

**THE SAMPLE EXPERIENCE:  
THE DEVELOPMENT OF A RICH MEDIA ONLINE  
MATHEMATICS LEARNING ENVIRONMENT**

by

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B.A. (Cognitive Science), Simon Fraser University, 1995

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# Abstract

This report documents the development of *Sample Architecture for Mathematically Productive Learning Experiences* (SAMPLE), a rich media, online, mathematics learning environment created to meet the needs of middle school educators. It explores some of the current pedagogical challenges in mathematics education, and their amplified impacts when coupled with under-prepared teachers, a decidedly wide-spread phenomenon. The SAMPLE publishing experience is discussed in terms of its instructional design, multidisciplinary workflow, and technical framework. Considerations for like ventures in the future are analyzed.

*To my best friend, Dennis, for his encouragement, trust, and good humour;  
and to my parents for their support, sacrifice and love.*

*“Please don’t take it amiss, good sirs, if there are more mistakes in this little book than there are grey hairs on my old head. What can I do? I’ve never had much to do with book-learning and the like before. May the fellow who dreamed it all up choke on his porridge! As you stare at those letters they start to look the same. Your eyes cloud over, just like someone had scattered grain all over the page. See how many misprints I’ve found! All I ask, if you find any of them, is that you pay no attention, and read them as if they were spelt correctly.”*

VILLAGE EVENINGS NEAR DIKANKA — NIKOLAI GOGOL, 1993

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# Chapter 1

## Introduction

The Sample Architecture for Mathematically Productive Learning Experiences (SAMPLE) project was a teacher-focused research initiative designed to develop stand-alone learning materials for a middle school curriculum. It entailed the development of a prototype that harnessed “rich media”<sup>1</sup> and communication technologies to provide educators, new and experienced alike, with more tools to cope with the demands in today’s classrooms. The long-term goal was to build on the knowledge gained from this experience and apply it in a larger setting to enhance learning in high school and post-secondary mathematics classes.

### 1.1 Mathematics Education Reform and SAMPLE

SAMPLE was conceived in the midst of mass reforms in mathematics education. The impetus for this project was influenced by three factors: 1) the growing importance of numeracy in society, 2) the current state of mathematics learning, and 3) the benefits emergent technologies can offer in the classroom. Around the world, the definition of literacy has expanded to include a quantitative aspect. Numeracy is being recognized by UNESCO and other agencies as an indispensable skill in everyday life, one that is intimately tied to an individual’s economic survival (Wagner, 2001). With the latest large-scale study, the International Adult Literacy and Skills Survey (IALSS)<sup>2</sup>, showing that 55.1% of the

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<sup>1</sup>The term “rich media” was first coined by Suzanne Brisendine of Intel in 1998 to refer to “technologies that created a richer surfing experience” (McCloskey, 2000) which included interactive digital technologies.

<sup>2</sup>The International Adult Literacy and Skills Survey 2003, released in 2005 by Statistics Canada, is the second round of the International Adult Literacy Survey (IALS). Internationally, the IALSS is known as

population aged 16 and above in Canada lacks basic numeracy skills, there is an urgency to make mathematics education more accessible and accommodating to the current crop of students. In fact, as numeracy becomes a priority and equity a guiding principle, major changes to pedagogical practices are needed.

## 1.2 Some Challenges and Trends in Mathematics Education

Measures put in place by policy makers to address some of the challenges in mathematics education include fundamental changes to pedagogy and school curriculum. These initiatives often entail curriculum renewal and more mandatory courses. However, these approaches are not without serious consequences to the integrity of the educational system as a whole. For example, more topics<sup>3</sup> are being included by curriculum designers. These extra requirements are problematic because they increase the load for both teachers and students, especially for those using the “spiral curriculum”<sup>4</sup>. Another consequence of such mathematics reforms is that, as a means to raise student participation and attainment in high level mathematics, mathematics education has become mandatory for more students for a longer period of time in school with a shift from a focus on excellence to one on the basics. Some researchers have attributed the gradual decline in student performance on national mathematics competitions to a cut in enrichment support<sup>5</sup>. Concomitantly, there is an increased emphasis on developing students’ problem-solving skills. Yet, teachers are already struggling with an over-reaching curriculum that is too large (W. H. Schmidt, McKnight, & Raizen, 1997, pp. 4) to deliver without sacrificing mentorship and individual student attention. To take into account factors, such as growing class sizes or under-prepared teachers, it becomes a daunting task for any educational system to implement such a broad mandate.

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the Adult Literacy and Lifeskills Survey (ALL). The IALS was conducted between 1994 and 1998 with 22 participating countries. The ALL survey had six countries participating in the first phase in 2003 and five countries in the second phase in 2005. These surveys are designed to measure adult literacy skills, such as prose, document, and quantitative literacies. Problem solving literacy was added to the ALL survey.

<sup>3</sup>Topics that are seen as vital for all to function in the technological society such as statistics and probability were added to the British Columbia curriculum.

<sup>4</sup>There are two prevalent curricula in use in the educational system: spiral and mastery. A spiral curriculum is one that covers a multitude of topics each school year and then builds on them in the subsequent years. A mastery curriculum covers a small number of topics in depth in the year they are taught and may or may not be built upon in subsequent years. Students are expected to master each topic before advancing to the next. British Columbia subscribes to a spiral curriculum.

<sup>5</sup>According to Professor George Bluman, BC students have been performing more poorly than their Ontario counterparts on the Euclid Mathematics Contest since 2001. (R. Schmidt, 2005)

### 1.3 The Use of Computer Technologies and SAMPLE

To mitigate some of the above challenges, there is a movement to incorporate technologies into the classroom. One of the advantages of using computer technologies is the ability to customize the learning environment for both educators and students. Lessons can be designed to take into account individual learning style, aptitude and performance. Another benefit of technologies is the capability to assist students in visualizing mathematical concepts through a series of interactive simulations and experiments. In fact, one of the outcomes of the mass reforms in mathematics education is to engage students in exploratory activities with manipulatives (both concrete and virtual) in order to ameliorate their problem-solving skills. Computer programs that employ rich media offer students modes of mathematical visualization that are often not feasible using traditional methods. As an added bonus, teachers can easily conduct and manage concurrent virtual mathematics experiments and visualization exercises in a classroom setting.

SAMPLE's role was to harness the capabilities of computer technologies to deliver a discovery-based learning environment that was tailored to, first and foremost, teachers in addition to students and parents. It was intended to address the needs of elementary and middle-school mathematics educators, who were often under-prepared, by providing easy-to-use lesson plans, interactive content and remedial resources in a scalable system.

### 1.4 About SAMPLE

The SAMPLE project<sup>6</sup> (see Figure 1.1) was supported by an Initiative on the New Economy grant from the Social Sciences and Humanities Research Council of Canada. SAMPLE's principal investigator was Dr. David M. Kaufman from the Learning and Instructional Development Centre at Simon Fraser University. Dr. Jonathan M. Borwein<sup>7</sup> and Dr. Carolyn R. Watters from the Faculty of Computer Science at Dalhousie University were the co-investigators. MathResources Inc. was a partner of the SAMPLE project. Research personnel were partly funded by the Natural Sciences and Engineering Research Council of Canada and the Canada Research Chairs program. Much of the research took place in the

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<sup>6</sup>To visit the SAMPLE site, use the login and password pairs: student/studentpass for student-level access; parent/parentpass for parent-level access; and teacher/teacherpass for teacher-level access.

<sup>7</sup>Dr. Borwein was in the Department of Mathematics at Simon Fraser University until the conclusion of the SAMPLE project.

CoLab, a facility funded by the Canada Foundation for Innovation and British Columbia Knowledge Development Fund. The development of SAMPLE began in the spring of 2002 and concluded in the fall of 2003. Dr. June Lester assisted with the grant proposal and initial project development. The author of this report was the project manager of SAMPLE. The pedagogy team was recruited from the Faculty of Education at Simon Fraser University. The mathematical technology team consisted of researchers from the Department of Mathematics at Simon Fraser University. The content management team was from the Faculty of Computer Science at Dalhousie University.



Figure 1.1: The SAMPLE Home Page: <http://sample.colab.sfu.ca>.

## 1.5 About This Report

This report is organized into three major sections. It begins with the rationale for SAMPLE considered in the context of the importance of numeracy skills and current pedagogical challenges that pertain to mathematics education. It then describes in detail the publishing experience of this multidisciplinary project that employs a variety of computer technologies.

SAMPLE's instructional design is discussed in terms of its target audience, project workflow, content design, and the organization of lessons in the SAMPLE portal. The technical aspects of SAMPLE are then considered in two parts: 1) the authoring of rich media mathematical learning objects<sup>8</sup>, and 2) the design of a learning management system. It concludes with a reflection and assessment of developing technology-based mathematics learning environments.

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<sup>8</sup>Learning objects are small self-contained software modules that are designed to be reusable in different learning environments.



## Chapter 2

# Rationale for SAMPLE: Reasons for Creating an Online Curriculum

One of SAMPLE's goals was to augment classroom activities by building an online learning community that allowed for the sharing of ideas and learning objects by both the teachers and students.

The strategy of SAMPLE was not to transfer traditional teaching materials into digital formats but to combine traditional teaching wisdom with the use of information and communications technologies (ICT). SAMPLE endeavoured to render the use of technologies as intuitive and seamless as possible so as to free the students and teachers from getting distracted from the actual content at hand.

In fact, SAMPLE equipped instructors with the necessary tools to incorporate their own materials into the lesson plans and combined technology training, curriculum integration, and student performance assessment in a cohesive manner. This approach was particularly useful because it empowered instructors with an easy-to-use system that they could quickly learn and build on to prepare lesson plans.

“In our experience supporting academics in making effective use of the Web for teaching and learning is best achieved by placing the academic in the role of a learner who develops technical skills on a need-to-know basis by discussing potential improvements in their own pedagogical practice.” (Littlejohn, Stefani, & Sclater, 1999, p. 30)

A rich media-based initiative also fits in with the prevailing pedagogy by using a constructivist approach, that is, one that creates a discovery-based environment for learning, and serves to teach literacy and numeracy in addition to engaging in technology diffusion. Technological advances allow for more innovative ways (e.g. graphics, applets, etc.) to bring mathematics to the classroom which in turn allow students to experiment more easily and develop intuitions at the same time.

## 2.1 The Growing Importance of Numeracy

“Citizens who cannot reason mathematically are cut off from whole realms of human endeavor. Innumeracy deprives them not only of opportunity but also of competence in everyday tasks.” (Kilpatrick, Swafford, & Findell, 2001, p. 16)

Mathematics is the language of science and technology<sup>1</sup>. It is well established in the literature that many-faceted digital divides<sup>2</sup> are developing and that upskilling the general population in numeracy may help improve a region’s competitiveness<sup>3</sup> (Ontario Task Force on Competitiveness, Productivity and Economic Progress, 2002) and preserve the standard of living of its people (Betcherman, 1997). The International Adult Literacy Survey (IALS) (Statistics Canada, 1995), for example, has found a strong linkage between numeracy and an individual’s economic security. In fact, mathematical understanding has become an increasingly important skill both in the workplace and in everyday life. Yet, in Canada, 55.1% of adults between 16 and 65 years of age have less than the desired level of numeracy (Statistics Canada, 2005, p. 27). Furthermore, according to a report by the US National Academy of Sciences, “three of every four Americans stop studying mathematics before completing career or job prerequisites.” ((US) National Research Council, 1989, pp. 1–2)

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<sup>1</sup>A report by the National Research Council examined the relationship between mathematical sciences and modern industries in the United States and made a strong case for strengthening mathematics education “from kindergarten through graduate school” (Glimm, 1991, p. v) as a means to ensure economic competitiveness.

<sup>2</sup>Leslie Regan Shade, currently at Concordia University, prepared an in-depth review (Shade, 2002) of the wide-ranging literature on the subject.

<sup>3</sup>According to the World Knowledge Competitiveness Index 2005, six Canadian regions were considered and all were ranked among the bottom half of the 125 knowledge-based regional economies in the study (Huggins, Izushi, & Davies, 2005). Canada is ranked seventh out of 61 nations in the IMD World Competitiveness Yearbook 2006 (IMD, 2006, p. 7) [cited by permission] and ranked 14th out of 117 nations on the World Economic Forum’s Growth Competitiveness Index 2005 (Porter, Schwab, & Lopez-Claros, 2005, p. 7).

A Statistics Canada study (Bordt, Broucker, Read, Harris, & Zhang, 2001, p. 12) shows that only 18.6% of all Canadian upper secondary students surveyed in the Third International Mathematics and Science Study (TIMSS) were enrolled in mathematics in 1995. Of all reporting countries, Canada has the lowest participation rate in mathematics. A dwindling interest in mathematics is also evident in the findings of that study. While 89% of Grade 4 students sampled were keen about mathematics, that enthusiasm diminished to 74% for Grade 8 students. For students in their final year of secondary school, only 61% remained interested in the subject. Of those who stopped studying mathematics, 72% reported the subject was too difficult and 48% found mathematics boring.

## 2.2 The Current State of Student Performance in Mathematics

Results from domestic and international surveys have been relied upon as indicators of student performance and they suggest that there is much room for improvement in our educational system.

According to Human Resources and Skills Development Canada, 47% of all employment between 1987 and 2003 required at least post-secondary education (Bergeron, Dunn, Lapointe, Roth, & Tremblay-Côté, 2004, p. 7) and that “six out of 10 jobs created during that period were in highly skilled occupations<sup>4</sup>,” many of which required advanced numeracy skills. In fact, it projects that, between 2004 and 2008, 66% of “new non-student jobs<sup>5</sup> are expected to require a post-secondary education or to be in the management group<sup>6</sup>” (Bergeron et al., 2004, p. 19). In British Columbia, the job forecast predicts that by 2013, 70% of all employment openings will require some form of post-secondary education (Ministry of Advanced Education, 2005, p. 6). Unfortunately, many youths are not even completing their secondary education. In 2002–2003, the pan-Canadian secondary school graduation

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<sup>4</sup>These include professional occupations in “natural and applied sciences (particularly computer and information systems professionals), in business and finance, and in social science, education and government services.” (Bergeron et al., 2004, p. 6)

<sup>5</sup>Non-student jobs accounted for 92.8% of all employment in 2003. Temporary student employment is excluded from this analysis of the permanent job market. (Bergeron et al., 2004, p. 16)

<sup>6</sup>The projection “assumed that most management positions require a high skill level.” (Bergeron et al., 2004, p. 19)

rate was only 74%<sup>7</sup> (Canadian Education Statistics Council, 2006, p. 215), far below some of the OECD countries<sup>8</sup> whose graduation rates are 90% or higher (OECD, 2005, p. 10). In fact, the graduation rate of 74% already takes into account after-typical-age students. The graduation rate drops to only 64% (Canadian Education Statistics Council, 2006, p. 237) if only typical-age students are reported.

How do academic abilities and student attitudes correlate with student performance in mathematics? International and domestic indicators are revealing a complex problem. In the Canadian National Longitudinal Survey of Children and Youth (NLSCY)<sup>9</sup>, researchers have identified mathematics achievement as one of several valid indicators of academic engagement (Norris, Pignal, & Lipps, 2003, p. 30). Other recent studies suggest that “engaged” learners are more likely to succeed in class and complete secondary school.

At first glance, the waning interest in mathematics in Canada does not seem to dampen mathematics achievement in middle school when mathematics performance of Canadian students is compared to the performance of students from other countries in an international context. Canadian students are above average in middle-school mathematics when compared to their international counterparts. For example, a recent study<sup>10</sup> placed 15-year-old British Columbia students’ mathematical performance around the Canadian average, in the top performing group among 41 surveyed countries, where Canada as a whole was part of an eight-nation cluster outperformed by only two other countries (Bussière, Cartwright, & Knighton, 2004). However, other results and findings are not as encouraging.

While some research has shown that Canadian (and in fact North American) students’ mathematics proficiency has seen a modest improvement over the last decade, educators are confronted with students having less-than-satisfactory performance and high attrition rates in mathematics at both the secondary and post-secondary levels. For example, in the 2004–2005 school year, 69% of the Nova Scotia students did not pass the Math 12 provincial exam (with an average of 41%) and 43% did not pass the Advanced Math 12 exam (with

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<sup>7</sup>This rate does not include Quebec and Ontario in the 2002–2003 reporting partly due to Ontario’s double graduating cohorts as a result of the elimination of Grade 13 in 2003. Quebec’s reporting included those in adult programs. For comparison purposes, the 2000–2001 pan-Canadian overall graduation rate was 75%.

<sup>8</sup>The countries with graduation rates at 90% or higher include Germany, Greece, Ireland, Japan, Norway and Switzerland.

<sup>9</sup>This joint project of Human Resources Development Canada (HRDC) and Statistics Canada was initiated in 1994 – 1995.

