocity alone:

$$P_g = \Pi(\dot{\theta}) \tag{9}$$

The combustion pressure can then be determined uniquely by the angular speed, obviating the need for the measurement of external load torque.

The function $\Pi(\dot{\theta})$ can be modelled to an arbitrary accuracy by many series expansions such as the Taylor series or by using kernel-type methods of which the radial basis function network may be regarded as a subclass. Provided that such series are capable of universal approximation the most appropriate series is selected by the degree of parsimony of representation that the series exhibits.

3 RADIAL BASIS FUNCTION NETWORKS

3.1 Architecture of a radial basis function network

A radial basis function (RBF) network is a linear model of a function. This model is constructed from a linear combination of the responses of a set of fixed non-linear basis functions. The input is applied to radial Gaussian functions so each hidden unit simulates the effect of overlapping and locally tuned receptive fields:

$$\phi_j(\boldsymbol{x}) = \mathrm{e}^{(-|\boldsymbol{x}-\boldsymbol{c}_j|)/\varphi^2} \tag{10}$$

The exponential decay of the Gaussian function as $\boldsymbol{x}$ moves away from the location of the centre, $\boldsymbol{c}_j$, ensures the local properties of the Gaussian RBF model. Figure 1 illustrates a univariate Gaussian with centre $\boldsymbol{c}_j = 0$ and radius $\varphi = 1$. The width, φ, of the Gaussian function parameters also affects the nature and scope of the hidden unit output. When a larger φ is used, the output is less sensitive to a change in the Euclidean distance and a better extrapolation between the centres can be obtained. However, for test data locations that are similar to training data sites the accuracy of the reconstructed pressure waveform is reduced, since the locality of the model representation is itself reduced. On the other hand, a smaller radius enables the network to predict a slight change in the speed signature but gives poor interpolation between the centres. This indicates that, to reach a certain waveform reconstruction accuracy over the range of engine operation, a large number of training centres must be used when the interpolation between the centres is not sufficiently smooth. An appropriate value for the Gaussian radius needs to be identified. This is usually set using heuristics, such as defining it to be half the Euclidean distance to the nearest N other centres.

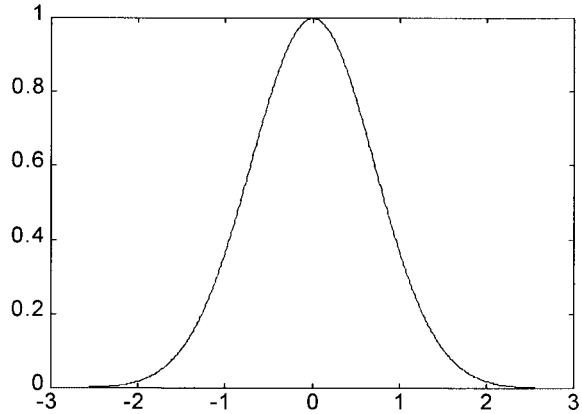

Fig. 1 Gaussian decay (width $\varphi = 1.0$)

The linear layer creates a mapping from the RBF outputs to the output units. It calculates linear combinations of the activation values of the radial basis functions. A two-stage supervised training process is used to set the free parameters in the network. Candidate centres, $\boldsymbol{c}_i$, are chosen for each of the m RBF neurons using a regularized forward selection algorithm. Since an RBF network is a linear network, the weights, w_{ij}, connecting the RBF neurons to the output units may then be calculated by a least mean squares minimization. As mentioned earlier, a regularization term may be introduced into the cost function or matrix inversion can be carried out using singular value decomposition. This avoids the danger of numerical instabilities associated with direct solution of the normal equation.

If each training data vector, $\boldsymbol{x}_1,\ldots\boldsymbol{x}_p$, is used as a centre, then the network will implement strict interpolation. This, however, is not usually desirable since the number of degrees of freedom in the network model will often greatly exceed the degrees of freedom of the underlying process that gave rise to the training data. In this case the problem is overdetermined and the network will likely fit many random features of the training data, resulting in poor generalization to unseen inputs. It is desirable, then, that a model is constructed with as few non-linear terms as possible. The aim is to be able to reconstruct the mapping (hypersurface) indicated by the underlying system with as parsimonious a model as possible.

