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Optical and Thrust Measurement of a Pulse Detonation Combustor with a Coaxial Rotary Valve

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Abstract

We developed a rotary valve for a pulse detonation engine (PDE), and confirmed its basic characteristics and performance. In a square cross-section combustor, we visualized a multi-shot of a pulse detonation rocket engine (PDRE) cycle at an operation frequency of 160 Hz by using a high-speed camera (time resolution: 3.33 μsec, space resolution: 0.4 mm) and a Schlieren method. The propellant filling process and the purge process were confirmed, and each process was modeled. Moreover, we confirmed the processes of detonation wave generation and burned gas blowdown. In addition, we investigated the impact of shortening the passage width of a combustor and negative-time ignition (ignition time is earlier than the end-time of the propellant filling process) on the deflagration-to-detonation transition (DDT) distance and time. The DDT distance did not depend on the passage width of a combustor and decreased under the negative-time ignition condition. With a passage width of 20 mm, the DDT distance decreased by 22% under the negative-time ignition condition to a minimum value (76 ± 8 mm). The DDT time from spark time reached a minimum value (69 ± 14 μsec) under the condition of a passage width of 10 mm and negative-time ignition. The detonation initiation time and the DDT distance were represented by the time until the flame expanded toward the tube-axis one-dimensionally from ignition (characteristic time). We also carried out thrust measurement using a PDRE system composed of a circular cross-section combustor and the newly developed valve. We obtained a stable time-averaged thrust in a wide range of operation frequency (40 Hz - 160 Hz) and confirmed the increase of specific impulse due to a partial-fill effect.
At a maximum operation frequency of 159 Hz, we achieved a maximum propellant-based specific impulse of 232 sec and a maximum time-averaged thrust of 71 N. (300 words)

**Keywords**

pulse detonation engine; pulse detonation rocket engine; rotary valve; deflagration-to-detonation transition

**Nomenclature**

- $A$: Cross-section area
- $C_t$: Experimental Constant
- $d$: Diameter of gas-supply port
- $d_e$: Effective diameter of purge gas-supply port
- $D_{CJ}$: Chapman-Jouguet detonation velocity
- $f_{exp}$: Experimental operation frequency
- $f_{set}$: Experimental setup operation frequency
- $F_{cal}$: Calculated time-averaged thrust
- $F_{exp}$: Experimental time-averaged thrust
- $F_{lc}$: Load cell output
\( g \) Gravitational acceleration

\( I_{sp, \text{cal}} \) Calculated propellant-based specific impulse

\( I_{sp, \text{exp}} \) Experimental propellant and purge gas-based specific impulse

\( I_{sp, \text{exp, all}} \) Experimental propellant-based specific impulse

\( I_{sp, \text{FF}} \) Propellant-based specific impulse with a full-filled propellant

\( I_{sp, \text{PF}} \) Propellant-based specific impulse with a partial-filled propellant

\( l_{\text{DDT}} \) DDT distance from the closed end \((l_{\text{DDT}} = x_{\text{DDT}} - x_{\text{wall}})\)

\( l \) Distance to the rotating center from the supply-port center (turning radius)

\( l_p \) Propellant fill length

\( l_i \) Purge gas fill length

\( \bar{l}_w \) Averaged distance of ignition point to the four walls parallel to the x-axis

\( m \) Experimental mass of one cycle

\( \dot{m} \) Experimental mass flow rate

\( M \) Molecular mass of gas

\( M_0 \) Shock Mach number on the wall before diffraction

\( M_1 \) Mach number at surface 1 of the abrupt expansion

\( M_2 \) Mach number at surface 2 of the abrupt expansion

\( M_w \) Shock Mach number on the wall after diffraction
\( p \) Absolute pressure

\( r \) Opening-ratio of the supply port

\( R \) Gas constant

\( S_{\text{input}} \) Spark input signal

\( S_{\text{output}} \) Spark output signal

\( t_{\text{DDT}} \) Time to DDT occurrence from the start time of propellant supply

\( t_{\text{ope}} \) Operation time interval

\( t_{\text{spark}} \) Spark time by spark plug from the start time of propellant supply

\( t_{\text{stop}} \) Stop time of propellant supply from the start time of propellant supply

\( \tau \) Open time of each gas supply port per one PDE cycle.

\( \tau_{c} \) Arrival time at which flame front arrives at \( \bar{l}_w \) from ignition time (characteristic time)

\( \tau_{CJ} \) Time at which the detonation wave reaches the open-tube end of a combustor

\( \tau_{\text{exhaust}} \) Time at which \( p_{\text{wall}} = p_0 \) from ignition time

\( \tau_{\text{ini}} \) Initiation time of the detonation wave from ignition time

\( \tau_{\text{plateau}} \) Time at which plateau in pressure history at the closed-end wall from ignition time

\( \tau_{\text{spark-DDT}} \) Time to DDT occurrence from ignition time (\( \tau_{\text{spark-DDT}} = t_{\text{DDT}} - t_{\text{spark}} \))

\( T \) Absolute temperature

\( \Delta t \) Time interval of the sequence of Schlieren photographs
\( u \)  
Gas-flow velocity inside a combustor

\( V \)  
Volume

\( W_{\text{tube}} \)  
Square width of a combustor in the visualization experiment

\( x_{\text{DDT}} \)  
DDT distance from the origin

\( x_{\text{rf}} \)  
Front boundary of the exhausting rarefaction wave

\( x_{\text{tip}} \)  
Jet-tip distance from the origin

\( Y \)  
Propellant mass fraction per total mass inside a combustor

Greek symbols

\( \alpha \)  
Success rate of generation of the detonation wave \((0.9 \, D_{\text{CJ}})\)

\( \beta \)  
Averaged-flame-velocity ratio \((\text{averaged-flame velocity} / D_{\text{CJ}})\)

\( \phi \)  
Equivalent ratio of propellant

\( \gamma \)  
Ratio of specific heat

\( \lambda \)  
Opening-time ratio of supply port per one PDE cycle

\( \theta_{w} \)  
Diffraction angle \((-90^\circ)\)

\( \rho \)  
Gas density

\( \xi \)  
Time-averaged total pressure loss ratio per one PDE cycle

\( \psi \)  
Gas fill fraction
Subscripts

- **a**: Ambient
- **b**: Burned gas
- **f**: Fuel
- **i**: Purge gas
- **o**: Oxidizer
- **p**: Propellant
- **s**: Supply
- **t**: Total

- **tube**: Detonation tube (combustor)
- **valve**: Between a rotary valve and a combustor
- **wall**: Closed-tube end wall of a combustor

- **1**: Inlet of the abrupt expansion (surface 1)
- **1st**: First PDE cycle
- **2**: Outlet of the abrupt expansion (surface 2)
- **2nd**: Second and higher PDE cycles

- **0**: Atmosphere
* Throat
1. Introduction

Many fundamental studies of detonation waves [1-3] and of heat engines using detonation waves [4] have been carried out. A pulse detonation engine (PDE) is such a typical heat engine. A PDE can obtain nearly constant thrust and perform mechanical work by the generation of a high-frequency detonation wave [5-18]. The PDEs are classified as air-breathing PDEs [19-22] or pulse detonation rocket engines (PDREs) [23-25]. With a simplified PDE setup, that is, when viscosity and heat conduction are neglected and a tube-geometry combustor is fully filled with propellant at ambient pressure and a detonation wave is initiated instantaneously, a PDRE using H$_2$-O$_2$ propellant can achieve a specific impulse of 190 sec and an air-breathing PDE using hydrogen fuel can achieve a specific impulse of 4200 sec, as achieved in experiments by Schuer et al. [26], Cooper et al. [27] and Wintenberger et al. [28], numerical analysis by Wintenberger et al. [29] and theoretical analysis by Endo et al. [30]. Moreover, according to Cooper et al. [31] and Sato et al. [32], a PDE can achieve a dramatically high specific impulse by performing a partial fill of the propellant. Numerical analysis by Morris et al. [23] and an experiment by Copper et al. [33] showed that the expansion effect of by nozzle became dominant, like a steady flow, if the ambient pressure around a PDE was sufficiently low. Kasahara et al. [34] showed the physical essence of the partial-fill effect in an experiment with a shock-tube-shaped ballistic pendulum and by numerical analysis. Not only the above-mentioned fundamental studies but also validations of PDE systems have been carried out as application studies.
For example, there has been a performance validation of a PDRE system (Todoroki) by Kasahara et al. [25] and a flight demonstration of a PDE system by Hoke et al. [35].

A PDE can generate high pressure without compression mechanisms, such as compressors and pistons, and a PDRE can achieve a higher thrust-weight ratio compared to a conventional rocket engine. The high thrust-weight ratio is achieved by reduction in the weight of the system, the large flow of propellant and the use of a high-frequency operation valve. A wide variety of valve systems for PDEs have been proposed by Hinkey et al. [36], Baklanov et al. [37], Golub et al. [38, 39] and Matsuoka et al. [40, 41]. Matsuoka et al. proposed an inflow-driven valve. Because this valve can convert the enthalpy of propellant into the kinetic energy of a piston and generate intermittent flow, external power is not necessary. Hinkey et al. proposed a rotary valve [36] suitable for higher-frequency operation and larger flow compared to a reciprocating valve such as a solenoid valve, because the rotary valve can generate intermittent flow by its rotation motion. Cutler et al. [42, 43] measured the thrust of a rotary-valved PDE at high operation frequency. When the operation frequency of the propellant supply was identical to the wave frequency in a combustor (optimum condition), a detonation wave was generated just after supplying of the propellant stopped, and 85% of the theoretical thrust was obtained at an operation frequency of 580 Hz. Studies of a rotary-valved PDE have been conducted by General Electric and Pratt & Whitney. However, many of the detailed thrust characteristics of the valve have not been made public.
During high-frequency operation, reduction of the deflagration-to-detonation transition (DDT) distance, $l_{DDT}$, is a critical issue. The DDT process is necessary to generate a detonation wave [1], because propellant is ignited by inputting a small amount of energy generated by a spark plug. Figure 1 shows the state inside a detonation tube (combustor) at the end of the propellant fill process. If the operation frequency increases with the constant mass flow rate, the propellant fill length, $l_p$, will decrease and the combustor volume can be reduced. However, Takeuchi et al. [44] reported that the specific impulse decreased when the length ratio ($l_{DDT} / l_p$) between the propellant fill length, $l_p$, and the DDT distance, $l_{DDT}$, decreased. They varied the initial fill pressure of the H$_2$-O$_2$ stoichiometric mixture and investigated the relation between the DDT distance and the specific impulse by a shingle-shot ballistic pendulum and found that the specific impulse rapidly decreased when the length ratio ($l_{DDT} / l_p$) was 0.9. Cooper et al [27] also varied the nitrogen-dilution ratio of the mixture and investigated the impact of DDT distance on the specific impulse using a shingle-shot ballistic pendulum. They found that the DDT distance increased as the nitrogen-dilution ratio increased, and the specific impulse decreased compared to a theoretical value. In addition, the specific impulse significantly decreased when the detonation wave did not occur. However, these studies were single-shot experiments. In contrast, in multi-shot thrust measurement experiments [26, 45, 46], the relationship between the DDT distance and the thrust performance was not investigated.
To operate at high frequency, we will need to carry out multi-shot thrust measurement with careful consideration of the impact of DDT distance on thrust performance.

