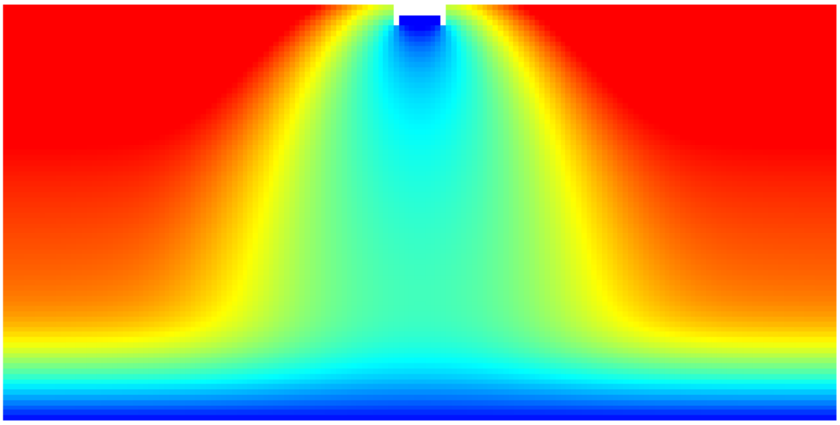
Modelling leakage from perched rivers using the unsaturated flow model VS2DTI

Groundwater Systems & Water Quality Programme

Internal Report IR/05/019



BRITISH GEOLOGICAL SURVEY

GROUNDWATER SYSTEMS & WATER QUALITY PROGRAMME

INTERNAL REPORT IR/05/019

Modelling leakage from perched rivers using the unsaturated flow model VS2DTI

C Jackson

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Summary

The objectives of the modelling work are two-fold. The first aim is to investigate and verify the adequacy of the relationship between water table elevation and river leakage that is used to represent river-aquifer interaction in saturated groundwater flow models, such as MODFLOW McDonald and Harbaugh, 1988). The second objective is to highlight some of the tasks involved in the process of constructing an unsaturated groundwater flow model to contribute to the development of BGS best practice guidelines for numerical modelling.

River-aquifer interaction is usually represented in saturated groundwater flow models by a simple Darcian type leakage mechanism, which depends on the head in the aquifer, the river stage and the hydraulic properties of the river. This representation of the river assumes that the channel is rectangular, that it banks are vertical and impermeable and, that a bed separates the surface water from the aquifer. This relationship between groundwater head and leakage from a river in a coupled surface water-groundwater system, which is implemented in MODFLOW and commonly applied in other saturated groundwater flow model codes, is discussed. To determine the accuracy of this simplified relationship, the unsaturated flow model code VS2DTI (Hsieh et al., 2000) is used to simulate a surface water-groundwater system in detail. The process of developing an unsaturated flow model using this software is described.

The shape of the curve describing the relationship between water table elevation and river leakage, which is included in MODFLOW and other saturated groundwater flow models, is similar to that which is simulated using the unsaturated flow simulation code, VS2DTI. Under perched conditions the leakage from a river reaches a maximum value as the water table is lowered. However, this maximum leakage rate cannot be assumed to occur immediately as the water table falls below the bed of the river. The assumptions that it is only the aquifer medium that governs the rate of flow to the water table and that this occurs through a saturated column with a width that is equivalent to that of the perched river is simplistic. The approaches to modelling perched rivers that are incorporated into saturated groundwater flow models based on these assumptions are likely to be adequate for regional scale models, in which the detailed flow pattern beneath the river is not of interest, but this will not be the case for the simulation of channel scale river-aquifer interaction processes.

The rate of leakage from a perched river is complex and governed by the following features:

- The river stage and water table elevation.
- The vertical hydraulic conductivity of the river bed *and* the aquifer.
- The relationship between pressure head and moisture content and between hydraulic conductivity-moisture content for the porous media.
- The horizontal hydraulic conductivity of the aquifer, which will affect the width of the column of vertically flowing water.
- The width and length of the river channel.
- The thickness of the river bed deposits.
- The shape of the river channel.

Further work is requires to understand the process of river aquifer interaction in different hydrogeological settings. This should involve a literature review of the research undertaken to date and the simulations of different types of river-aquifer systems, for example in which the shape of the river channel is more complex.

1 Introduction

This report has been produced as part of a short numerical modelling investigation into leakage between perched rivers and aquifers using the unsaturated zone simulation code VS2DTI (Hsieh et al., 2000). The objectives of the modelling work were two-fold. The first aim was to investigate and verify the adequacy of the relationship between water table elevation and river leakage that is used to represent river-aquifer interaction in saturated groundwater flow models. This would inform the further development of the groundwater flow models developed by BGS. The second objective was to highlight some of the tasks involved in the process of constructing an unsaturated groundwater flow model, the information from which, would form part of the best practice guidelines for numerical modelling being developed as part of the Digital Geoscience Spatial Model (DGSM) project.

2 Simulation of leakage from rivers

River-aquifer interaction is usually represented in saturated groundwater flow models by a simple Darcian type leakage mechanism, which depends on the head in the aquifer, the river stage and the hydraulic properties of the river. Figure 1 illustrates the conceptual model of a cross-section through a river, which is included in MODFLOW (Macdonald and Harbaugh, 1988), for example. Whilst MODFLOW is only one of many saturated groundwater flow codes, it is the most commonly applied and therefore is used to illustrate how river-aquifer interaction is implemented in numerical models. This representation of the river assumes that the river channel is rectangular, that it banks are vertical and impermeable and, that river bed deposits separate the surface water from the aquifer. The material composing the river bed is assigned a hydraulic conductivity (m day^{-1}) and thickness (m). Consequently, the flow between the river and the aquifer is vertical.

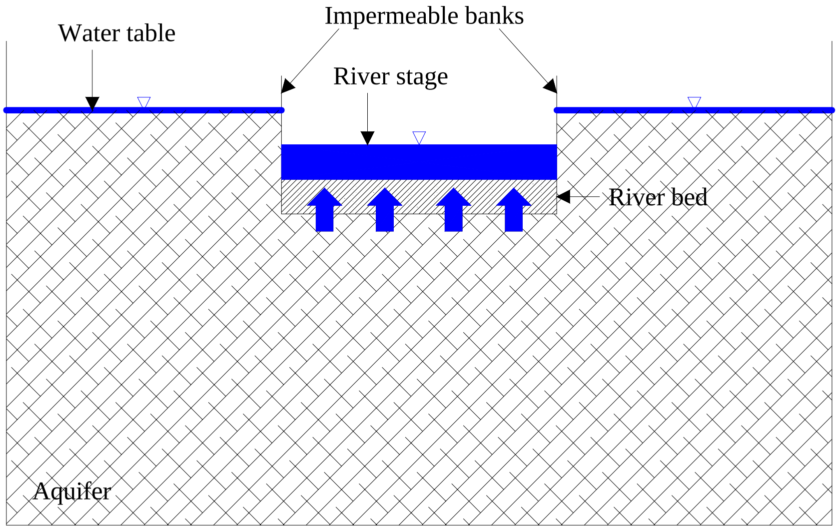


Figure 1 Numerical model conceptualisation of river-aquifer interaction for an influent river with the water table above the river stage

Figure 1 shows the condition in which the elevation of the water table is higher than the river stage. In this case the river receives groundwater from the aquifer at a rate expressed by the Darcian flow equation:

$$Q = K \times W \times L \times \frac{h_{River} - h_{Aquifer}}{B} \qquad \text{(Equation 1)}$$

where:

Q is the flow rate (m^{3}day^{-1}),

K is the hydraulic conductivity of the river bed (m day^{-1}),

W is the width of the river (m),

L is the length of the river reach (m),

B is the thickness of the river bed (m),

h_{River} is the river stage and,

$h_{Aquifer}$ is the head in the aquifer.

For convenience, and because it is difficult to define the thickness of a river bed and its associated hydraulic conductivity, the terms on the right hand side of Equation 1, other than the river stage and head in the aquifer, are combined into a single conductance term, C, with units of m^2day^{-1}. Consequently, Equation 1 becomes:

$$Q = C \times \left(h_{River} - h_{Aquifer}\right) \qquad \text{(Equation 2)}$$

If the head in the aquifer falls below the river stage, the hydraulic gradient is reversed and the river recharges the aquifer. Figure 2 shows the case in which the water table has fallen just below the river stage. Under these conditions, Equation 2 still applies, the linear relationship between head difference and river leakage is maintained and the numerical representation of this interaction process does not need to be modified in the saturated flow model code. This, however, is not the case when the water table falls below the base of the river bed and the river becomes *perched*.

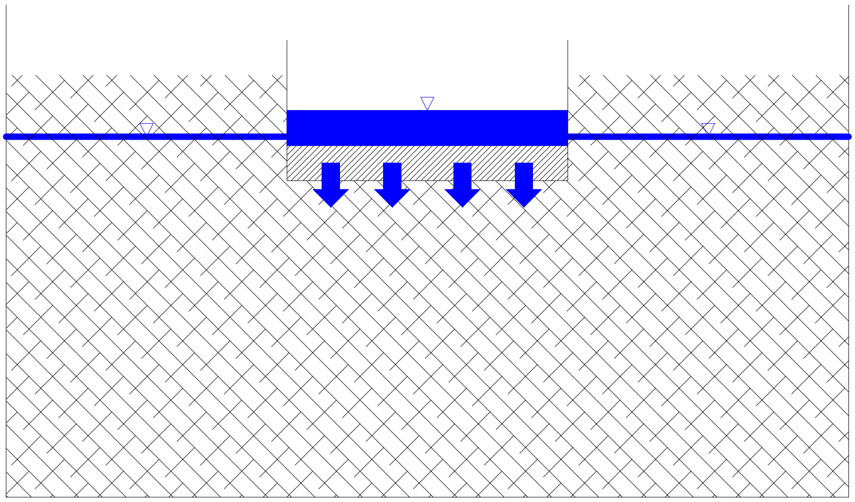


Figure 2 Numerical model conceptualisation of river-aquifer interaction for an effluent river with the water table just below the river stage

Figure 3 shows the situation in which the river has become perched due to the water table falling significantly below the bed of the river. In this case, the column of aquifer beneath the river may have become unsaturated causing the hydraulic conductivity to reduce. In order to determine the leakage from the river under these conditions it is assumed in MODFLOW, as it is in many saturated groundwater flow models, that the vertical hydraulic gradient beneath the river is equal to unity. This assumption is supported by considering the flow condition represented in Figure 4.