There is no limit to the number of inputs or outputs in the network. However, it should be borne in mind that networks with very many free parameters will require a great deal of training data to arrive at statistically significant values for the network parameters. The network then has the form

$$g_i(\boldsymbol{x}) = \sum_{j=1}^{m} w_{ij} h_j(\boldsymbol{x}) \tag{11}$$

or matrix notation

$$\mathbf{G} = \mathbf{HW} \tag{12}$$

In this case, $\{h_j(\cdot)\}1^m$ are the responses of the m non-linear layer neurons to the p vectors in the input training data, $\mathbf{X} = [\boldsymbol{x}_1,\ldots,\boldsymbol{x}_p]$, where $\boldsymbol{x} \in \Re Q$.

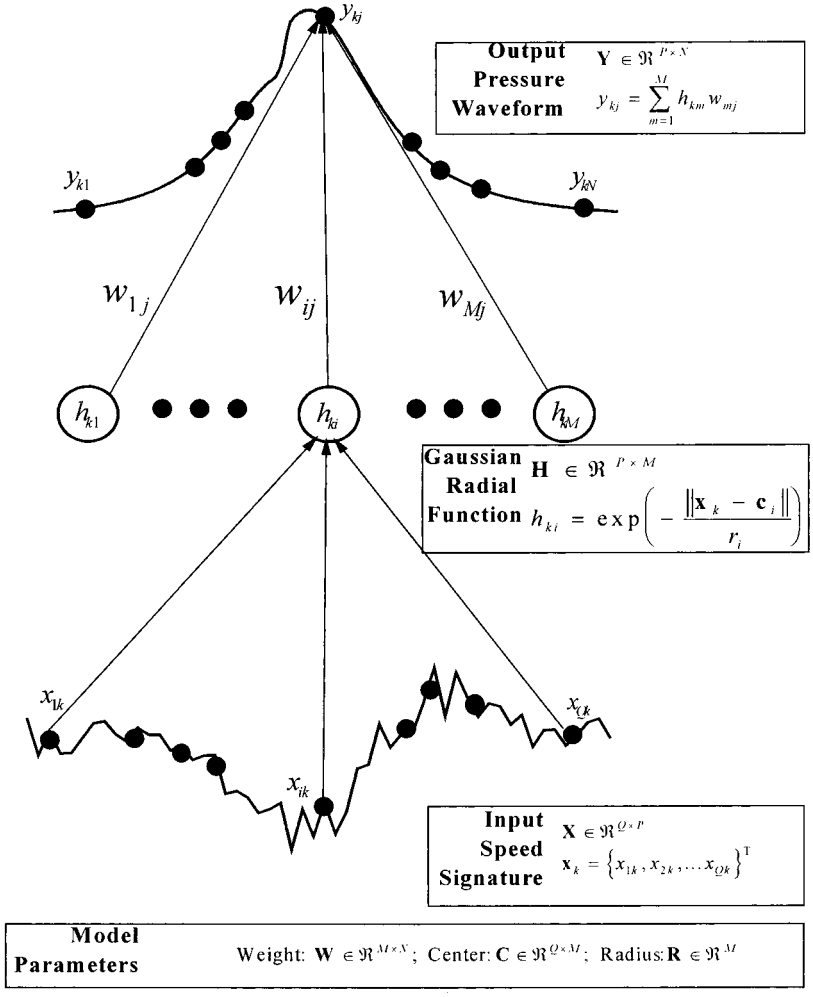

Fig. 2 RBF network for cylinder pressure waveform reconstruction from the angular speed signature of the crankshaft

3.2 Replicating cylinder pressure with an RBF network

If an RBF network is used to model cylinder pressure, where pressure and speed are represented by y and θ respectively, then equation (9) can be rewritten as

$$y_i(\theta) = \sum_{j=1}^{m} w_{ij} h_j(\dot{\theta}) \tag{13}$$

The detailed RBF model structure is presented in Fig. 2. It can be seen that a subspace is constructed by the outputs of the hidden layer, radial basis functions from which the target vectors may be linearly projected. The model input space is sparsely occupied, since only a small subset of $\boldsymbol{x} \in \Re^N$ can represent possible pressure waveforms. The compressed $\Re^M$ space generated in the middle layer is more densely occupied. The pressure is now expressed as a linear combination of a set of M fixed basis functions $h_i(\cdot)$, and the coefficients of the linear combinations, w_{ij}, are the network weights or model parameters. The following section describes the forward selection approach to performing the non-linear optimization which is required to derive values for the model parameters.