Direct visualization of each process of a PDE cycle (propellant filling, the DDT process and purging) during high-frequency operation is also very important. Many visualization experiments of the DDT process in stationary gas have been carried out [1, 47-50]. However, in multi-shot experiments, the DDT form differs greatly depending on various factors such as the mixing and propellant fill processes (supply pressure, mixing method, etc.), spark timing and purge process, so prediction of the DDT process by numerical calculation is difficult. Although direct visualization of each process of a PDE cycle is necessary, the time constant of the propellant fill process and purge process is significantly larger than the time constant of the DDT process. Not only high resolution time but also many recorded images are required for a high-speed camera to visualize the multi-shot PDE cycle.

To meet these requirements, we carried out a visualization experiment of a multi-shot PDE cycle at a high operation frequency of 160 Hz by using a rotary-valved PDRE system, a Schlieren method and a high-speed camera. We investigated all of the processes comprising the PDE cycle and examined the DDT mechanism of the cycle in detail. We confirmed the reduction of the DDT distance by negative-time ignition. We also carried out a thrust measurement experiment using similar PDRE system. We measured the mass flow rate, thrust and specific impulse, and estimated
the thrust performance of a rotary-valved PDRE. We changed the propellant fill length, $l_p$, by employing a wide range of operation frequencies using the rotation-speed control of the rotary valve, and we investigated the impact of the length ratio ($l_{DDT} / l_p$) between the propellant fill length, $l_p$, and the DDT length, $l_{DDT}$, on the thrust performance.

2. Coaxial rotary valve

2.1. Design of a coaxial rotary valve

Figure 2 shows a schematic diagram of the coaxial rotary valve developed in the present study. This single valve can supply three different gases (fuel, oxidizer and purge gas) into a combustor. The valve is composed of a top cover, a casing and a rotation disk. The rotation disk is inserted into the interspace of the top cover and the casing, and the external torque is input from the rotating shaft attached to the rotation disk.

Figure 3 shows a cross-section diagram of a coaxial rotary valve. Since three different gases are supplied into the valve, there is a risk of gas mixing in the valve and gas leakage from the sliding surface of the rotation disk. To avoid gas leakage from between the top cover and the disk and between the casing and the disk, we mounted O-rings around the supply and exhaust ports of the top cover and the casing, and the O-rings are in contact with the sliding surface of the rotation disk. Moreover, we spread silicon oil over the sliding surfaces of the rotation disk. We constructed a
labyrinth seal to keep gas from mixing in the valve, by making four annular grooves in the top cover and the casing between the gas ports, and the rotation disk has four annular projections corresponding to the annular grooves. The gas mixing in the valve could be minimized due to the silicon oil flow into the very small clearance of the labyrinth structure.

In a PDRE, three valves are needed, for fuel, oxidizer and purge gas. However, the unification of these valves is possible by using our newly designed valve while maintaining the mass flow rate. With this valve, a PDRE system can be simple and lightweight, and thus its thrust-weight ratio can be increased.

2.2. One PDE cycle with a coaxial rotary valve

Figure 4 shows a schematic of the rotation disk position during each process of a PDE cycle viewed from the top cover side. In Fig. 4, the top cover and the casing have three through-holes. These holes are supply ports of the top cover and exhaust ports of the casing. The gray areas of the rotation disk show long holes of the disk corresponding to each supply port. When the rotation disk rotates and these long holes overlap with supply ports, gas is supplied and exhausted. If a constant valve rotating speed can be maintained, the time interval during which gas is supplied can be determined. As shown in Fig. 4, the Cartesian coordinate system was chosen. The origin of this coordinate system is the center of the rotation disk. The positive direction of the x-axis is the
direction of gas blowout, and the rotation disk rotates clockwise as viewed from the top cover side. The heavy arrowed line OA is the baseline, which is fixed in the rotation disk.

State (0) is the state just before the start of supplying propellant (C$_2$H$_4$-O$_2$). Process (i) is composed of the state just after the start of supplying propellant, the state of the propellant supply ports being fully open and the state just before the end of supplying propellant. Process (ii) is composed of the state just after the end of supplying propellant and the state just before the start of supplying purge gas, and all of the supply ports are closed. Process (iii) is composed of the state just after the start of supplying purge gas, the state of the purge gas supply port being fully open and the state just before the end of supplying purge gas. During this process, the burned gas in a combustor is purged. In the visualization experiment, propellant was ignited by a spark plug for process (i) or process (ii). A rotation disk makes a half turn by the end of process (iii) from state (0), and in that time one PDE cycle is completed. A full 360-degree turn of the rotation disk thus produces two PDE cycles.

The time ratios of the processes comprising a PDE cycle are 45% (0° ≤ θ ≤ 81°, 180° ≤ θ ≤ 261°) for process (i), 32% (81° ≤ θ ≤ 139°, 261° ≤ θ ≤ 319°) for process (ii) and 23% (139° ≤ θ ≤ 180°, 319° ≤ θ ≤ 360°) for process (iii). We carried out experiments based on this time ratio.

During process (ii), the maximum torque was generated, because all of the supply ports were closed. The range of torque at process (ii) was approximately 0.34-0.45 Nm. The rated torque of the electromagnetic motor used in the experiment was 1.91 N, and the value was 24% of the maximum
torque of the valve. In addition, the maximum torque and the power consumption of the motor were 5.73 Nm and 600 W, respectively.

It takes finite lengths of time until the supply port reaches the fully-open state (opening process) or the fully-closed state (closing process). The lengths of time vary, because the distances between the origin and the center of each supply port are different. In the case of an operation frequency of 160 Hz, this time interval is 0.25 msec for fuel (C₂H₄), 0.36 msec for oxidizer (O₂) and 0.65 msec for purge gas (He). During the opening process (closing process), there is abrupt expansion in the flow passage due to the rotating disk, and total pressure loss occurs. Therefore, using the model of total pressure loss at the abrupt expansion of a flow passage [51], we estimated the pressure drop during the opening and closing processes. As shown in Fig. 5, the flow passage in the valve was closed partially by the sufficiently thin rotating disk, and the cross-section area was expanded abruptly from $A₁$ to $A₂$. The dashed line shown in Fig. 5 indicates the control volume. The flow passage inlet was surface 0, the interrupt surface of the rotating disk was surface 1 and the flow passage outlet was surface 2. First, we assumed that the flow was an inviscid quasi-one-dimensional steady flow and the fluid was a thermally and calorically perfect gas. Next, we assumed that the flow was accelerated to sound speed from surface 0 to surface 1 isentropically and that the pressure loss occurred between surface 1 and surface 2. Then, the total pressure ratio ($p_{t, 2}/p_{t, 1}$) between surface 1 and surface 2 was expressed as follows [51]:
If the start time of each gas supply was 0 sec, the time-averaged total pressure loss ratio, $\xi$, was defined as follows:

$$\xi = \frac{1}{\tau} \int_0^\tau \left( 1 - \frac{P_{t,2}}{P_{t,1}} \right) dt$$

(2)

where $\tau$ was the open time of each gas supply port per one PDE cycle. Next, we assumed that the pressure difference between surface 0 and surface 2 was significantly large and that the Mach number at surface 1 was $M_1 = 1$. The Mach number at surface 2, $M_2$, generally depended on the downstream condition of surface 2. However, we assumed $M_2 = 1$ because total pressure loss became the maximum value at $M_2 = 1$ [51].

Substituting $M_1 = M_2 = 1$ into Eq. (1), we rewrote the total pressure ratio, $p_{t,2}/p_{t,1}$, according to the ratio of the cross-section area, $A_1/A_2$, and the time-averaged total pressure loss, $\xi$, was given from the following equation:

$$\xi = \frac{1}{\tau} \int_0^\tau \left[ 1 - \frac{A_1}{A_2} \right] dt$$

(3)

In this case, open time, $\tau$, was expressed as follows:

$$\tau = \frac{\lambda}{f_{\text{set}}}$$

(4)

where $\lambda$ was the open-time ratio of the supply port per one PDE cycle ($\lambda_0 = 0.45$, $\lambda_f = 0.4$, $\lambda_i = 0.23$).

Here, $f_{\text{set}}$ was the experimental setup operation frequency. The cross-section area of surface 1, $A_1$,
was changed linearly from 0 mm$^3$ to 19.6 mm$^3$ (fully-opened state) during the opening process. Similarly, $A_1$ was changed linearly from 196 mm$^3$ to 0 mm$^3$ during the closing process. The cross-section area of surface 2, $A_2$, was constant at 19.6 mm$^3$ ($d = 5$ mm).

In this experimental setup, $\xi$ was not dependent on the operation frequency and was 0.09 for ethylene, 0.13 for oxygen and 0.23 for helium. If the distance to the rotating center from the supply-port center (turning radius, $l$) and opening-time ratio of the supply port per one PDE cycle, $\lambda$, increased and the supply port diameter, $d$, decreased, it was possible to decrease the total pressure loss ratio, $\xi$. For example, if the oxygen turning radius, $l_o$, of this rotary valve ($\lambda_o = 0.45$, $d_o = 5$ mm, $l_o = 27.4$ mm) increased to $l_o = 70$ mm, the pressure loss ratio, $\xi$, decreased to $\xi = 0.05$ from 0.13.