Figure 4 shows the condition where the regional water table has fallen far enough below the base of the river so that only a narrow column of saturated aquifer exists beneath the bed of the river. In this case if the water table is assumed to be near vertical in the vicinity of the river, the contours of total groundwater head are near horizontal and consequently, the vertical head gradient is approximately equal to one. Macdonald and Harbaugh (1988) state:

> *"… that further lowering of the regional water table will not increase this gradient. Thus, once a condition similar to that in Figure 4 is established, seepage from the stream is independent of further head decline in the aquifer. The situation shown in Figure 4 is itself an oversimplification of field conditions, which may often involve complex patterns of saturated and unsaturated material beneath the stream. In all situations, however, seepage from the stream must at some point become independent of head in the aquifer, as that head continues to decline."*

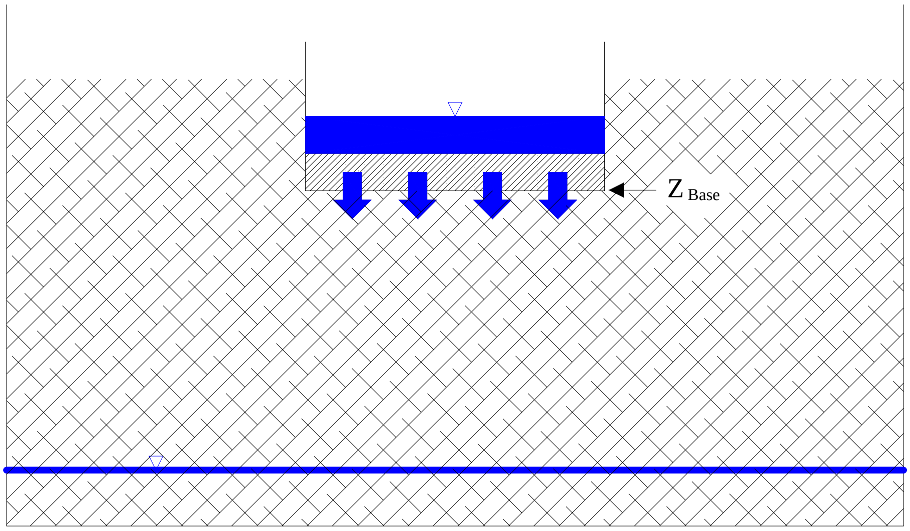


Figure 3 Numerical model conceptualisation of river-aquifer interaction for a perched river with the water table significantly below the river stage

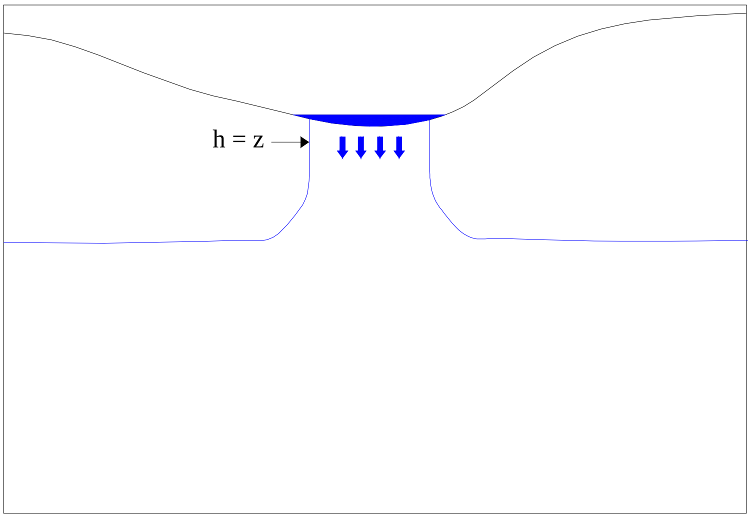


Figure 4 Schematic diagram of assumptions of perched river conditions implemented in MODFLOW

Using this assumption, Macdonald and Harbaugh (1988), derive an approximate Darcian type equation, which describes the leakage from the river under perched conditions. This leakage is quantified and represented in MODFLOW using the following argument:

- The hydraulic gradient beneath the aquifer is equal to unity when the river is perched.
- Consequently, the flow from the river to the aquifer is governed by the hydraulic properties of the *aquifer* beneath the river bed.
- This flow can thus be expressed by $Q_{Limit} = K_{Aquifer} \times W \times L$, where Q_{Limit} is the maximum flow between the river and the aquifer, which occurs under perched conditions, $K_{Aquifer}$ is the vertical hydraulic conductivity of the aquifer beneath the river bed and, W and L are the width and length of the river, respectively.
- To enable the correct representation of this maximum flow in the numerical model, suppose that the difference in head between the river stage and the base of the river bed drives this flow under perched conditions (Figure 3).

- An equivalent conductance, C_{Limit}, of the *river bed* (which applies under perched conditions only) can then be calculated by equating Q_{Limit} and the head difference $(h_{River} - Z_{Base})$.
- Thus, based on Darcy's equation, $C_{Limit} = Q_{Limit} / (h_{River} - Z_{Base})$ (Equation 3)

Equations 2 and 3 are used to represent river-aquifer interaction under different water table elevations in MODFLOW and other saturated groundwater flow models. Equation 2 represents the mechanism when the water table is above the base of the river bed and Equation 3 is used when it is below it. As described above, the conductance terms in the two equations are different and depend on either the river bed or the aquifer hydraulic conductivity.

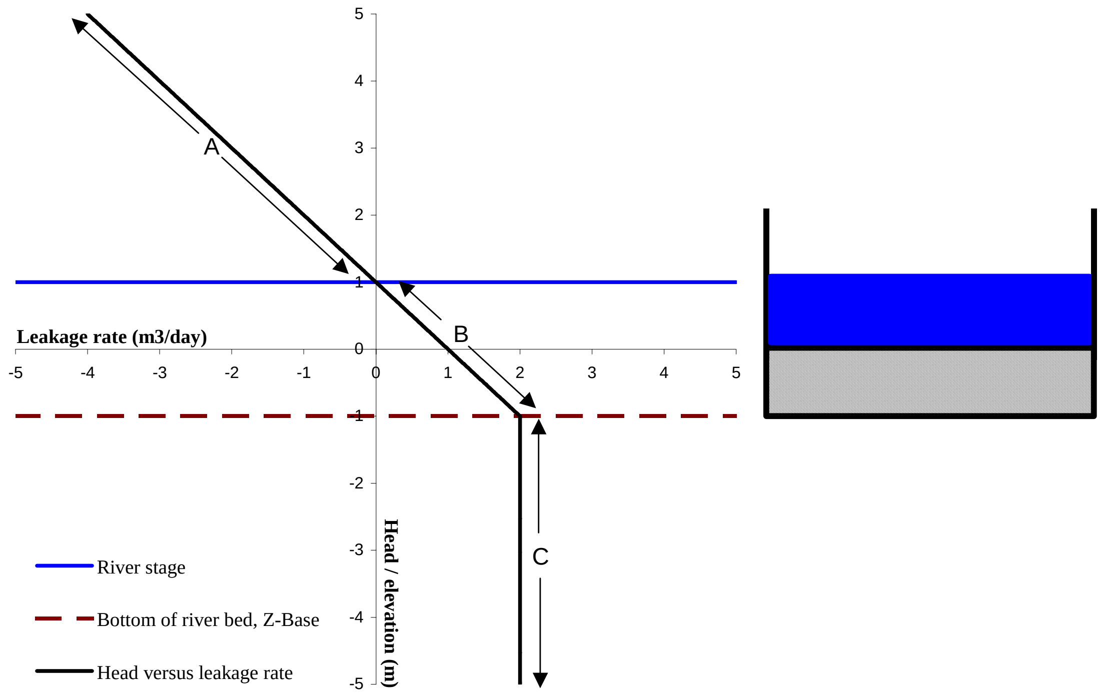


Figure 5 Groundwater head versus river leakage relationship included in MODFLOW

Equations 2 and 3 describe the curve plotted in Figure 5, which shows the relationship between river leakage and water table elevation; in this case the conductance of the river bed is equal to 1 day^{-1}. The three sections, A, B and C, shown in the figure represent the three types of flow condition which can operate in the aquifer. Section A represents the condition when the water table is above the river stage and the river is influent. Section B represents the condition when the elevation of the water table is between the river stage and the base of the river bed and river water leaks to the aquifer. Section C represents the conditions when the river is *perched* and the water table is below the base of the river bed.

In this section the derivation of the relationship between groundwater head and leakage from a river in a coupled surface water-groundwater system has been derived. This simple relationship is implemented in MODFLOW and commonly applied in other saturated groundwater flow model codes. In addition to being included in numerical models, this curve is often presented in the literature e.g. Rushton (2003). Whilst Equation 3 is commonly used to represent perched conditions in numerical models, it incorporates the simplifying assumptions that the hydraulic gradient beneath the river is equal to unity and that the width of the saturated column is equivalent to the width of the river. Though the accuracy of the representation of the flow behaviour beneath the river channel is not likely to be important in regional groundwater models, where the near channel flow pattern is of little interest, if models are to be used to simulate river-aquifer interaction in detail, more accurate numerical models may be required. To determine if this is the true, an assessment of the errors that can be produced when using the simplified relationships discussed above is required. This can be performed by simulating surface water-groundwater systems using unsaturated flow models to produce the groundwater head versus river leakage curve plotted in Figure 5.

The use of an unsaturated flow model to investigate the relationship between river leakage and groundwater head is described in the following sections of this report. This has been undertaken to assess if the shape of the curve shown in Figure 5 is accurate and to examine if the maximum leakage rates that the unsaturated models predict are in agreement with those calculated using Equation 3.

The unsaturated flow model code used to simulate the coupled river-aquifer system is VS2DTI. This is a graphical software package, developed by the USGS (Hsieh et al., 2000) for simulating fluid flow and solute transport in variably saturated porous media. The process of developing an unsaturated flow model using this software is described briefly in the next section.

3 Introduction to VS2DTI

3.1 BACKGROUND

VS2DTI is a Windows based graphical software package for simulating flow and solute transport in unsaturated porous media in one or two dimensions. Strictly, VS2DTI is a graphical pre-processor and post-processor that is used to construct numerical models and analyse the results of simulations. The interface is used to run the simulation code VS2DT (Healy, 1990; Lappala et al., 1987), which is a finite-difference model that solves the Richards (1931) equation for fluid flow and the advection dispersion equation for solute transport. Richards equation is expressed as:

$$\nabla \cdot K(\psi)\,\nabla H = C(\psi)\frac{\partial \psi}{\partial t}$$

where $K(\psi)$ is the unsaturated hydraulic conductivity [LT^{-1}], $C(\psi)$ is the specific moisture capacity [L^{-1}], H is the total head [L] and ψ is the pressure head [L].

The relationships between hydraulic conductivity and moisture content versus pressure head can be described using the methods presented by Brooks and Corey (1964), Haverkamp et al. (1977) or van Genuchten (1980). The solute transport processes that can be simulated include advection, dispersion, first-order decay, adsorption and ion exchange (Hsieh et al., 2000).

3.2 INSTALLING VS2DTI

VS2DTI can be downloaded, at no cost, from the U.S. Geological Survey's web site (http://water.usgs.gov/software/ground_water.html) as part of VS2DI. In addition to VS2DTI, the flow and solute transport package, VS2DI also contains a similar code for simulating flow and energy transport in variably saturated media, VS2DHI. The installation procedure is simple and the software is accompanied by a clear set of help files in html format.

3.3 CONSTRUCTING AN UNSATURATED FLOW MODEL

The VS2DTI pre-processor interface and an example model are shown Figure 6. This shows a model used to simulate the flow through a dam fill. A regular finite difference mesh is superimposed onto the model domain, defined by the near triangular polygon representing the dam.

The total head is specified as 10 m along the left hand model boundary and as 0 m in the bottom right hand corner of the model. The right hand model boundary is represented as a seepage face, the condition along which cannot be defined a priori. During the simulation either a no-flow condition will be specified along this boundary or the pressure head will be set to zero. The hydraulic properties of the dam are shown in Figure 6.

The process of construction of an unsaturated flow model using VS2DTI is similar to that undertaken when developing a saturated flow model. However, because of the relationships between pressure head and moisture content and between moisture content and hydraulic conductivity, some additional parameters have to be defined. The process of constructing an unsaturated model using VS2DTI is described in detail in Appendix 1.

The resulting pressure head profiles at four times during the simulation of the dam are shown in Figure 7. These show the movement of the saturation front through the dam until steady-state conditions are reached. The initial condition is based on an equilibrium profile in which the pressure head is set to the negative of the elevation head.