<sup>10</sup>The Programme for International Student Assessment (PISA) is a study conducted by the Organisation for Economic Co-operation and Development (OECD) of its member countries.

an average of 54%) (Province of Nova Scotia, Department of Education, 2005, pp. 7-8). In Ontario, only 27% of Grade 9 applied math students met the province's standards in 2005 (Education Quality and Accountability Office, 2005, p. 45). In fact, the PISA report confirms that a very troubling trend is emerging:

“PISA 2003 divides students according to the highest of the six proficiency levels at which they can usually perform tasks correctly. . . . The small minority who can perform the most complex and demanding tasks are ranked at Level 6; those who can only perform very simple tasks are at Level 1. Students unable even to complete these tasks are said to be “below Level 1”. . . . Only 4 per cent of students in the combined OECD area . . . can perform the highly complex tasks required to reach Level 6. . . . About a third of OECD students can perform relatively difficult tasks at Levels 4, 5 or 6. . . . About three-quarters of OECD students can perform at least mathematical tasks at Level 2. . . . Eleven per cent of students in OECD countries are not capable even of Level 1 tasks. These students may still be able to perform basic mathematical operations, but were unable to utilize mathematical skills in a given situation, as required by the easiest PISA tasks.” (OECD, 2004, p. 8).

The report reveals that 10% of Canadian students are at Level 1 or below. For Prince Edward Island, our worst performing province, 18% of the students are in this category. However, our North American counterparts fared even worse. More than one-quarter of the students tested in the United States and two-thirds in Mexico are performing at Level 1 or below (Bussière et al., 2004, pp. 25–26).

At the post-secondary level, educators are noticing a competency gap despite the extremely high admission standards. Among those students who are admitted into post-secondary institutions, many lack the basic skills to perform satisfactorily in their first-year mathematics courses and are required to take remedial courses at universities all across Canada. Many universities and colleges have been, or are in the process of, implementing mathematics placement tests and remedial support for students. For example, at University of Manitoba, first-year students are required to take two semesters of remedial math courses (“Failing our students: Dumbed down curriculum needs an overhaul”, 2004). In fact, the problem of under-prepared freshmen has become so widespread that various forms of mathematics placement tests and remedial support are being tried and implemented by

institutions such as University of Victoria, Simon Fraser University, University of Ottawa, Carleton University, and Ryerson University (S. Schmidt, 2005).

Some educators have attributed the inadequate preparation of incoming high school students partly to the stringent admission requirements for post-secondary institutions. These critics contend that taking a rigorous mathematics course could potentially lower the students' grade average, thus affecting their chances of admission. Others reported that "grade inflation" was another factor.

Some might argue that perhaps these students would not have been accepted into universities had they taken their mathematics courses in high school. On the other hand, perhaps a better high school curriculum could have provided the needed support for the students to perform satisfactorily in mathematics both at high school and at the universities, in turn alleviating some of the fear students have about these courses. Others suggest that the current curriculum and admission system may simply delay the remedial help students needed to succeed. Whichever the case may be, secondary schools are graduating a significant number of students who lack the basic mathematics foundation to undertake and succeed in mathematics courses at the post-secondary level and this problem is too serious to be ignored. One possible solution may point to early intervention at the secondary level.

The Council of Ministers of Education, Canada has been conducting the School Achievement Indicators Program (SAIP) to evaluate student achievement since 1993.

According to the SAIP 2001 Mathematics Assessment, the achievement of both the 13-year-old and 16-year-old students failed to meet the expectations of a pan-Canadian panel of educators and non-educators (Council of Ministers of Education, Canada, 2002, pp. 30–32). For example, less than half (Canadian Education Statistics Council, 2003, p. 90) of the 16-year-old students demonstrated math problem solving skills at the desired Level 3. In essence, the percentage of Canadian students expected to achieve at or above each of the five performance levels as set out by the framework and criteria and by the questions asked in the assessment did not materialize.

Similar studies and reports have prompted calls to reform mathematics instruction. The US National Council of Teachers of Mathematics (NCTM) recommended five major shifts to combat these weaknesses in the education system, all of which point towards a cohesive discovery-based learning strategy that is aimed at increasing students' problem solving skills (National Council of Teachers of Mathematics, 1991, p. 3). Mathematics education is seriously in need of revitalization.

## 2.3 Findings and Shifts in Pedagogical Models in Mathematics

“In reality, no one can *teach* mathematics. Effective teachers are those who can stimulate students to *learn* mathematics. Educational research offers compelling evidence that students learn mathematics well only when they *construct* their own mathematical understanding.” ((US) National Research Council, 1989, p. 58)

Recent efforts to improve the educational system have produced new understanding in learning models and the many factors that affect learning outcomes. For example, there is a shift in pedagogical practices, from an instructivist learning model to a constructivist one. Many factors, such as disengagement, that affect student learning have been identified. While some of these factors apply to most school subject areas, many other factors, such as mathematics anxiety and under-qualified teachers, are specific to mathematical learning and contribute to diminished learning outcomes. Each of these topics is discussed in more detail below.

### 2.3.1 Instructivist vs. Constructivist Learning Models

Many educators hold the opinion that “most students do not learn what teachers teach. Instead they retain explanations personally constructed to account for phenomena in the rational universe.” (Yager, 2000, p. 19) In fact, this perception is so widespread that some (Connell, 1999; Corless, 1995) say that “mathematics is not taught, it is learned.”

There are two popular schools of thought on how mathematics education should be conducted. The traditional approach is frequently referred to as an “instructivist” learning model. An instructivist environment provides unidirectional communication and is often characterized by rote-learning or direct instruction. In this setting, students are asked to learn by memorizing procedures through mechanical repetitions as practice. Critics claim that the instructivist approach often leads to passive learning styles where students are to absorb knowledge via reading, seeing and listening.

The standards, issued by the NCTM, advocate a constructivist approach and have served to mobilize a modal change in knowledge dissemination in the classroom. According to the NCTM, instruction must focus on assisting students to develop thinking strategies. Some

research has shown that a constructivist environment encourages reciprocal communication. This means a change from a teacher-centred “instructivist” model to a student-centred “constructivist” regime (Diaz & Bontenbal, 2000). A concrete example of reciprocal communication could be that students, through the use of visualization and chat tools, discovered and communicated alternative solutions that were equivalent and equally valid to those presented by the teacher. The teacher would then be in a position to provide guidance that would take into account the particular knowledge the students had gained in the discovery process. In a constructivist setting, students are exposed to a multitude of contexts during the learning process. They are encouraged to first explore the relationship between newly presented information and their own prior knowledge, and then to construct new knowledge and understanding. The students’ individual learning styles are also taken into account. Proponents of this theory claim that a constructivist classroom not only contributes to active learning on the students’ part and builds critical thinking skills, but also the nature of this model fosters collaboration and cooperation (Anderson, 1997). To build a constructivist learning environment requires a serious pedagogical shift, and even more importantly, a commitment to fundamental cultural change.

## 2.4 Factors that Affect Learning Outcomes

How does one reconcile outstanding performance demonstrated by Canadian students on international assessments with poor scholastic examination results and high attrition rate in secondary mathematics enrollment? Critics have charged that curricular design may be at the root of the problem which may explain the rising number of under-prepared freshmen requiring, often mandatory, remedial assistance at the post-secondary level.

“Other education experts, however, said the main reason high school students lose interest in math and science because of weak teachers and dry curriculum.”  
(Sokoloff, 2002b)

From working with mathematics educators, SAMPLE researchers have learned that, in the everyday classroom, mathematics anxiety and disengagement are two prevalent problems that affect student learning outcomes and enrollment. Furthermore, low enrollment and scholastic achievement in mathematics courses are often traced back to curricular deficiencies (W. H. Schmidt et al., 1997), and a lack of qualified teachers or mentorship. There is ample



evidence that mathematics anxiety affects learning and contributes to disengagement and attrition as outlined in the following section.

### 2.4.1 Mathematics Anxiety

Mathematics anxiety is a common phenomenon<sup>11</sup> and a serious problem afflicting many students, parents<sup>12</sup> and teachers alike (Zaslavsky, 1994; Tobias, 1978). In the Dreyfus Gender Investment Comparison Study<sup>13</sup> conducted in 1996 on 1287 adults between the ages of 18 and 80, only 32% of the respondents were comfortable with high school mathematics (Welsh, 1997).

A survey conducted in 1992 on 9,093 students by researchers at University of Florida found that more than one-quarter of the respondents reported needing help to cope with math anxiety (Probert & Vernon, 1997). Countless post-secondary lecturers of mathematics education, at home (Cohen & Leung, 2004; Seaman, 1998) and abroad (Milgram, 2005; Alderson, 1999; Cornell, 1999), can attest to the high-level of mathematics anxiety reported by pre-service teachers<sup>14</sup>. It has been documented that 52% of primary teachers in Australia had “negative feelings about teaching mathematics.” (Carroll, 1999) This phenomenon is familiar to Canadian educators as well.

“At a recent orientation assembly [at University of Western Ontario], we asked our in-coming group of 440 elementary preservice teachers how they felt about mathematics. When asked to raise their hands if they loved mathematics, 15–20 hands went up. When asked to raise their hands if they hated mathematics, a sea of hands filled the auditorium.” (Gadanidis & Namukasa, 2005)

In a large-scale review of 151 studies on the subject, it was found that among college students of different majors, “the highest [mathematics] anxiety levels occurred for students

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<sup>11</sup>Marilyn Burns, a prominent U.S. mathematics educator, claimed that “more than two-thirds of American adults fear and loathe mathematics” (Burns, 1998).

<sup>12</sup>A recent study randomly surveyed 500 adults each in Massachusetts and Washington States and found that while only 14% admitted to having mathematics anxiety, about 40% of respondents with children reported that it was more difficult to help their children with mathematics than with other subjects. More than half of the parents who did not help their children with mathematics cited personal incompetence or complex curriculum as the reason (Mass Insight Education and Research Institute, 2004).

<sup>13</sup>This study was conducted by Dr. Christopher L. Hayes of the National Center for Women and Retirement Research (NCWRR) at the Long Island University.

<sup>14</sup>The impact of under-prepared and math anxious pre-service and in-service teachers is discussed in more details in the Teachers’ Qualifications section.

preparing to teach in elementary school” (Hembree, 1990, p. 42). The same research also found that “there is no compelling evidence that poor performance causes mathematics anxiety,” and in fact that “higher achievement consistently accompanies reduction in mathematics anxiety . . . treatment can restore the performance of formerly high-anxious students to the performance level associated with low mathematics anxiety.”

Mathematics anxiety impairs learning and can be debilitating for learners (Shore, 2005) with devastating consequences. Research has found that “low-anxious students tend to perform better on standardized achievement tests than high-anxious students” (Heinrich & Spielberger, 1982, p. 155) and that “attitudes, including math anxiety, affect one’s opportunities to gain math competence, and an individual’s overall competence is one of two major influences on performance.” (Ashcraft & Kirk, 2001, p. 236). In fact, the same research has shown that “math anxiety disrupts the on-going, task-relevant activities of working memory, slowing down performance and degrading its accuracy.” As a matter of fact, poor performance and mathematics avoidance in the classroom have been attributed to mathematics anxiety. Fortunately, researchers have found that intervention such as mathematics confidence workshops can help students of varying aptitude achieve “significant, long-lasting, self-reported improvement in math performance and in the ability to learn and use mathematics, as well as a reduction in math and test anxiety.” (Probert & Vernon, 1997, p. 6)

#### **2.4.2 Enrollment, Disengagement and Attrition**

In light of the changing work-place demands of a knowledge-based and information-rich society, labour force readiness has become a major concern of Canadian policy makers. Human Resources Development Canada and Statistics Canada jointly developed the Youth in Transition Survey (YITS), a longitudinal survey designed to investigate the relative success of youths as they progressed from school to training and to work. One of the determining factors of whether one stays in school is inextricably linked to school engagement.

Statistics Canada defines engagement, both academic and social, in terms of a student’s identification with and participation in the respective context and measures engagement on a scale based on responses to a series of questions, such as “I complete my assignments” or “People at school are interested in what I had to say” (Bushnik, Barr-Telford, & Bussière, 2004, p. 37). The survey showed a correlation between disengagement and attrition.

“Relative to high school graduates, dropouts revealed attitudes and behaviours indicative of less academic engagement in school.” (Bowlby & McMullen, 2002)

Many factors affect school engagement, including students’ social and economic background (Bowlby & McMullen, 2002). The YITS (Bushnik et al., 2004, p. 13) has found that 35% of dropouts were disengaged by age 15 and 19.9% of all dropouts reported being bored at school. The same survey revealed that most high school dropouts left school because of school-related reasons with being “bored or not interested” ranked highest on the list. Findings from the National Longitudinal Survey of Children and Youth (NLSCY) underscores the relationship between academic engagement and achievement.

“The academic engagement measure had a reasonable degree of predictive and concurrent validity, correlating moderately with the measures of academic achievement<sup>15</sup> and social engagement. . . . Academic and social engagement each comprise participation and identification.” (Norris et al., 2003, p. 30, 33)

According to the 2005 British Columbia Graduate Transition Survey (Ministry of Education, 2005), teachers’ “moral support, motivation/discipline, and practical help” were collectively ranked by respondents as the single most important school factor (at 59%) in aiding students to reach graduation. Courses, on the other hand, were cited (at 28%) as the main hindrance. Students claimed that courses were “too advanced or demanding” and made it difficult to reach graduation.

At schools, educators are reporting a high number of “school leavers” and declining interests in mathematics courses. According to Statistics Canada, only 18.6% of all upper secondary students surveyed in the TIMSS were enrolled in mathematics in 1995, “the lowest participation rate in mathematics of all the countries reporting.”<sup>16</sup> The same study shows that students in lower grades tend to have a more positive attitude towards mathematics than those in upper grades. For example, 89% of grade 4 students reported “liking or enjoying” mathematics compared to 74% in grade 8 and 61% in the last year of high school.

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<sup>15</sup>Three measures of academic achievement were used: the mathematics computation scale score, the teacher’s rating of academic achievement, and the parent’s rating of academic achievement. (Norris et al., 2003, p. 30)

<sup>16</sup>Twenty countries reported their students’ mathematics participation rate (in order of student mathematics participation): Russian Federation, Hungary, France, Cyprus, Slovenia, Czech Republic, Lithuania, Italy, Australia, Denmark, Austria, New Zealand, Sweden, South Africa, Norway, United States, Iceland, Switzerland, Netherlands, and Canada (Bordt et al., 2001, p. 12).

Of the upper secondary students who dropped mathematics courses, more than 70% found mathematics difficult.

“The most common reason for not taking mathematics courses was that students found mathematics difficult. Nearly two-thirds of all the students surveyed (63.6%) thought that mathematics was not an easy subject. For those not currently taking mathematics, this figure rose to 72.1%” (Bordt et al., 2001, p. 6)

Perhaps the most troubling finding is that 59.9% of those who perceived themselves to have good aptitudes in the subject dropped mathematics before grade 12. In fact, 55.5% of all upper secondary students surveyed indicated that they would not like a job involving mathematics. This rate jumps to 69.7% for those who are no longer taking mathematics courses. Disengagement and attrition have some far-reaching consequences especially when students become disengaged and withdraw from mathematics courses. Many students are dissuaded from pursuing their careers as professionals after poor performance in mathematics.

“Youth who had dropped out by the age of 17 were much less engaged in school when they were 15 – both socially and academically – than were those who had either continued in school or had already graduated.” (Bushnik et al., 2004, p. 13)

It must be said that not all “at risk” students can be re-engaged. However, for those students whose interest in learning has not totally diminished, support from mentors is essential. Research by Meece et al. suggests that “performance expectancies predict subsequent math grades, whereas the perceived importance of mathematics predicts course enrollment intentions”. The study also found that mathematics anxiety has “indirect effects on subsequent performance and enrollment intentions.” In fact, it was determined that “students who assigned more importance to achievement in mathematics reported less math anxiety,” and the researchers concluded that “teachers can help enhance students’ valuing of math in several ways, including explicitly relating the value of math to students’ everyday lives, making math personally meaningful, and counseling students about the importance of mathematics for various careers.” (Meece et al., 1990, p. 69) Authors of the

SAIP 2001 report further underline the critical role attitudes play in one's success, noting the importance of perseverance in mathematical learning:

“Student attitudes toward mathematics show a pattern of relationships with achievement. Negative associations are found for perceived difficulty of mathematics and attribution of low mathematics marks to bad luck. The strong pattern of positive associations for persistence at a difficult mathematics problem until it is solved suggests an element of internal motivation on the part of higher-achieving students. More generally, the results for other similar items reveal a pattern that might be interpreted as fatalism or external motivation on the part of low-achieving students and internal motivation on the part of higher-achieving students.” (Council of Ministers of Education, Canada, 2002, p. 80)

Mentorship therefore plays an important role in engaging and retaining students and such early intervention requires well-supported teachers.