3. Visualization experiment of a PDRE

3.1. Experimental setup and conditions

The inside of a combustor in a multi-shot PDE cycle was visualized using the rotary valve described in the previous section, a high-speed camera and a Schlieren method. Figure 6 shows a schematic diagram of the experimental apparatus (passage width is $W_{tube} = 20$ mm). As shown in Fig. 6, the Cartesian coordinate system was chosen, and these coordinates were the same as the coordinates in Fig. 4. The center axis of the rectangular combustor was the $x$-axis, and the direction of the open-tube end from the origin was in the positive direction of the $x$-axis. The cross-section of
the combustor was square. The position of the closed-tube end wall was $x_{\text{wall}} = 49$ mm, and the position of the open-tube end was 176 mm. The purge gas was supplied to the combustor at the closed end. The central axis of the purge gas port was the $x$-axis, and the diameter of the purge gas supply port was $d_i = 10$ mm. The position coordinates of the oxidizer-port center were $x = 59$ mm, $y = W_{\text{tube}}/2$ mm, $z = 0$ mm, and the position coordinates of the fuel-port center were $x = 59$ mm, $y = -W_{\text{tube}}/2$ mm, $z = 0$ mm. The diameters of the propellant supply ports were $d_o = d_i = 5$ mm. Oxidizer and fuel were mixed by opposing jets, and the combustor was filled with the mixture. The position coordinates of the spark plug were $x = 86$ mm, $y = W_{\text{tube}}/2$ mm, $z = 0$ mm. The locations of each supply port and the spark plug were largely similar to the locations of the ports and spark plug in the PDRE system used in the thrust experiment. The observation area of the combustor was $49$ mm $\leq x \leq 161$ mm, $-W_{\text{tube}}/2$ mm $\leq y \leq W_{\text{tube}}/2$ mm, and the area was $112W_{\text{tube}}$ mm$^2$. The observation window was made of 20-mm-thick silica glass. Pressure gauges were mounted in the pipe between the rotary valve and the combustor, and the pressure between the rotary valve and the combustor (upstream pressure, $p_{\text{valve}}$) was measured. We confirmed the valve operation by monitoring $p_{\text{valve}}$ of each gas and the pressure rise by combustion.

The propellant supply pressure was $p_{s,p} = 2$ MPa. The orifice was mounted upstream of the fuel supply port to control the equivalent ratio. The purge gas supply pressure was set to $p_{s,i} = 1.2$ MPa. The minimum pipe diameter was 4 mm for oxygen and purge gas. This diameter was the orifice
diameter of the solenoid valve installed upstream of the valve. The operation time interval, $t_{\text{ope}}$, was
controlled by this solenoid valve. The operation time interval was $t_{\text{ope}} = 30$ msec under all of the
conditions, and about 5 repetitions of a PDE cycle were completed. Table 1 shows the experimental
conditions. We carried out a visualization experiment under four conditions that were combinations
of two combustor passage widths ($W_{\text{tube}} = 20$ mm, 10 mm) and two kinds of spark timing
(positive-time ignition condition and negative-time ignition condition). When the time at state (0)
was $t = 0$ msec, the end-time of propellant supply was constant at $t_{\text{stop}} = 2.8$ msec, because the
experimental setup operation frequency was constant at $f_{\text{set}} = 160$ Hz. Based on this time, we used
the term “positive-time ignition” when the spark was made at a positive time compared to the base
time ($t_{\text{stop}} < t_{\text{spark}}$) and the term “negative-time ignition” when the spark was made at a negative time
compared to the base time ($t_{\text{stop}} > t_{\text{spark}}$). In addition, the spark time was $t_{\text{spark}} = 3.6 \pm 0.2$ msec under
the positive-time ignition condition and $t_{\text{spark}} = 2.6 \pm 0.2$ msec under the negative-time ignition
condition. Under the negative-time ignition condition ($t_{\text{stop}} > t_{\text{spark}}$), the spark was made while
propellant was being supplied to the combustor.

We used a Schlieren method. The $x$-axially-changed density and $y$-axially-changed density were
visualized by knife-edges. The high-speed camera used in this experiment was the FASTCAM SA5
(made by Photron, Ltd.), the interframe gap and the exposure time were 3.33 µsec and 370 nsec,
respectively.
3.2. Monitoring result of one PDE cycle

Figure 7 shows the spark output signal, $S_{\text{output}}$, upstream pressure of the combustor, $p_{\text{valve, o}}$ (oxidizer), $p_{\text{valve, f}}$ (fuel), $p_{\text{valve, i}}$ (purge gas), spark input signal, $S_{\text{input}}$, and supply-port opening ratio, $r_o$ (oxidizer), $r_f$ (fuel), $r_i$ (purge gas) in one PDE cycle under the positive-time ignition condition (cycle no. V-1-3). $S_{\text{input}}, r_o, r_f, r_i$ was calculated based on the time $t = 0$ msec at which $p_{\text{valve, o}}$ began to rise. In addition, Fig. 8 shows $S_{\text{output}}, p_{\text{valve, o}}, p_{\text{valve, f}}, p_{\text{valve, i}}, S_{\text{input}}, r_o, r_f, r_i$ under the negative-time ignition condition (cycle no. V-2-1).

In Figs. 7 and 8, the propellant supply ports first began to open at state (0). These ports opened gradually, with the opening ratio reaching $r_o = r_f = 1$ at $t = 0.25$ msec for fuel ($\text{C}_2\text{H}_4$) and at $t = 0.36$ msec for oxidizer ($\text{O}_2$). From $t = 0$ msec to $t = 1.5$ msec, the upstream pressure of oxidizer, $p_{\text{valve, o}}$, and the upstream pressure of fuel, $p_{\text{valve, f}}$, increased gradually, and the pressures rose to almost constant at $t = 2.0$ msec. After that, the pressures gradually decreased after the end-time of propellant supply, $t_{\text{stop}} = 2.8$ msec. At the spark time, $t_{\text{spark}}$, the spark output signal, $S_{\text{output}}$, was measured. This result showed that $S_{\text{input}}$ was turned on at the same time. Under the positive-time ignition condition (cycle no. V-1-3), the spark time was $t_{\text{spark}} = 3.59$ msec ($t_{\text{stop}} < t_{\text{spark}}$), and the spark was done when $p_{\text{valve, o}} = 0.23$ MPa (47% of the maximum). In contrast, under the negative-time ignition condition (cycle no. V-2-1), the spark time was $t_{\text{spark}} = 2.55$ msec ($t_{\text{stop}} > t_{\text{spark}}$), and the spark was done when
the \( p_{\text{valve},o} = 0.41 \) MPa (89% of the maximum). The spark was made when the propellant was supplied to the combustor or just after the end of propellant supplying. In Fig. 7, during \( 3.8 \text{ msec} \leq t \leq 4.2 \text{ msec} \), all of \( p_{\text{valve}} \) rapidly increased. In Fig. 8, during \( 2.8 \text{ msec} \leq t \leq 3.2 \text{ msec} \), all of \( p_{\text{valve}} \) rapidly increased. This pressure rise was due to the arrival of a retonation wave at the pressure gauge ports. After that, the purge gas supply port began to open from \( t = 4.8 \text{ msec} \), and the supply-port opening ratio of the purge gas port became \( r = 1 \) after 0.65 msec. The upstream pressure of purge gas, \( p_{\text{valve},i} \) gradually began to increase when the purge gas supply port began to open. \( p_{\text{valve},i} \) decreased before it became constant, because the fully open state was 10% of time interval of process (iii). The valve reached state (0) when the purge gas supply port closed completely. In process (i) at \( t > 6.25 \text{ msec} \), \( p_{\text{valve},o} \) and \( p_{\text{valve},r} \) increased again. We confirmed a multi-shot PDE cycle at 160 Hz by monitoring the upstream pressure of the combustor, \( p_{\text{valve}} \), and the spark output signal, \( S_{\text{output}} \).

### 3.3. Visualization of one PDE cycle

Figure 9 shows a sequence of Schlieren photographs illustrating one PDE cycle under the positive-time ignition condition (cycle no. V-1-3), and Fig. 10 shows a sequence of Schlieren photographs illustrating one PDE cycle under the negative-time ignition condition (cycle no. V-2-1). We reversed the color to make these images more visible. Here, \( t = 0 \text{ msec} \) was state (0). The time interval of the sequence of Schlieren photographs was \( \Delta t = 0.15 \text{ msec} \).
3.3.1. State (0)

Although all of the supply ports were closed at state (0) in Figs. 9 and 10 \((t = 0.00 \text{ msec})\), the jet and the jet impinging plane of the previous cycle were observed. In Figs. 9 and 10, the jet diameters around the center of the oxidizer supply port \((x = 59 \text{ mm}, y = +10 \text{ mm}, z = 0 \text{ mm})\) and the center of fuel supply port \((x = 59 \text{ mm}, y = -10 \text{ mm}, z = 0 \text{ mm})\) were approximately 5.6 mm, as shown by the arrows at \(t = 0.00 \text{ msec}\) in Figs. 9 and 10. These jet diameters were nearly equal to the diameter of the propellant supply ports, \(d_o = d_f = 5 \text{ mm}\). This diametral jet injected toward the negative direction of the \(y\)-axis for oxidizer and the positive direction of the \(y\)-axis for fuel.

3.3.2. Process (i)

The period \(0.15 \text{ msec} \leq t \leq 2.70 \text{ msec}\) in Figs. 9 and 10 shows the sequence of Schlieren photographs illustrating process (i). In this process, the impinging plane between the oxidizer jet and the fuel jet was observed. The center of the impinging plane oscillated in the \(y\)-axis at the center position of the propellant supply port \((x = 59 \text{ mm})\). Just after the start of supplying propellant \((t = 0.15 \text{ msec})\), the center position of the impinging plane was \(x = 59 \text{ mm}, y = 4 \text{ mm}\) in Fig. 9 and \(x = 59 \text{ mm}, y = -2 \text{ mm}\) in Fig.10. After that, the center position of the impinging plane moved in the \(-y\)
direction. This could have been because the supply pressures were the same but the oxidizer mass flow rate was 3 times higher than the fuel mass flow rate.

The propellant filling in process (i) was modeled as follows. First, all of the gases were assumed to be thermally and calorically perfect gases. If the oxidizer was isentropically accelerated from a supply tank and the oxidizer was choked at the throat (4 mm) of the solenoid valve, the oxidizer mass flow rate was obtained by the following equation:

\[ \dot{m}_o = \frac{\lambda_o}{\sqrt{R_o T_{s,o}}} \left( \frac{2}{\gamma_o + 1} \right)^{\frac{\gamma_o+1}{\gamma_o-1}} \]  

where \( \lambda_o \) was the opening-time ratio of the oxidizer supply port per one PDE cycle (\( \lambda_o = 0.45 \)), \( p_{s,o} \) was the internal pressure of the oxidizer supply tank (\( p_{s,o} = 2.0 \) MPa), \( A_{o}^* \) was the throat area of the oxidizer pipe (\( A_{o}^* = 12.6 \) mm\(^2\)), \( R_o \) was the oxidizer gas constant (\( R_o = 259.8 \) J/(kg·K)), \( T_{s,o} \) was the internal temperature of the oxidizer supply tank (\( T_{s,o} = 283.15 \) K) and \( \gamma_o \) was the ratio of specific heat of the oxidizer (\( \gamma_o = 1.39 \)). As determined using Eq. (5), the oxidizer mass flow rate was \( \dot{m}_o = 28.5 \) g/sec. Here, we assumed the propellant was a stoichiometric mixture (\( C_2H_4+3O_2 \)), and the fuel mass flow rate was \( \dot{m}_f = 8.3 \) g/sec, so the propellant mass flow rate was \( \dot{m}_p = \dot{m}_o + \dot{m}_f = 36.8 \) g/sec.