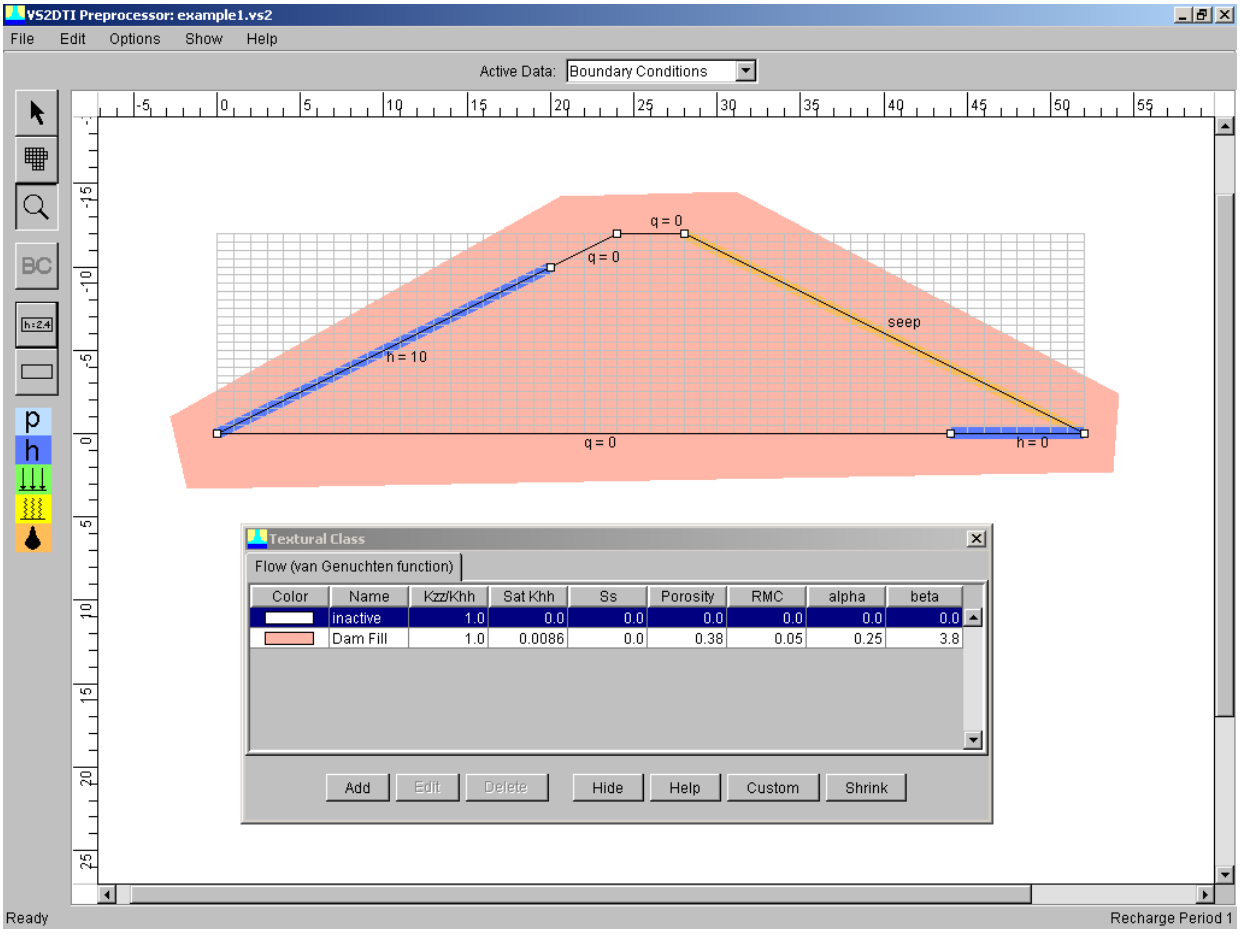


Figure 6 Example of the construction of a VS2DTI model of a dam

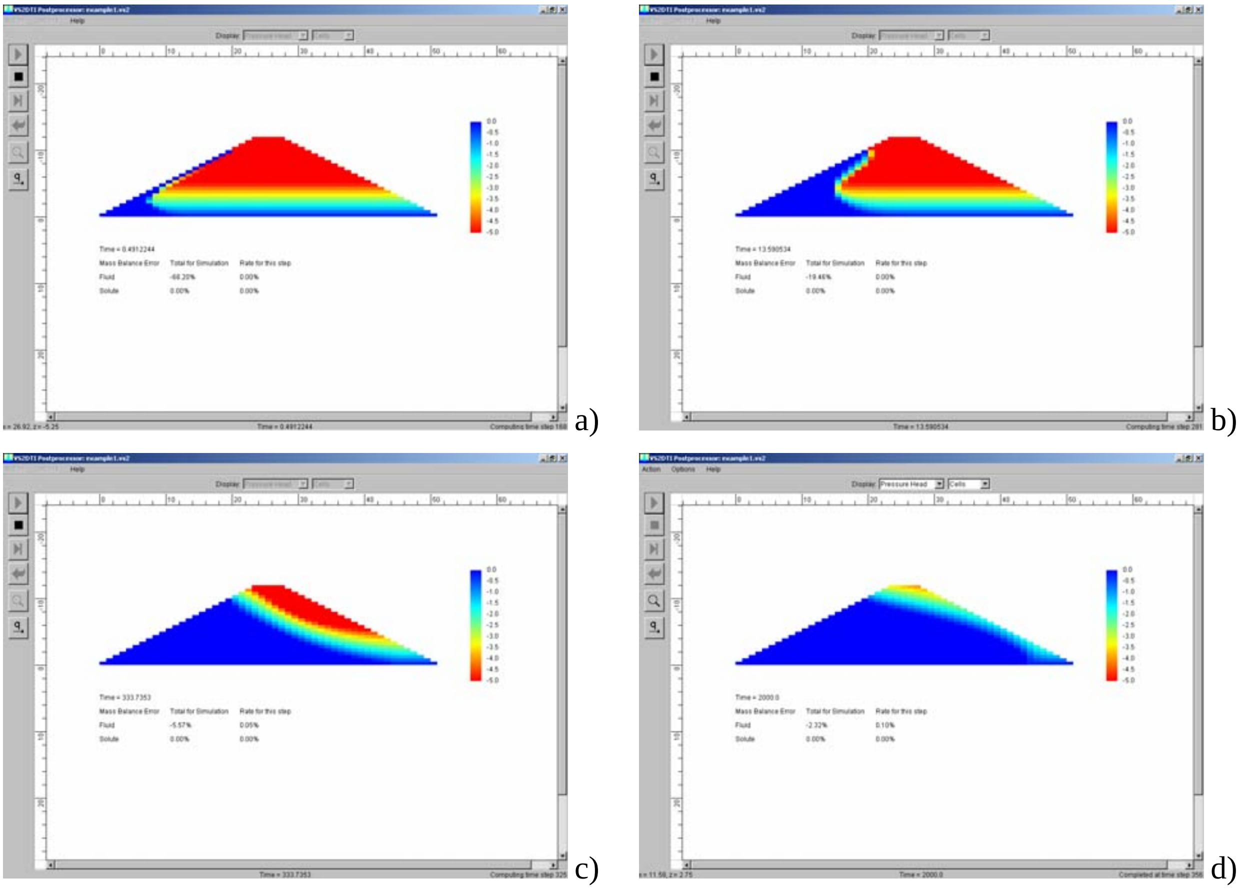


Figure 7 Example VS2DTI post-processor output of simulated pressure head in dam after approximately (a) 0.5 days, (b) 14 days, (c) 334 days and (d) 2000 days

4 Unsaturated zone modelling of perched rivers

4.1 INTRODUCTION

As described in Section 2 one of the objectives of the modelling was to use VS2DTI to derive the relationship between river leakage and the water table elevation to assess if the shape of the curve shown in Figure 5 is accurate. This involved examining if the maximum leakage rates that the unsaturated models predict are in agreement with those calculated using Equation 3, which is commonly applied in saturated groundwater models. This numerical modelling could then be used to quantify the errors involved in the simplistic saturated flow modelling methods, which are derived by assuming that the hydraulic gradient beneath the river is equal to unity and that the width of the saturated aquifer column is equal to the width of the river.

4.2 MODEL STRUCTURE

To fulfil these objectives a number of similar models were built representing cross-sections through a river-aquifer system. The models are split into two groups. In the first group the bed of the river is assigned the same hydraulic properties as the aquifer material, with the saturated hydraulic conductivity set to 1 m day^{-1}. In the second group the hydraulic conductivity of the river bed deposits is one-tenth of that of the aquifer medium for all pressure heads; the saturated hydraulic conductivities of the river bed and aquifer are 0.1 m day^{-1} and 1 m day^{-1}, respectively. In each of the two groups, eight models are built which are identical except for the elevation of their bottom boundary, which represents the water table. Models are constructed with water table elevations of -2, -3, -4, -5, -10, -15, –20 and -25 m. The river stage is set to 0 m in all models.

The structure of one of the models, in which the elevation of the model base (equivalent to the water table elevation) is set to –25 m is shown in Figure 8. All models are 40 m wide and use a regular 0.25 m square mesh. The river is 2 m wide. The bed of the river is represented by two layers of eight cells, which are assigned a saturated hydraulic conductivity of either 1 m day^{-1} or 0.1 m day^{-1} (Figure 9). The porosity is set to 25% throughout the model domain.

Each model simulates an 864,000-second (10-day) period from an initially saturated condition by the end of which steady conditions have been reached. Observation points are defined along a vertical line below the river, at each of which the total head, pressure head and moisture content are recorded during the simulation period. Because there is a limit on the maximum number of observation points that can be used, observation points could not placed at every node in the vertical in the thicker models.

