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著者	SUMITA Ushio, ZUO Jinshui
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Sumita Ushio and Zuo Jinshui

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UNIVERSITY OF TSUKUBA

Tsukuba, Ibaraki 305-8573
JAPAN

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Ushio Sumita , Jinshui Zuo

Graduate School of Systems and Information Engineering, University of Tsukuba, 1-1-1,Tenoudai,Tsukuba, Ibaraki, 305-8573, Japan
(E-mail address: sumita@sk.tsukuba.ac.jp;ksa@sk.tsukuba.ac.jp)

Abstract

One of the advantages of the mobile access to the Internet can be found in that it allows one to conduct information search in a ubiquitous manner. Accordingly, the time for net surfing required to gain necessary information through the Internet would be reduced when the mobile access is combined with the ordinary PC access appropriately. As the mobile devices continue to advance rapidly, it then becomes quite important to find a way to assess the impact of the mobile access to the Internet on the performance of e-commerce. Among such performance indicators, of particular interest would be the conversion rate, where the conversion means that a customer takes a firm action at the website desired by the management of the e-commerce, e.g. to become a member, to place an order and the like. The purpose of this paper is to develop and analyze a mathematical model for describing the information search process through the Internet with or without the mobile access. The analytical framework enables one to assess the impact of the mobile access on the information search completion time and the conversion rate. Numerical examples are given, demonstrating the effectiveness of the

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computational procedures developed in this paper.

Key words: PC access, Mobile access, Information search completion time, Conversion rate

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1. Introduction

Independent studies have shown that the mobile access to the Internet has been evolving quite rapidly. Accoroding to Juniper Research [4], the number of mobile Internet users would grow from 577 million in 2008 to 1700 million in 2013 in the world. In [5], iSuppli reports that the number of mobile phone users would reach 4 billion in the world by 2010. The mobile phone access to the Internet has been already widely spread in Japan and Korea, while PDA is mostly used in the United States for the mobile access to the Internet, see Roto [14]. Khan, Weishaar and Polinsky[6] point out that, although the mobile phone penetration in the United States is high with 84% of Americans using mobile phones, the mobile search market is in the early adoption stage. However, according to Neilsen[10] and Osterman[12], this industry is expected to grow drastically and would become important for marketing as well as enhancement of workforce productivity.

Table 1.1 below compares the characteristic features of the mobile access to the Internet with those of the PC access.

Table 1.1

	Mobile Access	*PC Access*
Screen Size	*Small*	*Large*
Nonumber of Windows	*Single*	*Multiple*
Navigation	*Slow*	*Fast*
Speed	*Low*	*High*
Size of Message	*Small*	*Big*
Mobility	*Yes*	*No*

As can be seen from Table 1.1, the capability of the mobile access to the Internet is still rather limited. However, the strong advantage of the mobile access over the PC access can be found in that it allows one to conduct information search in a ubiquitous manner. Accordingly, the time for net surfing required to gain necessary information would be reduced when the mobile access to the Internet is combined with the ordinary PC access to the Internet appropriately.

The impact of the mobile access to the Internet on e-businesses has been addressed by many researchers including Roto [14], Kim [9] and Chae and Kim [2] to name only a few. In addition, Barwise [1] and Hammond [3] discuss the emerging trend of m-commerce in the foreseeable future. Wu and Hisa [22] propose a hypercube model for analyzing e-commerce innovation and impact including m-commerce. Siau, Sheng and Nah [16], and Park and Fader [13] analyze m-commerce from the perspective of consumers, while Thompson and Garbacz [21] discuss the effects of e-commerce and m-commerce on global productive efficiency. Concerning the mobile access vs. the PC access to the Internet, Roto[15] discusses how information search on a mobile phone differs from information search through the PC access. Okazaki and Hirose[11] examine the media displacement-reinforcement among traditional media, PC Internet and mobile Internet. These works are mostly empirical, qualitative or static in their analytic nature. To the best knowledge of the authors, no mathematical models exist in the literature for analyzing how the mobile access would facilitate the information search process through the Internet.

In e-commerce, one of the key performance indicators would be the conversion rate, where the conversion means that a customer takes a firm action at the website desired by the management of the e-commerce, e.g. to become a member, to place an order and the like. Before deciding whether or not to make a conversion, individual customers would gather information through the Internet. In order to analyze the conversion rate along the time axis, it is then necessary to understand how long it would take for a customer to complete information search. The purpose of this paper is to develop and analyze a mathematical model for understanding the impact of the mobile access to the Internet on the information search completion time. This in turn enables one to assess how the mobile access to the Internet would enhance the conversion rate.

More specifically, we consider a website for e-commerce, where it is assumed that individual customers would search certain information through net surfing inside as well as outside the website within the Internet before deciding whether or not to make a conversion. This information search would be completed when the cumulative amount of information gathered through net surfing exceeds a prespecified level z. Upon completion of information search, each customer decides to make a conversion with probability p and not to make a conversion with probability $1 - p$. Given N customers at time 0, of interest then would be how many of them would finish information search and make a conversion by time t.

The structure of the paper is as follows. In Section 2, a mathematical model is formally introduced for describing the information search process through the Internet with or without the mobile access. Section 3 is devoted to analysis of the information search completion time $T_{PC:z}$ with only the PC access, $T_{M:z}$ with only the mobile access and $T_{B:z}$ with both the mobile access and the PC access, deriving the double Laplace transforms of their probability density functions with respect to the threshold level z and the information search completion time t. The means and variances are also obtained explicitly. In Section 4, the double transforms are inverted into the real domain under the assumption of exponential information gathering and deterministic search time. Section 5 presents numerical examples, demonstrating the effectiveness of the computational procedures developed in this paper for assessing the impact of the mobile access on the information search completion time and the conversion rate. Some concluding remarks are given in Section 6.