Next, if the propellant flow in the combustor was an incompressible one-dimensional steady flow, the propellant flow velocity in the combustor was expressed as follows:
where the propellant pressure in the combustor was \( p_a = 0.1013 \text{ MPa} \), the propellant temperature was \( T_a = 283.15 \text{ K} \), the propellant gas constant was \( R_p = 268.2 \text{ J/(kg·K)} \) and the cross-section area of the combustor was \( A_{\text{tube}} = 400 \text{ mm}^2 \) (\( W_{\text{tube}} = 20 \text{ mm} \)). As determined using Eq. (6), the propellant-flow velocity in the combustor was \( u_p = 69 \text{ m/sec} \). The front of the propellant flow is indicated by the dashed line in the period \( 0.00 \text{ msec} \leq t \leq 1.35 \text{ msec} \) in Figs. 9 and 10. At \( t = 0.90 \text{ msec} \) in Figs. 9 and 10, the strong light-dark distributed structure was observed over the entire combustor compared to the light-dark distributed structure at \( t = 0.15 \text{ msec} \) in Figs. 9 and 10. From these results, we estimated that the \( x \)-axial velocity of the propellant filling was about 100 m/sec faster than \( u_p \). This difference was thought to have occurred because the actual flow was 3-dimensional, and the internal temperature of the combustor was raised in a multi-shot operation and the apparent flow velocity was increased due to the increased propellant volume.

3.3.3. Process (ii)

The period \( 2.85 \text{ msec} \leq t \leq 4.80 \text{ msec} \) in Figs. 9 and 10 shows the sequence of Schlieren photographs illustrating process (ii). During the period \( 2.85 \text{ msec} \leq t \leq 3.45 \text{ msec} \) in Fig. 9 (before the spark time \( t_{\text{spark}} = 3.59 \text{ msec} \)), jets from the propellant supply ports and jet impinging plane were observed. However, the light-dark distributed structure was weak compared to the strong light-dark
distributed structure observed in propellant filling process (process (i)). These jets could be the remaining gas in the pipe or a small amount of leaked gas from the sliding surface of the rotation disk.

During the period $4.65 \text{ msec} \leq t \leq 4.80 \text{ msec}$ in Fig. 9 (after the spark time $t_{\text{spark}} = 3.59 \text{ msec}$) and the period $3.45 \text{ msec} \leq t \leq 4.80 \text{ msec}$ in Fig. 10 (after the spark time $t_{\text{spark}} = 2.55 \text{ msec}$), jets from the propellant supply ports were observed. These jets could be burned gas or oxidizer and fuel, the supply of which to the combustor had been stopped temporarily due to the inside of the combustor being under high pressure from the generation of the detonation wave. Under the positive-time ignition condition in Fig. 7, the oxidizer upstream pressure just before the pressure rise resulting from the retonation wave was $p_{\text{valve, o}} = 0.12 \text{ MPa}$. In contrast, under the negative-time ignition condition in Fig. 8, the oxidizer upstream pressure just before the pressure rise resulting from the retonation wave was $p_{\text{valve, o}} = 0.29 \text{ MPa}$ (242% of the pressure in the positive-time ignition condition), indicating that the density of the negative-time-ignition gas, the supply of which to the combustor was stopped temporarily, was higher than the density of the positive-time-ignition gas. Moreover, the weak light-dark distributed structure was observed at $t = 4.80 \text{ msec}$ in Fig. 9, and this distributed structure was similar to the light-dark distributed structure in process (i). The appearance of the distributed structure indicates that the combustor was filled with propellant. This light-dark distributed structure can be discriminated clearly at $4.95 \text{ msec} \leq t \leq 5.25 \text{ msec}$ in process (iii). In
contrast, during the period $3.15 \text{ msec} \leq t \leq 4.80 \text{ msec}$ in Fig. 10, the above-mentioned jet impinging plane was observed, but the light-dark distributed structure was not observed. We think that the burned gas became the ignition source and the diffuse combustion occurred just after the oxidizer-fuel impingement.

3.3.4. Process (iii)

The period $4.95 \text{ msec} \leq t \leq 6.00 \text{ msec}$ in Figs. 9 and 10 shows the sequence of Schlieren photographs illustrating process (iii). Unlike the propellant, which was supplied from the direction perpendicular to the $x$-axis, the purge gas was supplied from the center of the purge gas supply port ($x = 49 \text{ mm}, y = 0 \text{ mm}, z = 0 \text{ mm}$) toward the positive direction of the $x$-axis.

The diameter of the purge gas supply port ($d_i = 10 \text{ mm}$) is shown by the arrows at $t = 4.95 \text{ msec}$ in Figs. 9 and 10, and as the figures show, the jets of oxidizer and fuel drifted toward the positive direction of the $x$-axis. We think this was due to the helium jet that was injected from the purge gas supply port.

The purge process was modeled in the same way as the propellant fill process. From Eq. (5), the purge gas mass flow rate was $\dot{m}_i = 3.3 \text{ g/sec}$. In this process, $\lambda_i$ was the opening-time ratio of the purge gas supply port per one PDE cycle ($\lambda_i = 0.23$), $p_{s,i}$ was the internal pressure of the purge gas supply tank ($p_{s,i} = 1.2 \text{ MPa}$), $A_i^*$ was the throat area of the purge gas ($A_i^* = 12.6 \text{ mm}^2$), $R_i$ was the
gas constant of the purge gas \((R_i = 2078.5 \text{ J/(kg·K)})\), \(T_{s,i}\) was the internal temperature of the purge gas supply tank \((T_{s,i} = 283.15 \text{ K})\) and \(\gamma_i\) was the ratio of the specific heat of the purge gas \((\gamma_i = 1.67)\).

From Eq. (6), the purge gas-flow velocity at the supply port \((x_{\text{wall}} = 49 \text{ mm})\) was \(u_i = 244 \text{ m/sec}\).

Here, we assumed that the purge gas pressure and temperature in the combustor was \(p_a = 0.1013 \text{ MPa}\) and \(T_a = 283.15 \text{ K}\), respectively. The Reynolds number at the purge gas supply port was approximately 21000, and the purge gas jet that was injected into the combustor could be a disturbed flow. Here, the characteristic linear dimension was the diameter of the purge gas supply port \((d_i = 10 \text{ mm})\), the mean velocity was the purge gas jet velocity \((u_i = 244 \text{ m/sec})\) and the kinematic viscosity was 116 mm\(^2\)/s.

If the purge gas jet was an incompressible two-dimensional turbulent jet, the trajectory of the jet tip on the \(x\)-axis, \(x_{\text{tip}}\), was expressed as follows by Abraham [52]:

\[
x_{\text{tip}} = \frac{4.2d_iu_i}{16\pi^{0.5}C_i} + x_{\text{wall}}
\]

where \(u_i\) was the purge gas jet velocity at the purge gas supply port \((u_i = 244 \text{ m/sec})\), \(C_i\) was the experimental constant \((C_i = 0.0161 [53])\), \(d_e\) was the effective diameter. This diameter was given as follows:

\[
d_e = d_i \sqrt{\frac{\rho_i}{\rho_a}}
\]

where \(d_i\) was the diameter of the purge gas supply port \((d_i = 10 \text{ mm})\), \(\rho_i\) was the density of the helium jet at atmospheric pressure \((\rho_i = 0.17 \text{ kg/m}^3)\) and \(\rho_a\) was the density of ambient gas, and the
ambient gas under positive-time ignition in Fig. 9 was the propellant at ambient pressure and
temperature ($\rho_i = 1.33 \text{ kg/m}^3$), and the ambient gas under negative-time ignition in Fig. 10 was a
constant-pressure-combustion gas under ambient pressure ($\rho_a = 0.09 \text{ kg/m}^3$). The trajectory of the jet
tip obtained from Eq. (7) is indicated by the dashed line during the period 4.95 msec $\leq t \leq 6.00$ msec
in Fig. 9 and during the period 4.95 msec $\leq t \leq 5.25$ msec in Fig. 10. Under the positive-time ignition
shown in Fig. 9, the combustor was filled with the burned gas, and the front of the helium jet was not
observed. This could be because the density difference between the burned gas and the helium jet
was small, and the boundary surface between the burned gas and the helium jet was not observed by
the Schlieren method. However, the propellant was pushed by the helium jet, and the front position
of this boundary surface was identical to $x_{\text{tip}}$ to within 10%. In contrast, under the negative-time
ignition shown in Fig. 10, the front of the helium jet was observed. The front position of the helium
jet was identical to $x_{\text{tip}}$ to within 10%. This model was applied to the free jet, but the model was
basically matched under the experimental condition of the present study.

3.4. Processes of ignition, DDT and blowdown of burned gas

3.4.1. Processes of ignition and DDT

Figure 11 shows the processes of ignition, DDT and blowdown of burned gas with
high-resolution-time Schlieren photographs under the positive-time ignition condition (cycle no.
V-1-3). We reversed the color to make these images more visible. The time in Fig. 11 was \( \tau (t-t_{spark}) \).

Here, the spark time was \( t_{spark} = 3.63 \) msec. The time interval of the sequence of Schlieren photographs was \( \Delta t = 0.02 \) msec during the period \( 0.00 \) msec \( \leq \tau \leq 0.16 \) msec, \( \Delta t = 0.00333 \) msec during the period \( 0.16 \) msec \( \leq \tau \leq 0.30000 \) msec and \( \Delta t = 0.03 \) msec during the period \( 0.30000 \) msec \( \leq \tau \leq 0.57 \) msec, respectively. The processes of ignition, DDT and blowdown of burned gas under the positive-time ignition condition went through the following stages:

At \( \tau = 0.00 \) msec, the propellant was ignited by the spark plug mounted on the side surface of the combustor (\( x = 86 \) mm, \( y = 10 \) mm, \( z = 0 \) mm).

During the period \( 0 \) msec \( < \tau \leq 0.16 \) msec, the deflagration wave (flame front, FF) propagated spherically. After that, the one-dimensional deflagration wave propagated in the open-tube end and the closed-tube end of the combustor after the deflagration wave arrived at the three walls (\( y = -10 \) mm, \( z = \pm 10 \) mm). Moreover, the compression wave (CW) was generated in front of the FF with increasing of the flame propagation velocity.