### 2.4.3 Teachers' Qualifications

“Too often, elementary teachers take only one course in mathematics, approaching it with trepidation and leaving it with relief. Such experiences leave many elementary teachers totally unprepared to inspire children with confidence in their own mathematical abilities. What is worse, experienced elementary teachers often move up to middle grades (because of imbalance in enrollments) without learning any more mathematics.” ((US) National Research Council, 1989, p. 64)

Teachers of mathematics often have no special training in the subject matter. According to TIMSS 2003, of the Grade 4 mathematics teachers surveyed in Ontario, 63% reported to have primary/elementary education majors with no specialization in mathematics. In fact, only 6% reported to have a primary education major and a major or specialization in mathematics; 5% reported to have a mathematics or science major without a major in primary education. For Grade 8 mathematics teachers surveyed in Ontario, 15% reported to have a major in mathematics education and 12% reported to have a mathematics major. Mathematics teachers in Ontario elementary schools are not required to be certified specialist in the subject.

According to a report in the *Ottawa Citizen*, “it’s possible to become a teacher in the province of Ontario with no high school math credits past the compulsory Grade 10 (Laucius, 2004). The same teacher could be teaching mathematics at the elementary level. The Ontario College of Teachers stipulates that “every Ontario teacher must be qualified in at least two consecutive divisions,” (e.g., Primary/Junior) and that there are four divisions, namely, “Primary (Grades K–3), Junior (Grades 4–6), Intermediate (Grades 7–10) and Senior (Grades 11–12)” (Ontario College of Teachers, 2006, p. 27). In order to teach either the Primary or Junior Division, a teacher must receive the Basic Qualification certification which entails knowing the Ontario curriculum for the respective division. For example, a teacher certified for the Junior Division is expected to know “the Ontario curriculum which includes Grades 4–6, in all subject areas” for “program development, planning, implementation and assessment and evaluation” (Ontario College of Teachers, 2001, p. 4). In other words, teachers certified for the Primary/Junior Divisions are allowed and “may be asked to teach everything” (including mathematics) (Ontario College of Teachers, 2005, p. 4).

An anecdotal account reported in the *National Post* echoes the concern about teacher qualifications in British Columbia by a mathematics professor:

“I am at the moment teaching a class of 100 students who will be elementary school teachers in 18 months and I would guess that no more than a half of the students in the class are beyond the level that the test makers say most 13-year-olds should be at.” (Sokoloff, 2002a)

The said course was Math 190, Principles of Mathematics for Teachers, at Simon Fraser University. According to the BC College of Teachers, the teacher education program prescribes one compulsory university mathematics course<sup>17</sup> in addition to a methodology course. In British Columbia, teaching assignments are made at the discretion of the individual school board where a certified elementary school teacher is usually hired as the primary teacher for a given class and can be expected to teach all but specialty subjects (e.g., French immersion, band/music or English as a Second Language) (British Columbia College of Teachers, 2006b, 2006a).

Unfortunately, these findings are not isolated instances and are actually supported by other reports. In the Ontario Education Quality and Accountability Office (EQAO) Provincial Report on Achievement (1996–97), it was found that most Grade 6 teachers have no

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<sup>17</sup>e.g., Math 335 at University of British Columbia or Math 190 at Simon Fraser University.

formal training in “teaching or assessing mathematics” and many Grade 3 and 6 teachers expressed discomfort teaching or assessing mathematics (Education Quality and Accountability Office, 1997, pp. 21, 39). It was noted in the SAIP 2001 that 13-year-old students who were taught by subject teachers performed better in the problem solving component of the mathematics assessment than those who were instructed by homeroom teachers (Council of Ministers of Education, Canada, 2002, p. 86). It is no wonder that about one in five 13- and 16-year-olds students is receiving supplementary assistance such as tutoring in mathematics (Council of Ministers of Education, Canada, 2002, p. 56).

Various studies (Cohen & Leung, 2004; Uusimaki & Kidman, 2004; Levine, 1996) and support groups such as the Math Empowerment Workshops (Cohen, 2003; Cohen & Green, 2002) at University of Toronto have shown that appropriate intervention can reduce mathematics anxiety in the participants.

Another teacher qualification that is sometimes lacking in the classroom is computer training. Since the use of ICT has become an integral part of the mathematics curriculum, teachers who lack computer training further hinder learning in the classroom. While all levels of government are committed to the use of ICT in the classroom, the implementation of ICT in everyday instructions can be problematic and it varies across jurisdictions. In fact, teachers are reporting that they are lacking preparation time for computer-based lesson planning and “in most provinces, teachers’ lack of knowledge or skills in using computers for instructional purposes was cited as a major obstacle in schools representing more than 50% of enrolments.” (Canadian Education Statistics Council, 2003, p. 73)

#### 2.4.4 Cumulative Effect

“In the long run, it is not the memorization of mathematical skills that is particularly important – without constant use, skills fade rapidly – but the confidence that one knows how to find and use mathematical tools whenever they become necessary. There is no way to build this confidence except through the process of creating, constructing, and discovering mathematics.” ((US) National Research Council, 1989, p. 80)

Student outcomes are affected by teacher-specific characteristics, including mentorship. It is said that “proficiency in most of mathematics is not an innate characteristic; it is achieved through persistence, effort, and practice on the part of students and rigorous and

effective instruction on the part of teachers.” (California State Board of Education, 1999, p. v) Proper mentorship requires qualified teachers and holds the key to student retention.

Mathematically anxious teachers with few resources are less likely to be in a position to provide mentorship for their students, especially those who require more support. To counter this problem, it is important to change attitudes. It is also important to adjust instructional styles to effect a change of culture. Some researchers (Burris, Heubert, & Levin, 2004) have suggested that what disadvantaged “at risk” students need is not only remedial help but also enrichment support as these students must learn at an accelerated pace to catch up to their peers. This seemingly counter-intuitive strategy was demonstrated to be quite fruitful by Henry M. Levin, an education and economics professor at Columbia University, in his Accelerated Schools Project (Burris et al., 2004; Levin, 1995).

There are many structural issues inherent in mathematics education that warrant attention and a coordinated response from all involved. In order to re-engage at-risk students, adequate guidance and mentorship must be made available from educators who are not preoccupied with anxiety or an over-reaching curriculum. Mathematics educators who are anxious or under-prepared need to seek intervention, such as mathematics empowerment workshops, or to upgrade their professional qualifications in ICT and mathematics. Clearly, some of these issues are beyond the scope of SAMPLE.

The role of the SAMPLE project, however, was to facilitate mathematical learning in this context. In order to improve learning outcomes, SAMPLE tackled some of the issues outlined above by specifically targeting several crucial areas in the educational system. By using SAMPLE, educators had at their disposal all the necessary resources in the form of prepared lesson plans and online remedial assistance specifically tailored for those who may be under-prepared or needing a refresher course on the subject matter. (These features are explained in the Project Description of this report.) SAMPLE applied principles of the constructivist learning model in its designs by creating a discovery-based learning environment, one that would allow students to develop intuition in problem solving. SAMPLE was also designed with the capacity to embrace both remedial and enrichment approaches. The customized environment was conducive to learning. In essence, SAMPLE allowed students to work at their own pace and respected their individual learning styles. Better learning contributes to an increase in engagement and a drop in attrition. Through the use of SAMPLE, teachers may be able to find more time to provide mentorship and successful instructions in class.



## Chapter 3

# Project Description

The SAMPLE prototype website was an interdisciplinary collaborative publishing project that brought together researchers from three domains: mathematics, education and computer science. The goal of SAMPLE was to build a framework that would foster mathematical learning in the classroom through the use of innovative technologies. In doing so, this project allowed researchers to gain experience with design structure and to use SAMPLE as a test bed to investigate how to integrate technology with mathematical learning as opposed to how to develop more technology for mathematics education.

This project was mainly concerned with harnessing the dynamic nature of rich media, such as interactivity, for the delivery of learning resources. The prototype was intended to be a complete sample unit with limited scope. A geometry topic, *Shape and Space*, was selected from a senior elementary school level mathematics curriculum and served as the central focus of this project. The pedagogy team was responsible for devising a unit plan of 14 lessons. The mathematical technology team, comprising mathematics researchers, assisted with the creation of learning objects, as directed by the pedagogy team, to meet the objectives of each lesson plan. The content management team, made up of computer scientists, provided input on the theoretical architecture of the web portal structure.

### 3.1 Part I: Instructional Design

SAMPLE's web-based learning environment was developed with the NCTM standards in mind. The SAMPLE web portal, informed by the mathematics education reform to move towards constructivist instruction, employs a student-centred approach.

Many students who discontinue their mathematics education attribute anxiety to be their chief obstacle. This is particularly true when materials are delivered in a passive fashion where students try to learn by rote and drill without achieving real comprehension (Cornell, 1999; Levine, 1996; Williams, 1988; Stodolsky, 1985). Contemporary learning theories suggest that knowledge is not transmitted from an authority to a learner but that active construction takes place for the acquisition of knowledge and reconstruction for the recall of information. The constructive process is enhanced when multimodal input are used in the learning context (Anderson, 1997). Rich media make it possible for SAMPLE to present learning materials in a variety of contexts, such as independent, collaborative or teacher-directed, to suit different learning styles, such as textual, aural or visual. Interactive content offers an opportunity to add another dimension to learning and allows students to acquire mathematical concepts through experimentation and visualization. By using interactive content, students are then able to conjecture and construct their own understanding before internalizing the knowledge. For many, learning through discovery solidifies understanding and, in turn, increases confidence and reduces mathematics anxiety.

### **3.1.1 Target Audience**

SAMPLE provides support for three groups of users: teachers, students, and parents. As such, the SAMPLE portal offers content via three streams to reflect a varying degree of utility by each respective user group. The content provided to each group differs in terms of function, access level and content sophistication.

#### **3.1.1.1 Teacher Users**

Educators are the primary users envisioned for the SAMPLE web portal. It is intended to support instruction in several ways. One of the three main areas of support SAMPLE provides is remediation of concepts. For those who lack the background or are otherwise unfamiliar with a particular topic, SAMPLE acts as a training tool and offers teachers the opportunity to learn or re-acquaint themselves with the concepts prior to devising a lesson plan. Another major support SAMPLE provides is to facilitate the lesson planning process itself. SAMPLE acts as a knowledge base with a repertoire of learning objects (e.g., applets, sound bites, or video clips) that teachers can readily incorporate into their lesson plan. These learning objects can be used independently or jointly with other resources the teachers may

have compiled. The option to print some of the content assists the teachers with class presentation. The third support SAMPLE offers is the ability to add learning objects to the repository via the use of a web-based interface to a database. That is, SAMPLE acts as a learning management system by giving teachers the flexibility to organize learning objects and other resources developed in-house or contributed by others and to build new lesson plans. In addition, SAMPLE also offers student assessment and monitoring tools as a part of a comprehensive system. Teacher users have access to all sections in SAMPLE.

#### 3.1.1.2 Student Users

SAMPLE offers students three levels of learning resources – remedial, curriculum, and enrichment – for each lesson. For each of these levels, there are functionalities that enable both independent and collaborative learning. For example, the descriptive content (akin to lecture notes with diagrams and definitions) is tailored for independent learners. Self checks such as pre-tests in the *Are You Ready?* section and post-tests in the *How Did You Do?* section are designed to assess a student’s level of knowledge. Based on the results of the tests, the teacher can guide a student to follow the links to lessons of a prerequisite unit for remedial support or a more advanced unit for enrichment on the same topic. Enrichment materials are also found in the form of challenge questions and problems in lessons where appropriate. Student users do not have access to teacher- or parent-specific sections.

#### 3.1.1.3 Parent Users

The SAMPLE project has invested some of its effort to encourage parental participation in the students’ learning process. This goal is mainly accomplished by repurposing some of the content in a form appropriate for parents to teach or refresh themselves with some of the main mathematical concepts in each of the lessons. Parent users have access to all student content in addition to *Parents’ Notes*.

### 3.1.2 Project Workflow

The lessons in SAMPLE were based on the provincial standards as set out in the British Columbia Integrated Resource Package (Province of British Columbia, Ministry of Education, Skills and Training, 1996). Project staff consulted the IRP for prescribed learning outcomes, suggested instructional strategies, suggested assessment strategies and learning

resources. While the project was guided by IRP's *Math 7, Shape and Space* in its scope, the content may be suitable for use by students between grades 5 and 7 outside of British Columbia depending on the jurisdiction. In addition, the NCTM standards were used as guiding principles for much of the pedagogical approach. The division of labour is described in the workflow below.

### 3.1.2.1 The Pedagogy Team

The pedagogy team was made up of experienced mathematics school teachers and researchers recruited by the Faculty of Education at Simon Fraser University. The team reviewed literature and available mathematics software, conducted content selection and authoring of lesson plans.

The members of this team had several key responsibilities in the authoring process. As education experts of the subject matter, they assumed the roles of designers, teachers, writers, editors, and users of SAMPLE. They were instrumental in determining the overall conceptual organization of the content.

The pedagogy team selected *Shape and Space* as the major topic that formed the basis for the geometry unit. Each lesson covered a subtopic and was built with its target audience in mind: teachers, students and parents. In fact, three user modes were offered and the level of access was determined at the time the users logged into the system. For students, every lesson revolved around four major sections: *Play, Explore, Challenge* and *Learn*, with assessment and collaborative sections: *Self-Check* and *Share Ideas*. Parents had access to the students' section in addition to the *Parents' Notes* section which provided supplementary resources to those who wished to assist the students in a non-classroom setting and to gain an understanding of the learning materials. Teachers had access to all sections including the *Teachers' Den*. The *Teachers' Den* section was a collection of tools that enabled teachers to review concepts, produce lesson plans and presentations, and develop new lessons. It comprised several subsections: *Teachers' Notes, Question Bank, Chat Log* and *Math Links*. In all, 14 lessons were planned and produced in several stages.

As instructional designers, the team defined the feature and functionality requirements of the sections. For example, a *unit overview map* was included for all lessons. As each unit was intended for non-linear traversals, the map showed how one lesson related to another and served as a breadcrumb for navigation purposes. The team also provided input on how the user environment should be implemented. For instance, each lesson was set up

to always default to the *Play* section to encourage discovery-based learning. The pedagogy team requested the inclusion of assessment tools to gauge student progress and collaborative tools to facilitate and monitor the actual learning process.

As teachers, the team researched and consulted various resources to build a comprehensive constructivist strategy, like a blueprint, before drafting lesson plans. The team produced a wish list of mathematical learning objects to be used in the *Play* and *Learn* sections by providing detailed specifications of how these objects should behave and how they were supposed to meet the learning objectives.

As writers and editors, the descriptive content used throughout the lessons (e.g., in the *Learn* section) was written entirely by teachers on the pedagogy team. The task of writing was shared among the teachers. Each teacher was assigned several lessons, with each lesson being completed from start to finish by the same individual for consistency.

The assessment section (e.g., tests such as *Are You Ready?*) was another area that was authored by teachers. Subsections in the *Self-Check* section, *Look Back* and *Go Forward*, were placeholders that connected students to either a prerequisite or advance unit on the same topic. As mentioned earlier, the BC school system subscribes to a spiral curriculum where students are introduced to a wide number of topics in lower grades. These topics are sequenced and taught over several years with increasing sophistication and deeper understanding. Some critics (W. H. Schmidt et al., 1997) of the spiral curriculum attribute student boredom, attrition and teacher burn out to a breadth-rich depth-poor curriculum. The *Look Back* and *Go Forward* subsections addressed and mitigated some of the shortcomings of a traditional spiral curriculum by offering remedial support on one hand and enrichment opportunities on the other. In other words, the placeholder *Look Back* subsection was designed to theoretically connect to prerequisite lessons on the same topic if a refresher course were deemed necessary. For example, if a Grade 5 student had performed unsatisfactorily on one of the self-tests, *Are You Ready?* or *How Did You Do?*, the *Look Back* subsection would link the student to the Grade 4 lessons that formed the foundation for the Grade 5 lessons. Similarly, the *Go Forward* subsection would serve to bring a student who was particularly keen to an advanced lesson of the same topic. As SAMPLE was a prototype that covered one unit for one grade, the *Look Back* and *Go Forward* placeholders were included for illustrative purposes.

In short, by providing a more tailored approach to the needs of individual students, teachers' workload could be kept manageable while student interest could be maintained.

As editors and users, the pedagogy team also proofread the marked up content, evaluated the rich media learning objects, provided feedback on usability issues, and conducted field tests. (They were not familiar with the web-publishing process.)

### 3.1.2.2 The Mathematical Technology Team

Before any of the lesson plan was ready, the mathematical technology team began the work of producing an infrastructure that met with the navigation requirements of the pedagogy team. This included programming the basic elements of the SAMPLE site, such as the log-in screen, the unit overview map, the side menus, the header graphics, and the user interface. As the lesson plans gradually became available, the mathematical technology team commenced the laborious process of marking up the descriptive content. At the same time, the team was also responsible for generating high-quality mathematical illustrations and graphics for use in worksheets. As these worksheets were meant to be usable both on screen and in print, they were laid out and made available as pdf files for accessibility.