2. Model Description

In this section, we develop a mathematical model for describing the net surfing process of individual customers without or with the mobile access to the Internet. The model enables one to assess the impact of the mobile access to the Internet on information search completion time, which in turn provides a basis for evaluating how the conversion rate would be improved via the mobile access.

Let Y_j be the random duration available for net surfing by a customer on the j-th day, $j = 1, 2, \cdots$, where Y_j are assumed to be independently and identically distributed (i.i.d.). We call Y_j the j-th search period. In what follows, we suppress non-search time along the time axis, i.e. "at time t" means "at the point in time" at which the cumulative search time has reached t.

A customer is of Type PC if the customer has only the PC access to the Internet. Let $X_1(j)$ be the amount of information gained by the customer through the PC access to the Internet during the j-th search period. Naturally, the amount of information that can be gathered through net surfing would depend on the time that can be spent for net surfing. However, net surfing over different search periods may be done independently. Accordingly, throughout the paper, we assume that $X_1(j)$ and Y_j are correlated for each j, but the vectors $[X_1(j), Y_j]$ are i.i.d. with respect to j. The information search is completed whenever the cumulative amount of information gained through net surfing exceeds a prespecified level z. More specifically, let $N(t)$ be the renewal process associated with $(Y_j)_{j=1}^{\infty}$, and define

$$S_{PC}(t) \stackrel{\text{def}}{=} \sum_{j=1}^{N(t)} X_1(j) \quad . \tag{2.1}$$

The information search completion time $T_{PC:z}$ for a Type PC customer given the threshold level z is then obtained as

$$T_{PC:z} \stackrel{\text{def}}{=} inf\{t : S_{PC}(t) > z\} \quad . \tag{2.2}$$

A customer is of Type M if the customer has only the mobile access to the Internet. Let $X_2(j)$ be the amount of information gained by the Type M customer through the mobile access to the Internet during the j-th search period. The cumulative amount of information gained by a Type M customer through net surfing by time t is then given by

$$S_M(t) \stackrel{\text{def}}{=} \sum_{j=1}^{N(t)} X_2(j) \quad . \tag{2.3}$$

The information search completion time $T_{M:z}$ for a Type M customer given the threshold level z is obtained as

$$T_{M:z} \stackrel{\text{def}}{=} inf\{t : S_M(t) > z\} \quad . \tag{2.4}$$

Tpye B customers are those who utilize the Internet through both the PC access and the mobile access. Let $X_1(j)$ and $X_2(j)$ be the amount of information gathered by a Type B customer via the PC access and the mobile access respectively during the j-th search period Y_j. As before, it is assumed that $\underline{X}(j) = [X_1(j), X_2(j)]$ and Y_j are correlated for each j, but the vectors $[\underline{X}(j), Y_j]$ are i.i.d. with respect to j. Let $S_B(t)$ be the cumulative amount of information gathered through net surfing by a Type B customer by time t, i.e.

$$S_B(t) \stackrel{\text{def}}{=} \sum_{j=1}^{N(t)} Z_B(j) \quad ; \quad Z_B(j) = X_1(j) + X_2(j) \quad . \tag{2.5}$$

The search completion time $T_{B:z}$ for a type B customer given a threshold level z is then given by

$$T_{B:z} \stackrel{\text{def}}{=} inf\{t : S_B(t) > z\} \quad . \tag{2.6}$$

Of interest are the distributions of $T_{PC:z}$, $T_{M:z}$ and $T_{B:z}$, which enables one to assess the impact of the mobile access to the Internet on the information search completion time. This in turn provides a basis for evaluating how the conversion rate would be improved via the mobile access. In the next section, the Laplace transforms of the probability density functions (p.d.f's) of $T_{PC:z}$, $T_{M:z}$ and $T_{B:z}$ are derived explicitly and the respective means and variances are evaluated.

3. Analysis of Information Search Completion Time

The information search completion times $T_{PC:z}$ and $T_{M:z}$ for Type PC and Type M customers have a structure of the cumulative shock model associated with a renewal sequence discussed by Sumita and Shanthikumar [20]. For Type B customers, however, the model in [20] should be modified so as to accomodate a sequence of bivariate shocks $\underline{X}(j)$ associated with a renewal sequence Y_j. The purpose of this section is to derive the Laplace transform of the p.d.f of $T_{B:z}$ along with its mean and variance. We confirm that the counterparts for $T_{PC:z}$ and $T_{M:z}$ originally given in [20] would be obtained as a special case.

Let $F_{\underline{X},Y}(x_1, x_2, y)$ be the joint distribution function of the vector process $\underline{X}(j) = [X_1(j), X_2(j)]$ and Y_j defined by

$$F_{\underline{X},Y}(x_1, x_2, y) = P[X_1(j) \le x_1, X_2(j) \le x_2, Y_j \le y] \quad . \tag{3.1}$$

It is assumed that $F_{\underline{X},Y}(x_1, x_2, y)$ is absolutely continuous with joint p.d.f $f_{\underline{X},Y}(x_1, x_2, y)$ satisfying

$$F_{\underline{X},Y}(x_1, x_2, y) = \int_0^{x_2} \int_0^{x_1} \int_0^{y} f_{\underline{X},Y}(x_1', x_2', y')dy'dx_1'dx_2' \quad . \tag{3.2}$$