At \( \tau = 0.16333 \) msec, the unburned propellant was compressed by the closed-tube end and the CW, and the local explosion near the closed-tube end (LE_{left}) occurred with strong emission. During the period \( 0.16666 \) msec \( \leq \tau \leq 0.17666 \) msec, the fast combustion wave (FCW) with strong emission generated by the LE_{left} propagated toward the open-tube end along the top wall surface. This FCW temporarily became a fast shock wave (FSW) during the period \( 0.18000 \) msec \( \leq \tau \leq 0.18333 \) msec.
However, during the period $0.18666 \text{ msec} \leq \tau \leq 0.19333 \text{ msec}$, this FSW became the FCW again by the unburned gas that existed along the bottom surface of the combustor. According to Yageta et al. [54], this fast wave was the combustion wave that propagated through the compressed unburned gas in a corner of the orthogonal wall, and this combustion wave velocity was reported to be greater than $D_{CJ}$ under certain conditions when the propellant was ignited at around the wall surface of a rectangular cross-section combustor. When the LE$_\text{left}$ occurred ($\tau = 0.16333 \text{ msec}$), the local explosion near the position $x = 86 \text{ mm}$, $y = -10 \text{ mm}$ (LE$_\text{right}$) occurred, and the FCW by the LE$_\text{right}$ propagated with strong emission toward the open end of the tube along the bottom wall surface during the period $0.16666 \text{ msec} \leq \tau \leq 0.17666 \text{ msec}$. The velocity of FCW with strong emission was greater than $D_{CJ}$. We think the generating mechanism of this FCW by the LE$_\text{right}$ was the same as that of the combustion wave by the LE$_\text{left}$. This FCW wave caught up with the FF, and the transition to the detonation wave (DW) occurred during the period $0.18000 \text{ msec} < \tau < 0.18333 \text{ msec}$ (initiation of the DW).

During the period $0.18333 \text{ msec} \leq \tau \leq 0.19000 \text{ msec}$, the DW propagated toward the top wall surface ($y = 10 \text{ mm}$) and the open-tube end side (+x direction). The retonation wave (RW) generated by the DDT propagated toward the closed-tube end side (-x direction).

At $\tau = 0.18666 \text{ msec}$, the DW was reflected off the top wall ($y = +10 \text{ mm}$), and the transverse wave (TW) was generated. In $\tau \geq 0.18666 \text{ msec}$, this TW was reflected off the top and bottom walls
repeatedly. During the period $0.24666 \text{ msec} < \tau \leq 0.25666 \text{ msec}$ in Fig. 11, the propagation velocity of the TW was 1295 m/sec, and this velocity was nearly identical to the sound speed of the Chapman-Jouguet state (1283.8 m/sec [55]). At $\tau = 0.24000 \text{ msec}$, the RW reflected at the closed-tube end wall, and reflected retonation wave (RRW) propagated toward the open-tube end side during the period $0.24333 \text{ msec} \leq \tau \leq 0.31333 \text{ msec}$.

Figure 12 shows the processes of ignition, DDT and blowdown of burned gas with high-resolution-time Schlieren photographs under the negative-time ignition condition (cycle no. V-2-1). The spark time was $t_{\text{spark}} = 2.55 \text{ msec}$. The time interval of the sequence of Schlieren photographs was $\Delta t = 0.02 \text{ msec}$ during the period $0.00 \text{ msec} \leq \tau \leq 0.10 \text{ msec}$, $\Delta t = 0.00333 \text{ msec}$ during the period $0.10 \text{ msec} \leq \tau \leq 0.24666 \text{ msec}$ and $\Delta t = 0.03 \text{ msec}$ during the period $0.24666 \text{ msec} \leq \tau \leq 0.54666 \text{ msec}$. The processes of ignition, DDT and blowdown of burned gas under the negative-time ignition condition were qualitatively identical to the processes under the positive-time ignition condition. However, the early flame propagation velocity was fast ($0.00 \text{ msec} \leq \tau \leq 0.10 \text{ msec}$), and the distance between the FF and the CW was short compared to the positive-time ignition condition.

Under the negative-time ignition condition, the LE_{left} and the LE_{right} occurred at $\tau = 0.11000 \text{ msec}$. The FCW by LE_{left} was not observed, because the compressed-unburned gas in the corners of the orthogonal wall did not exist, and the FCW by LE_{right} was not observed because the location of the
LE_right was very close to the FF. The RW propagating toward the closed-tube end and the TW propagating toward the ±y direction were observed as in the positive-time ignition condition.

3.4.2 Shock wave

During the period $0.28000 \text{ msec} \leq \tau \leq 0.57 \text{ msec}$ in Fig. 11 and the period $0.22333 \text{ msec} \leq \tau \leq 0.54666 \text{ msec}$ in Fig. 12, a shock wave (SW) that propagated from the open-tube end side to the closed-tube end was observed. We think that this SW did not propagate into the combustor, because the reflected SW was not observed at $\tau = 0.48 \text{ msec}$ in Fig. 11 or at $\tau = 0.4266 \text{ msec}$ in Fig. 12. The DW was separated into the combustion wave and the SW at the open-tube end, and this SW diffracted, indicating that this SW could be the SW that propagated along the outer wall of the experimental apparatus shown in Fig. 6.

The Mach number on the wall of the SW after diffraction along the two-dimensional wall was expressed as follows [56]:

$$M_w = M_0 \exp\left(\frac{\theta_w}{\sqrt{n}}\right) \quad (9)$$

where $M_w$ was the Mach number on the wall of the SW after diffraction, $M_0$ was the Mach number on the wall of the SW before diffraction, $\theta_w$ was the diffraction angle of the wall and $\theta_w = -90^\circ$. In this experimental apparatus, $n$ was defined by the ratio of specific heat, as follows:

$$n = 1 + \frac{2}{\gamma} + \frac{2\gamma}{\sqrt{\gamma^2 - 1}} \quad (10)$$
It was assumed that all of the diffraction SW propagated through the air in this experiment, and \( n \) became 5.0743 from Eq. (10) (ratio of the specific heat of air \( \gamma_a = 1.4 \)). In Fig. 6, the first SW diffraction occurred at the open-tube end of the combustor \((x = 176 \text{ mm})\). If the SW velocity before diffraction was identical to the velocity of the DW, the Mach number on the wall of the SW before the first diffraction was \( M_0 = 7.8 \) under the positive-time ignition condition in Fig. 11, and \( M_0 = 6.9 \) under the negative-time ignition condition in Fig. 12. After the first diffraction, the SW propagated toward the \( \pm z \) direction along the open-tube-end \((x = 176 \text{ mm})\), and the second SW diffraction occurred at \( x = 176 \text{ mm}, z = \pm 38 \text{ mm} \). This diffraction SW propagated toward the closed-tube end side from the open-tube end. From Eq. (9), the Mach number on the wall of the SW after the second diffraction was \( M_w = 1.9 \) under the positive-time ignition condition in Fig. 11 and \( M_0 = 1.7 \) under the negative-time ignition condition in Fig. 12. The Mach number of the SW observed during the period \( 0.28000 \text{ msec} \leq \tau \leq 0.57 \text{ msec} \) in Fig. 11 was 1.1 (propagation velocity of the SW was 380 m/sec), and this Mach number was 58% of the model Mach number \((M_w = 1.9)\). The Mach number of the SW observed during the period \( 0.22333 \text{ msec} \leq \tau \leq 0.51666 \text{ msec} \) in Fig. 12 was 1.1 (propagation velocity of the SW was 377 m/sec), and this Mach number was 65% of the model Mach number \((M_w = 1.7)\). This difference between the experiment and the model was thought to be due to the fact that the actual SW diffraction occurred three-dimensionally relative to the two-dimensional model. The decay effect of the SW was strong, and the Mach number on the wall of the SW after diffraction
decayed strongly compared to the model. Moreover, the propagation velocity of the SW decayed with increasing the propagation distance, but this effect was not considered in this model. These SWs are important as the interference phenomena between combustors when a number of combustors are used (as in a multi-tube PDE).

3.5. Detailed analysis of the DDT process

3.5.1 x-t diagram of wave trajectory

Figure 13 shows the $x$-$t$ diagram of the combustion wave and the shock wave as shown in Fig. 11 (under the positive-time ignition condition) and the time-history of the pressure at the closed-tube end wall, $p_{\text{wall}}$, which was obtained by using the model of Endo et al. [30]. Figure 14 shows the $x$-$t$ diagram of the combustion wave and the shock wave as shown in Fig. 12 (under the negative-time ignition condition) and the time-history of the pressure at the closed-tube end wall, $p_{\text{wall}}$. In addition, the vertical time was $\tau (t - t_{\text{spark}})$ and the spark time was $t_{\text{spark}} = 3.59 \mu\text{sec}$ under the positive-time ignition condition and $t_{\text{spark}} = 2.55 \mu\text{sec}$ under the negative-time ignition condition. The $x$-$t$ diagram in Fig. 13 shows the FF, FCW and FSW resulting from LE$_{\text{left}}$, the FCW resulting from LE$_{\text{right}}$, the RW, the reflected retonation wave before the rarefaction wave pass-through (RRW1) and the reflected retonation wave after the rarefaction wave pass-through (RRW2) reflected from the closed-tube end wall, and the SW observed in Fig. 11. The $x$-$t$ diagram in Fig. 14 shows the FF, the
FSW resulting from LE_{left}, the RW, the RRW1 and RRW2 reflected at the closed-tube end wall, and the SW observed in Fig. 12.

In the $x$-$t$ diagram of Figs. 13 and 14, $x_{rf}$ was the trajectory of the front boundary of the exhausting rarefaction wave, which propagated toward the closed-tube end from the open-tube end after the DW reached the open-tube end [30]. The lines in the $x$-$t$ diagrams of Figs. 13 and 14 show the fitted lines obtained by using the least-squares method, and each propagation velocity is described in the figures. In addition, the reflected retonation wave was divided into the RRW1 and RRW2. In Figs. 13 and 14, $p_{\text{wall}}$ was calculated in consideration of measured $D_{\text{CJ}}$ and the stoichiometric C$_2$H$_4$-O$_2$ mixture. Moreover, in the model, the DW was generated immediately after the ignition, so we conformed the calculated time, $\tau_{\text{CJ}}$, at which the DW reached the open-tube end of the model to the experimental $\tau_{\text{CJ}}$.

$p_{\text{wall}}$ began to decay when the front boundary of the exhausting rarefaction wave, $x_{rf}$, arrived at the closed-tube end. The time until $p_{\text{wall}}$ decayed to an ambient pressure (atmosphere pressure $p_0$) was $\tau = 641 \, \mu\text{sec}$ under the positive-time ignition condition in Fig. 13 and $\tau = 584 \, \mu\text{sec}$ under the negative-time ignition condition in Fig. 14.

