

# Filtering, smoothing and prediction using a control-loop spectral factorization method for coloured noise

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## SUMMARY

A method for the linear least-squares estimation of random signals contaminated with random noise that uses a new method of spectral factorization is shown. It is shown that the optimal filter can be written entirely in terms of the two spectral factors of signal plus noise and noise-alone, and can be applied to the general case of coloured and white additive noise. The method of spectral factorization used is novel and uses control-system methodology. Copyright © 2012 John Wiley & Sons, Ltd.

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## PRELIMINARIES AND NOTATION

$q^{-1}$	Backward shift operator: $y_{k-1} = q^{-1}y_k$ .
$z^{-1}$	Transform variable corresponding to the backward shift operator.
$E[\cdot]$	Expectation operator.
$\xi_k, \mu_k$	White-noise processes.
$y_k, s_k$	Signal and signal plus.
$\varepsilon_k$	White innovations process.
$\Phi_{ys}(z)$	Z-transform cross-spectral density transform between signal and signal plus noise.
$\Phi_{ss}(z)$	Z-transform spectral density transform of signal plus noise.
$\Lambda(z^{-1}), Z(z^{-1})$	Spectral factor and normalized spectral factor of signal plus noise.
$W_n(z^{-1})$	Noise-alone spectral factor.
$\mathcal{F}, \mathcal{F}^{-1}$	FFT and its inverse.

Polynomials of degree  $n$  written as  $a(z^{-1}) = a_0 + a_1z^{-1} + a_2z^{-2} + \dots + a_nz^{-n}$  are assumed to be strictly minimum phase with roots  $|z| < 1$  inside the unit circle. Often, the argument  $z^{-1}$  is missed for brevity that is  $a(z^{-1}) = a$ . Conjugate polynomials  $a(z) = a^*$  are assumed to be strictly non-minimum phase with roots outside the unit circle.

## 1. INTRODUCTION

The problem of estimating a random signal contaminated with random noise is one that has numerous practical applications. These include, but are not restricted to noise reduction from speech [1], automobile noise-reduction problems [2] and biomedical applications [3]. The basic theory of course is not new and dates back to the time of Wiener (continuous-time) [4] and Kolmogorov (discrete-time) [5]. The basic formulation however is just the beginning when trying to

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find a solution to such problems as it was found that the computation of these filters in the general case of additive coloured noise is not trivial. It is usual to assume that the noise characteristics can be obtained separately by some measurement method when the signal is absent. Thus, for speech signals, where the signal is absent during silence periods during words or natural pauses, these methods are particularly suited. With this assumption, it is then possible to estimate the noise model separately using various methods. Such methods include the use of recursive parameter estimation. For example, in the case of additive white noise, extended recursive-least-squares (ELS) has been used [6] and for coloured noise ELS and the use of Diophantine equations have been relatively successful [7–9]. However, Diophantine equations are the equivalent of solving Riccati equations, and the solution can be computationally intense, as can the estimation of model parameters using RLS. Another approach is to use Kesptrum (complex cepstrum) analysis [10] or even Kalman filtering techniques [11]. With the exception of the Kesptrum method, most of these approaches do not distinguish the separate filtering, smoothing and prediction problems with one single approach.

## 2. SIGNAL AND NOISE MODEL

Consider a signal  $y_k$  corrupted with coloured noise  $v_k$  giving a noisy signal as shown in Figure 1.

$$s_k = y_k + v_k \quad (1)$$

The signal and coloured noise are assumed to be generated from the following stable, minimum-phase transfer functions driven by uncorrelated white-noise.

$$y_k = W_s(q^{-1})\xi_k \quad (2)$$

$$v_k = W_n(q^{-1})\mu_k \quad (3)$$

Here, the colouring and signal transfer functions can be modeled as pole-zero, all-pole or all zero (FIR) transfer functions without losing any generality. For example, we can put  $W_s(z^{-1}) = \frac{\tilde{c}(z^{-1})}{\tilde{a}(z^{-1})}$  and  $W_n(z^{-1}) = \frac{\tilde{b}(z^{-1})}{\tilde{e}(z^{-1})}$  or represent them both as single polynomials. The white-noise processes  $\mu_k$  and  $\xi_k$  are assumed to have variances  $\sigma_\xi^2, \sigma_\mu^2$ , respectively. Using this model, any combination of white and coloured noise terms can be incorporated so that Figure 1 becomes quite general. We do however specify that the signal cannot be pure white-noise that is that there must be some dynamics within the transfer function  $W_s(z^{-1})$  to make practical sense. The z-transform spectrum of the signal plus noise is

$$\Phi_{ss} = W_s W_s^* \sigma_\xi^2 + W_n W_n^* \sigma_\mu^2 \quad (4)$$

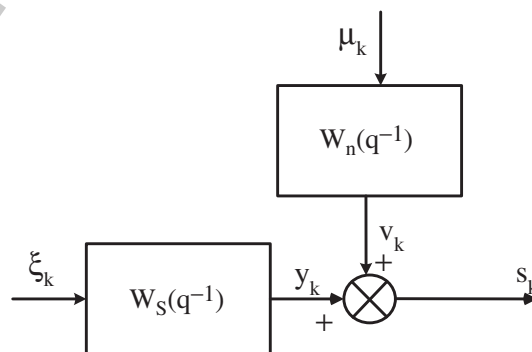


Figure 1. Signal plus additive coloured noise.

01 Similarly, the z-transform cross-spectral density between signal and coloured noise is

$$02 \quad \Phi_{ys} = W_n W_n^* \sigma_\xi^2 \quad (5)$$

04 and the z-transform of the spectrum of the signal plus noise process when the signal is absent  
05 (a so-called noise-alone period) is given by

$$06 \quad \Phi_{vv} = W_n W_n^* \sigma_\mu^2 \quad (6)$$

08 This is clearly of importance and will be used later, but tells us that the noise signal must be  
09 separately measurable as is often the case in such problems.