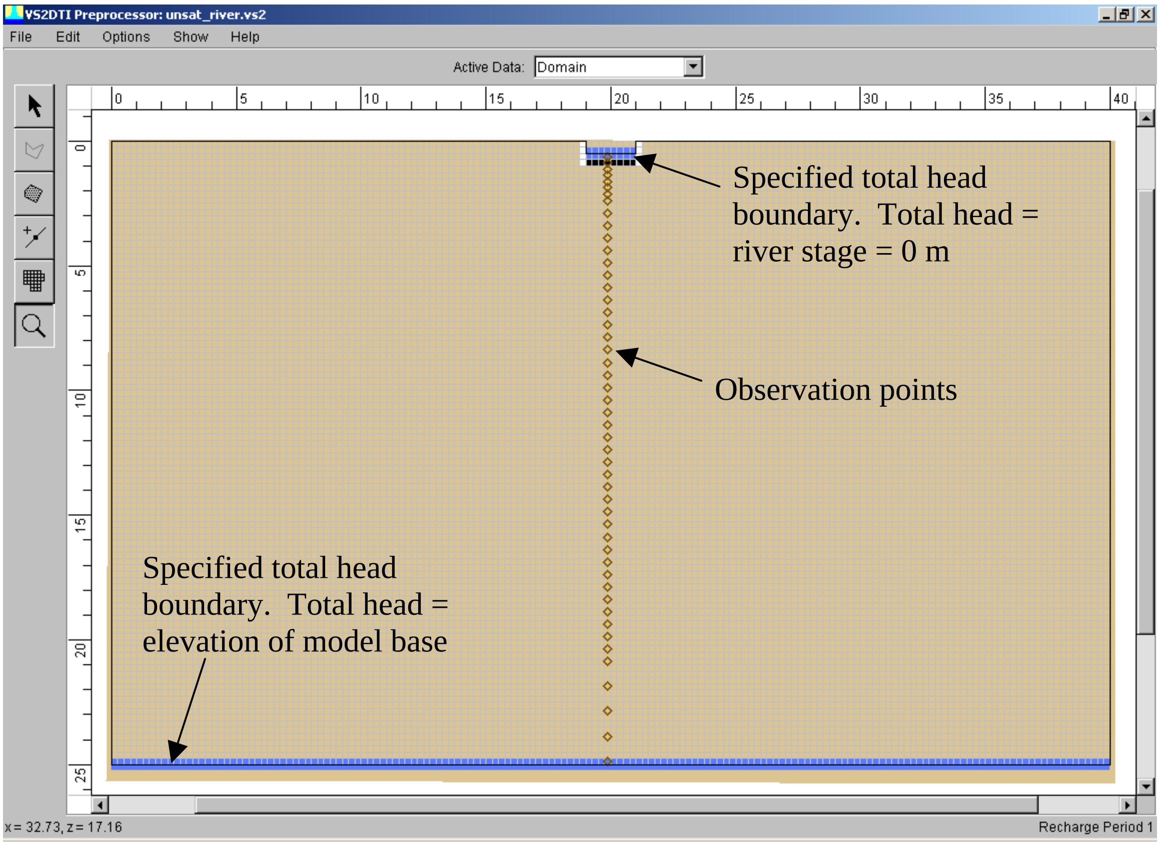


Figure 8 Structure of VS2DTI river model with the water table set at –25 m

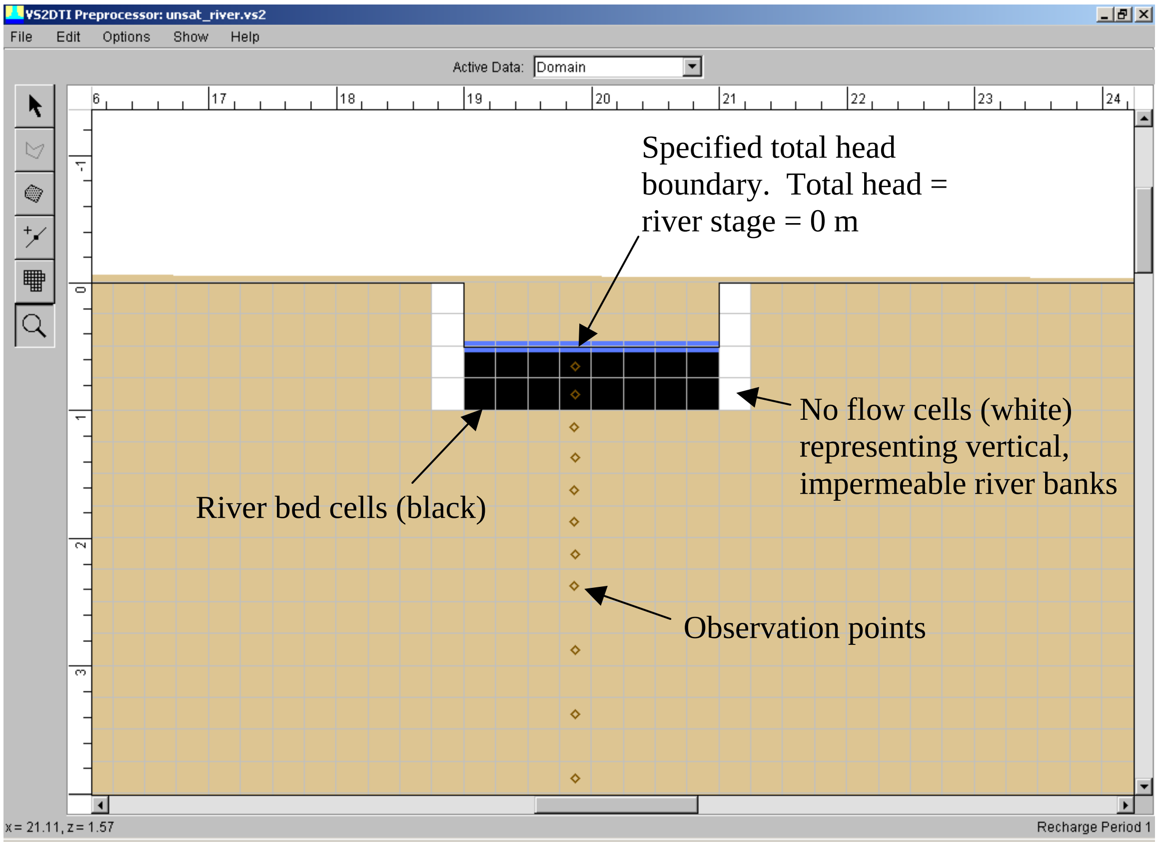


Figure 9 Structure of the VS2DTI model in the vicinity of the river

4.2.1 Porous media hydraulic properties

To define the relationships between pressure head and hydraulic conductivity and between pressure head and saturation, the van Genuchten model is adopted. This model defines the saturation and hydraulic conductivity as functions of the pressure head using the following expressions:

Unsaturated conditions - pressure head, $\psi < 0$:

$$S_e = \frac{1}{\left(1+|\alpha\psi|^{\beta}\right)^{\gamma}} \qquad K_r = \frac{\left(1-C\cdot D^{-\gamma}\right)^2}{D^{\gamma/2}}$$

$$C = |\alpha\psi|^{\beta-1} \qquad D = 1+|\alpha\psi|^{\beta}$$

Saturated conditions - pressure head, $\psi \geq 0$:

$$S_e = 1 \qquad K_r = 1$$

where:

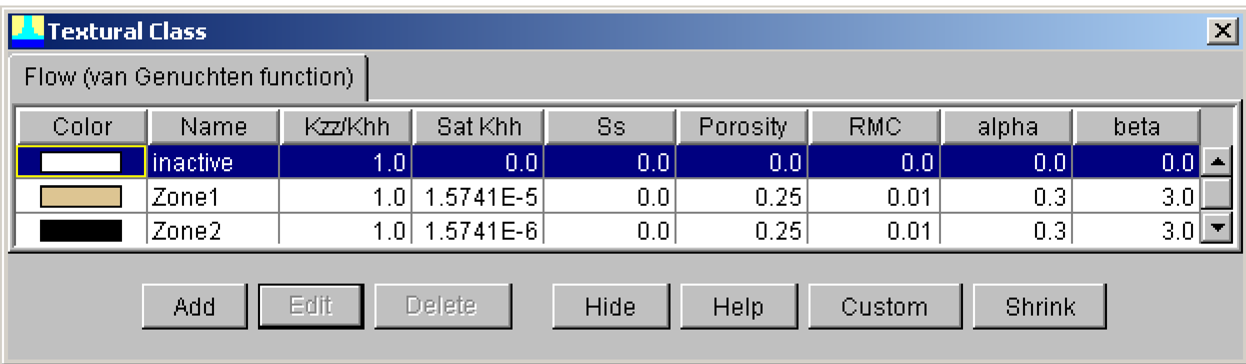
S_e is the effective saturation (a dimensionless parameter),

K_r is the relative hydraulic conductivity (a dimensionless parameter),

$\gamma = 1 - 1/\beta$ and,

α and β are the van Genuchten parameters derived from least squares fitting of the above equations to field data.

Figure 10 shows the van Genuchten parameters used in the models developed in this investigation. In all the river cross-section models the values of α and β are 0.3 and 3.0, respectively, which are appropriate for a sandy medium when compared to values derived empirically from field samples (Stephens, 1996). Values for the van Genuchten parameters have been defined for different porous media but are not representative of hydraulic properties.



Color	Name	Kzz/Khh	Sat Khh	Ss	Porosity	RMC	alpha	beta
	inactive	1.0	0.0	0.0	0.0	0.0	0.0	0.0
	Zone1	1.0	1.5741E-5	0.0	0.25	0.01	0.3	3.0
	Zone2	1.0	1.5741E-6	0.0	0.25	0.01	0.3	3.0

Figure 10 Textural class parameters used to define river bed and aquifer hydraulic properties (Hydraulic conductivity values are defined in m^3/sec)

The relationships between pressure head and its two dependent variables, saturation and relative hydraulic conductivity, specified using the van Genuchten model are shown in Figure 11 for $\alpha = 0.3$ and $\beta = 3.0$. This shows the rapid drop in saturation and hydraulic conductivity with increasingly negative pressure heads. The relationship between relative hydraulic conductivity and saturation specified using these parameters values is shown in Figure 12.

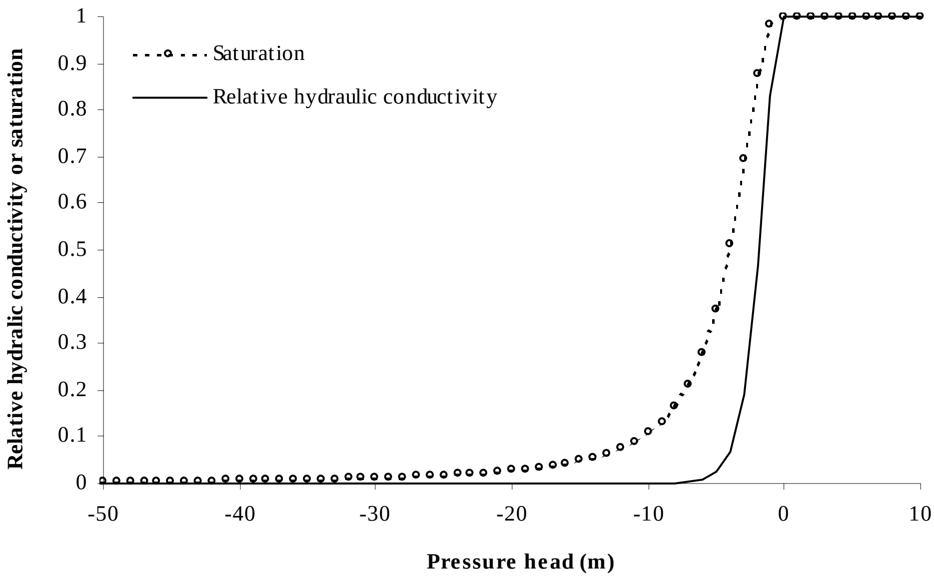


Figure 11 Relationship between (i) pressure head and saturation and (ii) pressure head and relative hydraulic conductivity included in the VS2DTI river cross-section models

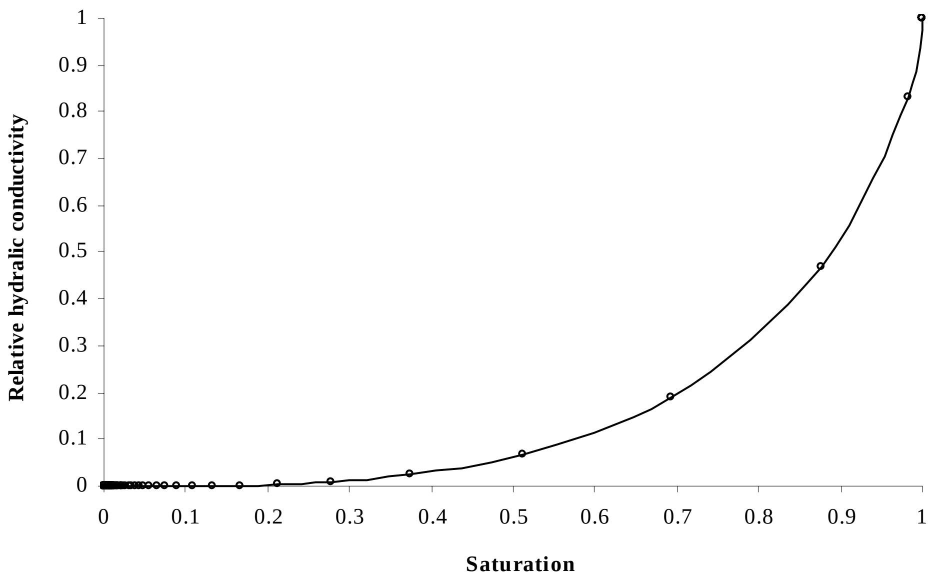


Figure 12 Relationship between saturation and relative hydraulic conductivity included in the VS2DTI river cross-section models

4.3 RESULTS

4.3.1 Simulated steady-state pressure head distributions

Figures 13 to 28 show the pressure head distributions in each of the sixteen models at the end of the ten-day simulation period by which time steady-state conditions have been reached. The dark blue colours represent area of the system where the pressure head is near to zero and the moisture content is near saturation. The red colours represent high suctions (larger negative pressure heads) where the moisture content is near to the residual moisture content for the porous medium. The sixteen figures are grouped into eight pairs, each of which has the same water table elevation. In the first of each pair, the figure relates to the model in which the river bed and aquifer hydraulic conductivity are the same. In the second figure of the pair, the results are shown for the model in which the river bed hydraulic conductivity is one-tenth that of the aquifer. These figures provide little quantitative information but are useful for visualising the flow pattern and pressure head distribution which is similar to the moisture content distribution. The following observations can be made about these sixteen pressure head distributions:

- Pressure heads are higher (less negative) beneath the river when the river bed hydraulic conductivity is assigned the higher value of 1 m day^{-1}.
- Near saturated conditions are maintained just beneath the river in models with the higher bed conductivity.
- The lower river bed hydraulic conductivity results in less leakage to the aquifer for the same water table elevation and consequently, a lower moisture content in the aquifer.
- The bell-shaped pressure head distribution indicates that flow occurs in both the horizontal and vertical direction.
- There is a vertical column, from the river to the water table, which is fully saturated in only two of the models. The aquifer only stays saturated beneath the river in the models in which the river bed deposits have the same hydraulic conductivity as that of the aquifer and in which the elevation of the water table is either –2 m or –3 m.