In Figs. 11 and 12, $x_{rf}$ in the $x$-$t$ diagram of Figs. 13 and 14 and the following rarefaction wave were not observed. The continual density reduction by the rarefaction wave could not be observed by the Schlieren optical system of this experiment. However, the propagation velocity of RRW was changed by the interference between the rarefaction wave and the RRW. In addition, under the
positive-time ignition condition in Fig. 13, the propagation velocity of the RRW1 before the interference was 1235 m/sec, and the propagation velocity of the RRW2 after the interference was 1524 m/sec. The propagation velocity of the RRW increased by 23% due to the interference between the rarefaction wave and the RRW. Under the negative-time ignition condition in Fig. 14, the propagation velocity of the RRW also increased by 26% due to the interference between the rarefaction wave and the RRW. The rarefaction wave appeared to be similar to the rarefaction wave of the model propagated into the combustor in the visualization experiment. Since the fitted lines of the RW in Figs. 13 and 14 passed through the DDT point, it appears that the RW occurred near the DDT point. Moreover, under the positive-time ignition condition in Fig. 13, the point at the intersection of the closed-tube end wall ($x_{wall} = 49$ mm) and the fitted line of the RW indicated the arrival time at which the RW reached the closed-tube end wall, and this time was $\tau = 237 \mu$sec. If the propagation velocity of the RW (1483 m/sec) was constant upstream of the combustor, the predicted arrival time at which the RW reached the pressure gauge for the purge gas from the closed-tube end was 93 $\mu$sec (pipe length was 138 mm) in Fig. 13, and the time interval from the spark time was $\tau = 330 \mu$sec. In contrast, in Fig. 7, the time interval between the spark time, $t_{spark}$, and the time at which $p_{valve, i}$ began to rise by the RW was 350 $\mu$sec, and this time interval was nearly identical (within 6%) to the time obtained from Fig. 13 (330 $\mu$sec). In the same way, the closed-end arrival time of the RW from the spark time, $t_{spark}$, was $\tau = 309 \mu$sec in Fig. 14, and the time interval between
the spark time, $t_{\text{spark}}$, and the time at which $p_{\text{valve},i}$ began to rise by the RW was 300 µsec in Fig. 8. This time interval was nearly identical (within 3%) to the time obtained from Fig. 14 (309 µsec). From these results, we conclude that the pressure rises of all $p_{\text{valve}}$ in Figs. 7 and 8 were due to the RW.

The DDT distance, $l_{\text{DDT}}$, and the spark-DDT time, $\tau_{\text{spark-DDT}}$, are defined in the $x$-$t$ diagram of Figs. 13 and 14. The length, $x_{\text{DDT}}$, and time, $t_{\text{DDT}}$, were averaged between two points at which the propagation velocity of FF became greater than 90% of the $D_{CJ}$ ($D_{CJ} = 2376.4$ m/sec under stoichiometric C$_2$H$_4$-O$_2$), and the DDT distance, $l_{\text{DDT}}$, was defined as the difference between the average length, $x_{\text{DDT}}$, and the position of the closed-tube end, $x_{\text{wall}}$ ($l_{\text{DDT}} = x_{\text{DDT}} - x_{\text{wall}}$). The spark-DDT time, $\tau_{\text{spark-DDT}}$, was defined as the difference between the averaged time, $x_{\text{DDT}}$, and the spark time, $t_{\text{spark}}$ ($\tau_{\text{spark-DDT}} = t_{\text{DDT}} - t_{\text{spark}}$).

In Figs. 13 and 14 was the averaged distance of the ignition point to the four walls ($x_{\text{wall}} = 49$ mm, $y = -W_{\text{tube}}/2$ mm, $z = \pm W_{\text{tube}}/2$ mm) parallel to the $x$-axis. This averaged distance was $\bar{I}_w = 18.7$ mm when the passage width, $W_{\text{tube}}$, was 20 mm and $\bar{I}_w = 13.7$ mm when the passage width was 10 mm. The point at the intersection of the averaged distance, $\bar{I}_w$, with the FF was defined as the characteristic time, $\tau_{\text{ch}}$, and the point at the intersection of the averaged distance, $\bar{I}_w$, with the fitted line of the DW was defined as the initiation time of the DW, $\tau_{\text{ini}}$. 
3.5.2 DDT distance and spark-DDT time

The processes of ignition, DDT and blowdown of burned gas under the positive-time ignition condition were qualitatively identical to the processes under the negative-time ignition condition. However, the early flame propagation velocity and the DDT distance and time were different. Table 1 shows the DDT distance, \( l_{\text{DDT}} \), and the spark-DDT time, \( \tau_{\text{spark-DDT}} \). The DDT distance depended heavily on the spark time, not on the passage width, \( W_{\text{tube}} \). The averaged DDT distance, \( l_{\text{DDT, ave}} \), (78.5 mm) under negative-time ignition conditions (V-2 and V-4) decreased by 20% compared to the averaged DDT distance, \( l_{\text{DDT, ave}} \), (98 mm) under positive-time ignition conditions (V-1 and V-3), and the minimum value of the averaged DDT distance, \( l_{\text{DDT, ave}} \), was 76\pm8 mm under the condition of V-2 (\( W_{\text{tube}} = 20 \) mm, negative-time ignition condition). In contrast, the spark-DDT time, \( \tau_{\text{spark-DDT}} \), decreased under the condition of V-4 (\( W_{\text{tube}} = 10 \) mm, negative-time ignition condition), and the minimum value of the averaged spark-DDT time, \( \tau_{\text{spark-DDT, ave}} \), was 69\pm14 \( \mu \)sec under the condition of V-4. In summary, The DDT distance, \( l_{\text{DDT}} \), was decreased by negative-time ignition, and the spark-DDT time, \( \tau_{\text{spark-DDT}} \), was reduced by shortening the passage width of the combustor and negative-time ignition.

3.5.3 Relation between characteristic time, detonation initiation time and DDT distance

Figure 15 shows the relation between the characteristic time, \( \tau_c \), and the initiation time of the DW,
\( \tau_{\text{ini}} \). Under the condition \( W_{\text{tube}} = 20 \text{ mm} \), the averaged propagation velocity of the early combustion wave from the ignition point to \( \bar{l}_w (\bar{l}_w / \tau_c) \) was 162 m/sec under the positive-time ignition condition and 207 m/sec under the negative-time ignition condition. The propagation velocity of the early combustion wave increased by 28% under the negative-time ignition condition. We confirmed that the averaged propagation velocity of the early combustion wave increased due to the disturbance and flow in the combustor. The time ratio was \( \tau_{\text{ini}} / \tau_c = 1.4 \pm 0.32 \). Thus, the DDT process did not occur unless the flame reached the averaged distance, \( \bar{l}_w \). In addition, the trends of the characteristic time, \( \tau_c \), and the initiation time of the DW, \( \tau_{\text{ini}} \) were similar to the spark-DDT time, \( \tau_{\text{spark-DDT}} \). In this case, \( \tau_c \) and \( \tau_{\text{ini}} \) decreased under the condition of \( W_{\text{tube}} = 10 \text{ mm} \) and the negative-time ignition condition because the required time interval until the combustion wave became the one-dimensional propagation mode became short under the condition of \( W_{\text{tube}} = 10 \text{ mm} \), and the averaged propagation velocity of the early combustion wave was fast under the negative-time ignition condition. Figure 16 shows the relation between the DDT distance, \( l_{\text{DDT}} \), and the characteristic time ratio, \( \tau_{\text{ini}} / \tau_c \). The characteristic time ratio was smaller than \( \tau_{\text{ini}} / \tau_c = 1.4 \) under the negative-time ignition condition, and the DDT distance was proportionate to the characteristic time ratio. This result suggested that the DDT distance depended heavily on the turbulence intensity of the propellant at the spark time, not on the passage width, \( W_{\text{tube}} \). In the high-frequency-operation PDRE, the shortening of the initiation time of the DW and the DDT distance is important. Not only the techniques of the turbulence
enhancement of the flame by the Shchelkin spiral and the orifice plates but also development of a
turbulence enhancement system of the propellant itself is important.

4. Thrust measurements of a PDRE

4.1 Experimental setup and conditions

We carried out thrust measurements using the rotary valve described in the previous sections and measured the mass flow rate, thrust and specific impulse. Figure 17 shows schematic diagrams of the thrust experiment apparatus and the cross-section of the PDRE. In the top schematic of Fig. 17, the rotation disk is rotated by the electromagnetic motor, and the rotation shaft has the spark trigger (trigger shaft). The proximity sensor senses the spark trigger and sends the spark input signal, \( S_{\text{input}} \) to a spark plug. The apparatus (PDRE and motor) is mounted on the rail guide and can slide along it. When thrust is generated, the apparatus slides on the rail guide, and the equivalent load is loaded to the load cell. The spring is mounted between the load cell and the apparatus to smooth the impulse generated by the DW.

In the lower schematic in Fig 17, the Cartesian coordinate system was chosen similar to the setup of the visualization experiment. \( \text{C}_2\text{H}_4\text{-O}_2 \) propellant and helium for purge gas were used. The diameter of each supply port was \( d = 5 \text{ mm} \), and the orifice was installed upstream of the fuel supply port to control the equivalent ratio. Pressure gauges were mounted in the pipe between the rotary
valve and the combustor to confirm the valve operation. The position of the closed-tube end was $x_{\text{wall}} = 26 \text{ mm}$, and the open-tube end of the combustor was $x = 1071 \text{ mm}$. The purge gas (He) was supplied into the combustor from the closed-tube end. The central axis of the purge gas port was the $x$-axis. The position coordinates of the oxidizer-port center were $x = 35 \text{ mm}, y = 14 \text{ mm}, z = 0 \text{ mm}$, and the position coordinates of the fuel-port center were $x = 35 \text{ mm}, y = -14 \text{ mm}, z = 0 \text{ mm}$. The oxidizer and fuel were mixed by opposing jets, and the combustor was filled with the mixture. The position coordinates of the spark plug were $x = 56 \text{ mm}, y = 14 \text{ mm}, z = 0 \text{ mm}$. The inner diameter of the combustor was expanded from 28 mm to 38 mm by tapering (angle of aperture of 15°). The total length and volume were 1045 mm and 0.98 L, respectively. Four ion probes were installed in the combustor at intervals of 25 mm from $x = 109 \text{ mm}$ to measure the combustion propagation velocity.