At the core of SAMPLE was the rich media content. While some of the media objects (e.g., video clips, sound files, communication tools, etc.) were readily sourced under the direction of the pedagogy team, suitable interactive learning objects were much harder to come by in general. In fact, the development of learning objects according to specifications was by far the most interesting and challenging aspect of the entire project. It provided the mathematics technology team the opportunity to turn some very imaginative ideas into actual stimulating instructional tools. Visualization played a key role in getting geometry concepts across to students. To illustrate the rotation or translation of shapes in 2D or 3D using computers required programmers with sophistication and ingenuity in both mathematics and computer science. It became an even more daunting endeavour when one needed to add interactivity to the equation.

### 3.1.2.3 The Content Management Team

In parallel development with the prototype website was a proof-of-concept script-based (i.e., Perl/CGI/DBI) application (see Figure 3.31) to aggregate content and generate web pages from a Microsoft Access database (Kellar et al., 2003). The content management team, based at Dalhousie University, was focused on ensuring content in the database could be added or modified online. Through an applet interface, web-savvy teachers could administer

the content. This aspect of the project is further discussed in the Design of a Learning Management System section with examples provided in Part II: Technical Aspects of SAMPLE.

### 3.1.3 Content Design

SAMPLE's main contribution was the creation of a constructivist environment for instruction. This approach entailed the provision of a channel for discovery in the learning process that goes beyond what a typical textbook would offer. It was not merely a matter of replicating a textbook in digital format.

Mathematical concepts are often best explained by actual demonstrations. For example, a popular exercise used in the traditional classroom by teachers to illustrate the property of *triangle inequality* is to give students pieces of dry spaghetti of varying lengths and ask the students to note which combinations of these pieces cannot form a triangle. Students are then guided to recognize the pattern and to arrive at the conclusion that the length of the longest side of the triangle must be less than the sum of the lengths of the other two sides. To perform this exercise in a classroom is often messy and requires students to have good manual dexterity and coordination. In SAMPLE, an applet has been designed to model concepts like the *triangle inequality*. By allowing students to vary the length of each side of a triangle, the applet simulates onscreen the shortening or lengthening of each side and shows how the triangle would collapse when the total length of two shorter sides approach the length of the longer side. The same concept is then reinforced in the textual content in the *Learn* section. Not only are the students then in a position to articulate the new principle, both parents and teachers are empowered with an easy-to-use tool that can be repurposed in guiding students through similar questions in assignments (see Figure 3.1).

New interactive content must be carefully crafted to achieve the learning objectives and yet leverage the technologies available. One of the challenging aspects of online content design is building relevant and effective interactive learning objects that reinforce the mathematical concept displayed on screen. SAMPLE is intended to fill a gap in the learning spectrum by providing alternatives to illustrate concepts with visualization activities.

### 3.1.4 The General Structure of SAMPLE

SAMPLE organized its learning materials by units. Each unit was centred around a mathematical topic that was developed through a series of lessons. Each lesson contained some of

Lesson D1  
Constructing Triangles  
Teacher  
Play  
Learn  
Talk Through  
Self Check  
Parents' Notes  
Teachers' Den  
Share Ideas  
Help

**PLAY**  
Explore | Challenge

4.0  
15.0  
13.0

4.0  
13.0  
15.0

**EXPLORE**

2. Try using the following side lengths for one of the triangles:  $a = 4\text{cm}$ ,  $b = 14\text{cm}$ , and  $c = 8\text{cm}$ . What happens? Can you find different numbers that make the same thing happen (you may want to write them down on paper)? Why is this happening? Start keeping a record of the numbers that produce the same error. Try to record the numbers in some sort of order and see if you can spot a pattern.

[Go Back](#) | [Answer](#) | [Next](#)

**ANSWER**

2. Not every combination of three sides will make a triangle, however. There is a basic property of the triangle that must not be broken. It is called the triangle inequality property and it states that the longest side cannot be longer than the two shorter sides added together. This is because the two shorter sides added together is as far as they can reach if they are put end-to-end. If the longest side is longer than this then the triangle cannot be constructed.

Figure 3.1: An Applet on Triangle Inequality.

the following sections: *Play*, *Explore*, *Challenge*, *Learn*, *Talk Through*, *Self-Check*, *Parents' Notes*, *Teachers' Den*, *Share Ideas* and *Help*. The prototype website, designed for a Grade 5 curriculum, offered three levels of support: remedial, curriculum and enrichment.

### 3.1.4.1 Site Navigation

Lessons could be accessed via the simplified site map on the top left corner of the site (see Figure 3.2). The map showed not only how the current lesson related to other lessons in the unit, it also indicated possible paths of proceeding through the unit.



The screenshot shows a web browser window displaying an interactive applet. The browser's address bar contains the URL `http://sample.colab.sfu.ca/teacher/D3/index.html`. The page header includes the title "SHAPE AND SPACE" and a logo of a person's head. Below the header, it states "Funding provided by Social Sciences and Humanities Research Council (SSHRC)".

The main content area is titled "PLAY" and features a grid with a polygon drawn on it. Above the grid are four buttons: "Add a vertex", "Remove a vertex", "Make Regular", and "Hide grid". To the left of the grid is a sidebar with navigation options: "Lesson D3", "Naming and Classifying Polygons", "Teacher", "Play", "Learn", "Talk Through", "Self Check", "Parents' Notes", "Teachers' Den", "Share Ideas", and "Help".

Below the grid are two sections: "CHALLENGE" and "ANSWER". The "CHALLENGE" section contains a diagram of a regular pentagon inscribed in a circle and asks the user to draw a regular octagon. The "ANSWER" section contains a diagram of a regular octagon inscribed in a circle and explains the method of drawing spokes from the center to the vertices.

Figure 3.2: The D3 *Play* Applet with the *Challenge* Section.

### 3.1.4.2 The *Play* Section

In a typical lesson, students would first encounter the *Play* section. This section was central to the discovery-based curriculum. The *Play* section contained an interactive applet and students were encouraged to experiment with it and to formulate some generalizations from working with it. For example, in Figure 3.2, students were given the opportunity to investigate the relationship between vertices and sides of polygons. Visually, students were introduced to concave and convex polygons simply by working with this applet. (See Figure 3.13 for the pedagogy team's specification and objectives for this learning object.) Once the students were ready to continue, they were directed to venture into the *Explore* and the

*Challenge* sections, which were linked directly from within the *Play* section. The *Explore* or *Challenge* section occupied two adjacent panes directly under the *Play* window. The pane on the left hand side was for questions; the pane on the right was for *Hints* (available in the *Challenge* sections only) and *Answers*. The usability consideration for presenting *Explore* or *Challenge* in this layout was to ensure that the *Play* applet was always visible and available to the students as a guiding tool. (Schools with small computer displays might require some scrolling.) The *Challenge* section presented questions that required a deeper understanding of the concepts.

Navigation links were built into the question and answer panes to enable students to move from one question to the next with ease. The generation of these applets were further elaborated under the “Learning Objects” section.

#### **3.1.4.3 The *Learn* Section**

The descriptive content of each lesson in the *Learn* section introduced and defined new terms and explained concepts in detail (see Figure 3.3). Rich media objects, such as images of real-life examples, figures, tables, and animation, were presented. Often, additional applets were made for the *Learn* section to reinforce certain aspects of concepts introduced in the *Play* section. For example, in Figure 3.3, different polyhedra were introduced and an applet showing the unfolding of five Platonic Solids was included to illustrate the properties of these regular polyhedra. This applet allowed students to rotate a polyhedron (to be selected from the pull-down menu) at any stage during its unfolding so that they could observe the polyhedron from different perspectives. Immediately below the applet was a table that summarized the properties (in terms of shape of faces, number of faces at each vertices, number of vertices, number of faces and number of edges) that defined each of the five Platonic Solids.

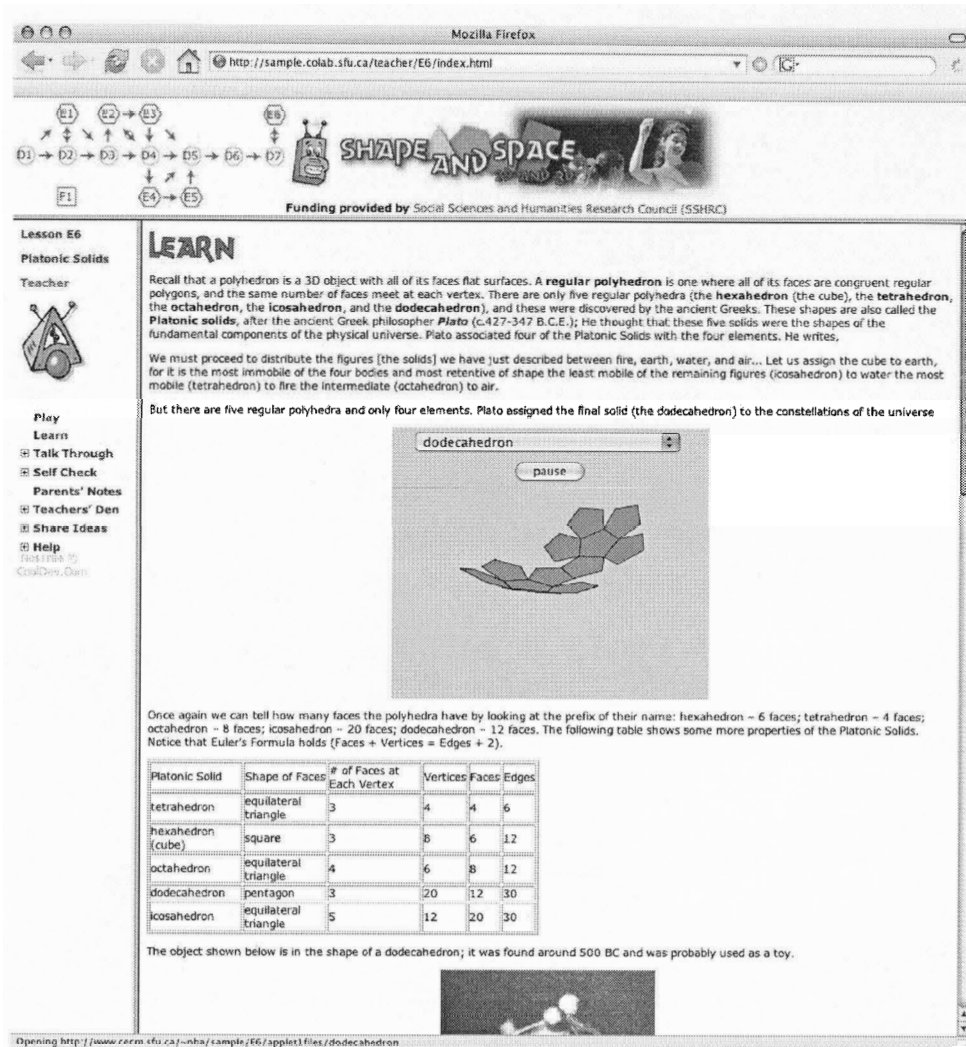


Figure 3.3: The E6 Learn Section.

### 3.1.4.4 The Talk Through Section

A placeholder was made in the *Talk Through* section (see Figure 3.4) to accommodate students with an aural learning style. As a proof-of-concept, a recording of two scripts, a full-script version and an interactive version, was made only for lesson E2. The full-script version was based on the *Learn* section and was included with the prototype. The full-script recording was configured to launch once the *Talk Through* section finished loading so that students could follow along with the content on screen. The small control panel on the

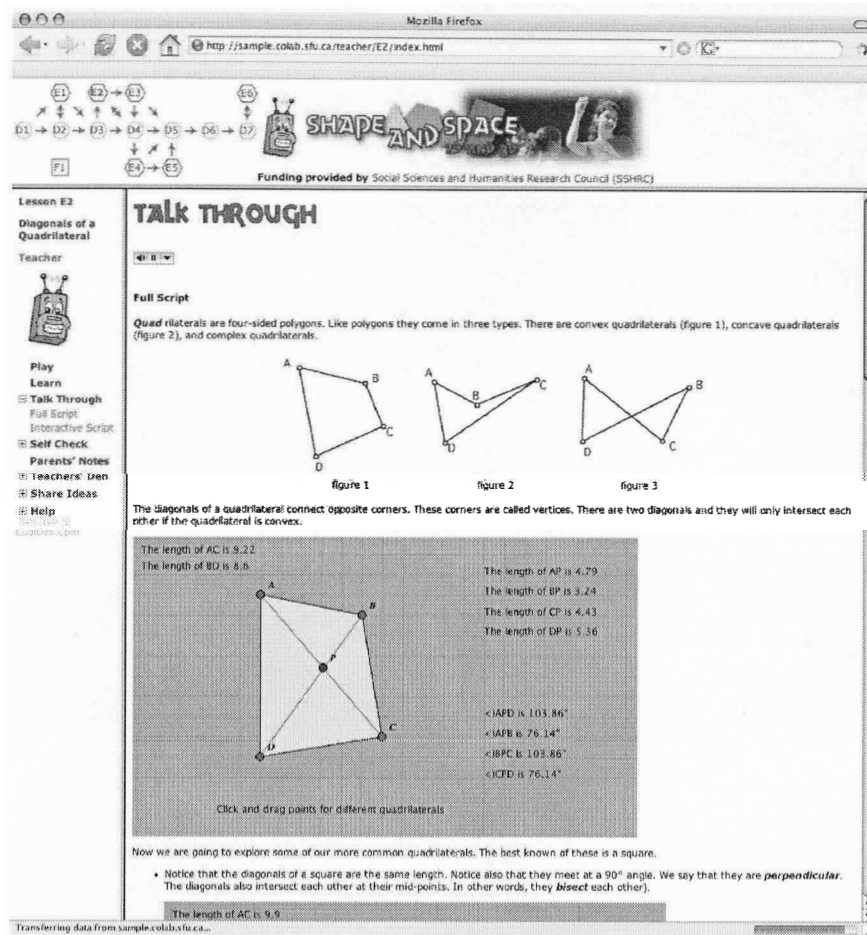


Figure 3.4: The E2 *Talk Through* Section.

top of the full-script version allowed students to adjust the volume, pause or resume the recording. The interactive version required well-timed and coordinated on-screen activity (which would be a rich media project in itself) and it was beyond the scope of this project. The interactive version was reserved for the next stage of the development process.

### 3.1.4.5 The *Self-Check* Section

The *Self-Check* section comprised four sub-sections. The *Are You Ready?* (see Figure 3.5) and *How Did You Do?* sections provided web-based testing for students to gauge their understanding and progress. Figure 3.5 showed questions that students were expected to be

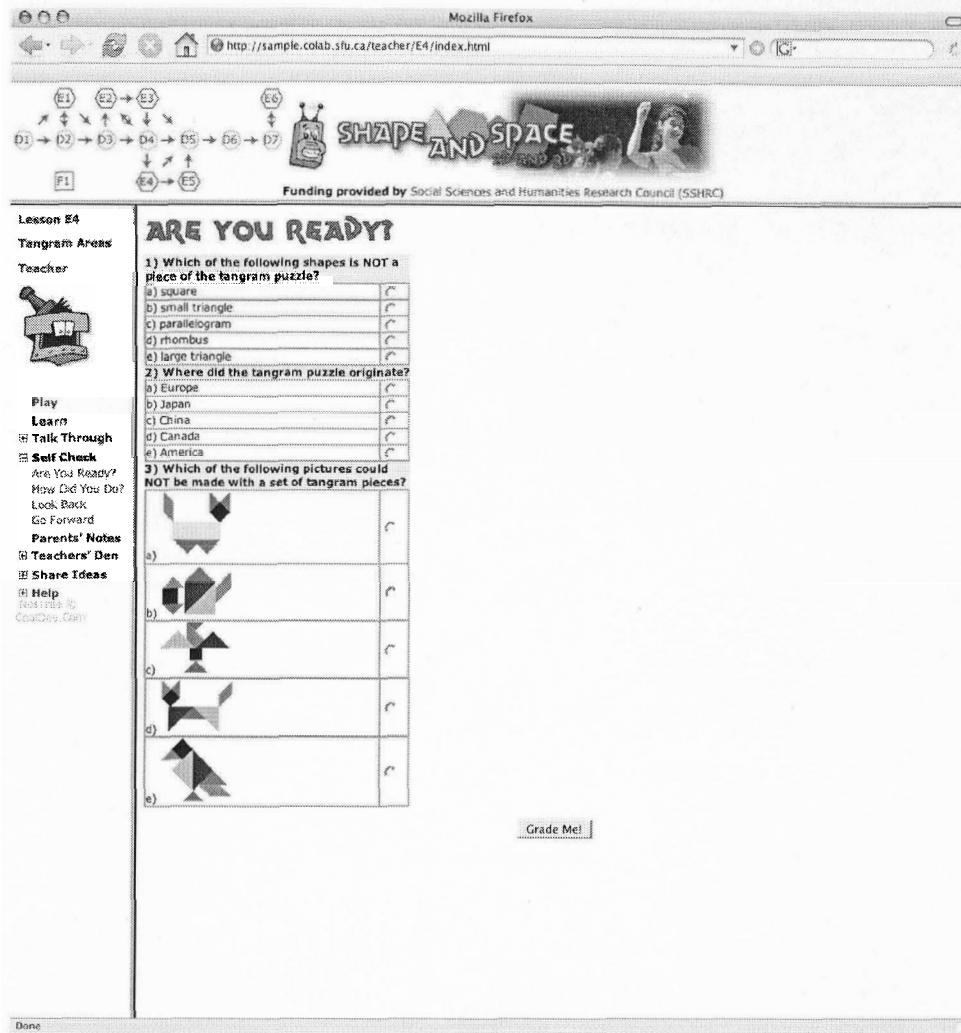


Figure 3.5: The E4 Self Check.

able to answer correctly at the beginning of the lesson. These questions were often presented with diagrams. For example, Figure 3.5 showed that students were tested on their knowledge of the tangram and were asked to determine which of the pictures could not be constructed using pieces from a tangram. The *Look Back* and *Go Forward* were placeholders for linking to a prerequisite or advanced unit on geometry. These sub-sections used multiple-choice questions with automatic grading and solutions provided immediately after the testing.