4 DERIVATION OF MODEL PARAMETERS

4.1 Cost functions

It is usual to define some form of 'merit' or 'cost' function for the model given a particular set of weight coefficient estimates, $\mathbf{W}$. This cost function is based on the square distance of the model predictions from the measured output vectors, $\mathbf{Y} = [\boldsymbol{y}_1, \ldots, \boldsymbol{y}_p]$, where $\boldsymbol{y} \in \Re^N$. Such a cost function is shown in the following equation for a model with N outputs:

$$\aleph^2 = \sum_{j=1}^{N} \sum_{i=1}^{P} \left[y_{ji} - \sum_{k=1}^{M} w_{jk} h_k(\boldsymbol{x}_i) \right]^2 \tag{14}$$

Translating this into matrix notation, an equation is obtained for this cost function:

$$\mathbf{C}(\mathbf{W}) = \tfrac{1}{2}(\mathbf{Y} - \mathbf{HW})^{\mathrm{T}}(\mathbf{Y} - \mathbf{HW}) \tag{15}$$

The freedom of the model to fit many functions is due to the freedom to pick different values for the weights, the basis functions and any parameters of the basis functions such as their width. In an RBF network, it is usual that the basis functions are set during the first stage of training and remain fixed. This means that a linear training rule may be applied to calculate the weights matrix $\mathbf{W} = [w_{ij}]$. Such a linear network can have its weights set by a linear least squares technique.

The optimum selection for the weights in $\mathbf{W}$ occurs when all elements of the matrix $\mathbf{C}(\mathbf{W})$ are minimized:

$$\frac{\partial \mathbf{C}}{\partial \mathbf{W}} = \mathbf{H}^{\mathrm{T}}\mathbf{HW} - \mathbf{H}^{\mathrm{T}}\mathbf{Y} \tag{16}$$

This occurs when the first differential of $\mathbf{C}(\mathbf{W})$ with respect to the weight matrix $\mathbf{W}$ is the zero matrix:

$$\mathbf{H}^{\mathrm{T}}\mathbf{HW} = \mathbf{H}^{\mathrm{T}}\mathbf{Y} \tag{17}$$

Solved for $\mathbf{W}$ this gives

$$\hat{\mathbf{W}} = (\mathbf{H}^{\mathrm{T}}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{Y} \tag{18}$$

$(\mathbf{H}^{\mathrm{T}}\mathbf{H})^{-1}$ is normally evaluated using Gauss–Jordan elimination. However, in many applications the solution to the normal equation is very close to singular. A zero pivot element may be encountered during the elimination procedure, giving rise to fitted parameters w_{ij} that are very large and cancel out almost precisely when the fitted model is evaluated at the training data points. However, these predictions are unstable and invalid away from the support points. However, these predictions are unstable and invalid away from the support points. As mentioned, this difficulty can be avoided by using singular value decomposition to calculate the weights matrix $\mathbf{W}$ (substitute $\mathbf{Y} = \mathbf{HW} = \mathbf{USV}^{\mathrm{T}}\mathbf{W}$) or by perturbing matrix $\mathbf{H}$ to $\mathbf{H} + \lambda\mathbf{I}$, as is achieved by regularization (discussed in Section 4.3).

If the basis functions are allowed to vary, then the network parameters would become non-linear and an iterative numerical procedure such as that of Levenberg-Marquardt [**16**] would have to be employed to determine the weights. Such procedures tend to be time consuming and suffer from local minima problems.