11 The strictly minimum phase, stable spectral factor of signal plus noise is defined as  $\Lambda$

$$12 \quad \Phi_{ss} = \Lambda \Lambda^* \quad (7)$$

14 The spectral factor  $\Lambda$  in (7), however, is not unique because there are an infinity of such spectral  
15 factors, which exist that can generate the same spectrum merely by rescaling the driving white noise  
16 variance. However, we can further simplify by having a *uniquely defined* spectral factor  $Z$

$$17 \quad \Phi_{ss} = ZZ^* \sigma_\varepsilon^2 \quad (8)$$

$$19 \quad \Lambda = Z\sigma_\varepsilon \quad (9)$$

21 where  $Z(0) = 1$ . It is this type of spectral factor that we use throughout the paper.

22 A unique innovations model can now be written, which generates the same spectrum as (8 or 7)

$$24 \quad s_k = Z(q^{-1})\varepsilon_k \quad (10)$$

26 where  $\varepsilon_k$  is a unique white innovations sequence of variance  $\sigma_\varepsilon^2$ . We also note that from (6), clearly  
27  $W_n$  is a spectral factor of the noise-alone model. When white-noise is added to the coloured noise  
28 model, then it too can have its own innovations model and we arrive back at the basic model used  
29 in Figure 1, but where the driving noise  $\mu_k$  is replaced by another white-noise innovations term.  
30 Hence, the model of Figure 1 is quite general if we assume in the same spirit as (10), but for the  
31 noise-alone transfer function, that  $W_n(0) = 1$ . It is important to use such innovation models, as  
32 without them the spectral factor is not unique and any constant can be absorbed into the driving  
33 noise variance.

## 34 2.2. Wiener filter

36 The optimal Wiener estimator minimises the mean-square error

$$37 \quad E[e_k^2] = E(y_k - \hat{y}_k)^2 \quad (11)$$

39 where the estimate  $\hat{y}_k = H(q^{-1})s_k$  and the Wiener estimator is found from the classical solution  
40 [10, 12]

$$42 \quad H(z^{-1}) = \left[ \frac{\Phi_{ys}(z)}{\Lambda(z)} \right]_+ \frac{1}{\Lambda(z^{-1})} \quad (12)$$

44 The + notation applied to within the square brackets denotes a suitable interval  $\mathfrak{R}_f = \{0, 1, 2, \dots\}$ ,  
45  $\mathfrak{R}_s = \{-d, -d+1, -d+2, \dots\}$  or  $\mathfrak{R}_p = \{d, d+1, d+2, \dots\}$  for some integer delay  $d > 0$   
46 depending on whether a filter, smother or predictor is required

48 Writing (4) as

$$49 \quad \Phi_{ss} = \Phi_{ys} + \Phi_{vv} \quad (13)$$

51 then using (8) and (9), we can write (12) as

$$52 \quad H(z^{-1}) = \left[ Z(z^{-1})\sigma_\varepsilon - \frac{\Phi_{vv}(z)}{Z(z)\sigma_\varepsilon} \right]_+ \frac{1}{Z(z^{-1})\sigma_\varepsilon} \quad (14)$$

in which using (6) becomes

$$H(z^{-1}) = \left[ Z(z^{-1}) - \frac{W_n(z^{-1})W_n(z)\sigma_\mu^2}{Z(z)\sigma_\xi^2} \right]_+ \frac{1}{Z(z^{-1})} \quad (15)$$

Observe that when  $W_n(z^{-1})$  is written as an innovations model, we must have  $W_n(0) = 1$  and any non-unity term is absorbed into the white driving noise variance of the coloured-noise model. We now consider three possible cases.

(i) **Filtering**

This corresponds to instantaneous estimation  $\hat{y}_{k/k}$  that is information up to and including time  $k$ . the interval  $\mathfrak{R}_f = \{0, 1, 2 \dots\}$  and the estimator has the transfer function

$$H(z^{-1}) = 1 - \left[ \frac{W_n(z^{-1})W_n(z)}{Z(z)} \right]_+ \frac{1}{Z(z^{-1})} \frac{\sigma_\mu^2}{\sigma_\xi^2} \quad (16)$$

This solution involves the two spectral factors: when there is signal  $Z$  and when signal is absent  $W_n$ .

(ii) **Smoothing**

This corresponds to the fixed-lag smoothed estimate  $\hat{y}_{k/k+d}$  that is an estimate of the signal at time  $k$  with information up to and including time  $k+d$ . The interval  $\mathfrak{R}_s = \{-d, -d+1, -d+2 \dots\}$ ,  $d > 0$  and the estimator has the transfer function

$$H(z^{-1}) = 1 - \left[ \frac{W_n(z^{-1})W_n(z)}{Z(z)} \right]_+ \frac{1}{Z(z^{-1})} \frac{\sigma_\mu^2}{\sigma_\xi^2} \quad (17)$$

which is identical to (16) except that the observation interval is different. The difference does not show in the form of (17) and further simplification using innovation representations must be found.

(iii) **Prediction**

This corresponds to the predicted estimate  $\hat{y}_{k/k-d}$  that is an estimate of the signal at time  $k$  with information up to and including time  $k-d$ . The interval  $\mathfrak{R}_p = \{d, d+1, d+2 \dots\}$ ,  $d > 0$  and the estimator has the transfer function

$$H(z^{-1}) = \left[ Z(z^{-1}) \right]_+ \frac{1}{Z(z^{-1})} - \left[ \frac{W_n(z^{-1})W_n(z)}{Z(z)} \right]_+ \frac{\sigma_\mu^2}{\sigma_\xi^2} \frac{1}{Z(z^{-1})} \quad (18)$$

Now from (16)–(18), the classical Wiener filter is expressed entirely in terms of the two unique spectral factors,  $Z$  and  $W_n$ , and the problem becomes one of providing innovation models for the three estimators and that of estimating the spectral factors themselves. The innovation models give rise naturally to the removal of the  $[\cdot]_+$  brackets. The ratio  $\frac{\sigma_\mu^2}{\sigma_\xi^2}$  also needs to be found. Therefore, a technique for the direct estimation of the spectral factors needs to be devised.