From (2.5), the joint distribution function of $Z_B(j)$ and Y_j denoted by $G_{Z_B,Y}(z,y)$ can be obtained as

$$\begin{aligned} G_{Z_B,Y}(z,y) &\stackrel{\text{def}}{=} P[Z_B(j) \le z, Y_j \le y] = P[X_1(j) + X_2(j) \le z, Y_j \le y] \\ &= \int_0^y \Big\{ \int_0^z \Big\{ \int_0^{z'} f_{\underline{X},Y}(z' - x, x, y')dx \Big\} dz' \Big\} dy' \quad . \end{aligned} \tag{3.3}$$

Let $g_{Z_B,Y}(z,y)$ be the joint p.d.f of $Z_B(j)$ and Y_j. One then has

$$g_{Z_B,Y}(z,y) \stackrel{\text{def}}{=} \frac{\partial^2}{\partial z \partial y} G_{Z_B,Y}(z,y) = \int_0^z f_{\underline{X},Y}(z-x,x,y)dx \quad . \tag{3.4}$$

Throughout the paper, Laplace transforms with respect to t, z or both are denoted by a circumflex or a double circumflex. The Laplace transform of $g_{Z_B,Y}(z,y)$ is denoted accordingly by

$$\hat{\hat{g}}_{Z_B,Y}(w,s) \stackrel{\text{def}}{=} \int_0^\infty \int_0^\infty e^{-sy-wz} g_{Z_B,Y}(z,y)dzdy \quad . \tag{3.5}$$

For notational convenience, the following functions are also introduced.

$$f_{X_1}(x_1) = \int_0^\infty \int_0^\infty f_{\underline{X},Y}(x_1,x_2,y)dx_2dy \tag{3.6}$$

$$f_{X_2}(x_2) = \int_0^\infty \int_0^\infty f_{\underline{X},Y}(x_1,x_2,y)dx_1dy \tag{3.7}$$

$$f_Y(y) = \int_0^\infty \int_0^\infty f_{\underline{X},Y}(x_1,x_2,y)dx_1dx_2 \tag{3.8}$$

$$f_{X_1,Y}(x_1,y) = \int_0^\infty f_{\underline{X},Y}(x_1,x_2,y)dx_2 \tag{3.9}$$

$$f_{X_2,Y}(x_2,y) = \int_0^\infty f_{\underline{X},Y}(x_1,x_2,y)dx_1 \quad . \tag{3.10}$$

$$
\begin{aligned}
\hat{\varphi}_{X_1}(w) &\stackrel{\text{def}}{=} \int_0^\infty e^{-wx_1} f_{X_1}(x_1)dx_1 && (3.11)\\
\hat{\varphi}_{X_2}(w) &\stackrel{\text{def}}{=} \int_0^\infty e^{-wx_2} f_{X_2}(x_2)dx_2 && (3.12)\\
\hat{\varphi}_{Y}(s) &\stackrel{\text{def}}{=} \int_0^\infty e^{-sy} f_{Y}(y)dy && (3.13)\\
\hat{\hat{\varphi}}_{X_1,Y}(w,s) &\stackrel{\text{def}}{=} \int_0^\infty \int_0^\infty e^{-wx_1-sy} f_{X_1,Y}(x_1,y)dx_1dy && (3.14)\\
\hat{\hat{\varphi}}_{X_2,Y}(w,s) &\stackrel{\text{def}}{=} \int_0^\infty \int_0^\infty e^{-wx_2-sy} f_{X_2,Y}(x_2,y)dx_2dy \quad . && (3.15)
\end{aligned}
$$

Let the distribution functions of $S_B(t)$ and $T_{B:z}$ be defined by

$$V_B(z,t) = P[S_B(t) < z] \quad ; \quad W_B(z,t) = P[T_{B:z} \le t] \tag{3.16}$$

respectively. Absolute continuity of $F_{\underline{X},Y}(x_1,x_2,y)$ assures that $T_{B:z}$ has the probability density function given by

$$w_B(z,t) = \frac{\partial}{\partial t} W_B(z,t) \quad . \tag{3.17}$$

We define

$$
\begin{aligned}
\hat{\hat{V}}_B(w,s) &= \int_0^\infty \int_0^\infty e^{-st-wz} V_B(t,z)dzdt \quad , && (3.18)\\
\hat{\hat{w}}_B(w,s) &= \int_0^\infty \int_0^\infty e^{-st-wz} w_B(z,t')dzdt' \quad . && (3.19)
\end{aligned}
$$

One easily sees that there exists a dual relationship between $S_B(t)$ and $T_{B:z}$ specified by

$$V_B(z,t) = P[S_B(t) < z] = P[T_{B:z} > t] = \overline{W}_B(z,t) \quad , \tag{3.20}$$

where $\overline{W}_B(z,t) = 1 - W_B(z,t)$ is the survival function of $T_{B:z}$. The following theorem then holds.

Theorem 3.1.

$$\hat{\hat{V}}_B(w,s) = \frac{1-\hat{\varphi}_Y(s)}{ws\{1-\hat{\hat{g}}_{Z_B,Y}(w,s)\}} \quad , \quad Re(s) \geq 0 \quad .$$

Proof. Since $V_B(z,t)$ is the probability that the cumulative amount of $Z_B(j)$ has not exceeded the level z for $0 \leq j \leq N(t)$, by conditioning on the first renewal time Y_1 and using the regenerative property of the paired process $[\underline{X}(j), Y_j]$ at Y_1, one sees that

$$V_B(z,t) = \overline{F}_Y(t) + \int_0^t \int_0^z g_{Z_B,Y}(x,y) V_B(z-x, t-y)\, dx dy \quad . \tag{3.21}$$

By taking the Laplace transform of both sides of (3.21) with respect to z and t, it can be seen that

$$\hat{\hat{V}}_B(w,s) \quad = \quad \frac{1-\hat{\varphi}_Y(s)}{ws} + \hat{\hat{g}}_{Z_B,Y}(w,s)\hat{\hat{V}}_B(w,s) \quad .$$

This equation can be solved for $\hat{\hat{V}}_B(z,s)$, completing the proof. □

The information search completion time $T_{B:z}$ has the dual relationship with $S_B(t)$ given in (3.20). The Laplace transform $\hat{\hat{w}}_B(w,s) = E[e^{-sT_{B:z}}]$ is then easily found from Theorem 3.1.

