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Nucleon decay matrix elements with the Wilson quark action: an update *

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We present preliminary results of a new lattice computation of hadronic matrix elements of baryon number violating operators which appear in the low-energy effective Lagrangian of (SUSY-)Grand Unified Theories. The contribution of irrelevant form factor which has caused an underestimate of the matrix elements in previous studies is subtracted in this calculation. Our results are 2∼4 times larger than the most conservative values often employed in phenomenological analyses of nucleon decay with specific GUT models.

1. Introduction

Nucleon decay is an important consequence of Grand Unified Theories (GUTs), which can give strong constraints on such theories. Unfortunately, theoretical estimations of the lifetime and branching ratios suffer from large uncertainties. One of the sources of the uncertainties is the value of hadronic matrix elements from the nucleon to pseudo scalar meson through baryon number violating operators which appear in the low-energy effective Lagrangian of GUTs.

Conventionally these matrix elements are estimated by rewriting them in terms of the three-quark annihilation amplitude $\langle 0|O|p\rangle$ using the soft-pion theorem. There have been several lattice calculations for $\langle 0|O|p\rangle$ [1–3]. They have not yielded a definitive result, however. Moreover, the validity of the soft-pion theorem for the nucleon decay process has not been clarified.

A pioneering lattice work to calculate $\langle \pi^0|O|p\rangle$ directly was made in [3]. We also made an effort to advance it in several fronts [4]. After this study, we have found that previous lattice calculations for the matrix elements including ours had a subtle point which caused an underestimate of the matrix elements. The problem is that the previous results contain the contribution of an irrelevant form factor, which vanishes after using the equation of motion of the out-going lepton and setting its mass to zero. This contribution can not be subtracted using our previous data, essentially because we did not measure all necessary components of the three-point function.

In this article we report results of a new simulation in which we measure all the necessary components of the three-point correlation function to disentangle the relevant form factor from the irrelevant one.

As in our previous calculation, we evaluate matrix elements of all dimension six baryon number violating operators classified according to $SU(3)\times SU(2)\times U(1)$ [5,6], so as to cover various GUT models and decay processes. Also the calculations are made at physical quark mass and physical momentum in order to take account of the $N \to K$ process which is expected to be the

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dominant mode for SUSY-GUTs.

2. Calculational method

We wish to calculate the matrix element of a class of three quark operators between the nucleon N and a pseudoscalar meson PS. Using Lorentz and parity invariance, one finds that the matrix elements are described by two form factors,

$$\begin{aligned} & \langle PS|(q_1 C P_{R/L} q_2) P_{R/L} q_3|N\rangle \\ = \;& A(q^2) P_{R/L} N + B(q^2) P_{R/L} \not{q} N, \end{aligned} \qquad (1)$$

where q_i represents a quark field, q is a four momentum of the out-going lepton and N and $P_{R/L}$ stands for the nucleon spinor and chiral projection operator, respectively.

Multiplying by the lepton spinor and using the equation of motion, one can see that the second term is proportional to lepton mass. We call A and B as relevant and irrelevant form factor, respectively, as the latter does not contribute when the lepton mass is neglected.

To extract the form factors, we form a ratio,

$$\frac{\langle J_{PS}(t_y) O_\gamma(t_x) \bar{J}_{N,\gamma'}(0)\rangle}{\langle J_{PS}(t_y) J^\dagger_{PS}(t_x)\rangle \langle J_N(t_x) \bar{J}_N(0)\rangle} \sqrt{Z_{PS}}\sqrt{Z_N}, \quad (2)$$

where γ and γ' are spinor indices, and Z_{PS} (Z_N) is the residue of the two-point function of PS meson (nucleon). This ratio has the following asymptotic form,

$$\begin{pmatrix} A + q^0 B & 0 \\ B\vec{q}\cdot\vec{\sigma} & 0 \end{pmatrix}, \qquad (3)$$

in a 2×2 block notation.

It is important that the upper component is a linear combination of the two form factors and the lower component is proportional to the irrelevant term. In order to disentangle the relevant form factor from the irrelevant one, both components are needed, while, in the previous studies, only the upper component was measured. It should be noted that we can not follow this procedure for the case $\vec{q}$=0.

To compare our results with the predictions of the tree-level chiral Lagrangian [8,9], we also calculate the three-quark annihilation amplitude,

$$\langle 0|(uCd_R)u_L|p\rangle \;=\; \alpha N_L, \qquad (4)$$

$$\langle 0|(uCd_L)u_L|p\rangle \;=\; \beta N_L, \qquad (5)$$

which are obtained from the two-point function.

3. Numerical simulation

Our calculation is carried out in quenched QCD at β=6.0 with the Wilson quark action on a $28^2\times 48\times 80$ lattice. We analyze 100 configurations at the hopping parameter K=0.15620, 0.15568, 0.15516, 0.15464. The lattice scale fixed by m_ρ=770 MeV in the chiral limit (K_c=0.15714(1)) is a^{-1}=2.30(4) GeV, and the point for the strange quark estimated from m_K/m_ρ=0.644 is given by K_s=0.15488(7).