### 3. IDENTIFICATION OF THE SPECTRAL FACTORS

We wish to identify a moving-average (MA) model of a stochastic process. This will give us a finite-impulse response (FIR) approximation to the spectral factor. Although it would be possible to use ELS and estimate a pole-zero innovations model, such models are known to suffer from stability problems when non-stationary data are used. Provided the order of the model is high enough, it will be able to cope with signals and noise, which have been generated from pole-zero (ARMA) type models. Let an  $n$ th-order MA model be defined as the output  $y_k$  of a FIR filter, whose input is stationary zero-mean white-noise  $\xi_k$  of variance  $\sigma_\xi^2$ . It is assumed that  $\xi_k$ , the driving noise is unmeasurable. This type of identification is often called ‘blind’ system identification because only the output is measurable. If the input were also measurable, then much faster methods could be used such as the least-mean squares algorithm [13].

Let such a MA model be described as

$$y_k = \sum_{j=0}^n a_j \xi_{k-j}, k = 1, 2, \dots \quad (19)$$

The object is to estimate the coefficients  $a_i, i = 1, 2, \dots, n$  solely from measurements of  $y_k$ . The estimation of the coefficients of an autoregressive model is a linear process involving the solution of the Yule-Walker equations. In this case of a MA model, the problem is more difficult and results in nonlinear equations. The finite length autocovariance coefficients at the output of the filter is given by [14]

$$r_{yy}(k) = \sigma_\xi^2 \sum_{i=0}^n -a_{i+|k|} a_i, k = -n \dots -2, -1, 0, 1, 2, \dots, n \quad (20)$$

Now the z-transform periodogram of (20) is found via

$$\begin{aligned} P_{yy}(z) &= \sum_{i=-n}^n r_{yy}(i) z^{-i} \\ &= a(z^{-1}) a(z) \sigma_\xi^2 \end{aligned} \quad (21)$$

Here  $a(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_n z^{-n}$  has all its roots within the unit circle in the z-plane and  $P_{yy}(z)$  and the coefficients of the Laurent series  $r_{yy}(k)$  are the coefficients of the autocovariance function. A faster method of estimating the periodogram and hence the Laurent series is via the FFT. When calculating the autocovariance, exponential smoothing can be used. For an autocovariance vector  $\mathbf{Y}_{yy} = [r_{yy}(0), r_{yy}(1) \dots r_{yy}(n)]$ , then  $\mathbf{Y}_{yy} = \mathfrak{F}^{-1}[|X_k|^2 / N]$  where  $\mathbf{X}_k = \mathfrak{F}[x_k]$  and  $x_k$  is the time-series. A smoothed autocovariance vector  $\mathbf{R}_{yy}$ , is then found from

$$\mathbf{R}_{yy}(j) = \beta \mathbf{R}_{yy}(j-1) + (1-\beta) \mathbf{Y}_{yy}(j), j = 1, 2, \dots \quad (22)$$

where  $0 < \beta < 1$  is a forgetting factor introduced to track time-varying changes in the model and  $j$  is an FFT frame number. If the forgetting factor is chosen to be too close to 1, then good smoothing will result but the tracking will not be as good. Conversely, if the forgetting factor is too low, then good tracking and a less smooth autocovariance vector will result. Overlapping short term FFTs can also be used [15]. Hence, from measurements of the system output and computation of the autocovariance function, we arrive at an estimate of (20), which is well known to be factorable into the form of (21). Therefore, from the autocovariance terms in (20), we can form the Laurent series (21) and perform a spectral factorization to arrive at  $a(z^{-1})$ . Note the normalisation of the spectral factor  $a(0) = 1$  and that the zeroth element of this polynomial has been absorbed into the variance  $\sigma_\xi^2$ .

Now a Laurent series can be easily factorized using many methods, but here we use a feedback method used in earlier work [16, 17], which is shown next. The technique uses negative feedback to force the output of a negative-feedback system to be the same as its input. In this case, the input 'signal' is one-half of the Laurent series, and convolution takes effect in the feedback path as illustrated in Figure 2 next.

F2

**Algorithm 3.1: Feedback spectral factorization**

Initialize vectors  $\mathbf{W}_0, \mathbf{V}_0$  of length  $n+1$  to some initial values, (possibly zero). Assume that the Laurent series vector  $\mathbf{L}$  is already known or can be estimated from (20) (or its FFT equivalent). We only require any one-half of the symmetric Laurent series including the zeroth term. Hence, write  $\mathbf{L} = [\hat{r}_{yy}(0), \hat{r}_{yy}(1), \dots, \hat{r}_{yy}(n)]$ . Use a constant gain (step-size)  $|\eta| < 1$ . Care must be taken not to make the gain too large as then the algorithm will become unstable. Too small a value will result in a slow rate of convergence.

*Step 1*

Form the convolution vector  $\mathbf{V}_k = \mathbf{W}_k^* \mathbf{W}_{-k}$  where its individual elements are found from the

elements of  $\mathbf{W}_k$  as  $v_i = \sum_{j=0}^{n-i} w_i w_{i+j}, i = 1, 2, \dots, n$

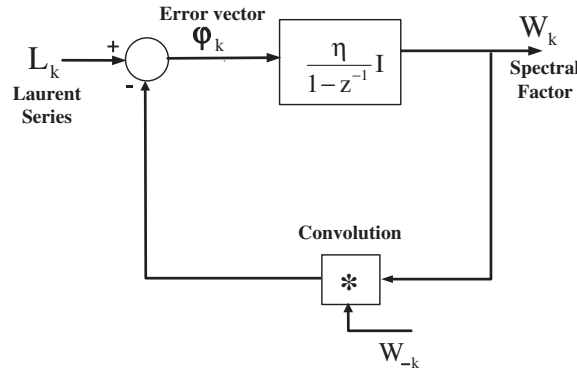


Figure 2. Spectral factorization using feedback.