Theorem 3.2.

$$\hat{\hat{w}}_B(w,s) = \frac{\hat{\varphi}_Y(s) - \hat{\hat{g}}_{Z_B,Y}(w,s)}{w\{1-\hat{\hat{g}}_{Z_B,Y}(w,s)\}} \quad , \quad Re(s) \geq 0 \quad .$$

Proof. From (3.20), one finds that $\hat{\hat{V}}_B(w,s) = \frac{\frac{1}{w} - \hat{\hat{w}}_B(w,s)}{s}$, so that $\hat{\hat{w}}_B(w,s) = \frac{1}{w} - s\hat{\hat{V}}_B(w,s)$. The theorem now follows from Theorem 3.1. □

By differentiating $\hat{\hat{w}}_B(w,s)$ with respect to s once or twice at $s = 0$ and taking the inverse with respect to w, the mean and the variance of $T_{B:z}$ can be obtained as given in the following corollary. Proof is mechanical and is omitted here.

Corollary 3.2.1. *Let* $\eta_{_Y}$ *and* $\eta_{_{Y_2}}$ *denote the mean and the second moment of* Y_j*, and let* $H_B(z)$ *be the renewal function associated with* Z_B*, i.e.* $H_B(z) \stackrel{def}{=} \sum_{n=1}^{\infty} G_{Z_B}^{(n)}(z)$*, where* $G_{Z_B}^{(n)}(z) = \int_0^z G_{Z_B}^{(n-1)}(z) dG_{Z_B}(z)$ *for* $n \geq 2$ *starting with* $G_{Z_B}^{(1)}(z) = G_{Z_B}(z) = G_{Z_B}(z, \infty)$*. One then finds that*

a) $E[T_{B:z}] = \eta_{_Y}\{1 + H_B(z)\}$

b) $$Var[T_{B:z}] = \eta_{_{Y_2}}\{1 + H_B(z)\} + 2\eta_{_Y} \int_0^z G_{Z_B}^{(n)}(z-x) E[Y|Z=x] dG_{Z_B}(x) \\ -\eta_{_Y}^2\{1 + H_B(z)\}^2.$$

The counterpart of Theorem 3.2 for the search completion time $T_{_{PC:z}}$ for Type PC customers or $T_{M:z}$ for Type M customers can be obtained by setting $X_2(j) = 0$ or $X_1(j) = 0$ respectively. More specifically, let the distribution function and the probability density function of $T_{_{PC:z}}$ and $T_{M:z}$ be defined by

$$W_{_{PC}}(z,t) = P[T_{_{PC:z}} \leq t] \quad ; \quad W_M(z,t) = P[T_{M:z} \leq t] \tag{3.22}$$

and

$$w_{_{PC}}(z,t) = \frac{\partial}{\partial t} W_{_{PC}}(z,t) \quad ; \quad w_{_M}(z,t) = \frac{\partial}{\partial t} W_M(z,t) \quad , \tag{3.23}$$

with the correspoding Laplace transforms with respect to t and z given by

$$\hat{\hat{w}}_{_{PC}}(w,s) = \int_0^\infty \int_0^\infty e^{-st-wz} w_{_{PC}}(z,t') dz dt' \quad , \tag{3.24}$$

$$\hat{\hat{w}}_{_M}(w,s) = \int_0^\infty \int_0^\infty e^{-st-wz} w_{_M}(z,t') dz dt' \quad . \tag{3.25}$$

From (3.14) and (3.15), the following theorem holds by replacing $\hat{\hat{g}}_{_{Z_B,Y}}(w,s)$ in Theorem 3.2 by $\hat{\hat{\varphi}}_{_{X_1,Y}}(w,s)$ and $\hat{\hat{\varphi}}_{_{X_2,Y}}(w,s)$, respectively.

Theorem 3.3.

$$a)\quad \hat{\hat{w}}_{PC}(w,s) = \frac{\hat{\varphi}_Y(s) - \hat{\hat{\varphi}}_{X_1,Y}(w,s)}{w\{1-\hat{\hat{\varphi}}_{X_1,Y}(w,s)\}} \quad , \quad Re(s) \geq 0$$

$$b)\quad \hat{\hat{w}}_{M}(w,s) = \frac{\hat{\varphi}_Y(s) - \hat{\hat{\varphi}}_{X_2,Y}(w,s)}{w\{1-\hat{\hat{\varphi}}_{X_2,Y}(w,s)\}} \quad , \quad Re(s) \geq 0$$

By differentiating $\hat{\hat{w}}_{PC}(w,s)$ and $\hat{\hat{w}}_{M}(w,s)$ with respect to s once or twice at $s=0$ and then inverting the results with respect to w, the next corollary follows as before.