### 3.1.4.6 The Parents' Notes Section

Parents were given an overview of each lesson in the *Parents' Notes* section (see Figure 3.6). Major concepts were reviewed and learning activities for students were outlined in the

The screenshot shows a web browser window with the URL `http://sample.colab.sfu.ca/parent/E3/index.html`. The page features a navigation menu at the top with icons for lessons E1 through E7 and F1. The main content area is titled "PARENTS' NOTES" and contains the following text:

There is a relationship between the diameter (the line through the centre) of a circle and the circumference (perimeter) of a circle. If you double the diameter you double the circumference, if you triple the diameter you triple the circumference, and so on. This means that the circumference of a circle with diameter 4 cm will be four times the circumference of a circle with a diameter of 1 cm. In order to discover this relationship your child must first learn how to measure the circumference of a circle – it can be approximated by squeezing it between two polygons – a circumscribed polygon (on the outside of the circle) and an inscribed polygon (on the inside of the circle).

The following worksheets will first introduce your child to the 'squeezing' method of approximating the circumference of a circle and then use this to explore the relationship between the diameter and the circumference.

**Worksheet 1: Sandwiched Between Two Polygons**

Take a look at the following diagram. The polygon on the outside of the circle is called a circumscribed polygon. The polygon on the inside of the circle is called an inscribed polygon. The perimeter of the circumscribed polygon will always be greater than the perimeter of the circle (circumference) and the perimeter of the inscribed polygon will always be less than the circumference of the circle. The circumference is sandwiched in between the perimeters of the two polygons. This fact can be used to approximate the circumference of the circle.

Approximate the circumference of the first circle by measuring the perimeter of the circumscribed and inscribed polygons. For the other two circles create your own circumscribed and inscribed polygons and find an approximation of the circumference of the circle.

The diagram shows a circle with a circumscribed polygon (a square) and an inscribed polygon (a square). Below the diagram are two circles of different sizes, one smaller and one larger, for which the user is instructed to create their own polygons.

Figure 3.6: The E3 Parents' Notes.

worksheets. The screen version of the worksheets was accompanied by high-resolution pdf files for printing purposes. This option was provided because while some activities could be described solely in words, other activities required precise diagrams both for illustrative purposes and as actual templates for students to cut out and prepare models. This section was available only to users with access as parents and teachers. As such, the language used in this section was adjusted accordingly to suit the audience.

### 3.1.4.7 The Teachers' Den Section

The *Teachers' Den* section consisted of four sub-sections to provide teachers with a solid set of resources and support: *Teachers' Notes*, *Question Bank*, *Chat Log*, and *Math Links*. In the *Teachers' Notes* (see Figure 3.7), a complete set of notes was included for the lesson

The screenshot shows a Mozilla Firefox browser window displaying the URL `http://sample.colob.sfu.ca/teacher/D6/index.html`. The page features a navigation menu on the left with options like 'Lesson D6', '3D Shapes with Specific Faces', 'Teacher', 'Play', 'Learn', 'Talk Through', 'Self Check', 'Parents' Notes', 'Teachers' Den', 'Question Bank', 'Chat Log', 'Math Links', 'Share Ideas', and 'Help'. The main content area is titled 'TEACHERS' NOTES' and includes a 'Workshests' section with four bullet points: 1. that prisms and pyramids are examples of polyhedra (geometric solids with all of their faces flat surfaces), 2. that prisms and pyramids are named according to the shape of their base, 3. there are relationships between the number of faces, edges, and vertices of certain polyhedra, and 4. prisms and pyramids can be drawn systematically by following steps. Below this is a 'Computer Based Environment' section with a description of the interactive environment. At the bottom, there is a section titled '1. Prisms and Pyramids' with a definition of a polyhedron. The page also features a 3D visualization of a pyramid and a navigation menu at the top.

Figure 3.7: The D6 *Teachers' Notes*.

plan. In addition, the activities for the computer-based environment and the non-computer-based environment were fully explained. The major concepts of the lesson were described in detail with the *Explore* and *Challenge* questions placed appropriately within the context of the explanation. Worksheets that were available in the *Parents' Notes* were included in this sub-section.

*Question Bank*, the second sub-section of the *Teachers' Den*, was a compilation of supplementary questions, activities and additional resources for use in the classroom by the teachers. The solutions of each question were also included with definitions, diagrams, tables and explanation as appropriate (see Figure 3.8).

The screenshot shows a Mozilla Firefox browser window displaying the URL <http://sample.colab.sfu.ca/teacher/E5/index.html>. The page header features a navigation diagram with nodes E1-E6 and D1-D7, and a logo for 'SHAPE AND SPACE' with the text 'Funding provided by Social Sciences and Humanities Research Council (SSHRC)'. The main content area is titled 'QUESTION BANK' and contains the following text and diagrams:

**Lesson E5**  
**Pattern Block Areas**  
**Teacher**

**QB1:**  
 If the area of the triangle is one, what is the area of the hexagon, the trapezoid, and the blue rhombus?

**QB1A:**  
 You can see below that the area of the hexagon, trapezoid, and blue rhombus can be represented in terms of triangles. This way of representing the shapes makes finding areas much easier.

Hence, if the triangle has an area of 1 then the area of the hexagon is 6, the area of the trapezoid is 3, and the area of the blue rhombus is 2.

**QB2:**  
 If the area of the triangle is 1, what is the area of the square and the skinny rhombus?

**QB2A:**

It can be seen from the pictures on the left that three triangles cover the same area as one square and one triangle (two half triangles). So that means that the square has the same area as two triangles, or an area of 2. From the pictures on the right it can be seen that a square and a triangle have the same area as two skinny rhombi and a triangle. Because both pictures have a triangle in them it means that the remaining shapes must have the same area. So, the square has the same area as two skinny rhombi and because a square has an area of 2 then skinny rhombus has an area of 1.

**QB3:**

Figure 3.8: The E5 *Question Bank*.



*Chat Log* was essentially a transcript of the chat application, *Chat-N-Time*<sup>1</sup>, that students were encouraged to use to collaborate and exchange ideas (see Figure 3.9). This was



Figure 3.9: The *Chat Log*.

in response to some prevalent recommendations<sup>2</sup> to remedy students' inability in communicating and formalizing mathematical concepts and terms by encouraging students to put their thoughts in writing. Teachers could use the log to monitor student interactions and articulation of concepts, evaluate which aspects of the lesson students might find challenging,

<sup>1</sup>A description of *Chat-N-Time* is provided in the *Shared Ideas* section.

<sup>2</sup>According to a recent Ontario report, students could benefit from keeping a math journal so that they can practise expressing mathematics concepts in writing (Education Quality and Accountability Office, 2005, p. 47).

and develop a set of remedial or enrichment supplementary resources for the classroom. The log recorded student interactions for all lessons in the chat room and was not associated with any particular lesson.

The fourth sub-section, *Math Links*, was a set of hyperlinks to common resources, from the provincial curriculum to websites of other educators teaching the same courses.

### 3.1.4.8 The *Share Ideas* Section

There were two sub-sections to *Share Ideas*. *Chat-N-Time* was a free chat room application adapted for the SAMPLE project. This chat program was used to allow students to communicate in real-time (see Figure 3.10). Students were able to log on and communicate both

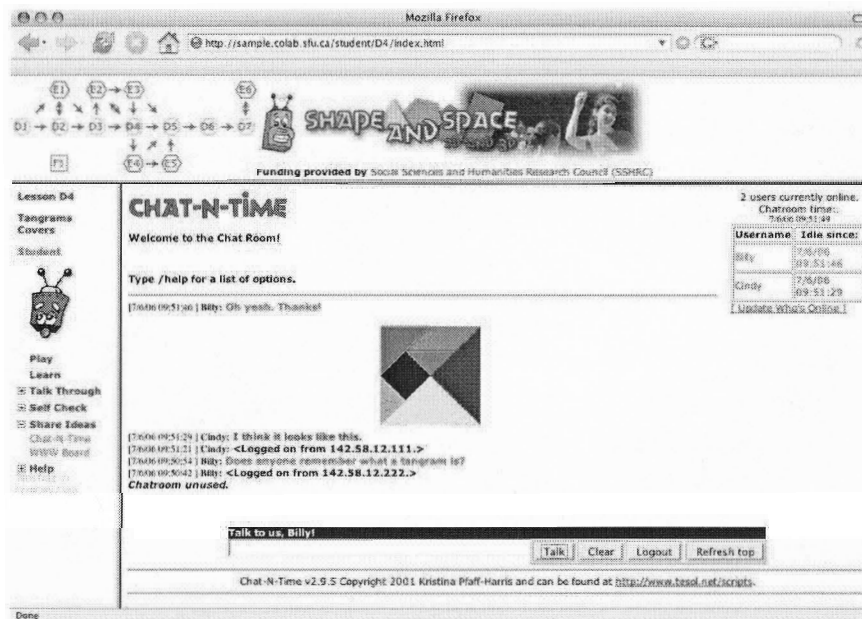


Figure 3.10: The *Chat-N-Time* Section.

with text and with images. In fact, *Chat-N-Time* is still a free Perl program available from the “Scripts for Educators”<sup>3</sup> website, a repertoire of simple, ready-made and user-friendly utility programs (e.g., interfaces for conducting surveys, quizzes, etc.) written for educators and has been serving the online community for the last decade.

<sup>3</sup><http://www.tesol.net/scripts>.

The *WWW Board* is another sub-section (see Figure 3.11). It is also a free Perl script provided by another online archive, Matt's Script Archive<sup>4</sup>. This bulletin board allowed students and teachers to post notices or questions when the rest of the class was not necessarily logged on at the same time. It acted as a forum and allowed for threading and more elaborate questions and answers. This application made it possible for users to have more time to construct a detailed question or response, if required.

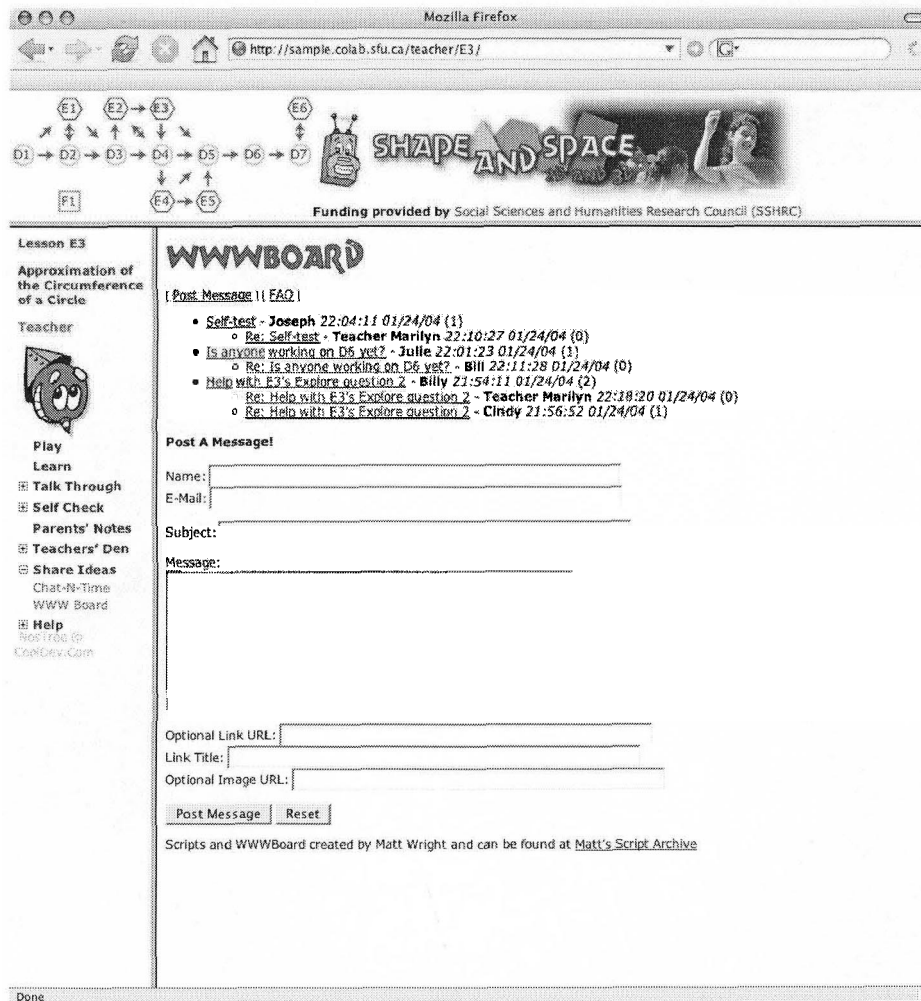


Figure 3.11: The *WWW Board* Section.

<sup>4</sup><http://www.scriptarchive.com/wwboard.html>.

### 3.1.4.9 The *Help* Section

There were mainly two sub-sections in *Help*. The *FAQ* sub-section was a placeholder. The *Search* sub-section was driven by Google (see Figure 3.12). The search could be limited to just the SAMPLE site or expanded to the entire world wide web.

## 3.2 Part II: Technical Aspects of SAMPLE

### 3.2.1 Authoring of Rich Media Mathematical Learning Objects

Two main sources of learning objects were employed in this project. The first kind was applets generated through proprietary geometry software, Cinderella<sup>5</sup>, MathResources<sup>6</sup>, JavaView<sup>7</sup>, and Geometer's Sketchpad<sup>8</sup>. The second kind, free or open-source programs, was generated in-house or borrowed with permission (sometimes with modifications) from other educators because not all tools demanded by the educators were available through off-the-shelf software. This infrastructure gave the flexibility of mixing and matching of resources. Teachers with different software preferences could build the requisite learning objects using a variety of tools. The rich media platform allowed for a heterogeneous mix of repurposable and interoperable resources to be used on the website. Unlike books, SAMPLE was non-linear in nature.

#### 3.2.1.1 Permissions and Usage of Learning Objects

As SAMPLE is a non-profit educational research project, permissions to use the various learning objects were relatively easy to obtain. For proprietary software that have an applet export feature, users are usually allowed to share their work with other teachers and students. In fact, geometry software companies routinely grant users permission to publish applets created by the users on the web for non-commercial use. As for the custom-designed applets, they were all programmed by salaried researchers involved with SAMPLE and the university's copyright policy R30.01 governs such arrangements and provides for joint copyright ownership.

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<sup>5</sup><http://www.cinderella.de>.

<sup>6</sup><http://www.mathresources.com>.

<sup>7</sup><http://www.javaview.de>.

<sup>8</sup><http://www.keypress.com/sketchpad>.