*Step 2*Update the error vector  $\phi_k = L - V_k$ *Step 3*Update the weight vector  $W_{k+1} = W_k + \eta\phi_k$ *Step 4*Test for convergence  $J = \sum_{j=0}^n (l_j - v_j)^2 < \delta$ , for some small  $\delta$ , where  $l_j, j = 1, 2, \dots, n$  are theknown Laurent series coefficients of the vector  $L$ . Stop if convergent, else return to step 1. The elements of the vector  $W_k$  are now the spectral factor polynomial coefficients. The driving noise variance is estimated from the zeroth element of the vector  $\sigma^2 = W(0)^2$ .

## 4. INNOVATION-BASED ESTIMATION

The estimators (16)–(18) in the preceding section are of little computational use as they require the removal of the  $[\cdot]$  brackets. It is possible however to proceed further by separating the terms within the brackets. Because the model estimation method here results in FIR systems, it is pertinent to also represent the spectral factors as FIR filters and not as rational transfer functions. Therefore, we assume that  $W_n$  and  $W_s$  are both FIR filters represented by polynomials that are both strictly minimum phase of degree  $n$ . The degree of the spectral factors must be large enough to model signals and noise, which have pole-zero generating transfer functions in the form of ARMA systems. Depending on the application, the value of  $n$  could be anywhere from around 10 to maybe 60 or above. Because all three equations have a similar expression, consider the following two convergent power series expansions, using the fact that  $W_n(0) = Z(0) = 1$

$$\frac{W_n(z)}{Z(z)} = 1 + p_1z + p_2z^2 + \dots \quad (23)$$

and

$$W_n(z^{-1}) = 1 + \gamma_1z^{-1} + \gamma_2z^{-2} + \dots \quad (24)$$

The coefficients in (23),  $p_i, i = 1, 2, \dots$  are impulse-response coefficients of an uncausal sequence whereas the coefficients  $\gamma_i, i = 1, 2, \dots$  are impulse-response coefficients of a causal sequence.

Convolving (23) and (24) results in

$$c_k = \sum_{i=-\infty}^{\infty} p_i \gamma_{k-i} \quad k = \dots -2, -1, 0, 1, 2, \dots \quad (25)$$

01 Assume that both impulse responses have been truncated to a length  $\ell \gg d$ , which is chosen  
 02 depending on the application. Typically for most applications,  $\ell$  need not be more than 64 and at  
 03 least 16. Then, we can write (25) as

$$04 \quad c_k = \sum_{i=0}^{\ell} p_i \gamma_{k+i}, k = 0, 1, 2, \dots \quad (26a)$$

07 and

$$09 \quad c_k = \sum_{j=0}^{\ell} p_{-k+j} \gamma_j, k = -1, -2, \dots \quad (26b)$$

12 These are the coefficients of a Laurent series, which evaluate within the intervals  $\mathfrak{R}_f$ ,  $\mathfrak{R}_s$  or  $\mathfrak{R}_p$   
 13 to remove the []+ bracket thus

$$15 \quad \left[ \frac{W_n(z^{-1})W_n(z)}{Z(z)} \right]_+ = \sum_{i \in \mathfrak{R}} c_i z^{-i} \quad (27)$$

18 We now consider all three cases using the fact from (10)  $s_k = Z(q^{-1})\epsilon_k$ .

19 **(i) Filtering in innovations form**

20 The interval is  $\mathfrak{R}_f = \{0, 1, 2 \dots\}$ . Using (16) and (27) results in

$$22 \quad \hat{y}_{k/k} = s_k - \frac{\sigma_{\mu}^2}{\sigma_{\epsilon}^2} \sum_{i=0}^{\ell} c_i \epsilon_{k-i} \quad (28)$$

25 **(ii) Smoothing in innovations form**

26 The interval is  $\mathfrak{R}_s = \{-d, -d + 1, -d + 2 \dots\}$ ,  $d > 0$ . Using (17) and (27) results in

$$28 \quad \hat{y}_{k/k+d} = s_k - \frac{\sigma_{\mu}^2}{\sigma_{\epsilon}^2} \sum_{i=-d}^{\ell} c_i \epsilon_{k-i} \quad (29)$$

31 **(iii) Prediction in innovations form**

32 The interval is  $\mathfrak{R}_p = \{d, d + 1, d + 2 \dots\}$ ,  $d > 0$ . The coloured noise predictor has two  
 33 parts. The first part is similar to the white-noise predictor case. From (18), use the expansion

$$34 \quad Z(z^{-1}) = 1 + g_1 z^{-1} + g_2 z^{-2} + \dots \quad (30)$$

36 Using (27), (18) and (30) results in the overall result

$$38 \quad \hat{y}_{k/k-d} = \sum_{i=d}^{\ell} \left\{ g_i - \frac{\sigma_{\mu}^2}{\sigma_{\epsilon}^2} c_i \right\} \epsilon_{k-i} \quad (31)$$

41 In order to estimate the innovation sequence, we use the inverse of (10), which results in the  
 42 inverse of the strictly minimum-phase spectral factor of signal plus noise.

$$44 \quad \epsilon_k = \frac{1}{Z(q^{-1})} s_k \quad (32)$$

46 This gives us the white innovation process with the correct variance.

48 **5. ADAPTIVE FILTERING, SMOOTHING AND PREDICTION IN COLOURED NOISE**

49  
 50 The previously found innovation-based estimators can now be included in an adaptive algorithm.  
 51 However, this requires separate measurements of the noise-alone signal. Here we use a simple voice  
 52 activity detector (VAD) based on smooth variance estimation as a VAD. In this way, only one mea-  
 53 surement is needed as the noise signal comes from times of absence in the speech signal. Any VAD  
 54 could be used but it must be accurate, else speech will be removed along with the noise.