Corollary 3.3.1. *Let $\eta_{_Y}$ and $\eta_{_{Y_2}}$ be as in Corollary 3.2.1, and let $H_{PC}(z)$ and $H_M(z)$ be the renewal functions associated with X_1 and X_2 respectively. One then finds that*

$$a)\quad E[T_{PC:z}] = \eta_{_Y}\{1 + H_{PC}(z)\}$$

$$b)\quad E[T_{M:z}] = \eta_{_Y}\{1 + H_{M}(z)\}$$

$$c)\quad Var[T_{PC:z}] = \eta_{_{Y_2}}\{1 + H_{PC}(z)\} + 2\eta_{_Y}\int_0^z F_{X_1}^{(n)}(z-x)E[Y|Z=x]dF_{X_1}(x)$$
$$-\eta_{_Y}^2\{1 + H_{PC}(z)\}^2$$

$$d)\quad Var[T_{M:z}] = \eta_{_{Y_2}}\{1 + H_{M}(z)\} + 2\eta_{_Y}\int_0^z F_{X_2}^{(n)}(z-x)E[Y|Z=x]dF_{X_2}(x)$$
$$-\eta_{_Y}^2\{1 + H_{M}(z)\}^2$$

We note that Theorem 3.3 and Corollary 3.3.1 coincide with Theorem 1.A2 and Corollary 1.A3 of [20].

In principle, $W_B(z,t)$, $W_{PC}(z,t)$ and $W_M(z,t)$ can be evaluated by inverting $\hat{\hat{w}}_B(w,s)/s$, $\hat{\hat{w}}_{PC}(w,s)/s$ and $\hat{\hat{w}}_M(w,s)/s$ with respect to both w and s based on Theorems 3.2 and 3.3. Given the numbers of customers in Type B, Type PC and Type M, denoted by N_B, N_{PC} and N_M respectively, it then becomes possible to find the distribution of the number of customers who have made a conversion by time t. More specifically, for $Var \in \{B, PC, M\}$, let $K_{Var}(t)$ be the number of Type Var customers who have made a conversion by time t and define

$$P_{Var}(k,t) \stackrel{\text{def}}{=} P[K_{Var}(t) = k] \quad , k = 0, 1, \cdots N_{Var} \quad . \tag{3.26}$$

It can be seen that the following theorem holds. Proof is trivial and omitted.

Theorem 3.4. *For $Var \in \{B, PC, M\}$ and $k = 0, 1, \cdots N_{Var}$, one has*

$$P_{Var}(k,t) = \binom{N_{Var}}{k} \Big\{ p \cdot W_{_{Var}}(z,t) \Big\}^k \cdot \Big\{ 1 - p \cdot W_{_{Var}}(z,t) \Big\}^{N_{B_{Var}} - k} \quad .$$

Considering that a website manager may be interested in the time needed to achieve a desired level of conversion rate, we introduce the minimal time needed to have K customers with conversion by time t. Formally, we define

$$\tau_{_{Var}}(K) \stackrel{\text{def}}{=} inf \Big\{ t : K_{Var}(t) \geq K \Big\} \quad , \tag{3.27}$$

and the associated survival function is denoted by

$$\overline{D}_{_{Var}}(K,t) = P[\tau_{_{Var}}(K) > t] \quad . \tag{3.28}$$

The following theorem is then immediate from Theorem 3.4.

Theorem 3.5. *For $Var \in \{B, PC, M\}$ and $0 < K \leq N_{Var}$, one has*

$$\overline{D}_{_{Var}}(K,t) = \sum_{k=0}^{K-1} P_{Var}(k,t) \quad .$$

In general, the double inversion of $\hat{\hat{w}}_{_{Var}}(w,s)$ is laborious and cumbersome. In the next section, we overcome this difficulty by assuming that the time period available for net surfing in a day is deterministic, i.e. $Y_j = T$ for all j, and the amount of information gathered through net surfing consists of two parts: a part which is proportional to T and another part which is exponentially distributed and independent of T.

4. Inversion of Double Laplace Transforms for Exponential Information Gathering and Deterministic Search Time

In this section, we evaluate the distribution function of the information search completion time for Type B, Type PC and Type M customers explicitly based on the results of Section 3. This in turn would provide a computational vehicle for assessing the impact of the mobile access to the Internet on the conversion rate, to be addressed in Section 5. For this purpose, we assume that both $X_1(j)$ and $X_2(j)$ consist of two parts: a part independent of Y_j and another part proportional to Y_j. The former parts for $X_1(j)$ and $X_2(j)$ are denoted by $\hat{X}_1(j)$ and $\hat{X}_2(j)$ respectively. More formally, we define

$$X_1(j) \quad = \quad \hat{X}_1(j) + \alpha_{_{PC}} Y_j \quad ; \quad X_2(j) = \hat{X}_2(j) + \alpha_{_M} Y_j \quad , \tag{4.1}$$

where $\hat{X}_1(j)$, $\hat{X}_2(j)$ and Y_j are independent of each other, but $X_1(j)$ and $X_2(j)$ are not independent because of sharing Y_j.

It is natural to consider that the amount of information gathered via the PC access would be stochastically larger than that via the mobile access. This is so because the mobile access to the Internet is often limited in time in comparison with the PC access. Furthermore, the screen size and the memory capacity of mobile devices would be also less than those of PCs. Accordingly, we assume that

$$\hat{X}_1(j) \succ_{st.} \hat{X}_2(j) \text{ and } \alpha_{_{PC}} \geq \alpha_{_M} \quad , \tag{4.2}$$

where, for nonnegative random variables V and W, $V \succ_{st.} W$ means that $P[V > x] \geq P[W > x]$ for all $x \geq 0$. Although this assumption is not essential theoretically, we adopt it from an application point of view.