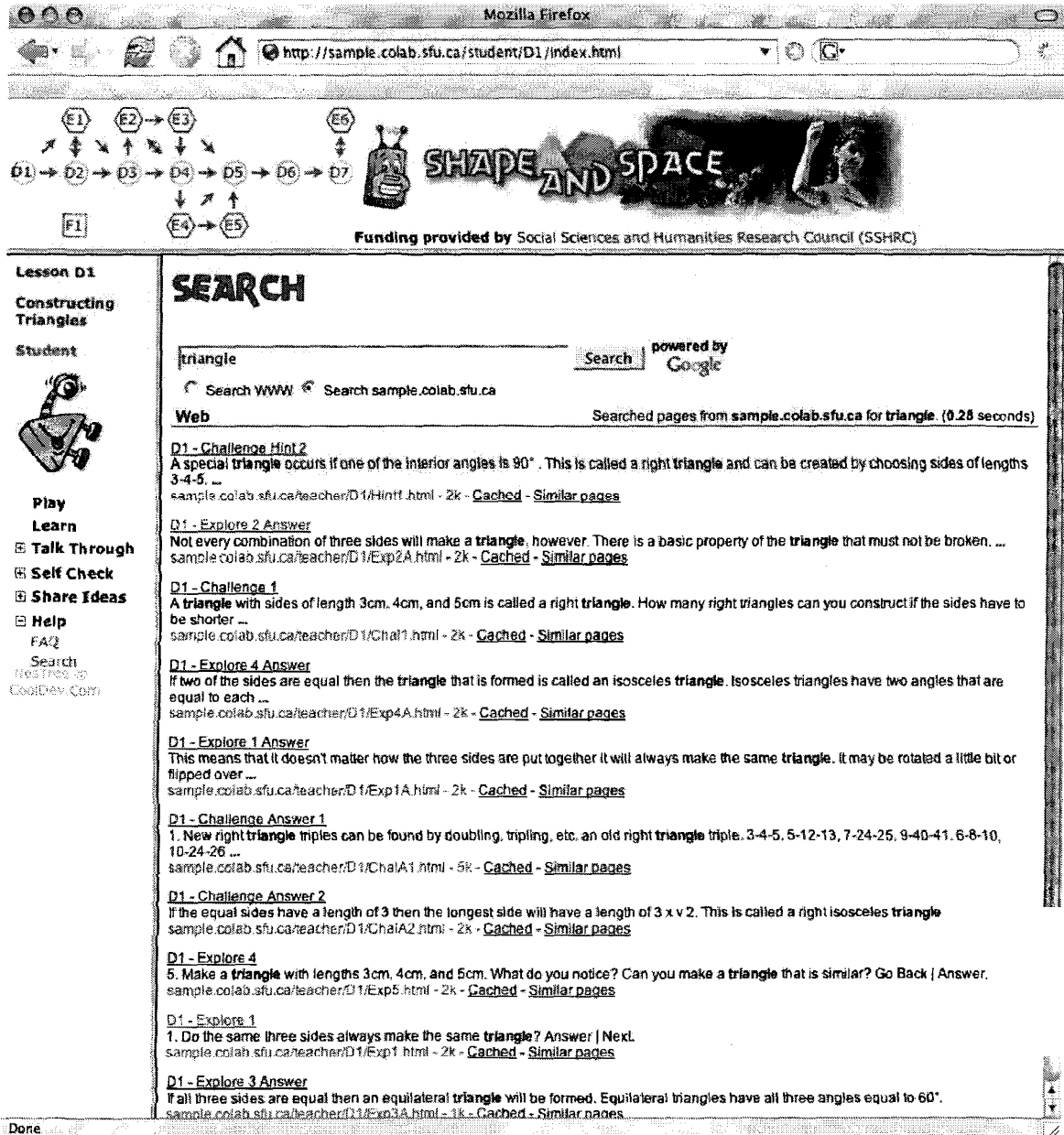


Figure 3.12: The Customized *Site Search* Provided by Google’s Public Service Search.

### 3.2.1.2 The Development of Learning Objects

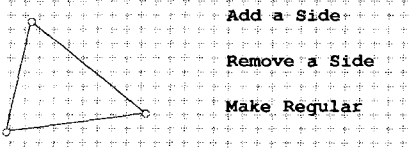
The process of developing learning objects involves close collaboration between teachers and programmers so that the end products would provide students the desired interactivity with the mathematical concepts. Mathematical learning objects are in general more challenging to develop than the typical ones used in other disciplines because of the more complex nature of modelling mathematical interactions. All of the interactive geometry learning objects were Java applets to allow for browser-independent publishing.

#### 3.2.1.2.1 In-house Custom-built Learning Objects

Some of the learning objects needed for SAMPLE were made specifically to meet our pedagogy team's requirements and required the longest development time in comparison to other learning objects used in SAMPLE. Each applet followed a similar workflow. For example, the pedagogy team would provide a simple specification such as the one below (see Figure 3.13) from lesson D3 and the mathematical technology team was asked to supply a learning object as close to it as possible. In this particular case, no common proprietary

**Play**

A scalene triangle is presented on a grid along with three buttons. The first button is called 'Add a Side'. This will add a side at a selected vertex. The student clicks the 'Add a Side' button and then selects a vertex – the vertex splits and a new side is added to the polygon. The second button is 'Remove a Side'. This will remove a side that is selected and merge the unconnected vertices together. This button is greyed out for a triangle. The last button is called 'Make Regular'. This button will take whatever polygon is on the screen and make it into a regular polygon. There needs not be any morphing – just a snap to a regular polygon with the same number of sides. I have some ideas on how to do this if you wish to hear them. In addition, every vertex on the polygon should be draggable. This will allow them to contort their polygons into complex and concave polygons. If the polygon they are manipulating is complex then the 'Make Regular' button will be greyed out.



applet #1

Figure 3.13: Pedagogy Team's Specification of the *Play* Section in Lesson D3, Naming and Classifying Polygons.

software had learning objects that would meet this requirement. As a result, the mathematical technology team actually custom-built the required learning object from scratch (see Figure 3.14). This learning object incorporated the specific functionalities requested by the pedagogy team.

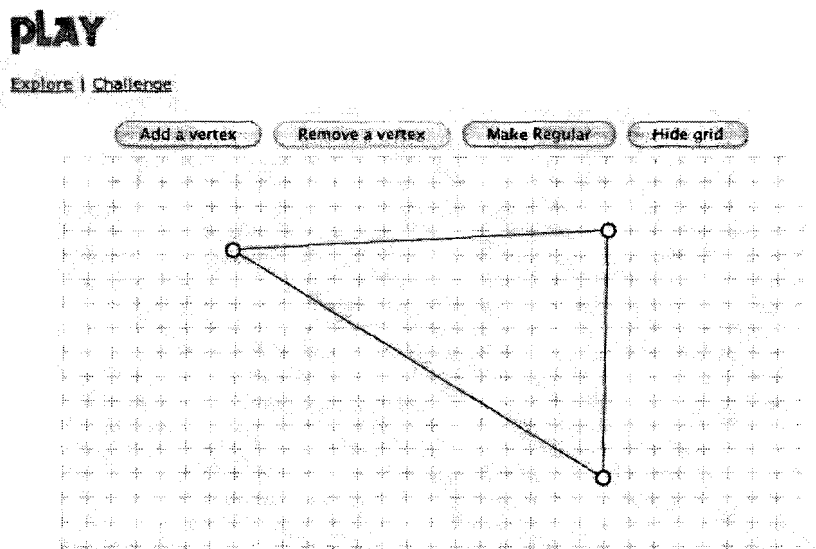


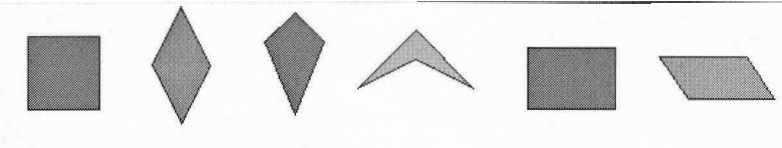
Figure 3.14: The Actual *Play* Section Built for Lesson D3.

Figure 3.15 is another example of a custom-built learning object when no suitable ones were available from the common geometry packages. The pedagogy team wanted an applet for students to learn about properties of polygons. What this entailed was to include a set of shapes and the associated attributes. For example, the applet would show a square from a collection of quadrilaterals if a student selected the following attributes from the pull-down menus of the applet: four sides, two parallel sides and four right angles. This learning object allows students to learn how to classify shapes based on these attributes. To meet the objectives of this learning object, an applet was built using Java to bring about the interactivity required (see Figure 3.16).

The custom-built mathematical learning objects were designed to meet the requirements as set out by the pedagogy team. In the next example created for lesson E3 (see Figure 3.17), one can see that the finished product (see Figure 3.18) not only met the specifications but also closely resembled the desired look-and-feel as illustrated in the mock-up generated by

**Play**

An applet appears that is really just a checklist of properties. As students check each of the attributes they will be shown all the shapes that satisfy the attributes selected. The more attributes they chose the less shapes are shown, until they have reduced it down to one shape. If there exists no shape for the attributes chosen then show nothing. No instructions are necessary. Allow at most one attribute in each row to be selected. I'm not sure how you will program this applet. I will submit an accompanying file that includes each of shapes and a list of all the attributes it satisfies.



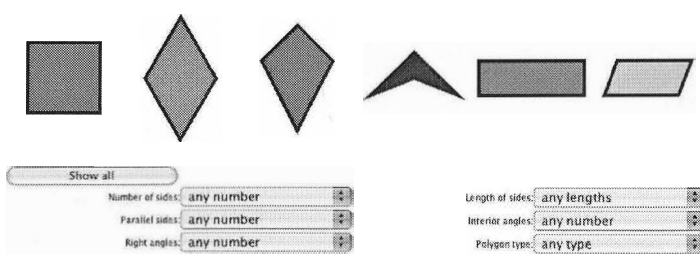
Number of Sides	3	4	5	6	7	8
Lengths of Sides	all sides equal	no sides equal	1 set of equal sides	2 sets of equal sides	3 sets of equal sides	4 sets of equal sides
Parallel Sides	no sets of parallel sides		1 set of parallel sides	2 sets of parallel sides	3 sets of parallel sides	4 sets of parallel sides
Interior Angles	all angles equal		no angles equal	2 angles equal		2 sets of equal angles
Right Angles	1 right angle		2 right angles	3 right angles		4 right angles
Type	concave			convex		complex

**{applet #1}**

Figure 3.15: Pedagogy Team’s Specification for the *Play* Section in Lesson D5, Precise Description of Shapes.

**PLAY**

[Explore](#) | [Challenge](#)



Number of sides: any number

Length of sides: any lengths

Parallel sides: any number

Interior angles: any number

Right angles: any number

Polygon type: any type

Figure 3.16: The Actual *Play* Section Built for Lesson D5.



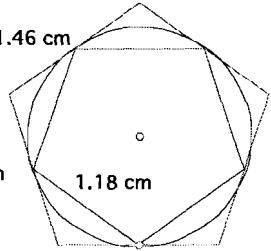
**Play**

Two regular pentagons are shown: one inscribed in a circle (in blue), and the other circumscribed around the same circle (in red). The circle has a default diameter of 2 but can be changed by the student. The side length of each pentagon is labeled and a calculation of the perimeter of each is displayed at the bottom of the screen. The student is asked to try entering a different number of sides. Once they do this, the program will construct a regular polygon using the number of sides indicated. It will inscribe and circumscribe that figure around a circle, and again display the two perimeters.

The program should be able to construct such a figure for any number of sides up to 20.

Diameter: 1  
Number of sides: 5

Perimeter of red =  $5 \times 1.46 \text{ cm} = 7.30 \text{ cm}$   
Perimeter of blue =  $5 \times 1.18 \text{ cm} = 5.90 \text{ cm}$



{applet #1}

Figure 3.17: Pedagogy Team’s Specification of the *Play* Section in Lesson E3, Approximation of the Circumference of a Circle.

**PLAY**

Explore | Challenge

Number of sides:

Diameter: 4.06  
Number of sides: 20  
Perimeter of red =  $20 \times 0.64 = 12.85$   
Perimeter of blue =  $20 \times 0.63 = 12.65$

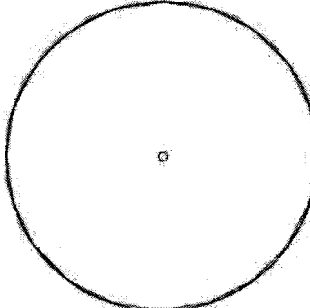


Figure 3.18: The Actual *Play* Section for Lesson E3.

the pedagogy team. Sometimes, the mathematical team was able to produce applets that not only met the requirements of the pedagogy team but also included significant enhancements in usability and functionality.

**3.2.1.2.2 Third-Party Software-assisted Generation of Learning Objects**

Some of the 3D learning objects (e.g., D6), were developed by taking advantage of existing geometry libraries. While the complexity of building these applets was simplified significantly, the process still required extensive development time.

The vision (see Figure 3.19) of the pedagogy team was realized in the following learning

**Play**

Two pull-down menus are presented. The student chooses one option from each menu and a polyhedron is created. The polyhedron will appear in the window and the student will be able to rotate it to see it at various angles. The student will also be able to remove any of the faces of the polyhedron by clicking on them. The faces will reappear at the top of the screen. Dotted lines will replace the removed faces on the solid.

-----Polyhedron goes here-----

Use the menu above polyhedron. Click object. Double click remove it.	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="padding: 5px;">Triangular !</th> <th style="padding: 5px;">Prism !</th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;">Rectangular</td> <td style="padding: 5px;">Pyramid</td> </tr> <tr> <td style="padding: 5px;">Pentagonal</td> <td style="padding: 5px;"></td> </tr> <tr> <td style="padding: 5px;">Hexagonal</td> <td style="padding: 5px;"></td> </tr> <tr> <td style="padding: 5px;">Octagonal</td> <td style="padding: 5px;"></td> </tr> <tr> <td style="padding: 5px;">Decagonal</td> <td style="padding: 5px;"></td> </tr> <tr> <td style="padding: 5px;">Dodecagonal</td> <td style="padding: 5px;"></td> </tr> </tbody> </table>	Triangular !	Prism !	Rectangular	Pyramid	Pentagonal		Hexagonal		Octagonal		Decagonal		Dodecagonal		to create a and drag to rotate the on any face to
Triangular !	Prism !															
Rectangular	Pyramid															
Pentagonal																
Hexagonal																
Octagonal																
Decagonal																
Dodecagonal																

Figure 3.19: Pedagogy Team’s Specification for the *Play* Section for Lesson D6, 3D Shapes with Specific Faces.

object (see Figure 3.20). The 3D geometry models needed to be viewed from different perspectives with the corresponding 2D models provided to indicate the relative location of the faces of the models. The development of this customized applet required research into suitable resources, e.g., the JavaView libraries, to bring the final product to fruition. JavaView is a 3D geometry viewer that allows dynamic viewing of mathematical models in online browsers. The interactive learning object was created in Java and used a subset

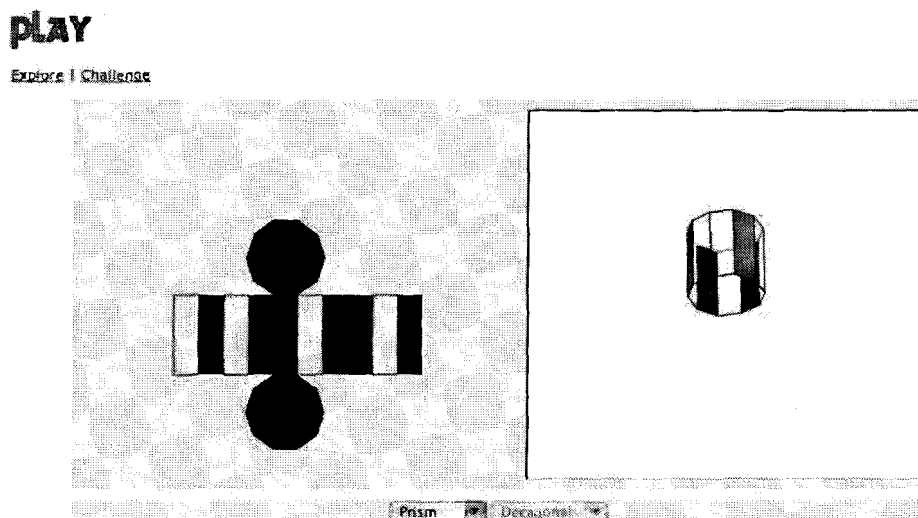


Figure 3.20: The Actual *Play* Section Built for Lesson D6, 3D Shapes with Specific Faces.

of JavaView's class libraries, or building blocks, for displaying geometrical models. For example, the functionality of the applet, such as the generation of the models, the selection of faces, etc., was programmed by the mathematical technology team and JavaView was used as a tool for users to visualize and manipulate the polyhedra on web pages.

Some third-party software applications allow users to export geometry constructions directly as interactive learning objects. All the users must know is how to use the applications to generate constructions and no knowledge of any programming language, (in this case, Java), is required to publish these exported applets. Proprietary applications are particularly helpful when the required learning objects are conventional and construction-based. Many such software applications, however, cater to a mathematically-sophisticated audience and provide a repertoire of advanced features and functionality.

The following specifications (see Figure 3.21) were used to create a learning object for scalene triangles. Figure 3.22 shows the interface one can use to produce an applet directly from Cinderella, a software application that supports Euclidean, hyperbolic and spherical geometry. The process is simple: one would construct a shape using Cinderella (and add labels, if required), select the *export to html* option and a web browser would be launched with all the parameters specified for publishing the learning object as an applet (see Figure 3.23).