Table 3 shows the experimental conditions of the thrust measurements. The experimental setup operation frequencies, $f_{\text{set}}$, were 40 Hz, 70 Hz, 100 Hz, 130 Hz and 160 Hz. The propellant supply pressure was $p_{s,p} = 2 \text{ MPa}$, and the purge gas supply pressure was $p_{s,i} = 2.5 \text{ MPa}$. The mass flow rate was constant because of the constant supply pressure, but the propellant fill length, $l_p$, was changed by changing the operation frequency. The operation time interval was $t_{\text{ope}} = 1500 \text{ msec}$, and the thrust measurement was carried out three times to ensure repeatability of the experiment. It is important when examining the valve operation to consider the increase in temperature due to the combustion. We installed an aluminum plate, 15 mm thick, between the valve and the combustor, as
shown in Fig. 17. We assumed that the temperature of the valve increased only in response to the heat conduction due to the temperature difference on surfaces of this plate (combustor side, valve side). If the temperature on the combustor-side surface (closed-tube end wall) was constant at 2000 K, the valve temperature increased by approximately 21 K after the operation time interval of 1500 msec. Here, the initial valve temperature was 300 K, the area of heat transfer was 616 mm$^3$ (inner diameter of the closed-tube end was 28 mm), the thermal conductivity of the aluminum plate was 120 W/(m$^\cdot$K), the specific heat at a constant pressure of the valve was 880 J/(kg$^\cdot$K) and the valve mass was 670 g. The width of the annular projections on the rotation disk serving as the labyrinth seal mechanism was 1.5 mm, and the thermal expansion of the projection was 0.7 $\mu$m due to the increase in temperature of 21 K (the linear expansion coefficient at 300 K was approximately $23.6\times10^{-6}$ 1/K). If the annular grooves of the top cover and casing expanded similarly, the total thermal expansion was 1.4 $\mu$m. This expansion was small compared to the clearance between the projections and grooves (20 $\mu$m), and the change in the valve characteristics due to the increase in temperature at an operation time interval of 1500 msec was small. In fact, the experimental operation frequency, $f_{\text{exp}}$, obtained by the spark output signal, $S_{\text{output}}$, was identical to the setup operation frequency, $f_{\text{set}}$, within 1%, and a variation of operation frequency was not confirmed. However, it is necessary to consider the change in the valve characteristics caused by the thermal expansion that would occur during long-time operation. If the temperature on the combustor-side surface
(closed-tube end wall) was constant at 2000 K, the clearance between the projections and grooves would be 0 μm during an operation time of 19 sec, and the torque would rapidly increase due to the contact of the rotation disk with the top cover and the casing. Moreover, the limited operation time for the O-ring was approximately 21 sec (the maximum operating temperature of the O-ring was 573 K), and the seal performance would decrease. However, it is also conceivable that the limited operation time would become short because of the heat transfer from the supply ports and the combustor. In the longer runs, the insertion of insulation between the valve and the combustor and the installation of a cooling system would be required.

For the calculation method of mass flow rate, thrust and specific impulse, see reference [41]. Finally, the propellant-based experimental specific impulse, $I_{sp, exp}$, was obtained from the time-averaged thrust, $F_{exp}$, the oxidizer mass flow rate, $m_o$, and the fuel mass flow rate, $m_f$, and gravitational acceleration, $g$, as follows:

$$I_{sp, exp} = \frac{F_{exp}}{g(m_o + m_f)}$$

We compared the experimental specific impulse to the partial filling model proposed by Sato et al. [32]. The equation of Sato et al. is expressed as follows, and this semi-empirical equation agrees rather well with the experiment value in the range of the propellant fill fraction $\Psi_p > 0.1$:

$$\frac{I_{sp, PW}}{I_{sp, exp}} = \frac{1}{\sqrt{Y}}$$


where $I_{sp,PF}$ is the propellant-based specific impulse when the combustor is partially filled with the propellant, and $I_{sp,FF}$ is the propellant-based specific impulse when the combustor is fully filled with the propellant. $Y$ is the mass fraction of the propellant mass per total mass inside the combustor. In a PDRE, an inert gas is generally required as the purge gas, and the specific impulse with the purge gas mass flow rate should be used as the PDRE’s specific impulse. However, the specific impulse obtained from the equation of Sato et al. was a propellant-based specific impulse, so the experimental propellant-based specific impulse was evaluated in the present study. The model of Sato et al. is shown schematically in Fig. 1. If the propellant, purge gas, burned gas and air exist inside the combustor, the propellant-based specific impulse can be expressed as follows:

$$I_{sp,PF} = I_{sp,PF} \sqrt{\frac{m_p + (m_i + m_b + m_a)}{m_p}}$$

(12)

where $m$ is the mass per one PDE cycle, and the subscript $p$ is propellant, $i$ is purge gas, $b$ is burned gas and $a$ is air. The $m_p$ and $m_i$ were obtained from the experimental mass flow rate and the operation frequency, $f_{exp}$. The calculated specific impulse was compared with the experimental specific impulse by changing the volume fraction of burned gas and ambient air. We assumed that the temperature and the pressure of each gas (propellant, purge gas and air) in the combustor were the ambient temperature (283.15 K) and atmospheric pressure (0.1013 MPa). The burned gas was the previous propellant which isentropically expanded from the Chapman-Jouguet state to the
atmosphere pressure [55]. \( I_{s, FF} \) of the stoichiometric \( C_2H_4-O_2 \) mixture was 171.2 sec [30]. The specific impulse at the second and higher PDE cycle, \( I_{s, PF, 2nd} \), was different from the first specific impulse, \( I_{s, PF, 1st} \), because the burned gas did not exist in the combustor at the first cycle. The calculated specific impulse, \( I_{s, cal} \), was determined using the following equation:

\[
I_{s, cal} = \left( f_{exp} - 1 \right) I_{s, PF, 2nd} + I_{s, PF, 1st} \frac{t_{pf}}{t_{exp} \cdot f_{exp}}
\]

(13)

The calculated thrust, \( F_{cal} \), can be obtained from the calculated specific impulse, \( I_{s, cal} \), and the measured propellant mass flow rate, \( \dot{m}_p \), as follows:

\[
F_{cal} = I_{s, cal} \rho g \left( \dot{m}_o + \dot{m}_i \right)
\]

(14)

The propellant fill fraction, \( \psi_p \), was defined as the propellant volume \( (V_o + V_i) \) of one cycle (at ambient temperature, \( T_a \) and pressure, \( p_a \)) divided by the detonation tube volume, \( V_{tube} \). If the propellant was the perfect gas and the combustor was filled with propellant at ambient temperature and atmosphere pressure, the propellant fill fraction could be described as follows:

\[
\psi_p = \frac{(V_o + V_i)}{V_{tube}} = \frac{T_a}{p_a V_{tube} f_{exp}} \left( R \dot{m}_o + R \dot{m}_i \right)
\]

(15)

Similarly, the purge gas fill fraction, \( \psi_i \), could be described as follows:

\[
\psi_i = \frac{V_i}{V_{tube}} = \frac{T \rho \dot{m}_i}{p_a V_{tube} f_{exp}}
\]

(16)

4.2 Results and discussion
Table 3 shows the experimental results. If the gases (oxidant, fuel and purge gas) were the perfect gas and the combustor was filled with the gases at ambient temperature and atmosphere pressure, the propellant fill length, $l_p$, and the purge gas fill length, $l_i$, in Table 3 represented the distance from the closed-tube end wall to the front of the gas. The propellant fill fraction, $\Psi_p$, was 0.15 at the maximum operation frequency of 160 Hz, so the equation of Sato et al. was applicable to the experimental results. The experimental propellant and purge gas-based specific impulse, $I_{sp, exp}$, was approximately 80% of the experimental propellant-based specific impulse, $I_{sp, exp}$. In Table 3, $\alpha$ shows the success rate of DW generation ($0.9 D_{CJ}$). In addition, $D_{CJ}$ was calculated by using the equivalent ratio of each experimental condition. The number of measurements was 20% of the total cycle (48 at 160 Hz). $\beta$ was the averaged flame velocity ratio (averaged flame velocity/$D_{CJ}$) measured between c and d of the ion probe. Under the condition of T-1, the success rate, $\alpha$, was low, but the flame accelerated to 70% of $D_{CJ}$ between c and d of the ion probe and the propellant fill fraction was high. The detonation wave appeared to be generated with high probability at downstream of d of the ion probe. Under the other conditions, the averaged flame velocity ratio, $\beta$, was over 100%. This may be because the DDT occurred near between c and d of the ion probe and the overdriven detonation wave passed through between c and d of the ion probe.

Figure 18 shows the propellant mass flow rates, $\dot{m}_p$, and the purge gas, $\dot{m}_i$. The mass flow rate was constant even if the operation frequency changed, because the opening time of the supply port
per unit time was constant. Figure 19 shows the propellant fill fractions, $\Psi_p$, and the purge gas fill fractions, $\Psi_i$. The mass flow rate was constant even if the operation frequency changed, as shown in Fig. 18, so the mass per one cycle decreased when the operation frequency increased. The minimum propellant fill fraction was $\Psi_p = 0.15$ at the maximum operation frequency of 160 Hz, and the fill length was approximately 179 mm (combustor length was 1075 mm). In the visualization experiment, the maximum DDT distance, $l_{DDT}$, was approximately 100 mm, and DDT distance, $l_{DDT}$, did not depend on the passage width, $W_{tube}$, so the minimum propellant fill length, $l_p$, could be sufficiently long relative to the DDT distance, $l_{DDT}$.

Figure 20 shows the load cell output, $F_{lc}$, under each set of experimental conditions, and $F_{lc}$ in Fig. 20 was closest to the averaged thrust of three experiments under the same condition. $F_{exp}$ in Fig. 20 shows the time-averaged thrust which was averaged by the operation time interval ($F_{lc} > 0$) and the load cell output, $F_{lc}$. It took a finite amount of time until the thrust became constant, because the PDRE was connected with the spring. The time-averaged thrust increased with increasing the operation frequency. Figure 21 shows the relationship between the propellant-based specific impulse, $I_{sp,exp}$, and the operation frequency, $f_{exp}$. Figure 22 shows the relationship between the time-averaged thrust, $F_{exp}$, and the operation frequency, $f_{exp}$. The closed circles (●) in Figs. 21 and 22 show the experimental values, and the open circles (○) in Figs. 21 and 22 show the calculated values when the volume fractions of burned gas and air were varied. The increasing rate of the calculated specific
impulse was lower when the volume fraction of burned gas increased. This was because the burned gas temperature was high and the burned gas density was low. \( \Phi \) in Figs. 21 and 22 shows the equivalent ratios calculated from the experimental mass flow rate. The lines in Figs. 21 and 22 are the fitted lines obtained by using the least-squares method, but the value under the condition of T-3 (100 Hz) was not included. The value under the condition of T-3 was small, and one of the reasons for this could be that not all of the propellant was used in the thrust because of the negative-time ignition condition \( (t_{\text{stop}} > t_{\text{spark}}) \), as shown in Table 2.