Let the distribution functions of $\hat{X}_1$ and $\hat{X}_2$ be denoted by $F_{\hat{X}_1}(x)$ and $F_{\hat{X}_2}(x)$ respectively, and let $F_{\underline{X},Y}(x_1, x_2, y)$ be the joint distribution function of $\underline{X}$ and Y. From (4.1), by conditioning on Y, one finds that

$$F_{\underline{X},Y}(x_1, x_2, y) = P[X_1 \leq x_1, X_2 \leq x_2, Y \leq y]$$

$$= \int_0^{min\{y, \frac{x_1}{\alpha_{PC}}, \frac{x_2}{\alpha_M}\}} F_{\hat{X}_1}(x_1 - \alpha_{PC}\tau) F_{\hat{X}_2}(x_2 - \alpha_M \tau) f_Y(\tau) d\tau \quad . \tag{4.3}$$

From (3.2), it then follows that

$$\begin{aligned} f_{\underline{X},Y}(x_1, x_2, y) &= f_{\hat{X}_1}(x_1 - \alpha_{PC} y) f_{\hat{X}_2}(x_2 - \alpha_M y) f_Y(y) \\ &\times \; I\Big\{0 \leq y \leq min\{\frac{x_1}{\alpha_{PC}}, \frac{x_2}{\alpha_M}\}\Big\}, \end{aligned} \tag{4.4}$$

where $I\{ST\} = 1$ if statement ST is true and $I\{ST\} = 0$ otherwise.

In order to facilitate the double conversion of $\hat{\hat{w}}_{Var}(w, s)$ for numerical tractability, we assume that $\hat{X}_1(j)$ and $\hat{X}_2(j)$ are exponentially distributed with respective probability density functions given by

$$f_{\hat{X}_1}(\hat{x}_1) = \lambda_{PC} e^{-\lambda_{PC}\hat{x}_1} \quad ; \quad f_{\hat{X}_2}(\hat{x}_2) = \lambda_M e^{-\lambda_M \hat{x}_2} \quad . \tag{4.5}$$

In addition, it is assumed that the available search time for each customer in each day is deterministic so that $Y_j = T$ for all j. This is equivalent to saying that the generalized probability density function of Y is given by the delta function, that is $f_Y(y) = \delta(y - T)$, where the delta function is defined as the unit operator for convolution, i.e. $u(x) = \int_0^\infty u(\tau)\delta(x - \tau)d\tau$ for any integrable function $u(x)$. Equation (4.4) can then be rewritten as

$$\begin{aligned} f_{\underline{X},Y}(x_1, x_2, y) &= \lambda_{PC} e^{-\lambda_{PC}(x_1 - \alpha_{PC} y)} I\{x_1 \geq \alpha_{PC} y\} \\ &\times \; \lambda_M e^{-\lambda_M (x_2 - \alpha_M y)} I\{x_2 \geq \alpha_M y\} \delta(y - T) \quad . \end{aligned} \tag{4.6}$$

Under these assumptions, we are now in a position to derive $\hat{\hat{w}}_{Var}(w,s)/s$ for $Var \in \{B, PC, M\}$. For notational convenience, the following functions are defined.

$$\zeta_B(w) \stackrel{\text{def}}{=} e^{-\alpha_B Tw} \frac{\lambda_{PC}}{w+\lambda_{PC}} \cdot \frac{\lambda_M}{w+\lambda_M}, \text{ where } \alpha_B = \alpha_{PC} + \alpha_M \quad ;$$

$$\zeta_{PC}(w) \stackrel{\text{def}}{=} e^{-\alpha_{PC} Tw} \frac{\lambda_{PC}}{w+\lambda_{PC}} \quad ; \quad \zeta_M(w) \stackrel{\text{def}}{=} e^{-\alpha_M Tw} \frac{\lambda_M}{w+\lambda_M} \quad . \tag{4.7}$$

From Theorems 3.2 and 3.3 together with Equations (4.1) through (4.6), one has the following theorem. Proof demands careful implementation of integrals involving the delta function, but it is rather mechanical and is omitted here.

Theorem 4.1. *For $Var \in \{B, PC, M\}$, let $\zeta_{Var}(w)$ be as in (4.7). Then $\hat{\hat{w}}_{Var}(w,s)/s$ is given by*

$$\frac{\hat{\hat{w}}_{Var}(w,s)}{s} = \sum_{k=0}^{\infty} \left\{ \frac{1}{s} \cdot e^{-(k+1)Ts} \right\} \left\{ \frac{1-\zeta_{Var}(w)}{w} \zeta_{Var}(w)^k \right\} \quad .$$

As discussed in Section 3, of central interest to this paper is the distribution function $W_{Var}(z,t)$ of the search completion time. In order to evaluate $W_{Var}(z,t)$ numerically, it is necessary to invert $\hat{\hat{w}}_{Var}(w,s)/s$ with respect to both w and s based on Theorem 4.1. For doing so, we first obtain $b_{Var}(z)$ by inverting $\zeta_{Var}(w)$ in (4.7). For $Var \in \{B, PC, M\}$, let $h_{Var}(z)$ be defined by

$$h_B(z) \stackrel{\text{def}}{=} \int_0^z \lambda_{PC} e^{-\lambda_{PC}(z-x)} \lambda_M e^{-\lambda_M x} dx = \frac{\lambda_{PC}\lambda_M}{\lambda_M - \lambda_{PC}} (e^{-\lambda_{PC} z} - e^{-\lambda_M z}) \quad ;$$

$$h_{PC}(z) \stackrel{\text{def}}{=} \lambda_{PC} e^{-\lambda_{PC} z} \quad ; \quad h_M(z) \stackrel{\text{def}}{=} \lambda_M e^{-\lambda_M z} \quad . \tag{4.8}$$