**Algorithm 5.1: Filtering ( $d=0$ ) and smoothing ( $d>0$ )***Step 1*

From measurements of the signal plus noise  $s_k$  and the noise alone  $v_k$ , estimate the autocovariance of both of these signals from the inverse FFT of their averaged periodograms. Arbitrarily select  $n$ , the order of the spectral factor polynomials. Estimate the normalized spectral factor polynomials,  $W_n(z^{-1})$  and  $Z(z^{-1})$ , respectively, by using the feedback spectral factorization method. Also find the corresponding innovation variances  $\sigma_\mu^2$  and  $\sigma_\varepsilon^2$  (Equations (6) and (8) by normalizing both spectral factor polynomials to have their zeroth term unity). For example, if the spectral factor of signal plus noise is found to be  $\Lambda(z^{-1})$  before normalization, then its innovation variance is  $\sigma_\varepsilon^2 = \Lambda^2(0)$ .

*Step 2*

Estimate the innovation white-noise sequence  $\hat{\varepsilon}_k$  from  $\hat{\varepsilon}_k = \frac{1}{Z(q^{-1})} s_k$

*Step 3*

Find the impulse response coefficients  $p_i, i = 1, 2, \dots, \ell$  from (23) and note that the  $\gamma_i, i = 1, 2, \dots, \ell$  coefficients are derived from the  $W_n(z^{-1})$  spectral factor already obtained.

*Step 4*

Perform the convolutions

$$c_k = \sum_{i=0}^{\ell} p_i \gamma_{k+i}, k \geq 0$$

$$c_k = \sum_{j=0}^{\ell} p_{-k+j} \gamma_j, k < 0$$

*Step 5*

The filtered ( $d=0$ ) or  $d$  steps smoothed estimate of the signal is found from

$$\hat{y}_{k/k+d} = s_k - \frac{\sigma_\mu^2}{\sigma_\varepsilon^2} \sum_{i=-d}^{\ell} c_i \hat{\varepsilon}_{k-i}$$

**Algorithm 5.2: Prediction ( $d>0$ )**

Steps 1–4 are identical to that of the filtering and smoothing case. Note that the normalized signal plus noise spectral factor is given as  $Z(z^{-1}) = 1 + g_1 z^{-1} + g_2 z^{-2} + \dots + g_n z^{-n}$ .

*Step 5*

Estimate the  $d>0$  steps ahead of predicted estimate from (31)

$$\hat{y}_{k/k-d} = \sum_{i=d}^{\ell} \left\{ g_i - \frac{\sigma_\mu^2}{\sigma_\varepsilon^2} c_i \right\} \hat{\varepsilon}_{k-i}$$

where  $g_i = 0$  for  $i > n$ .

## 6. ILLUSTRATIVE EXAMPLES

**Example 1. Stationary signal and additive noise**

First, consider an example where the noise can be measured directly. The signal and noise generating system is shown in Figure 3 next.

The signal-to-noise (SNR) ratio was measure as -6.5 dB. The spectral factors were estimated using order  $n = 10$ .

The noisy signal is shown in Figure 4. Its filtered, smoothed and predicted estimates are shown in Figures 5(a), (b) and (c), respectively.



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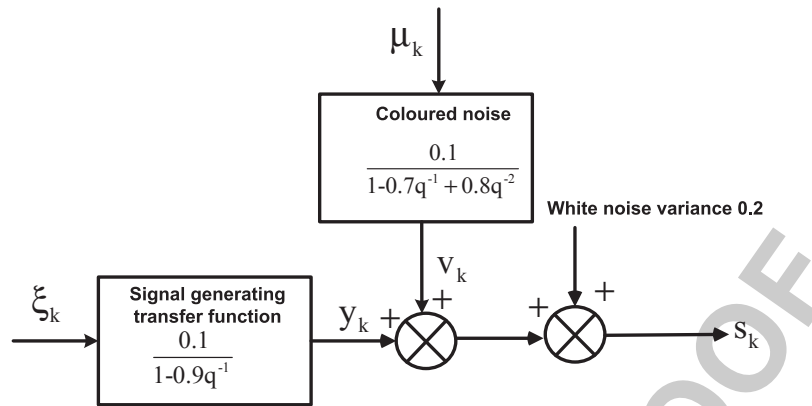


Figure 3. Illustrative example 1.

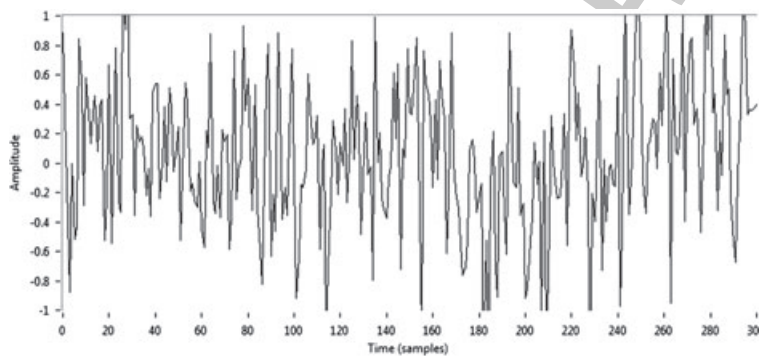


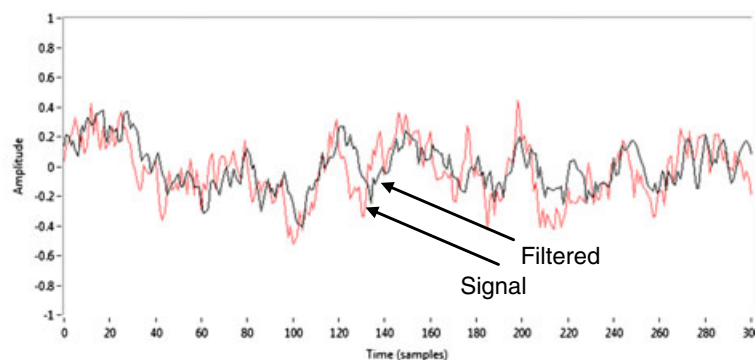
Figure 4. Signal plus coloured noise and white noise.