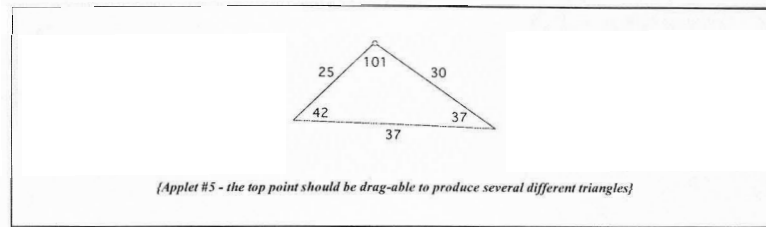


Figure 3.21: The Specification Provided by the Pedagogy Team on the Scalene Triangle Applet for the *Learn* Section of Lesson D1, Constructing Triangles.

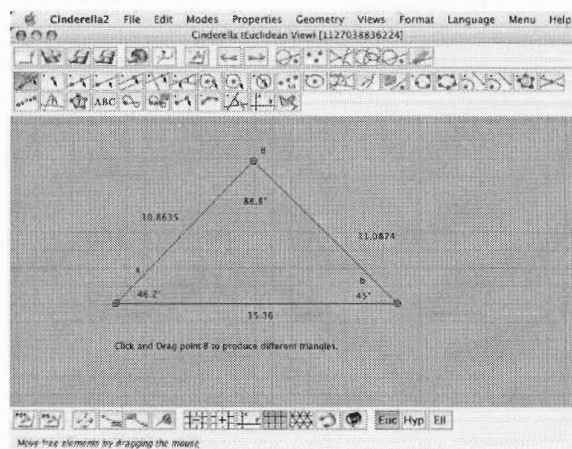


Figure 3.22: Cinderella's User Interface with its Many Features for Geometry Constructions.

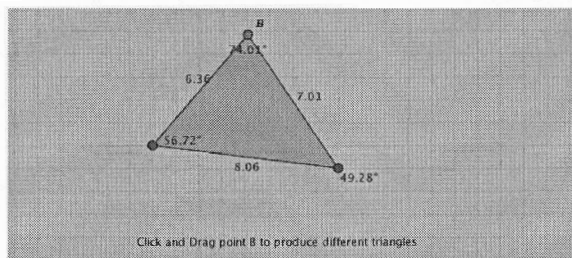


Figure 3.23: The Resulting Scalene Triangle Applet as Exported from Cinderella.

Another proprietary software application that offers the option of exporting applets is The Geometer's Sketchpad (see Figure 3.24). The pedagogy team was quite comfortable

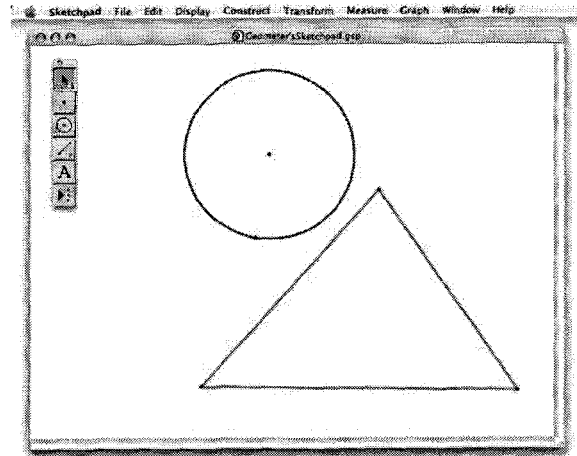


Figure 3.24: The User Interface of The Geometer's Sketchpad. Source: Dr. June Lester, by permission.

with this software and used it to draw and construct sketches which were published both as a part of the specification of the desired learning objects and as static images in the actual lessons. One interactive applet (see Figure 3.25) was generated using JavaSketchpad, the web companion to The Geometer's Sketchpad for generating dynamic activities online.

**PLAY**  
Explains / Challenge

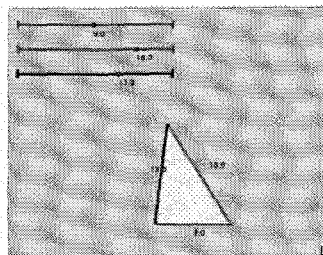


Figure 3.25: Exported from The Geometer's Sketchpad, the *Play* Section for Lesson D1, Constructing Triangles.

### 3.2.1.2.3 Modification and Adaptation of Existing Learning Objects

Often, some applets available had many of the qualities desired by the pedagogy team but did not completely address all of the objectives of the lesson plan. In these cases, the mathematical technology team would seek the permission from the applet authors for the use and modification of their programs.

The next applet, the Unfolding Polyhedra, (see Figure 3.26 for the specification and Figure 3.27 for the actual applet) is an example of such an arrangement. In this case,

**Play**

Two pull-down menus are presented. The student chooses one option from each menu and a regular polyhedron or its net is created. The polyhedron/net will appear in the window and the student will be able to rotate it to see it at various angles. Double clicking on the object will start an animation either of the object unfolding into its net, or of the net folding into the corresponding object.

-----Platonic solid or net goes here-----

<p>Make one selection above menus to polyhedron or its to rotate the object. object to see it or to see the net corresponding 3D object.</p>	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="padding: 2px;">Tetrahedron</th> <th style="padding: 2px;">!</th> <th style="padding: 2px;">Solid</th> <th style="padding: 2px;">!</th> </tr> </thead> <tbody> <tr> <td style="padding: 2px;">Hexahedron (Cube)</td> <td style="padding: 2px;"></td> <td style="padding: 2px;">Net</td> <td style="padding: 2px;"></td> </tr> <tr> <td style="padding: 2px;">Octahedron</td> <td style="padding: 2px;"></td> <td style="padding: 2px;"></td> <td style="padding: 2px;"></td> </tr> <tr> <td style="padding: 2px;">Icosahedron</td> <td style="padding: 2px;"></td> <td style="padding: 2px;"></td> <td style="padding: 2px;"></td> </tr> <tr> <td style="padding: 2px;">Dodecahedron</td> <td style="padding: 2px;"></td> <td style="padding: 2px;"></td> <td style="padding: 2px;"></td> </tr> </tbody> </table>	Tetrahedron	!	Solid	!	Hexahedron (Cube)		Net		Octahedron				Icosahedron				Dodecahedron				<p>from each of the create a regular net. Click and drag Double click on the unfold into its net, turn into the</p>
Tetrahedron	!	Solid	!																			
Hexahedron (Cube)		Net																				
Octahedron																						
Icosahedron																						
Dodecahedron																						

Figure 3.26: The *Play* Applet Specification for Lesson E6.

the customization was relatively trivial because the learning object being modified not only contained most of what was needed for the applet, it also supported more complex solids. The main concern was to ensure that the program could be configured properly on the server and that only the solids needed for the lesson were included in the applet.

François Labelle, the creator of the Unfolding Polyhedra, provided more advanced and interesting interactive applets on his website<sup>9</sup>. The following screenshot (see Figure 3.28)

<sup>9</sup><http://www.cs.mcgill.ca/~sqrt/unfold/unfolding.html>.

PLAY  
 Explore

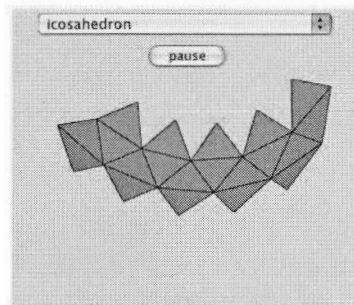


Figure 3.27: The Modified Version of the Unfolding Polyhedra.

shows Labelle's website with an example of an unfolding torus and options to other polyhedra that are more complex and are beyond the scope of a middle school curriculum.

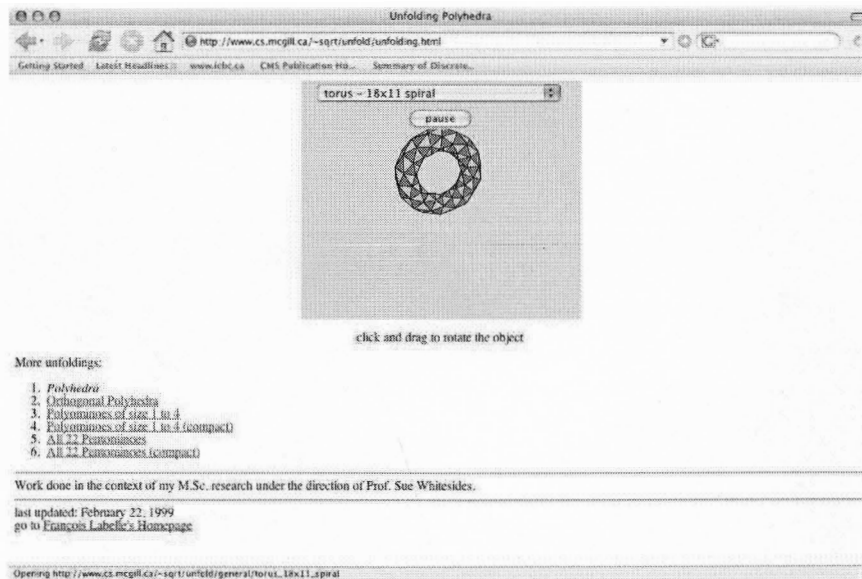


Figure 3.28: François Labelle's Applet Web Page.

There were some learning objects that met the pedagogy team's requirements completely without any modification and were used with permission. For example, the Pattern Block applet (see Figure 3.29) by Arcytech and the Tangram applet (see Figure 3.30) by MathResources Inc. were in this category.

During the project, the teams learned several valuable lessons. The pedagogy team presented the mathematical technology team with a wish list of mathematical learning objects and their specifications. While some of these learning objects were constructed and exported as applets by computer-savvy teachers using available geometry software, other objects on the wish list needed to be custom-built. As with other projects, clients who are less familiar with technologies are often not good judges of the level of difficulty of some of their requests and a knowledgeable intermediary is needed to educate and explain what is technically feasible and what cannot be done within the budget.

Another point about educators who are using technologies in the classroom is that many may possess basic web skills as users but not as content creators. While some may be able to acquire or generate more elementary rich media resources (e.g., text or image files for the Learn section of the site), many more may be unfamiliar with mark-up languages such as html to enter textual sections into the database (albeit via a graphical online interface), or be able to manipulate graphic or sound files. To accommodate users with a wide-ranging level of technical competence, it is extremely important to create an interface as user-friendly and seamless as possible. It is also critical to provide relevant professional development to those educators who may lack skills or confidence in using online tools.

More than half of all SAMPLE applets are custom-built. Some of them are created completely from scratch while others are built upon existing software libraries. It remains a difficult task for teachers to author applets completely on their own. The experience from the SAMPLE project shows that there are currently opportunities for specialists in mathematical technology to work closely with educators to systematically create a comprehensive set of innovative mathematical learning objects that befit a coherent discovery-based learning environment.

Indeed, quality and ready-made applets are at present in short supply. The authoring of interactive learning objects is a challenging proposition. Software-assisted authoring of applets is generally restricted in breadth though it may serve to fill some of the needs in the absence of ready-made learning objects. It must be said that many teachers have highly creative ideas for the classroom that do not fit in with the paradigm of standard geometry



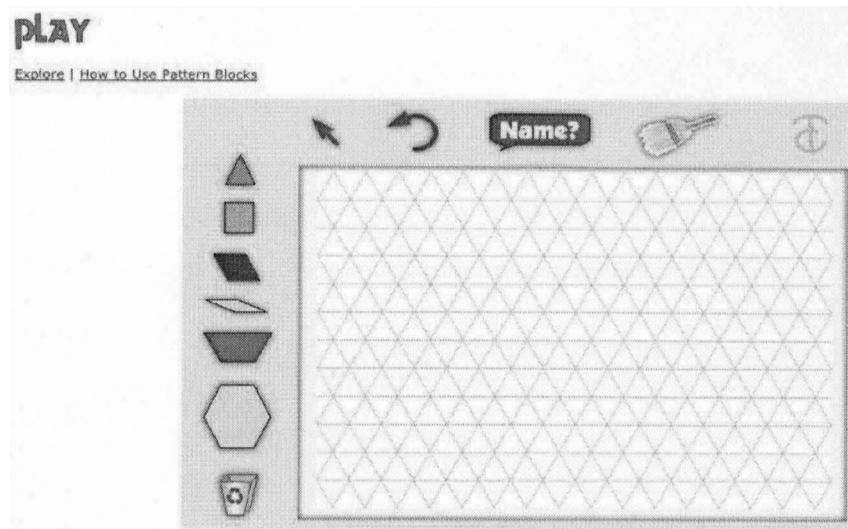


Figure 3.29: Pattern Blocks Applet by Arcytech.



Figure 3.30: Tangram Applet by MathResources Inc.

packages. To capture and realize these ideas will not be through the use of the export function of any particular software application. Moreover, unassisted authoring of novel and innovative applets requires significant programming skills and will be beyond the scope of most teachers for some time to come. The reliance on custom-made learning objects will remain strong in the near term.

Educational geometry software applications currently on the market are quite specialized and are designed to be used more as a stand-alone tool than as an integrated companion in classroom instruction. The cost of acquiring a multitude of third-party software applications can become expensive. Until software creators start putting in a concerted effort to build more powerful and sophisticated applications that can export a wide variety of interactive learning objects that are informed by classroom needs, a niche will continue to exist for textbook publishers to partner with mathematical technology specialists and instructional designers to bring forth more viable alternatives.

An example of a textbook-free alternative is the Interactive School Mathematics (ISM)<sup>10</sup> by MathResources Inc. ISM has been in development since the conclusion of the SAMPLE project and is informed by the same considerations. However, ISM's main focus is on developing lessons for a constructivist curriculum and defers to open-source applications to address communication (e.g., chat or forum), performance tracking (e.g., testing and paths) and learning content management needs. ISM addresses all of the issues mentioned above and utilizes similar elements like those employed in the SAMPLE framework while incorporating many sophisticated features and a more polished user interface. All of ISM's learning objects have a uniform look-and-feel and are custom-designed to fully integrate with the content of each lesson. It also comes with additional features such as a built-in journal, graphing calculator and glossary. ISM is entering the pilot-testing stage and looks promising to becoming a new classroom tool for educators.

### 3.2.2 Design of a Learning Content Management System

While the mathematical technology team in British Columbia was tagging the html content for display on the static prototype site, the content management team in Nova Scotia, a joint-effort between Dalhousie University and MathResources Inc., was in the process of developing a learning content management system that would eventually house the content.

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<sup>10</sup><http://www.mathresources.com/products/ism/index.html>.

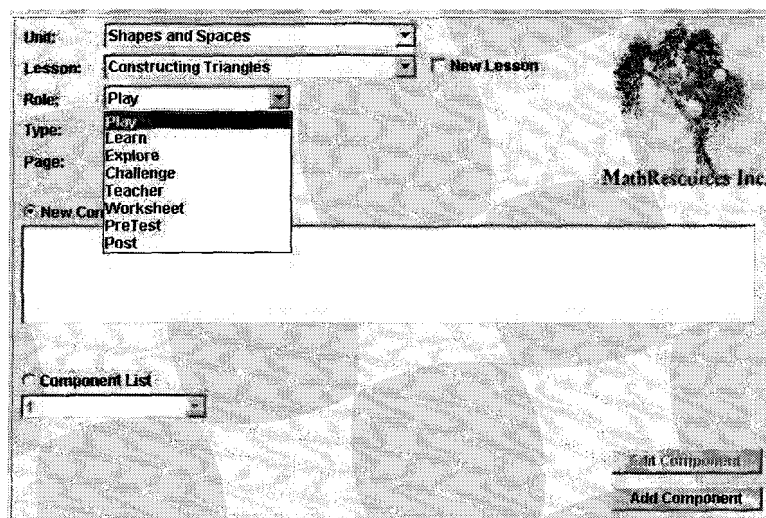


Figure 3.31: Interface of Prototype Database. Source: The SAMPLE project, by permission.

The content management team developed a script-based application that would dynamically generate web pages from a database. This content management system allowed teachers to input and modify content using a Java-based web interface. The content management team was responsible for developing a meta-tagging system that would effectively parse static web pages and separate rich media content from text-based content before depositing the content into the database as separate components. Once content was deposited into the database, a component identification number was assigned and could be repurposed as needed.

The following screen captures illustrate how users can input content into the prototype database using a custom-built web interface. Users, such as instructional technologists or teachers, must select the appropriate unit and lesson from the pull-down menus, and then the section (also known as “Role”) of the lesson in order to enter content into the database (see Figure 3.31). In other words, once the users have selected the particular unit and lesson, they must input the content and specify the associated role (e.g., the *Play* section), the type of content (e.g., text), the page number, and component number.

Figure 3.32 shows a list of file formats that the database can accommodate. Users can choose from text to many rich media file types (e.g., audio, video, applets, etc.) to include as a component.