It should be noted that the Laplace transform of $h_{_{Var}}(z)$ corresponds to the portion of $\zeta_{_{Var}}(w)$ without the exponential function. Since the real domain form of the function $exp\{-\alpha_{_{Var}}Tw\}$ is given as $\delta(z-\alpha_{_{Var}}T)$, one can find $b_{_{Var}}(z)$ by convolving it with $h_{_{Var}}(z)$, yielding

$$b_{_{Var}}(z) \quad = \quad h_{_{Var}}(z-\alpha_{_{Var}}T)I\{z \geq \alpha_{_{Var}}T\} \quad . \tag{4.9}$$

We also note that the Laplace transform $\{1-\zeta_{_{Var}}(w)\}/w$ corresponds to the survival function $\overline{B}_{_{Var}}(z)$ given by

$$\overline{B}_{_{Var}}(z) = \int_z^\infty b_{_{Var}}(\tau)d\tau = \int_{max\{z-\alpha_{_{Var}}T,0\}}^\infty h_{_{Var}}(\tau-\alpha_{_{Var}}T)d\tau \quad . \tag{4.10}$$

We are now in a position to describe $W_{_{Var}}(z,t)$ by inverting $\hat{\hat{w}}_{_{Var}}(w,s)/s$ based on Theorem 4.1. It can be seen that $\hat{\hat{w}}_{_{Var}}(w,s)/s$ is a separable function of w and s. Since $exp\{-(k+1)Ts\}/s$ can be inverted as $U(t-(k+1)T)$ where $U(t)$ is the step function defined by $U(t) = I\{t \geq 0\}$, the summation for $\hat{\hat{w}}_{_{Var}}(w,s)/s$ based on Theorem 4.1 should be taken over k in the range in which $t-(k+1)T \geq 0$.

For inverting the function of w in $\hat{\hat{w}}_{_{Var}}(w,s)/s$, we denote the k-fold convolution of a function $r(z)$ with itself by $r^{(k)}(z) = \int_0^z r^{(k-1)}(z-\tau)r(\tau)d\tau$ for $k = 1,2,\cdots$ with $r^{(0)}(z) = \delta(z)$. For the convolution of two different functions $r(z)$ and $q(z)$, we write $r * q(z) = \int_0^z r(z-\tau)q(\tau)d\tau$. With this notation, one finds from (4.7) and (4.9) that

$$b_{_{Var}}^{(k)}(z) = h_{_{Var}}^{(k)}(z-\alpha_{_{Var}}kT)I\{z \geq \alpha_{_{Var}}kT\} \quad , \tag{4.11}$$

whose Laplace transform is given by $\zeta_{_{Var}}(w)^k$. The inversion of $\hat{\hat{w}}_{_{Var}}(w,s)/s$ with respect to w can then be accomplished by convolving $b_{_{Var}}^{(k)}(z)$ with

$\overline{B}_{Var}(z)$ given in (4.10).

In summary, we have shown the following theorem. For notational convenience, we define

$$k_{Var}(z,T) \stackrel{\text{def}}{=} \lfloor \frac{z}{\alpha_{Var} T} \rfloor \quad ; \quad k(t,T) \stackrel{\text{def}}{=} \lfloor \frac{t}{T} \rfloor, \tag{4.12}$$

where $\lfloor a \rfloor$ is the largest integer which is less than or equal to a.

Theorem 4.2. *For $Var \in \{B, PC, M\}$, let $k_{Var}(z,T)$ and $k(t,T)$ be as in (4.12). One then has*

$$W_{Var}(z,t) \quad = \quad \sum_{k=0}^{min\{k_{Var}(z,T),k(t,T)\}} \overline{B}_{Var} * b_{Var}^{(k)}(z) \quad .$$

In order to evaluate $W_{Var}(z,t)$ in Theorem 4.2, it is necessary to compute convolutions $b_{Var}^{(k)}(z)$ and $\overline{B}_{Var} * b_{Var}^{(k)}(z)$ repeatedly. For this purpose, the Laguerre transform is employed. The Laguerre transform, originally developed by Keilson and Nunn[7], Keilson, Nunn and Sumita[8], and further studied by Sumita[17], maps a continuum function $f(x)$ into a sequence $(f_n^\sharp)$, thereby providing an alogrithmic basis for computing a variety of continuum operations through lattice operations. For two such functions $f(x)$ and $g(x)$, for example, the convolution $f * g(x)$ can be mapped into the lattice convolution $(f * g)_n^\sharp = \sum_{k=0}^n f_{n-k}^\sharp g_k^\sharp$. The reader is referred to Sumita and Kijima[18,19] for a succinct summary of theory and alogrithms of the Laguerre transform.

5. Numerical Examples

In this section, we illustrate how the mobile access to the Internet would affect the information search completion time, and consequently the conversion rate, through numerical examples. The basic set of the underlying parameter values for representing Type PC customers and Type M customers are set as in Table 5.1. It should be noted that the assumption in (4.2) is satisfied.

Table 5.1 : Basic Set of Parameter Values

parameter	λ_{PC}	λ_M	α_{PC}	α_M	z	T	N_{Var}	K	p
value	1.0	1.5	0.1	0.15	100	1.0	500	100	0.5

For Type B customers, as an anonymous referee pointed out, the use of a mobile device for information search through the Internet may replace the use of a PC for the same purpose. In order to reflect this point, we define the functional relationships among $\hat{\lambda}_{PC}$, $\hat{\lambda}_M$, $\hat{\alpha}_{PC}$ and $\hat{\alpha}_M$ for Type B customers as

$$\hat{\lambda}_{PC} = \lambda_{PC} + \frac{1}{\hat{\lambda}_M} \quad ; \quad \hat{\alpha}_{PC} = \alpha_{PC} \cdot \frac{1}{1+\hat{\alpha}_M} \quad . \tag{5.1}$$

Here, we consider a situation that an original Type PC customer with λ_{PC} and α_{PC} becomes a Type B customer by starting to use a mobile device. The degree of the use of the mobile device is characterized by $\hat{\lambda}_M > 0$ and $\hat{\alpha}_M > 0$, which results in degrading the use of PC where $\hat{\lambda}_{PC}$ is increased from λ_{PC} by adding $1/\hat{\lambda}_M$, and $\hat{\alpha}_{PC}$ is decreased from α_{PC} by multiplying $1/(1+\hat{\alpha}_M)$.