As expected, the smoothing filter gives the best estimate at the expense of a small time-delay. The smoothed estimate actually has a ‘rounded’ appearance, which is typical of a fixed-lag smoother. The predictor has less information to use and will always give the worst of the three estimates.

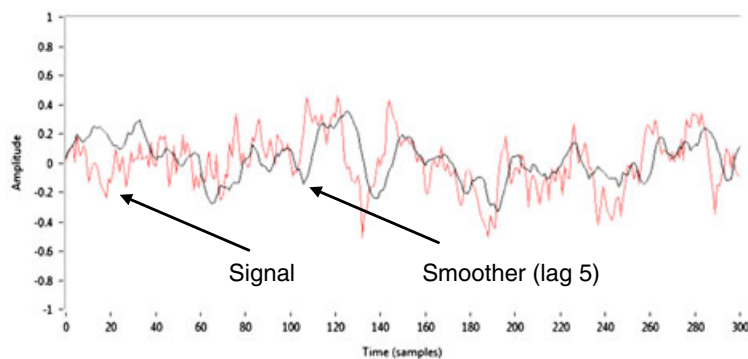
**Example 2: A speech signal and additive coloured noise**

The previous example is sufficient to show that the method works, but as an example, it is far too unrealistic. Most signals are nonstationary, and the noise on its own can be hard to obtain as a separate signal. We therefore consider a speech signal and additive factory noise. The factory noise was obtained from the Noisex corpus and is signal 021. A simple VAD was employed based on smoothed variance [18, 19] (see Appendix). The SNR over the whole sentence was measured as 8.6 dB. This is not that low but a VAD working with a single input cannot possibly work with too low an SNR, as it could filter the speech as well as the noise. The speech was segmented and treated as quasi-stationary over a short interval time-frame of 227 ms. This corresponds to 10 averages of FFTs of length 512 points sampled at 22.05 kHz. This interval can be reduced if averaging of the FFTs is done by overlapping instead of consecutive frames. The clean speech signal and the signal with additive factory noise are shown in Figures 6(a) and (b), respectively. Figure 6(c) shows the smoothed estimate of the signal when a lag of five steps was used. The FIR model order selected was 12.

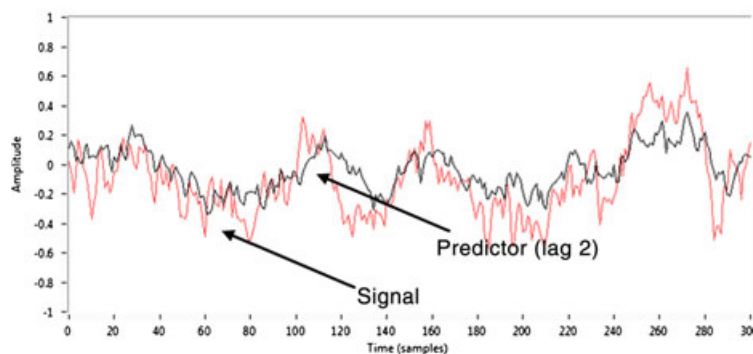
Although the adaptive Wiener filter and smoother works well to reduce the noise, the speech by no means sounded crystal clear. However, it may well be that, for certain applications, the estimated speech is passed to a speech recognition engine for possible isolated word recognition, and the human listener is removed from the process altogether. The periods during the previous noise-alone



(a)



(b)



(c)

Figure 5. (a) Signal and filtered estimate, (b) signal and smoothed ( $d = 5$ ) estimate and (c) signal and predicted ( $d = 2$ ) estimate.

sections of the speech in Figure 6(c) are virtually removed by the estimator in their entirety and have not been set to zero (which could be done since the VAD flags when they occur) as they appear on first sight.

## 7. CONCLUSIONS

A feedback control approach to the spectral factorization problem has been applied to the problems of adaptive filtering, smoothing and prediction of random signals. The algorithms solve the spectral factorization of both the signal plus noise and noise-alone averaged spectra. The usual assumption is made that the noise signal is obtainable as a separate signal. When applied to speech, we use a VAD to estimate periods of non-speech, and this is when the noise-alone spectral factor is updated. The