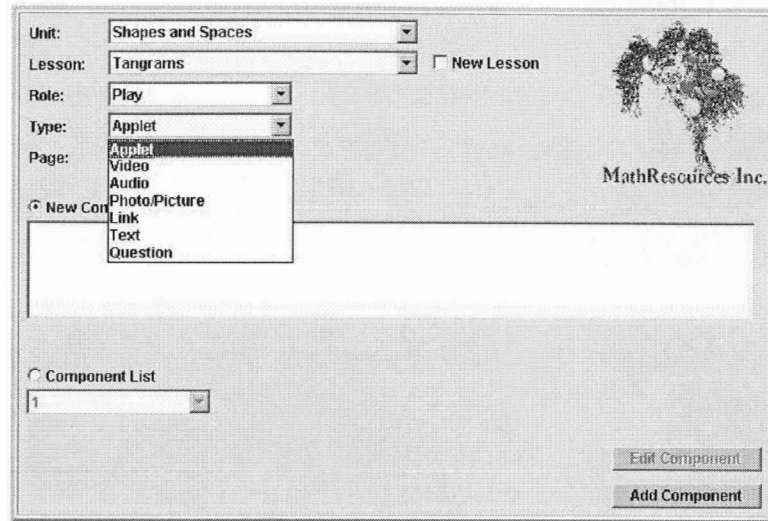


Figure 3.32: The Pull-down Menu of the Types of Components. Source: The SAMPLE project, by permission.

As with most sections, there are many components to each section and each component is treated as a small module. The object-oriented nature of the database structure requires the assignment of a component number to order each individual component, allowing for fine-grained identification, storage and retrieval (see Figure 3.33). When a component is added to the database, it is assigned a component identification number. Figure 3.34 shows a brand new component being entered into the database and Figure 3.35 shows a textual component entered with HTML mark-up tags.

Through the use of this interface, the content management team was able to demonstrate how this database can be deployed to serve up the mathematical content of SAMPLE. A detailed description of this aspect of the project has been published (Kellar et al., 2003).

### 3.2.3 Summary of the Technical Aspects of SAMPLE

SAMPLE's incorporation of communication tools, such as chat room and discussion board, allowed students to work collaboratively. In addition, access to the chat room's log and discussion board's postings allowed teachers to gauge the students' learning process and to determine which concepts may need to be reinforced in class. Standards, such as those

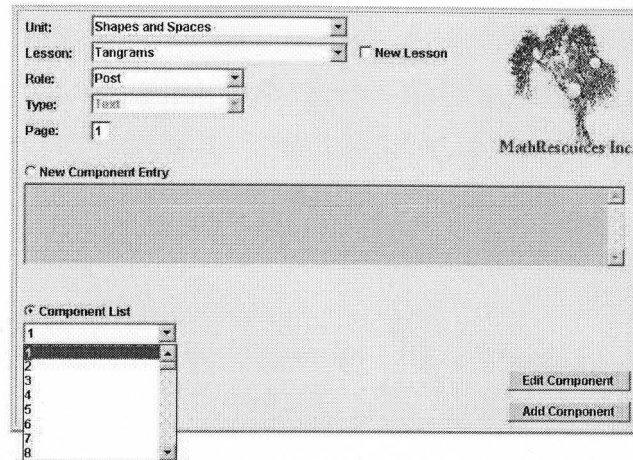


Figure 3.33: The Pull-down Menu of the Component list. Source: The SAMPLE project, by permission.

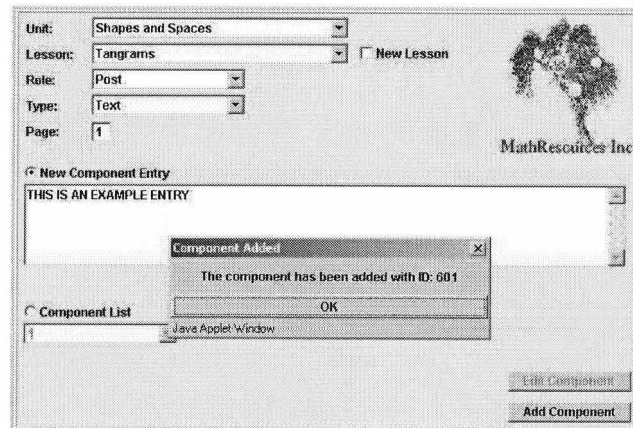


Figure 3.34: Confirmation Window for Component Submission. Source: The SAMPLE project, by permission.

developed by IMS Global Learning Consortium, Inc.<sup>11</sup>, for specifying learning objects are gradually being adopted.

<sup>11</sup><http://www.imsglobal.org>. IMS Global Learning Consortium is a non-profit body that is dedicated to developing common protocols and standards for learning technology.

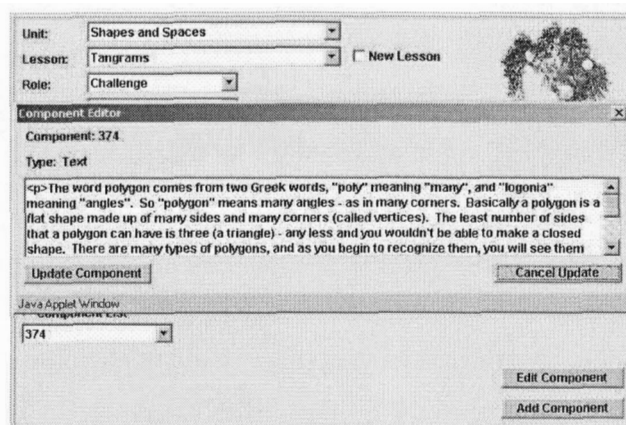


Figure 3.35: Textual Component Entered with HTML Mark-up Tags. Source: The SAMPLE project, by permission.

It should be pointed out that major initiatives for content management systems began to emerge around the same time SAMPLE was being developed. They range from several powerful and general-purpose open-source content management system applications, including Plone<sup>12</sup> and TikiWiki<sup>13</sup>, to more proprietary and domain-specific alternatives, such as Open Text Corporation's Livelink Learning Management System<sup>14</sup>. A new initiative called TheDump<sup>15</sup> (based on LON-CAPA<sup>16</sup>) is building a repository of mathematics resources for the K-12 level this summer.

It has subsequently been determined that an enterprise-level learning management system would be more suited to handle some of the planned and more complex monitoring features, such as activity tracking, performance grading, and assignment submissions, etc. For example, ATutor<sup>17</sup> is an open-source learning content management system that is compliant with the IMS standards and offers communication tools, such as chats and whiteboards, which hold the potential to further enhance collaborative learning.

<sup>12</sup><http://plone.org> (started in 2000).

<sup>13</sup><http://tikiwiki.org> (started in 2002).

<sup>14</sup><http://www.opentext.de/learning-management>.

<sup>15</sup><http://thedump.lon-capa.org/>.

<sup>16</sup>The LectureOnline-Computer-Assisted Personalized Approach (LON-CAPA) is an open-source content management system created by the College of Natural Science and Michigan State University.

<sup>17</sup><http://www.atutor.ca>.

### 3.2.4 A Brief Case Study of ISM and SAMPLE

ISM and SAMPLE started simultaneously and were examining in parallel different aspects of integrating technology with mathematical learning. MathResources Inc., whose expertise is in the development of mathematical software for schools, received a \$2 million (conditionally repayable) contribution from the Atlantic Innovation Fund in July 2002. The developers of ISM were chiefly concerned with issues around building standards-compliant instructional content and learning objects so that educators may easily manage the instructional process via the use of an online repository, such as a learning content management system.

SAMPLE's focus was in understanding how to build a framework to support mathematics learning in the classroom. Specifically, researchers were interested in exploring which elements would facilitate the learning process and how these elements could work together.

The outcomes of both projects have led to similar conclusions. Firstly, both projects are in support of a constructivist learning environment and have determined that the interactivity of ICT is an excellent avenue to promote the acquisition of mathematical knowledge. Secondly, researchers from both projects recognize the importance of mathematics education and the needs of educators to have access to a repository of mathematical tools.

Where the two projects differ is in how the learning objects were compiled and how such content was managed. SAMPLE used a heterogeneous mixture of learning objects and built its own repository. It was more concerned with exploring the nature of online learning than with standards compliance issues. MathResources Inc., on the other hand, has both the human resources and financial backing to streamline the design of learning materials. The result is a product with a unified feel and attention to emerging standards. It was certainly a wise move to focus on content development and leave content management issues to others. By being cognizant of standards for LCMS, MathResources Inc. is able to take advantage of ATutor and make the adoption of ISM a more convenient and attractive proposition. In summary, the two projects provide an interesting snapshot of an academic research project and a commercial software product.

## Chapter 4

# Conclusion

Middle school mathematics teachers are in need of support in the classroom to cope with high demands for numeracy from many fronts. Curriculum requirements must meet demanding numeracy standards needed to function in a knowledge-based society. Possessing knowledge of only basic arithmetic no longer suffices. As a result, students of varying abilities and motivation levels are expected to persist in mathematics courses to a more advanced level so that they can develop the understanding and problem-solving skills needed to make sound decisions in everyday life scenarios, from making financial investments to determining how much hardwood flooring is required for a home-renovation project<sup>1</sup>. This effort is premised on a successful outcome in early mathematics education. Mathematics educators, who must bear the brunt of these new demands and shifts in pedagogical models, are reliant on practical and innovative support.

SAMPLE was built to assist teachers in accessing the help they need in meeting the growing demands. It provided complete lesson plans for each of the topics in the unit with the option for advanced users to add or modify content. SAMPLE also incorporated remedial materials for under-prepared teachers needing a refresher course. Learner's characteristics, including aptitude, were taken into account and addressed through a customized environment so that students could catch up or skip ahead at their own pace depending on their grasp of the course materials. One salient feature of SAMPLE was the use of interactive learning objects. These applets were designed to model complex interactions

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<sup>1</sup>It was reported that one in three adults in England "cannot calculate the area of a room that is 21 by 14 feet, even with the aid of a calculator." (Department for Education and Employment, 1999, Chapter 1)



and concepts that were best explained visually. To maintain the students' interest in the content, innovative approaches were taken, including the use of rich media learning objects, to create a discovery-based learning environment. Such learning objects made it possible to realize creative ideas that may otherwise be too difficult to implement in the classroom.

## 4.1 The SAMPLE Experience

The realization of the SAMPLE prototype as a stand-alone and self-contained middle-school geometry unit was a fulfilling and rewarding experience. The multidisciplinary initiative not only had all the elements of traditional publishing, such as editing and proofreading, the authoring process entailed a significant use of technologies, much more so than would be common in the digital publishing of online journals and the like. This was largely a result of the technical nature of publishing interactive mathematical content. Indeed, it was very much a collaborative effort with room for creativity for all those involved. The SAMPLE project began with both the educators and programmers working jointly and in parallel right from the start. After the initial consultation of how the framework of SAMPLE should be, the educators went immediately to work on content authoring while the programmers worked laboriously to provide the structure to host the content. This collaborative process continued with frequent meetings and exchange of ideas in order to fine tune the many aspects of the prototype, including usability, navigation, additional features, etc. Once the first batch of instructional content became ready and the web framework was in place, the pedagogy team and the mathematical technology team began the work of compiling and developing the needed interactive learning objects. Simultaneously, the mathematical technology team also began marking up the content for inclusion on the website while the pedagogy team continued the preparation of lessons. The opportunity for the mathematical technology team to peer inside the process of instructional design and for the pedagogy team to be immersed in the culture of software design was invaluable and the result was a rare and mutually beneficial learning experience. The requirements specified by the pedagogy team were mostly satisfied by the mathematical technology team, sometimes with minor modifications, as motivated by enhanced usability or functionality, or as dictated by budgetary constraints and feasibility considerations. The content management team was busy preparing a database that would allow for the dynamic generation of lessons by repurposing content materials on the fly.

## 4.2 Outcomes of SAMPLE

SAMPLE was completed in the spring of 2003 and field tests involving more than three dozen pre-service and practising teachers, and parents were conducted in the summer of 2003 by the pedagogy team. The responses from teachers and parents were very positive and encouraging. Below is a summary of the responses.

“We asked 32 pre-service elementary teachers to review an earlier version of the program over a 2–3 hour period, and to complete an evaluation questionnaire. The results were summarized as follows: on the criteria of accessibility, responsiveness, visual appeal, readability and navigation, mean ratings on a four-point scale (from unsatisfactory to excellent) ranged from 3.0 to 3.5. On the criteria of interest, enjoyment and usefulness, mean ratings were 3.8, 3.7 and 3.7 respectively. User comments were extremely positive, with typical comments such as:

‘Interesting and makes you think.’

‘I think that it is going to be very helpful because they can actually play around with different concepts.’

‘I liked the teachers’ notes and parents’ notes as well as the flexibility to play with the shapes and seeing the results.’

We also asked three practicing teachers to review the prototype and to provide feedback. All were extremely positive, provided useful suggestions for improvement, and offered to participate in the next phase of the project. One senior secondary school teacher (and Mathematics Department head) wrote that:

“In a period which, as math teachers, we are increasingly challenged to provide ‘situated’ learning experiences which not only emulate genuine approaches to discovery but also allow the learner to progress at his own pace, Web-based activities such as those provided by this portal may provide the only economically feasible response. As a high school mathematics teacher whose assignment includes Calculus 12, I frequently encounter students who would benefit from both the potential for self-pacing and the emphasis on experimentation and visualization that characterize this software. Please include me (and my students)

as participants in research designed to test this approach at the senior high school or first year undergraduate levels (Stanway, 2003)”

Similarly, we asked three parents to try out the prototype with their children. Comments received were very encouraging, thus opening up a new avenue for teacher-parent collaboration in supporting students’ learning of mathematics.” (Kaufman, 2003, p. 20)

The SAMPLE project concluded with the field tests and the project’s objectives were met. Both the static prototype developed at Simon Fraser University and the database developed at Dalhousie University performed as intended. While full integration was demonstrated to be possible, the project stopped short of populating the content into the database and the two systems remained two separate, successful proofs-of-concept. This decision was made partly due to the availability of more advanced content management systems on the market and partly due to time constraints. In the fall of 2003, one of the co-investigators of SAMPLE transferred to Dalhousie University and the leadership for potential development of SAMPLE followed as well. In fact, SAMPLE was proposed as a suitable platform to continue further research investigation and many like-minded researchers in Atlantic Canada who are devoted to improving middle-school mathematics education, including the co-investigators of SAMPLE, have recently formed the Atlantic Community Math Network<sup>2</sup>.

### 4.3 Future Research

From the field tests, it was clear that there was much support for a constructivist learning environment and that SAMPLE was definitely a viable and an innovative proof-of-concept. Future research can build upon the work of SAMPLE by furthering the use of learning objects and discovery-based computer-driven environments in upper secondary and even post-secondary settings. As a next step to empower teachers as facilitators in the classroom, one can move toward a more mature and sophisticated learning management system that incorporates a web-based tracking system which can comprehensively capture students’ activities and present teachers with an understanding of how and when learning does or does not take place. To recap, a learning management system that delivers rich media-based learning objects can serve two major purposes: 1. to engage students in mathematics

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<sup>2</sup><http://www.aarms.math.ca/outreach>.

using a multitude of settings that suit individual styles and hence help them gain numeracy skills; and 2. to familiarize students with basic computer technologies in a practical manner at an early stage in their lives.

#### **4.3.1 Constructing a Better Learning Content Management System**

Researchers must be cognizant of the existing limitations in the current education system and try to effect change. More research is needed to look into the relationship between different teaching styles and the usage of online resources. As learning content management systems are still in their infancy for the elementary and secondary school market, there is room for consultation and identification of best practices. In so doing, more insight can be gained as to how the efficacy of a learning content management system can be optimized.

### **4.4 Some Final Comments About the Audience of SAMPLE**

From the literature review, there was much compelling evidence that a lack of literacy and numeracy skills may hamper one's life chances. A multi-disciplinary publishing endeavour that involves rich media will not solve problems on its own. It does, however, create a new reality that is conducive for all those concerned to explore more avenues to educate and engage the population with an important life-skill, numeracy, especially in the age of computer technologies. Allowing mathematics to be learned in such an environment would provide students not only with numeracy skills but also the necessary training to utilize the tools needed at work and at home.

The fact that some teachers have math anxiety and other teachers are not computer literate enough is evidence that there still exist hurdles in some regions to introduce computer-based learning environments into mathematics education. There is no question that middle school mathematics teachers need more professional development and support. This suggests that a more coordinated approach is needed to integrate numeracy, literacy and technology learning in the classroom.

SAMPLE was intended to be a supportive tool for teachers who may be under-prepared or apprehensive about mathematics. In the face of strong numeracy requirements, unsatisfactory assessment results of student performance, and educational reforms, a unified educational strategy is needed and SAMPLE can be only one part of the solution.

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