4.2 Forward selection of centres

In forward selection of RBF centres (see Chen [**17**]) a model is built from a set of regressors which are added one at a time from a finite set of possibilities. The criterion chosen to select the next centre to be added is usually a minimization of the squared error of the network outputs over some subset of the training data. Forward selection is a suboptimal heuristic procedure for picking the centres. Selection of an optimal subset would involve a combinatorial search, which would certainly be computationally prohibitive. There are $2^m - 1$ possible sets which may be chosen from a library of m original centres. Other approaches could be the use of directed search techniques such as genetic algorithms [**18**] using minimization of prediction error as the fitness function. Whatever the selection procedure used, termination occurs when the network error is reduced to an acceptable value. The criterion used for stopping may be the mean squared error or it may involve some form of bootstrap method such as generalized cross validation (see Section 4.5) to reflect that the model should generalize well to unseen data and to discourage bias.

Minimization of the squared error term can lead to overfitting of the training data, a danger highlighted above, even when relatively few regressors are used. It must therefore be combined with a technique such as regularization to build parsimonious networks that generalize well. Unfortunately, the methods highlighted will not counteract major deficiencies in the training data such as the presence of wild outliers which will significantly distort the weight optimization procedure because they encourage the network to fit the outlying data points. Regularization will help counteract this to some extent. Preprocessing of the data with a clustering algorithm such as k-means clustering or Kohonen's self-organizing feature map may be a more effective way of removing such outliers [**19**].

4.3 Regularization

Overfitting, whereby the network learns features that are specific to the training data, resulting in poor generalization, may be reduced by using a weight decay term in the network cost function. This technique is known to statisticians as ridge regression or as regularization within the neural network community. Take, for example, a physical process described by the function $y = \mathscr{J}(x)$. Knowledge of this process is limited to observation of its input–output behaviour, $\{(\boldsymbol{x}_i, \boldsymbol{y}_i)\}_{i=1}^{p}$. The aim is to reconstruct the entire function from examples of this behaviour. This is an ill-posed problem, but, by making some assumptions about the form of the function and its smoothness, it should be possible to derive a reasonable approximation to $\mathscr{J}(x)$. The hypersurface reconstruction problem is one of a generic class of problems known as inverse problems.

In regularization, the cost function [see equation (15)] is altered by the addition of a penalty term, $\Omega(\mathbf{Y})$, whose influence is tempered by a factor, λ:

$$\mathbf{C}(\mathbf{W}) = \tfrac{1}{2}(\mathbf{Y} - \mathbf{HW})^{\mathrm{T}}(\mathbf{Y} - \mathbf{HW}) + \lambda\Omega(\mathbf{Y}) \tag{19}$$

The penalty term controls the variance of the network and sets a balance between an *a priori* form for the model with a strong bias towards prior knowledge of

its form (suggested by relatively large values of λ) and *a posteriori* determination of the model shape based primarily on the training data.

In zero-order regularization, or weight decay as it is commonly referred to, the regularization term has the form

$$\Omega(\mathbf{Y}) = \lambda \mathbf{W}^{\mathrm{T}}\mathbf{W} \tag{20}$$

This term is a measure of the smoothness, or stability, of the desired solution. It penalizes large weight values, hence encouraging solutions with small weight values. The additional smoothness is achieved because it is more likely that large weights will be required to produce an extremely rough output function. One effect of such a ridge regression technique is to reduce the effective number of degrees of freedom in the model, making it less flexible and unable to fit as many functions. The advantage is that the model is more stable. If $\mathbf{C}(\mathbf{W})$ were to be minimized with respect to this term alone, then a solution would be arrived at that was very smooth but had nothing to do with the shape of the measured data. Other regularization terms may be used rather than equation (20), depending on *a priori* knowledge of the shape of the function [**16**].

The optimal solution for the weight matrix to minimize the altered network cost function [equation (19)] is given by the equation

$$\mathbf{W}_m = (\mathbf{H}_m^{\mathrm{T}}\mathbf{H}_m + \lambda \mathbf{I}_m)^{-1}\mathbf{H}_m^{\mathrm{T}}\mathbf{Y} \tag{21}$$

The introduction of regularization leaves one further parameter in the network training procedure for which a value must be sought. The aim is to choose a value for the regularization parameter, λ, that produces the lowest prediction error on unseen data, in that it does not lead to overfitting of the training data but still pays sufficient attention to its shape to set the model free parameters so that they reflect the general shape of the hypersurface. In Section 4.5 the way in which the model order may be estimated will be discussed; the criterion used in this paper is known as generalized cross validation (GCV). Since generalized cross validation is used to predict the network error on unseen data, this will also be used to set the regularization coefficient, λ.