We fix the nucleon source at $t = 0$, PS meson sink at $t = 29$ and move the operator between them. Matrix elements are evaluated at four spatial momenta $\vec{q}a$=(0,0,0), (π/14,0,0), (0,π/14,0), (0,0,π/24), injected in the PS meson sink. As mentioned in the previous section, it is not possible to disentangle the relevant form factor from the irrelevant one when $\vec{q}$=0. Therefore, we use two data with two finite momenta to interpolate to the physical momentum.

The form factor obtained by fitting the ratio of three-point function and two-point functions depends on the quark mass and the momentum squared. We distinguish u-d and s quark masses; the former (m_{ud}) is taken to the chiral limit, and the latter (m_s) interpolated to the physical s quark mass. We also interpolate the momentum square to the physical lepton mass. To do this, we fit the data with the following form,

$$c_1 + c_2\cdot q^2 + c_3\cdot(q^2)^2 + c_4\cdot m_{ud} + c_5\cdot m_s. \qquad (6)$$

Matrix elements are renormalized, with mixing included, by tadpole-improved one-loop renormalization factors [7] to the $\overline{\mathrm{MS}}$ scheme calculated at the scale $\mu = 1/a$. The quoted errors are only statistical, estimated by the single elimination jackknife procedure.

4. Results

We present a typical q^2 dependence of the relevant form factor in Fig. 1. We also plot a linear combination of the relevant and irrelevant form factors $A + q^0 B$ and the prediction from

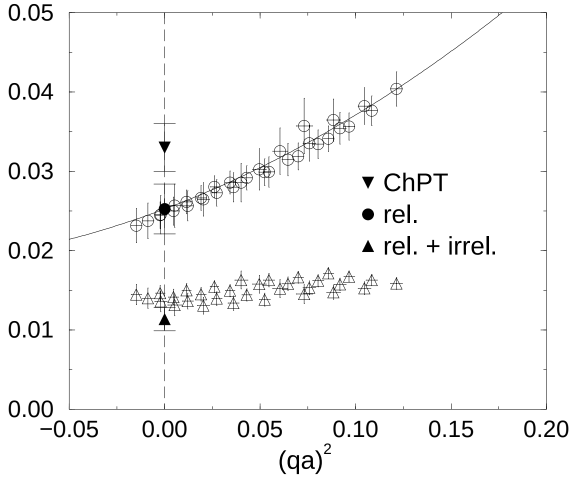

Figure 1. q^2 dependence of the relevant form factor $|A_\pi(q^2)|$ (circles) and a linear combination of the relevant and irrelevant form factors $|A_\pi(q^2)+q^0B_\pi(q^2)|$ (up-triangles), obtained from the matrix element $\langle\pi^0|(u_RCd_R)u_L|p\rangle$. The prediction from chiral Lagrangian is also shown (down-triangle).

tree-level chiral Lagrangian using $|\alpha|$=0.015(1) GeV^3 obtained in our simulation (we also obtain $|\beta|$=0.014(1) GeV^3). We see that the value of the relevant form factor is close to that from the chiral Lagrangian. As expected from analytical considerations with the chiral Lagrangian, there is a cancellation between the relevant and irrelevant form factors. We consider that this led to the inconsistency between the direct computation through three-point functions and the indirect computation using the tree-level chiral Lagrangian reported in previous lattice studies [3,4].

In Fig. 2, we plot our results for the relevant form factors at q^2=0. To compare our results with the most conservative estimation of the matrix elements, often employed in phenomenological analyses of nucleon decay with specific GUT models, we also plot the prediction from tree-level chiral Lagrangian with a choice of the parameters $|\alpha|$=$|\beta|$=0.003 GeV^3. Our results are about 2$\sim$4 times larger than the most conservative values, implying a stronger constraint on the parameter space of GUT models.

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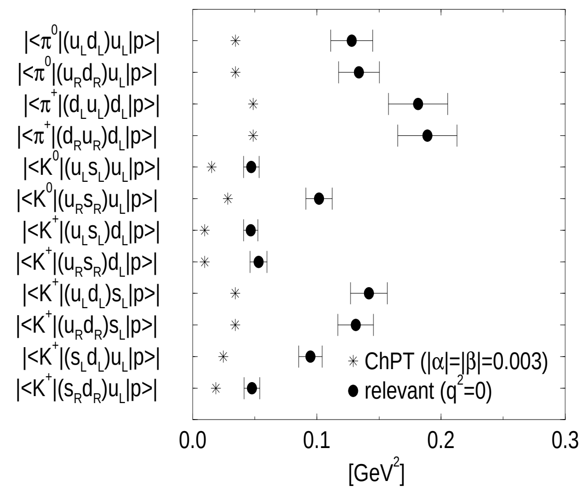

Figure 2. Relevant form factors at physical momentum $q^2 = 0$ from present calculation (filled circles) compared with the most conservative predictions of tree-level chiral Lagrangian (asterisks).

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