Water table elevation: –2 m

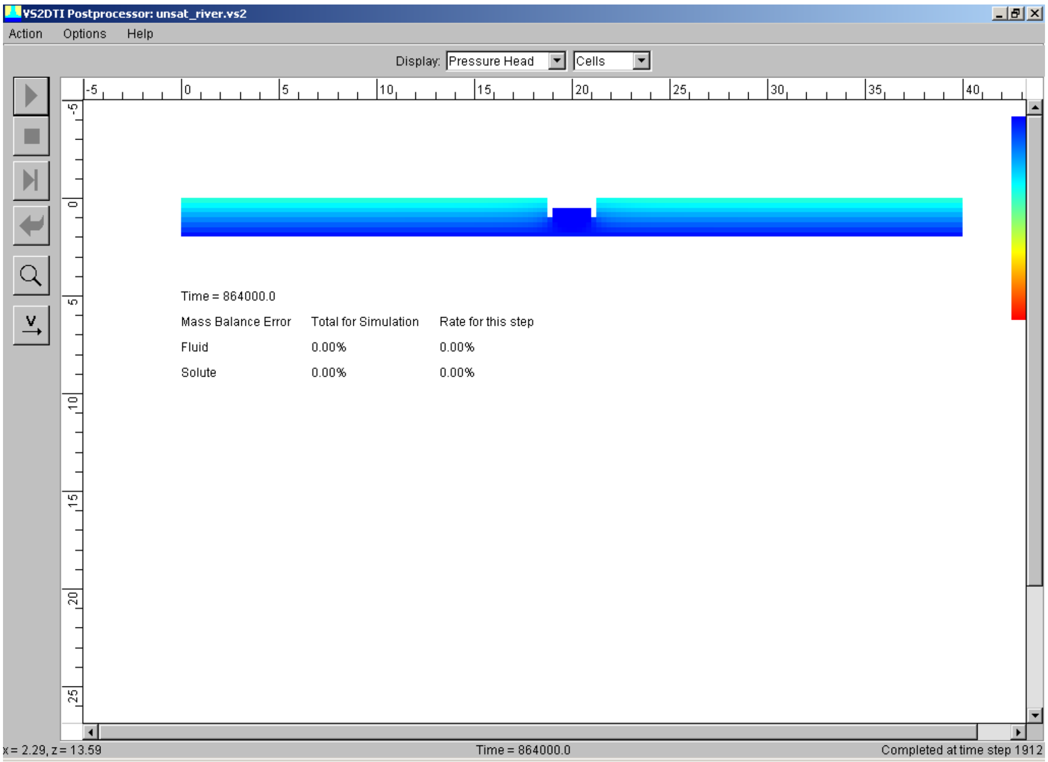


Figure 13 Simulated steady-state pressure head distribution. Water table elevation: –2 m. River bed and aquifer saturated hydraulic conductivity: 1 m day^{-1}

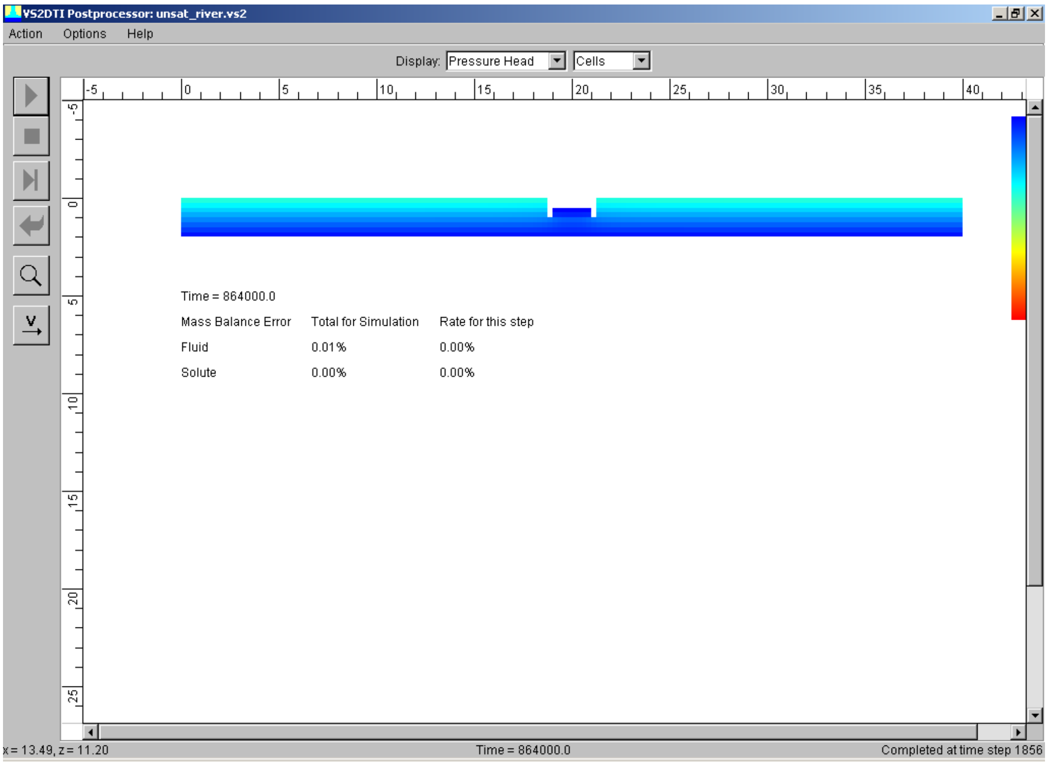


Figure 14 Simulated steady-state pressure head distribution. Water table elevation: –2 m. River bed saturated hydraulic conductivity: 0.1 m day^{-1}

Water table elevation: –3 m

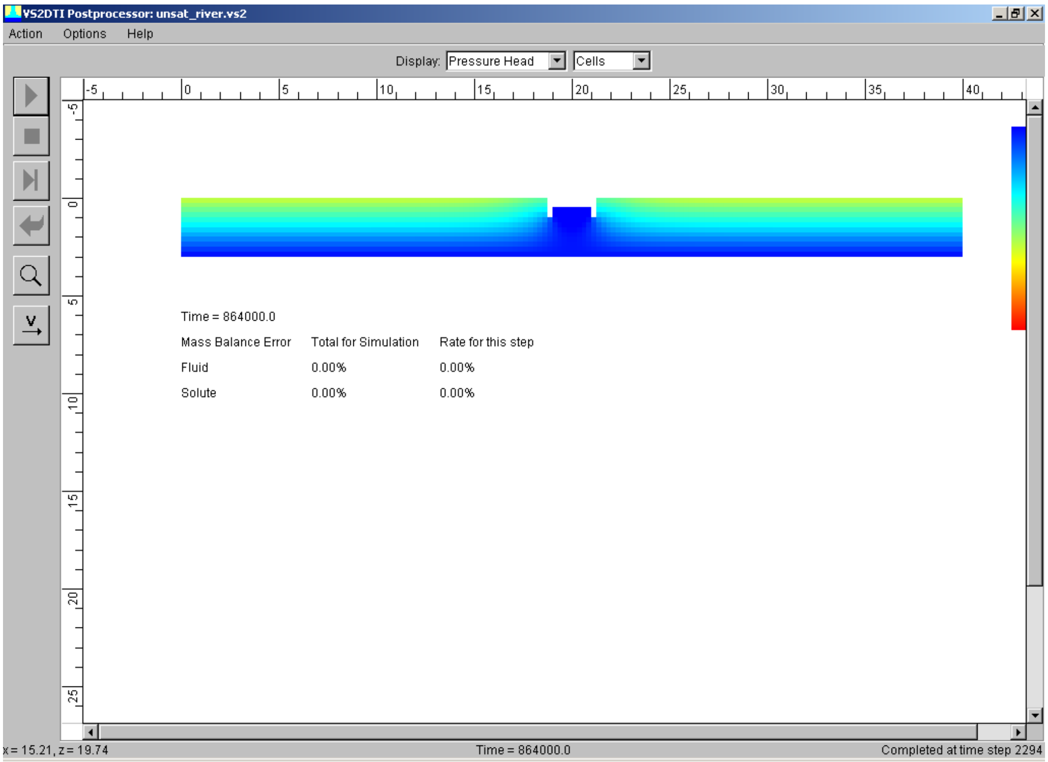


Figure 15 Simulated steady-state pressure head distribution. Water table elevation: –3 m. River bed and aquifer saturated hydraulic conductivity: 1 m day^{-1}

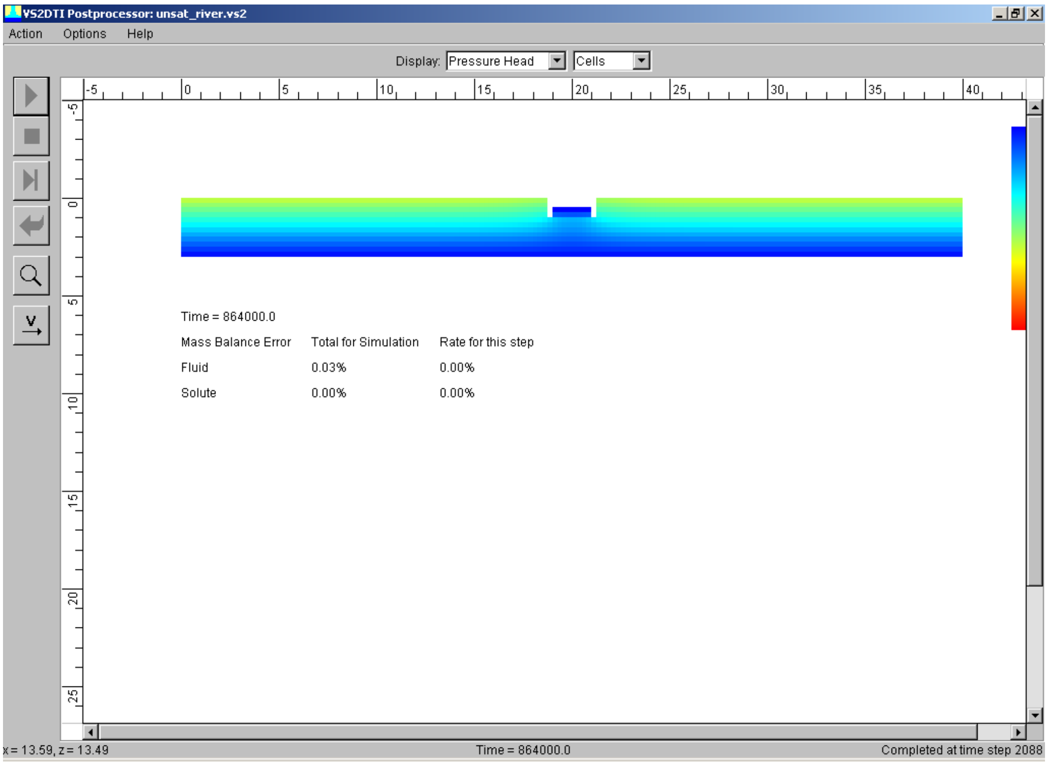


Figure 16 Simulated steady-state pressure head distribution. Water table elevation: –3 m. River bed saturated hydraulic conductivity: 0.1 m day^{-1}