In Fig. 21, the experimental specific impulse, \( I_{\text{sp, exp}} \), increased because the propellant mass fraction, \( Y \), decreased by increasing the operation frequency. Consequently, as shown in Fig. 22, the experimental time-averaged thrust, \( F_{\text{exp}} \), increased. In Fig. 21, the specific impulse ratio, \( I_{\text{sp, exp}} / I_{\text{sp, cal}} \), became between 88% and 93% when the volume fraction of burned gas was 86%. Considering the specific impulse loss in a multi-shot operation, we think that the experimental specific impulse was a reasonable value to use as the multi-shot PDREs specific impulse. In addition, the specific impulse ratio, \( I_{\text{sp, exp}} / I_{\text{sp, cal}} \), under the condition of T-3 was 77%. These results suggested that burned gas and air existed at the same volume fraction even if the operation frequency changed with the constant mass flow rate. However, it is possible that the specific impulse under high-frequency operation becomes smaller than the specific impulse under low-frequency operation (same propellant fill fraction) because the density in a combustor becomes low due to the increase in temperature. The
development of a device that uses the surrounding air such as an ejector is important to increase the specific impulse.

In addition, the maximum time-averaged thrust of 71 N was achieved under the condition of a propellant supply pressure of 2.0 MPa and an operation frequency of 159 Hz. The maximum specific impulse of 232 sec was achieved under the same condition. In the range of propellant fill length under the thrust measurement, the decay of the specific impulse due to the increase in length ratio ($l_{DDT}/l_p$) was not confirmed.

5. Conclusion

We developed a novel rotary valve for a PDE. This valve can supply three different gases (fuel, oxidizer and purge gas) to a combustor using one valve with a simple structure that allowed us to control the accurate supply timing. We investigated the basic performance of a rotary-valved PDRE by carrying out a multi-shot visualization experiment inside the combustor and taking multi-shot thrust measurements using a rotary-valved single-combustor PDRE.

Multi-shot of a PDE cycle (propellant filling, detonation wave generation and purging) at an operation frequency of 160 Hz was visualized by using a high-speed camera (time resolution: 3.33 μsec, space resolution: 0.4 mm) and a Schlieren method, and the pressure between the valve and the combustor and the ignition output signal were monitored. From the results of the visualization
experiment and monitoring, we confirmed the multi-shot operation of a PDE cycle at an operation
frequency of 160 Hz. From the sequence of Schlieren photographs, we confirmed the propellant
filling process and the purge process. The front velocity of the propellant filling process, which was
obtained from the sequence of Schlieren photographs, was in relatively good agreement with the
propellant-flow velocity obtained by using the model of an incompressible one-dimensional steady
flow. The tip trajectory of the purge gas jet was in good agreement with the tip trajectory of the jet
obtained by using the model of an incompressible two-dimensional turbulent jet. In the process of
DW generation, we confirmed the deflagration wave propagation, generation of the fast combustion
wave and the fast SW by local explosion, and the retonation wave generation. In the process of
burned gas blowdown, we confirmed the retonation wave reflection at the closed-tube end of the
combustor and the reflected retonation wave acceleration by the rarefaction wave.

We investigated the impact of shortening the passage width, \( W_{\text{tube}} \), and negative-time ignition
(ignition occurs before the end of propellant filling) on the deflagration-to-detonation transition
(DDT) distance and time. The DDT distance, \( l_{\text{DDT}} \), was decreased by negative-time ignition, and the
spark-DDT time, \( \tau_{\text{spark-DDT}} \), was reduced by shortening the passage width of the combustor and
negative-time ignition. The average DDT distance, \( l_{\text{DDT,ave}} \), (78.5 mm) under negative-time ignition
conditions (V-2 and V-4) was decreased by 20\% compared to the average DDT distance (98 mm)
under positive-time ignition conditions (V-1 and V-3), and the minimum value of the average DDT
distance was 76±8 mm under the condition of V-2 ($W_{\text{tube}} = 20 \text{ mm}$, negative-time ignition condition).

In contrast, the spark-DDT time, $\tau_{\text{spark-DDT}}$, was decreased under the condition of V-4 ($W_{\text{tube}} = 10 \text{ mm}$, negative-time ignition condition), and the minimum value of the average spark-DDT time was 69±14 μsec under the condition of V-4. Also, the detonation initiation time, $\tau_{\text{ini}}$, and the DDT distance, $l_{DDT}$, were characterized by the characteristic time, $\tau_c$, (time at which the flame reached the average distance, $\bar{l}_w$, from the ignition point).

Under the condition $W_{\text{tube}} = 20 \text{ mm}$, the average propagation velocity of the early combustion wave ($\bar{l}_w/\tau_c$) was 162 m/sec under the positive-time ignition condition and 207 m/sec under the negative-time ignition condition. The propagation velocity of the early combustion wave increased by 28% under the negative-time ignition condition. It was confirmed that the average propagation velocity of the early combustion wave increased due to the existing disturbance and flow in a combustor. The time ratio was $\tau_{\text{ini}}/\tau_c = 1.4 \pm 0.32$. Thus, the DDT process did not occur unless the flame reached the averaged distance, $\bar{l}_w$. In addition, the trend of the characteristic time, $\tau_c$, and the initiation time of the DW, $\tau_{\text{ini}}$, was similar to the spark-DDT time, $\tau_{\text{spark-DDT}}$. The characteristic time ratio, $\tau_{\text{ini}}/\tau_c$, was smaller than $\tau_{\text{ini}}/\tau_c = 1.4$ under the negative-time ignition condition, and the DDT distance, $l_{DDT}$, was proportionate to the characteristic time ratio. This result suggested that the DDT distance, $l_{DDT}$, depended heavily on the turbulence intensity of the propellant at the spark time rather than on the passage width, $W_{\text{tube}}$. 
We also carried out the thrust measurement in a PDRE system composed of a circular cross-section combustor and our proposed valve. The stable time-averaged thrust was obtained in a wide range of operation frequencies (40 Hz - 160 Hz), and the increase of the specific impulse due to a partial-filling effect was confirmed. At the maximum operation frequency of 159 Hz, we achieved the maximum propellant-based specific impulse of 232 sec and the maximum time-averaged thrust of 71 N.

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References


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103-110.


Table 1. Experimental conditions for visualization experiments and spark time, DDT distance and spark-DDT time

<table>
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<tr>
<th>Cycle number</th>
<th>Rectangular width of detonation tube</th>
<th>Stop time of propellant supplying</th>
<th>Spark time</th>
<th>DDT distance</th>
<th>Spark-DDT time</th>
<th>Averaged DDT distance</th>
<th>Averaged spark-DDT time</th>
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<td>V-1-1</td>
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<td>2.8</td>
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<td>187±25</td>
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<td>3.67</td>
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<td>165.0</td>
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<td>purge gas supply pressure</td>
<td>stop time of propellant supplying</td>
<td>spark time</td>
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### Table 3. Experimental results of thrust measurement experiments

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<th>Shot number</th>
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<th>Propellant mass flow rate</th>
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<th>Propellant fill length</th>
<th>Purge gas fill length</th>
<th>Propellant fill fraction</th>
<th>Purge gas fill fraction</th>
<th>Time-averaged thrust</th>
<th>Propellant-based specific impulse</th>
<th>Purge gas-based specific impulse</th>
<th>Success rate (0.9DCJ)</th>
<th>Averaged flame velocity ratio ( (\text{DCJ/velocity}) )</th>
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Fig. 1. Relationship between DDT distance and propellant fill length
Fig. 2. Schematic of a coaxial rotary valve
Fig. 3. Cross-section diagram of a coaxial rotary valve
Fig. 4. Rotation disk position during each process of a PDE cycle

(viewed from top cover side)
Fig. 5 Control volume of flow through the abrupt expansion
Fig. 6. Schematic diagram of the visualization experimental apparatus
Fig. 7. Spark output signal, upstream pressure of combustor, spark input signal and supply-port opening ratio

(positive-time ignition condition, cycle no. V-1-3)
Fig. 8. Spark output signal, upstream pressure of combustor, spark input signal and supply-port opening ratio
(negative-time ignition condition, cycle no. V-2-1)
Fig. 9. Sequence of Schlieren photographs of one cycle of a PDRE under the positive-time ignition condition (cycle no. V-1-3, $\Delta t = 0.15$ msec, FF: flame front, CW: compression wave, SW: shock wave)
Fig. 10. Sequence of Schlieren photographs of one cycle of a PDRE under the negative-time ignition condition (cycle no. V-2-1, $\Delta t = 0.15$ msec, SW: shock wave)
Fig. 11 Sequence of Schlieren photographs illustrating the ignition, DDT and blowdown of burned gas under the positive-time ignition condition

(Cycle no. V-1-3, CW: compression wave, DW: detonation wave, RW: retonation wave, LE<sub>left</sub>: local explosion at closed-tube end side, LE<sub>right</sub>: local explosion at open-tube end side, FCW: fast combustion wave, FSW: fast shock wave, TW: transverse wave, RRW: reflected retonation wave)
Fig. 12. Sequence of Schlieren photographs illustrating the igniton, DDT and blowdown of burned gas under the negative-time ignition condition

(Cycle no. V-2-1, CW: compression wave, DW: detonation wave, RW: retonation wave, LE\textsubscript{left}: local explosion at closed-tube end side, LE\textsubscript{right}: local explosion at open-tube end side, FSW: fast shock wave, TW: transverse wave, RRW: reflected retonation wave)
Fig. 13. $x$-$t$ diagram of the wave trajectory and pressure history of a closed-tube end wall under the positive-time ignition condition (cycle no. V-1-3)
Fig. 14. $x$-$t$ diagram of the wave trajectory and pressure-history of a closed-tube end wall under the negative-time ignition condition (V-2-1)
Fig. 15. Relationship between the characteristic time and the initiation time of a detonation wave.
Fig. 16. Relationship between the characteristic time ratio and the DDT distance
Fig. 17. Schematic diagrams of the thrust experiment apparatus and cross-section of a PDRE
Fig. 18. Mass flow rate versus operation frequency
Fig. 19. Fill fraction versus operation frequency
Fig. 20. Time-history of load cell output

(a) 40 Hz  (b) 70 Hz  (c) 100 Hz  (d) 130 Hz  (e) 160 Hz
Fig. 21. Propellant-based specific impulse versus operation frequency
Fig. 22. Time-averaged thrust versus operation frequency