Markov and Semi-Markov Processes & Related Fields 2011

The role of Kemeny's constant in properties of Markov chains

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Abstract:

In a finite m-state irreducible Markov chain with stationary probabilities $\{\pi_i\}$ and mean first passage times m_{ij} (mean recurrence time when $i = j$) it was first shown, by Kemeny and Snell (1960) that $\sum_{j=1}^{m} \pi_j m_{ij}$ is a constant, K, not depending on i. This constant has since become known as Kemney's constant, (Grinstead and Snell (1997). We consider a variety of techniques for finding expressions for K, derive some bounds for K, and explore various applications and interpretations of this result. Interpretations include the expected number of links that a surfer on the World Wide Web, located on a random page needs to follow before reaching a desired location (Levene and Loizou (2002), as well as the expected time to mixing in a Markov chain (Hunter (2006)). Various applications have been considered including some perturbation results (Hunter (2006), Catral, Kirkland, Neumann and Sze (2010)), mixing on directed graphs (Kirkland (2010)), and its relation to the Kirchoff index of regular graphs (Palacois (2010), Palacois and Renom (2010)). In a further extension,it is shown that typically the variances of the mixing times depend on i, (Hunter (2008)).

Keywords:

Markov chains,, Kemeny's constant, mixing times, perturbations, regular graphs

References:

M.Catral, S.J. Kirkland, M. Neumann and N.-S.Sze (2010) The Kemeny Constant for Finite Homogeneous Ergodic Markov Chains. *Journal of Scientific Computing*, 45, pp. 151-166.

C.M. Grinstead, J.L. Snell (1997), Introduction to Probability. American Mathematical Society, Providence.

J. J. Hunter (2006) Mixing times with applications to perturbed Markov chains. *Linear Algebra Applications*, 417, pp. 108-123.

J.J. Hunter (2008) Variances of First Passage Times in a Markov chain with applications to Mixing Times. *Linear Algebra Applications*, 429, pp. 1135-1162.

J. G. Kemeny, J.L. Snell (1960) Finite Markov Chains. Van Nostrand, New York.

S.J. Kirkland (2010) Fastest expected time to mixing for a Markov chain on a directed graph. *Linear Algebra Applications*, 433, pp. 1988-1996.

M. Levene and G. Loizou (2002) Kemeny's constant and the random surfer. *The American Mathematical Monthly*, 109, pp. 741-745.

J.L. Palacois (2010) On the Kirchhoff index of regular graphs. *International Journal of Quantum Chemistry*, 110, pp. 1307-1309.

J.L. Palacois and J. M. Renom (2010) Bounds for the Kirchhoff index of regular graphs via the spectra of their random walks. *International Journal of Quantum Chemistry*, 110, pp. 1637- 1641.