Water table elevation: –4 m

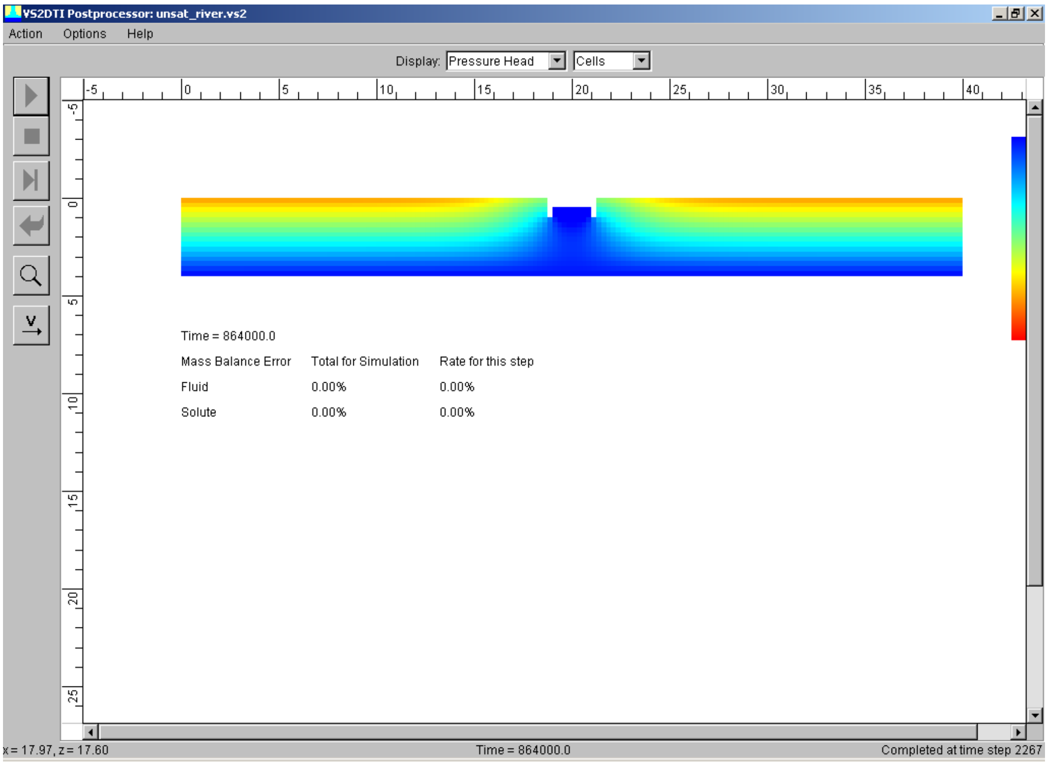


Figure 17 Simulated steady-state pressure head distribution. Water table elevation: –4 m. River bed and aquifer saturated hydraulic conductivity: 1 m day^{-1}

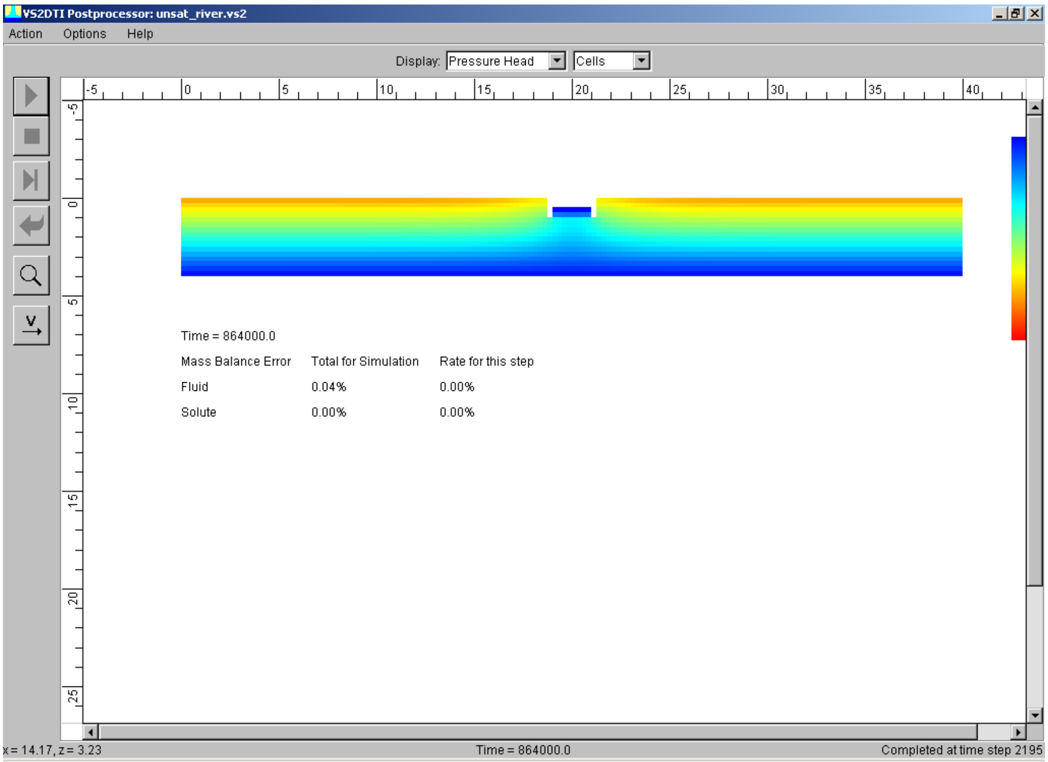


Figure 18 Simulated steady-state pressure head distribution. Water table elevation: –4 m. River bed saturated hydraulic conductivity: 0.1 m day^{-1}

Water table elevation: –5 m

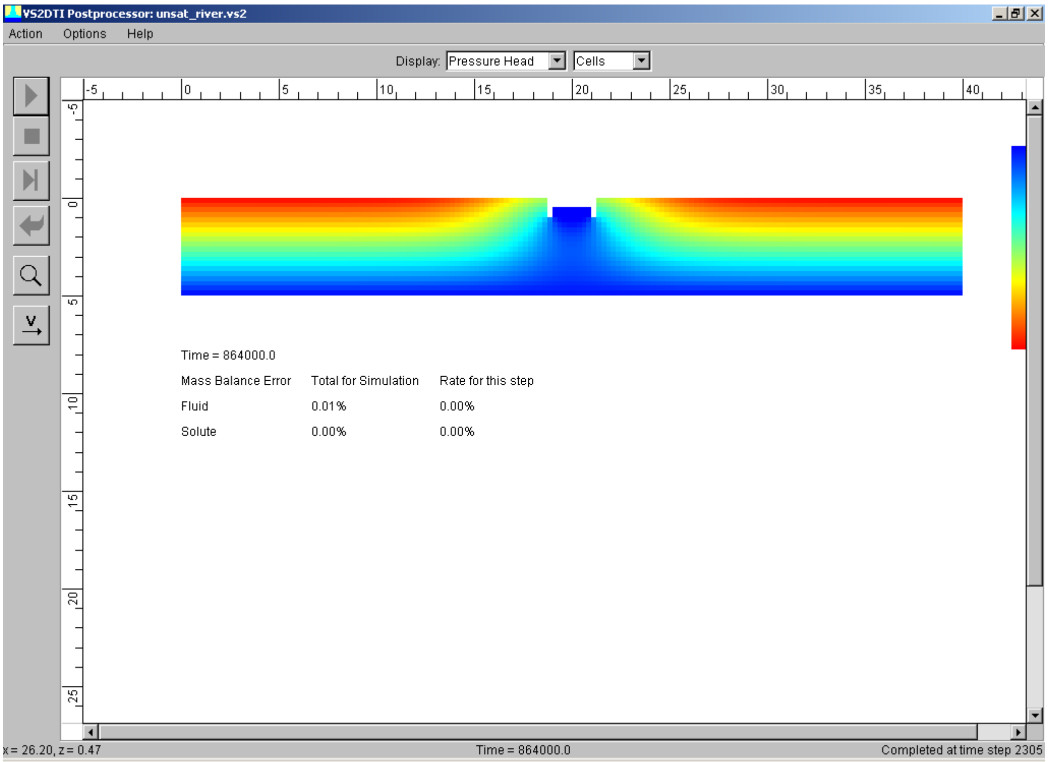


Figure 19 Simulated steady-state pressure head distribution. Water table elevation: –5 m. River bed and aquifer saturated hydraulic conductivity: 1 m day^{-1}

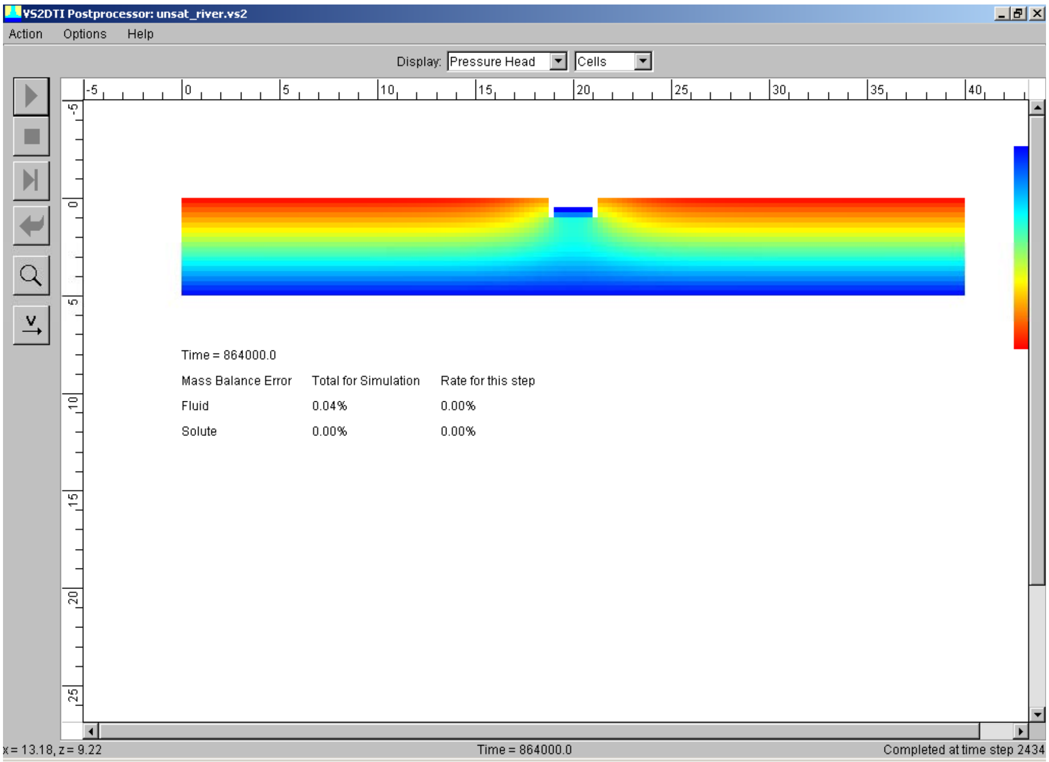


Figure 20 Simulated steady-state pressure head distribution. Water table elevation: –5 m. River bed saturated hydraulic conductivity: 0.1 m day^{-1}