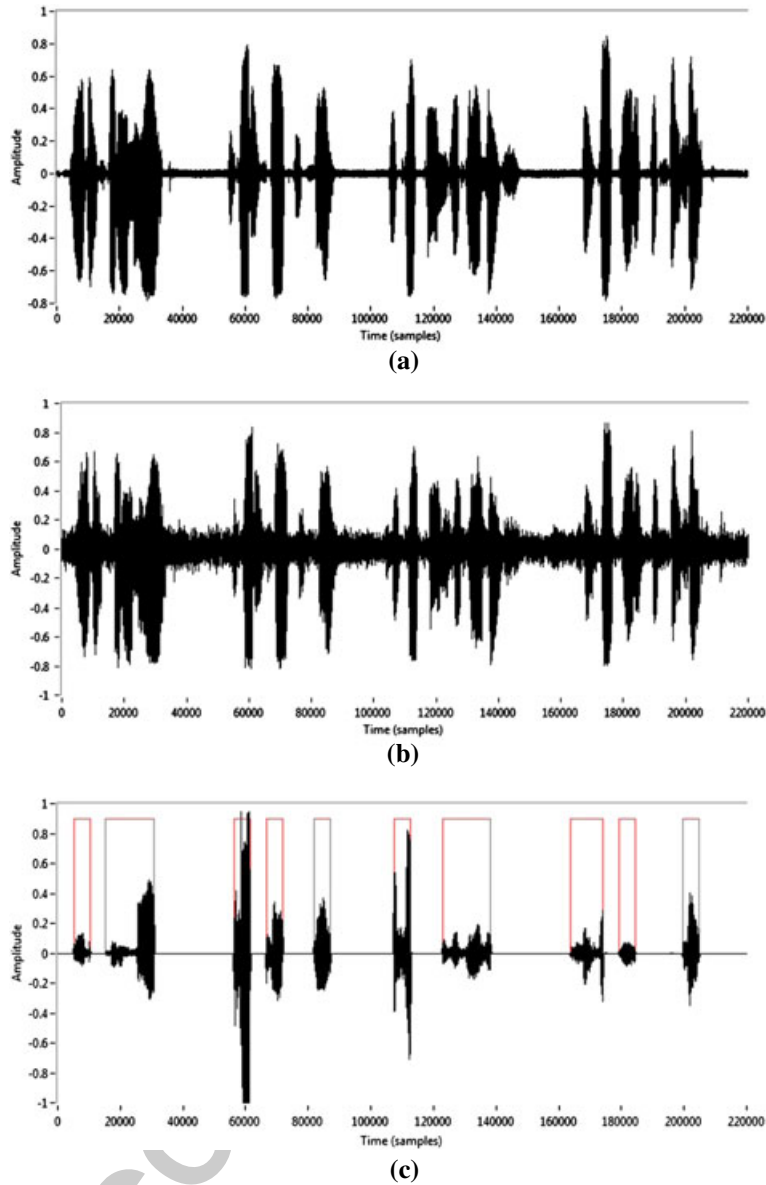


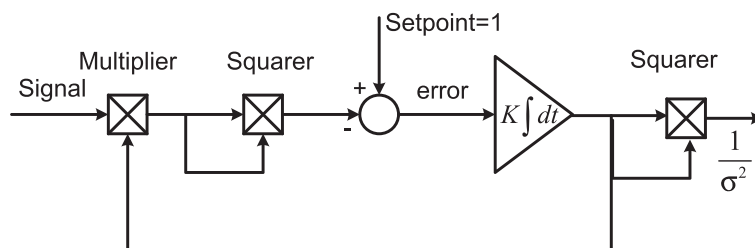
Figure 6. (a) Clean speech signal, (b) signal and additive factory noise (SNR 8.6 dB), (c) smoothed signal estimate and the VAD activity.

adaptive filters are implemented in innovation format and several examples were shown illustrating the effectiveness of the method, which is very much governed by the accuracy of the VAD.

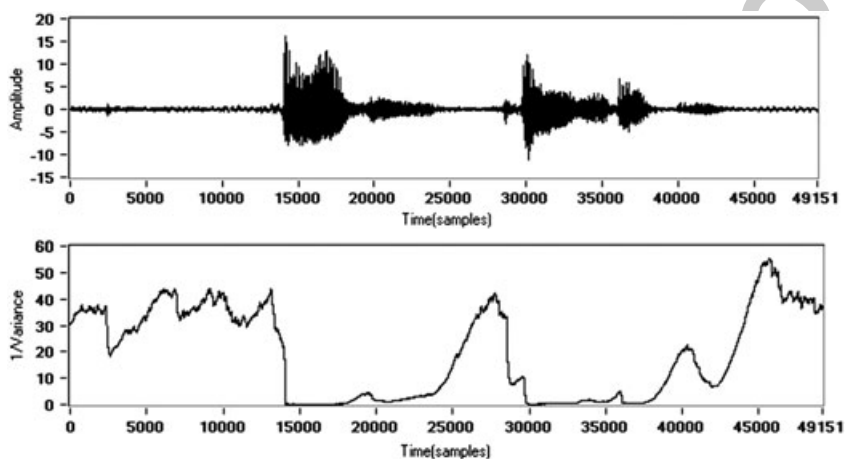
#### APPENDIX A: SIMPLE VOICE ACTIVITY DETECTOR

An estimator for inverse variance has already been discussed elsewhere [18]. The schematic is shown in the diagram next.

The operation is quite simple and relies on a feedback loop with an integrator, which drives the error to zero. It can be shown that for a zero-mean signal at the multiplier input, and provided the gain is not too large, the integrator output will be the smooth reciprocal rms (or standard deviation) of the signal. Squaring this value gives us reciprocal variance or reciprocal power. Because real signals always have some amount of additive noise, a division by zero is rarely, if ever, encountered



09 **Q5** Figure A.1. Inverse variance estimator.



26 Figure A.2. Clean speech signal and reciprocal variance output.

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30 when the signal value is low. For example, a typical speech signal and the reciprocal variance output is shown in Figure A.2 next.

31  
32 It can be seen that the control loop can easily be used as a voice-absence, and hence voice activity detector (VAD) by using a threshold to determine where speech or noise begins. Of course, the disadvantage of this kind of simple VAD is that it cannot work on SNRs that are too low as the distinguishing features of signal and noise are not so apparent. The method is sufficient, however, to illustrate the basic function of a VAD and can be quite accurate and smooth.