4.4 Fast forward selection and re-estimation of regularization coefficient

As stated, forward selection of centres in a radial basis function network involves picking centres one at a time from the training data set. These are added to an initially empty subset until the termination criterion (discussed in Section 4.5) is reached. This subset is maintained in the design matrix, $\mathbf{H}_m$, which, when the procedure begins, is the empty set:

$$\mathbf{H} = \varnothing \tag{22}$$

At the mth step in the procedure, the old design matrix, $\mathbf{H}_{m-1}$, is augmented with a new column

$$\mathbf{H}_m^{\mathrm{T}} = [\mathbf{H}_{m-1}\mathbf{f}_i] \tag{23}$$

where $\mathbf{f}_i$ is chosen from the columns of the full design matrix $\mathbf{F}$. This full design matrix is simply the non-linear function, $\phi(\cdot)$, applied to each item of the training data set, $\mathbf{X} = [\boldsymbol{x}_1,\ldots,\mathbf{x}_N]$:

$$\mathbf{F} = \begin{bmatrix} \phi_1(\boldsymbol{x}_1) & \cdots & \phi_1(\boldsymbol{x}_P) \\ \vdots & \ddots & \vdots \\ \phi_M(\boldsymbol{x}_1) & \cdots & \phi_M(\boldsymbol{x}_P) \end{bmatrix} \tag{24}$$

After adding a candidate column from the full design matrix and recalculating the weight matrix $\mathbf{W}_m$ from equation (21), the network cost function [equation (19)] can be re-evaluated. This cost function may be written as the sum of the network error over the training data error penalized by the roughness:

$$\begin{aligned}\mathbf{C}_i(\mathbf{W}_m) &= \tfrac{1}{2}(\mathbf{Y} - \mathbf{H}_m^i\mathbf{W}_m)^{\mathrm{T}}(\mathbf{Y} - \mathbf{H}_m^i\mathbf{W}_m) + \lambda \mathbf{W}_m^{\mathrm{T}}\mathbf{W}_m \\ &= \mathbf{Y}^{\mathrm{T}}\mathbf{P}_m\mathbf{Y}\end{aligned} \tag{25}$$

where the projection matrix $\mathbf{P}_m$ is given by

$$\mathbf{P}_m = \mathbf{I}_p - \mathbf{H}_m^i(\mathbf{H}_m^{i\,\mathrm{T}}\mathbf{H}_m^i + \lambda \mathbf{I}_m)^{-1}\mathbf{H}_m^{i\,\mathrm{T}}\mathbf{Y} \tag{26}$$

The criterion used to choose the best column, $\mathbf{f}_r^m$, from the full design matrix, $\mathbf{F}$, and corresponding to the best choice of centre is that the mth choice of best column should be selected according to

$$r_m = \underset{i}{\operatorname{argmin}}[\mathbf{C}_i(\mathbf{W}_m^i)] \tag{27}$$

This is equivalent to maximizing

$$\mathbf{C}_m(\mathbf{W}_m) - \mathbf{C}_{m-1}^i(\mathbf{W}_{m-1}^i) = \frac{\operatorname{trace}(\mathbf{Y}^{\mathrm{T}}\mathbf{P}_{m-1}\mathbf{f}_i)^2}{\lambda + \mathbf{f}_i^{\mathrm{T}}\mathbf{P}_{m-1}\mathbf{f}_i} \tag{28}$$

Recalculating the projection matrix, $\mathbf{P}_m$, p times for each iteration of the forward selection algorithm appears an onerous and time consuming task, but an update formula exists which requires far fewer operations; this is given in equation (29) (see reference [**19**] for details). This speed-up may be further advanced by orthogonalizing the design matrix:

$$\mathbf{P}_m = \mathbf{P}_{m-1} - \frac{\mathbf{P}_{m-1}\mathbf{f}_j\mathbf{f}_j^{\mathrm{T}}\mathbf{P}_{m-1}}{\lambda_j + \mathbf{f}_j^{\mathrm{T}}\mathbf{P}_{m-1}\mathbf{f}_j} \tag{29}$$