Water table elevation: –10 m

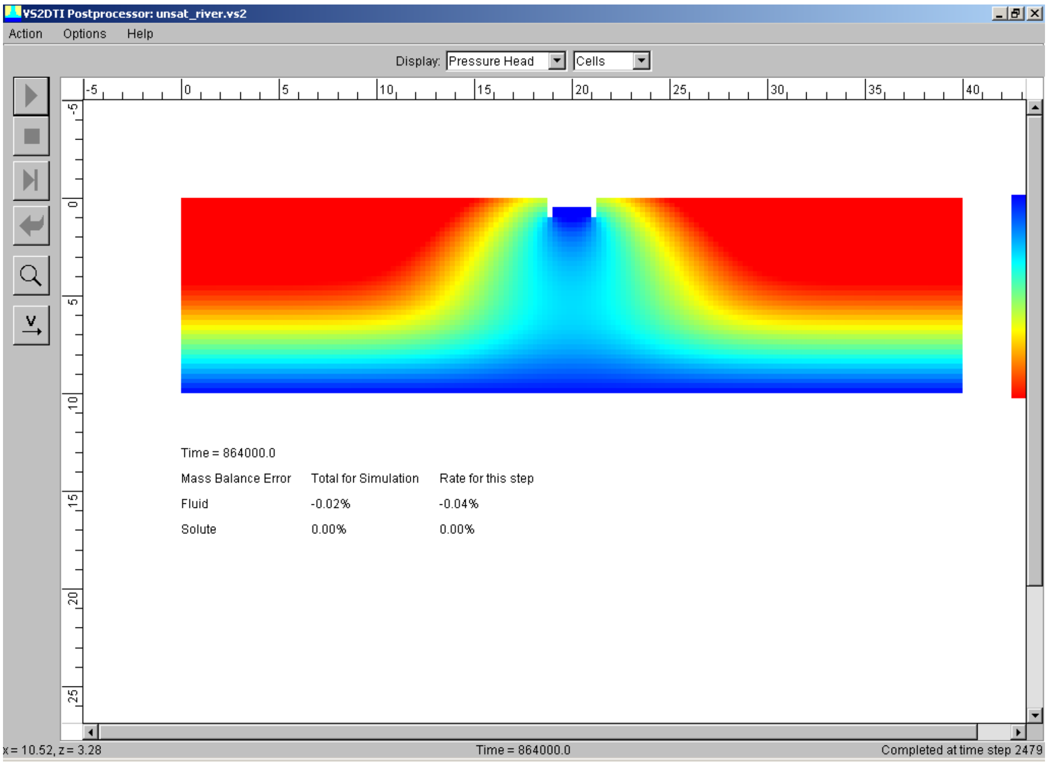


Figure 21 Simulated steady-state pressure head distribution. Water table elevation: –10 m. River bed and aquifer saturated hydraulic conductivity: 1 m day^{-1}

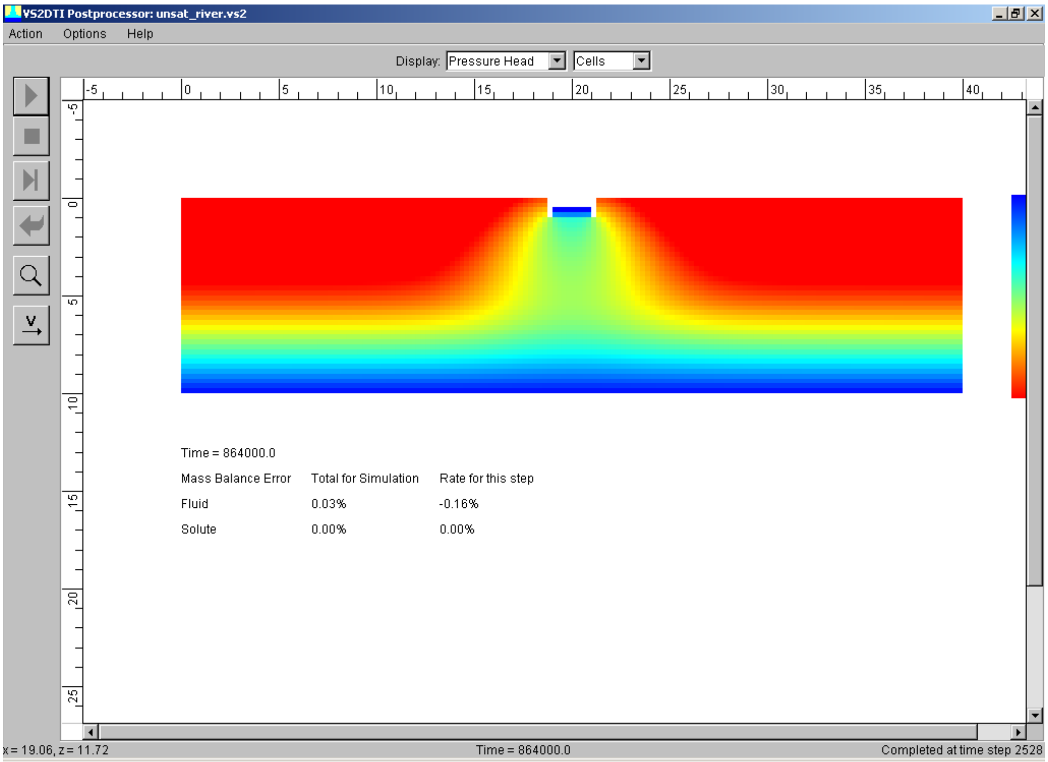


Figure 22 Simulated steady-state pressure head distribution. Water table elevation: –10 m. River bed saturated hydraulic conductivity: 0.1 m day^{-1}

Water table elevation: –15 m

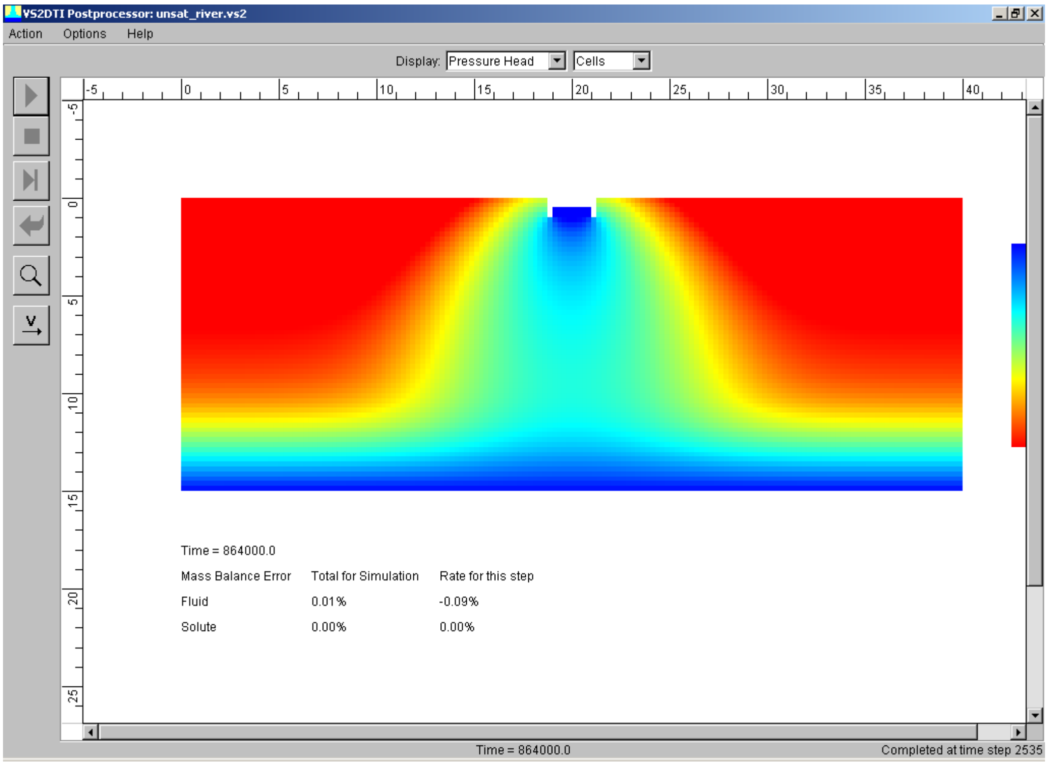


Figure 23 Simulated steady-state pressure head distribution. Water table elevation: –15 m. River bed and aquifer saturated hydraulic conductivity: 1 m day^{-1}

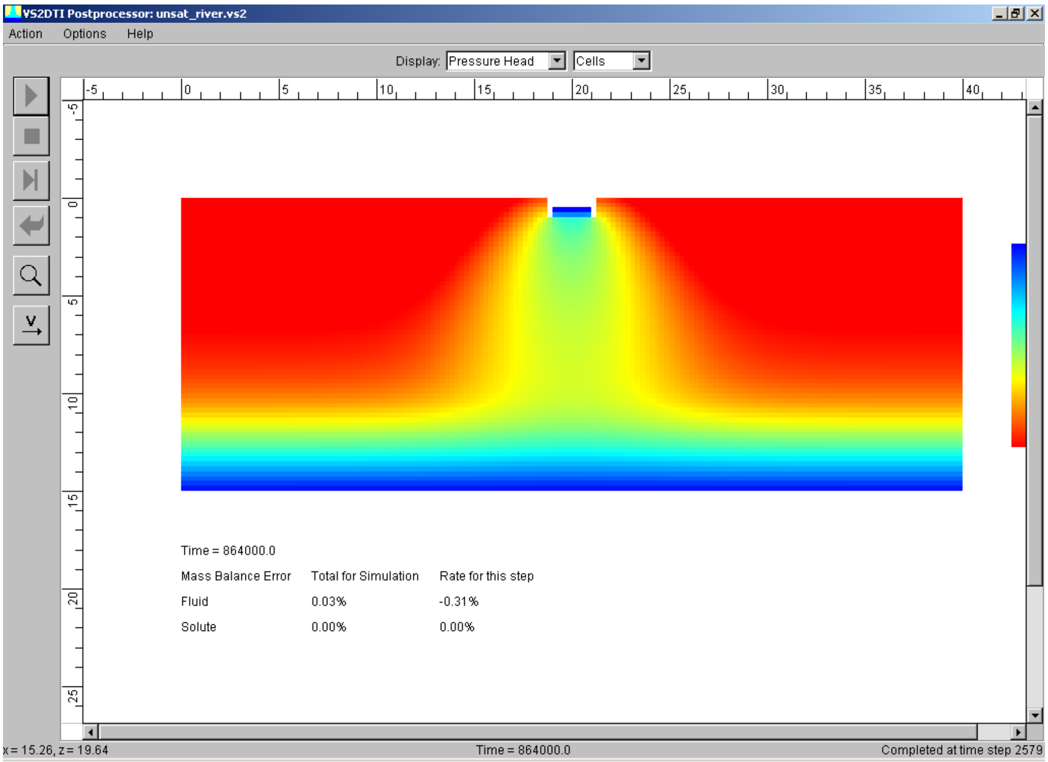


Figure 24 Simulated steady-state pressure head distribution. Water table elevation: –15 m. River bed saturated hydraulic conductivity: 0.1 m day^{-1}

Water table elevation: –20 m

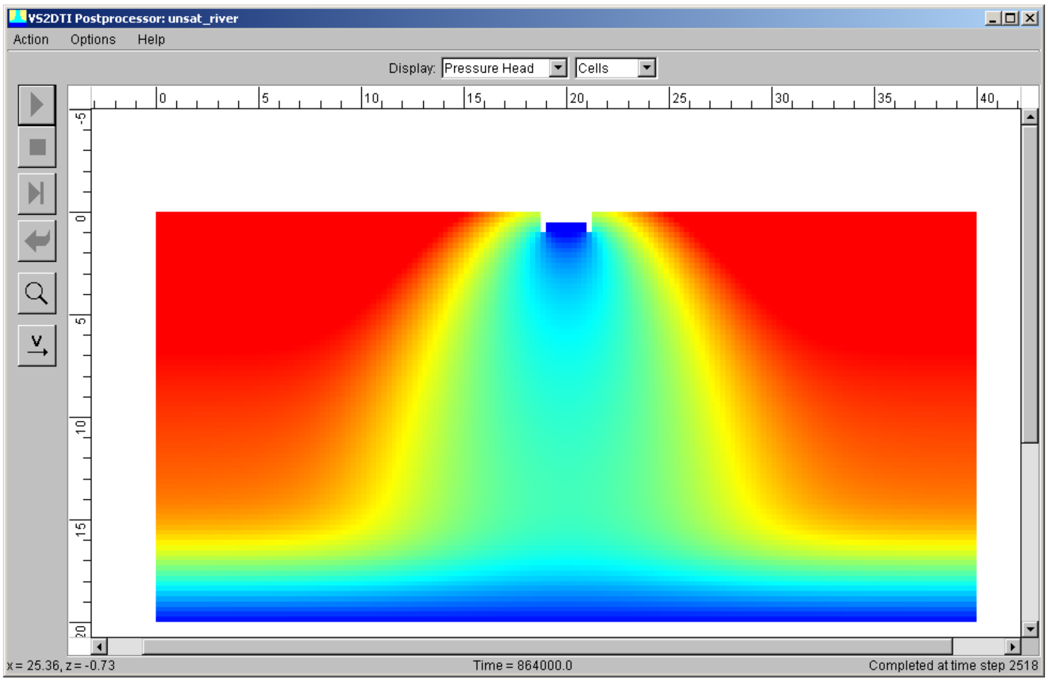


Figure 25 Simulated steady-state pressure head distribution. Water table elevation: –20 m. River bed and aquifer saturated hydraulic conductivity: 1 m day^{-1}