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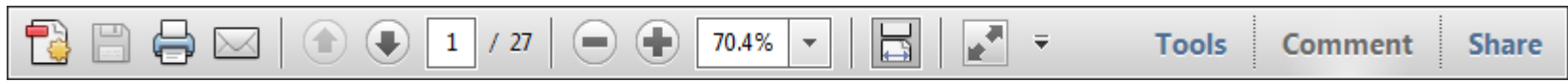
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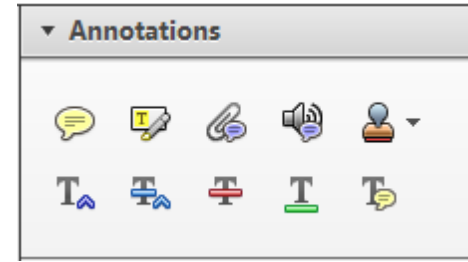
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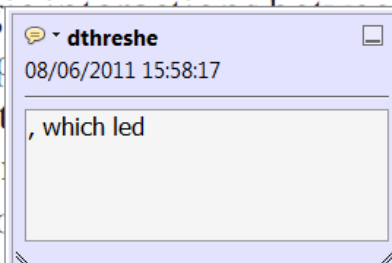


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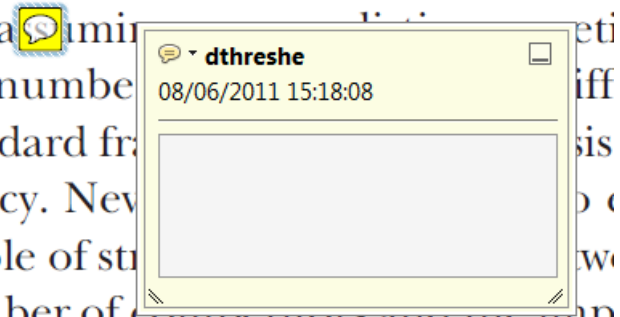


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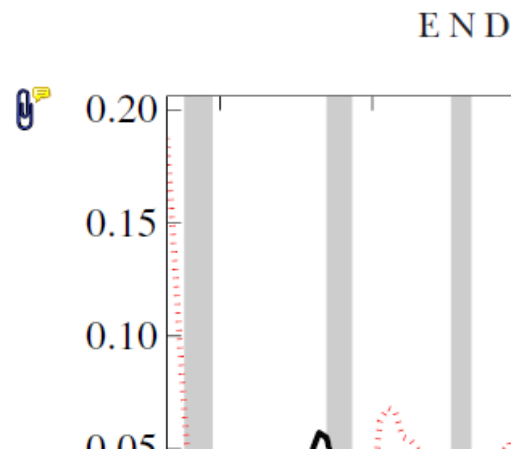
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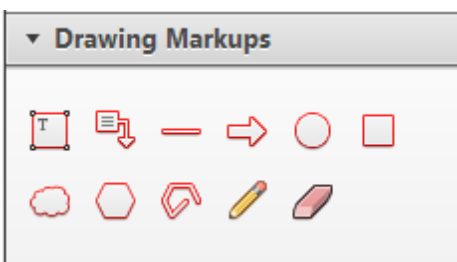


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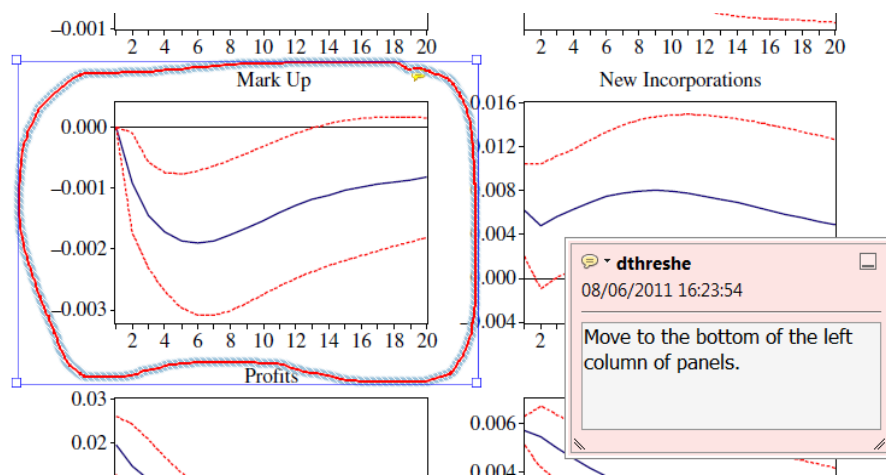


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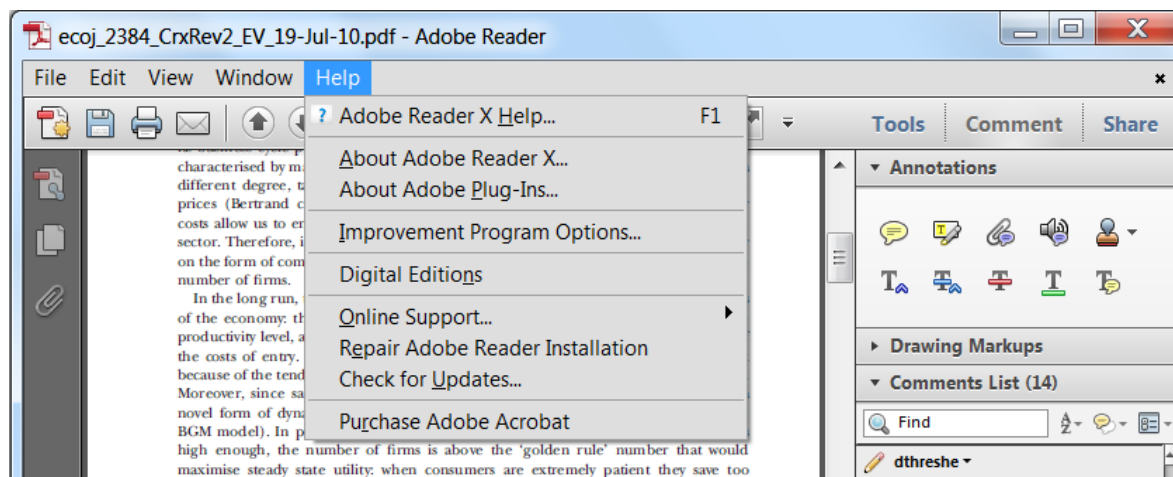
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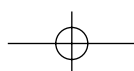
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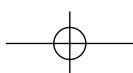
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