Orr [**19**] derives a re-estimation formula for λ which is based on minimization of the GCV error (see Section 4.5). By differentiating the formula for the GCV error [equation (31)] with respect to λ and setting the result to zero, a minimum value for GCV is obtained. The resultant re-estimation formula may then be applied each time a new centre is added to the network and is given by

$$\lambda := \frac{[\partial\ \operatorname{trace}(\mathbf{P}_m)/\partial\lambda]\mathbf{Y}^{\mathrm{T}}\mathbf{P}_m^2\mathbf{Y}}{\operatorname{trace}(\mathbf{P}_m)\mathbf{W}_m^{\mathrm{T}}(\mathbf{H}_m^{\mathrm{T}}\mathbf{H}_m + \lambda \mathbf{I}_m)^{-1}\mathbf{W}_m} \tag{30}$$

where

$$\frac{\partial \text{ trace}(\mathbf{P}_m)}{\partial \lambda} = \sum \frac{\mathbf{h}_j^{\mathrm{T}}\mathbf{h}_j}{(\lambda + \mathbf{h}_j^{\mathrm{T}}\mathbf{h}_j)^2}$$

$$\mathbf{Y}^{\mathrm{T}}\mathbf{P}_m^2\mathbf{Y} = \mathbf{Y}^{\mathrm{T}}\mathbf{Y} - \sum_{j=1}^{m} \frac{(2\lambda + \mathbf{h}_j^{\mathrm{T}}\mathbf{h}_j)(\mathbf{Y}^{\mathrm{T}}\mathbf{h}_j)^2}{(\lambda + \mathbf{h}_j^{\mathrm{T}}\mathbf{h}_j)^2}$$

$$\text{trace}(\mathbf{P}_m) = p - \sum_{j=1}^{m} \frac{\mathbf{h}_j^{\mathrm{T}}\mathbf{h}_j}{\lambda + \mathbf{h}_j^{\mathrm{T}}\mathbf{h}_j}$$

4.5 Model order selection

The model order selection criterion does not employ the mean squared error to judge the optimal model order, i.e. the number of regressors producing the lowest value for the mean squared error in the model predicted output. Rather it uses the generalized cross-validation (GCV) error, which is based not merely on the ability of the model to fit the training data but on future inputs which ideally are unseen as far as the fitting is concerned. This is also described in detail in reference [**19**]. GCV is a variant of cross-validation whereby the data are divided into two parts: one part is used for fitting the parameters and the other for assessing the model structure that performs best on the section of the data not used for fitting. The partitioning process is repeated many times, removing a different section of the data for testing (validation) each time. This helps to avoid the problem of overfitting mentioned previously. An analytical, closed-form expression exists for the GCV error of a linear network. This is given by

$$\hat{\sigma}^2_{\text{GCV}} = \frac{\mathbf{P}\mathbf{Y}^{\mathrm{T}}\mathbf{P}\mathbf{Y}}{[\text{trace}(\mathbf{P})]^2} \tag{31}$$

Other model order selection criteria exist such as the unbiased estimate of variance, the final prediction error and the Bayesian information criterion. Again, see reference [**19**] for more details. The forward selection procedure for addition of centres is terminated on the basis of the GCV error. After GCV has reached a minimum, forward selection continues for a further two centres to provide some assurance that it is not a local minimum in the optimization procedure that has been reached. However, since it is impossible to know for certain the location of local minima, the decision may well be suboptimal.

5 SUMMARY

RBF networks offer a flexible and robust approach to the typification of non-linear engine processes. The representation of the data is translucent in so much as the RBF neurons can be regarded as locally applicable submodels which are linearly combined to reflect the behaviour of the process over the training data range. Moreover, by the use of stabilization techniques, such as regularization, the model can be made robust. Part 2 of this paper deals in detail with the non-intrusive estimation of diesel engine cylinder pressure and presents experimental results which highlight the potential that this technique offers to the improvement of engine performance and condition monitoring.

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