In Figure 5.1, the survival functions $\overline{W}_{Var}(z,t)$ of the search completion time are plotted for $Var \in \{B_0, B, PC, M\}$. Here, the right-most curve $\overline{W}_M(z,t)$ corresponds to Type M customers who employ the mobile access exclusively. The next curve $\overline{W}_{PC}(z,t)$ describes the survival function of the search completion time for Type PC customers who utilize the Internet only through the PC access. The left-most curve $\overline{W}_{B_0}(z,t)$ represents Type B

customers who use both the mobile access and the PC access without replacement effects at all between the two different accesses. The three curves $\overline{W}_B(z,t)$ in between correspond to Type B customers where the mobile access replaces the PC access to some extent based on (5.1), and the effects of this replacement is weakened from the right to the left. The stochastic ordering among these search completion times is evident. For a user to complete information search with probability 0.7 or more, for example, 16 hours would be needed for a Type B_0 customer. This number is increased from 16 to 18, 25, 34, 78 and 99 as the curve moves from the left to the right.

Similar curves are depicted in Figure 5.2 for the time required to have 100 or more customers with conversion with the total population of 1500. For achieving the conversion rate of 1/15, with probability 0.7 or more, only 14 hours would be needed if all customers are of Type B_0. As before, this number is increased from 14 to 16, 21, 27, 55 and 78 as the curve moves from the left to the right.

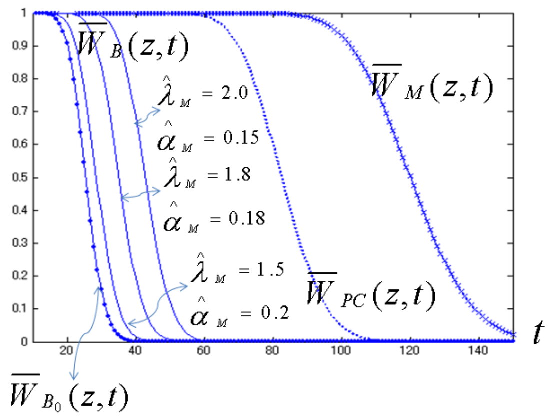

Figure 1: Survival Function of Information Search Completion Time

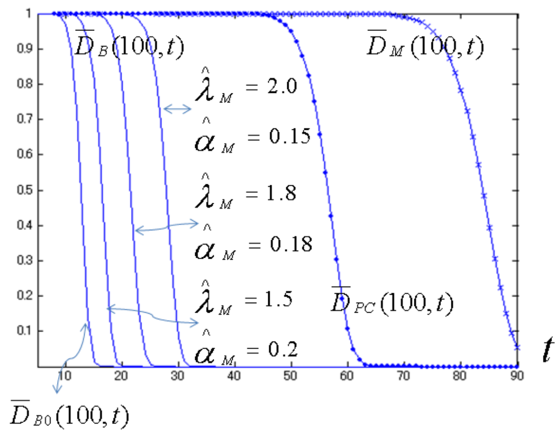

Figure 2: Survival Function of Time to Reach Conversion Rate of 1/15

6. Concluding Remarks

As the mobile devices continue to advance rapidly, it becomes quite important to find a way to assess the impact of the mobile access to the Internet on the performance of e-commerce. Among such performance indicators, of particular interest would be the conversion rate, where the conversion means that a customer takes a firm action at the website desired by the management of the e-commerce, e.g. to become a member, to place an order and the like. In this paper, a mathematical model is developed for describing the information search process through the Internet with or without the mobile access. The analytical framework enables one to assess the impact of the mobile access on the information search completion time and the conversion rate.

It is assumed that individual customers would search certain information through net surfing inside as well as outside the website within the Internet before deciding whether or not to make a conversion. This information search would be completed when the cumulative amount of information gathered through net surfing exceeds a prespecified level z. The information search completion time $T_{PC:z}$ with only the PC access, $T_{M:z}$ with only the mobile access and $T_{B:z}$ with both the mobile access and the PC access are analyzed, deriving the double Laplace transforms of their probability density functions with respect to the threshold level z and the information search completion time t. The means and variances are also obtained explicitly.

The double transforms are inverted into the real domain under the assumption of exponential information gathering and deterministic search time. This in turn enables one to compute the probability of how many of them would finish information search and make a conversion by time t, assuming that each customer decides to make a conversion with probability p and not to make a conversion with probability $1 - p$ upon completion of information search, and N customers are present at time $t = 0$. Numerical examples are given, demonstrating the effectiveness of the computational procedures developed in this paper for assessing the impact of the mobile access on the information search completion time and the conversion rate.

This type of analytical research for understanding the impact of the mobile access to the Internet on the performance of e-commerce is still in its infancy. The performance measures other than the conversion rate should be addressed. Furthermore, such performance analyses ought to be tied with economic analysis. It would be also necessary to establish a means to estimate the underlying parameters from real data. Such studies are in progress and will be reported elsewhere.

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