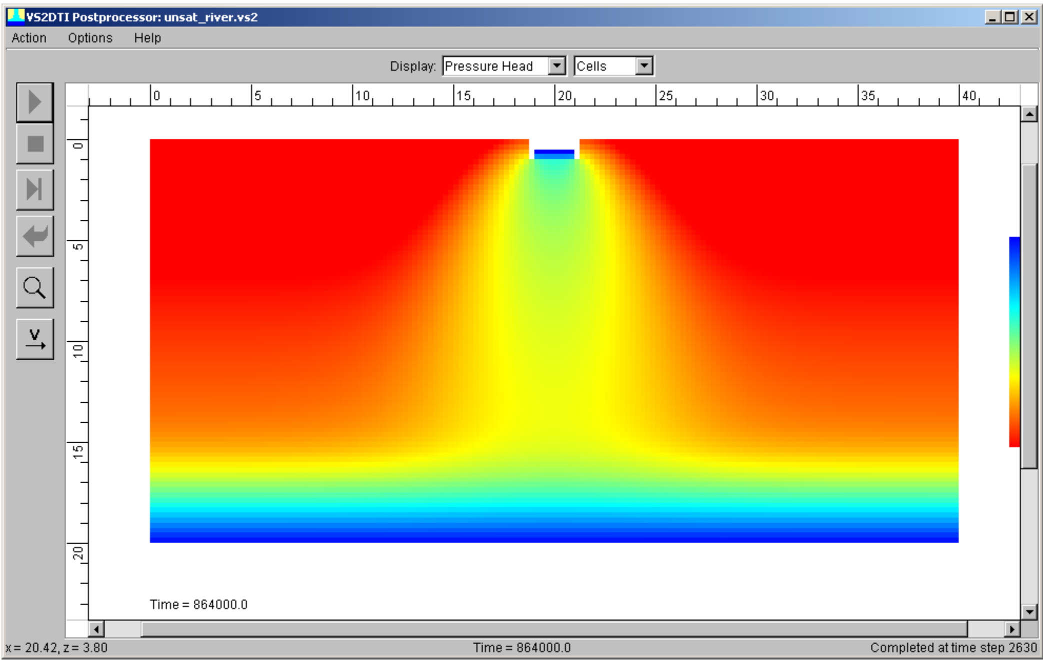


Figure 26 Simulated steady-state pressure head distribution. Water table elevation: –20 m. River bed saturated hydraulic conductivity: 0.1 m day^{-1}

Water table elevation: –25 m

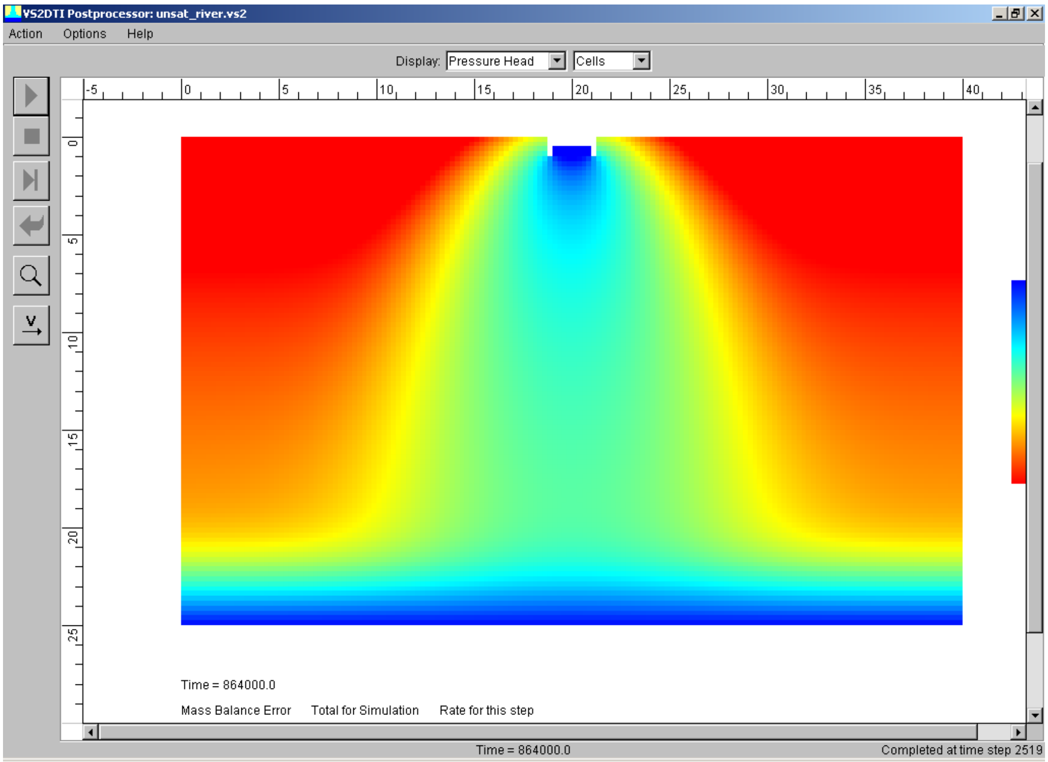


Figure 27 Simulated steady-state pressure head distribution. Water table elevation: –25 m. River bed and aquifer saturated hydraulic conductivity: 1 m day^{-1}

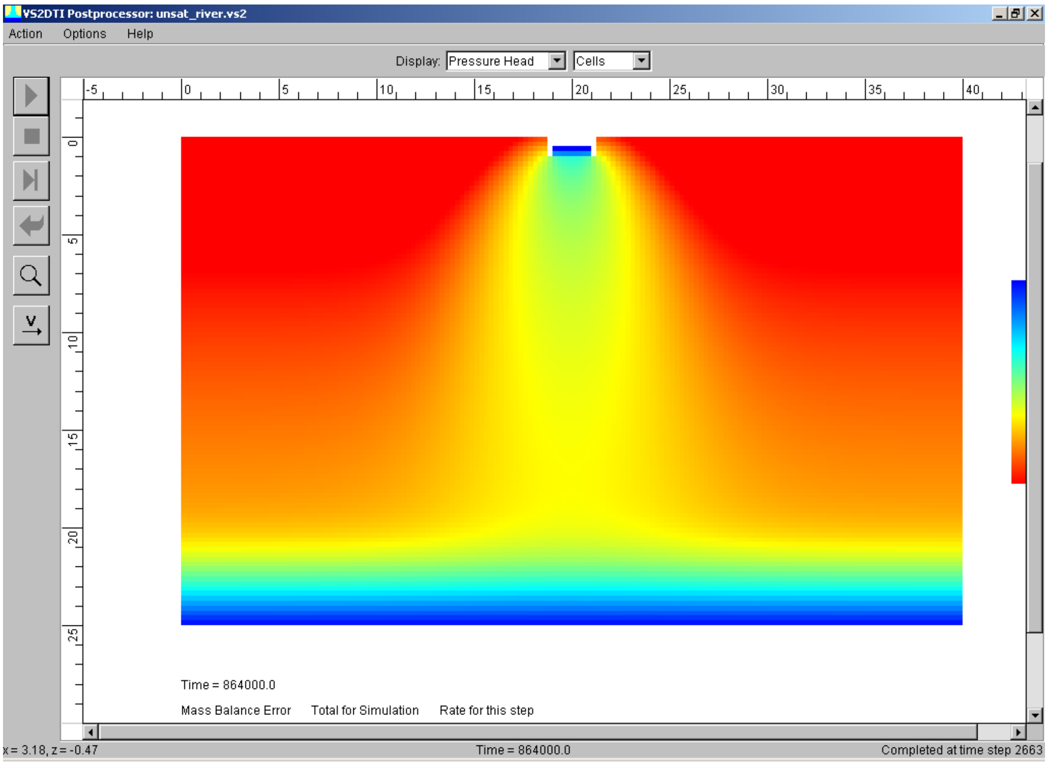


Figure 28 Simulated steady-state pressure head distribution. Water table elevation: –25 m. River bed saturated hydraulic conductivity: 0.1 m day^{-1}

4.3.2 Vertical variation of total head

In Figure 29 and 30 the total head is plotted at each of the nodes located on the vertical line directly beneath the centre of the river. The exact points at which aquifer state variables were monitored during each of the simulations are shown in Figure 8. The total heads are plotted at the end of the 10-day simulation period when steady conditions have been reached.

In Figure 29 the total head is plotted for the each of the eight models in which the river bed hydraulic conductivity is the same as that of the aquifer. In Figure 30 the total head is plotted for the each of the eight models in which the river bed hydraulic conductivity is one-tenth of that of the aquifer material. In both the figures the curve is plotted for each of the simulations with different water table (w.t.) elevations. The following observations can be made from these two figures:

- The total head reduces with depth beneath the river and flow is vertically downwards.
- The total head is equal to the river stage at the top of the profile and equal to the water table elevation at the bottom of each profile.
- Towards the top of each profile the rate of decrease of total head is greater than 1 m per metre depth. This indicates that pressure head decreases (becomes more negative) and moisture content reduces just below the river bed.
- Towards the bottom of each profile the rate of decrease of total head is less than 1 m per metre depth. This indicates that pressure head increases (becomes less negative) and moisture content rises near the water.
- The curvature of the total head versus elevation plots at the top and bottom of the profiles is greater in the models in which the river bed hydraulic conductivity is one-tenth of that of the aquifer material.
- In the models with lower water table elevations, parts of the central sections of each of the curves are straight lines or near straight lines (where the total head gradient is equal to one).

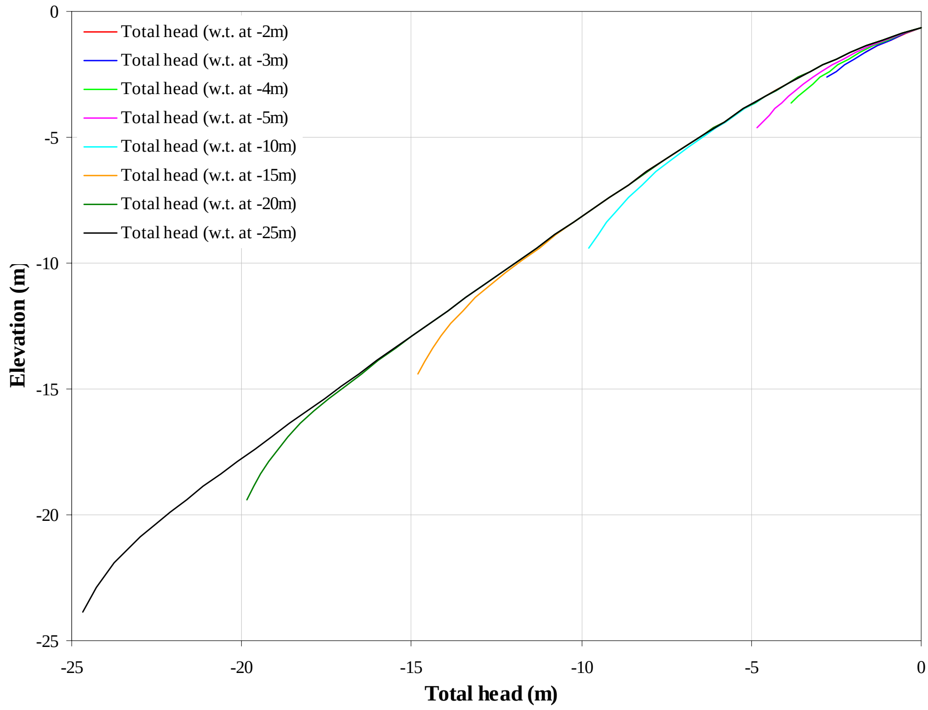


Figure 29 Vertical variation of total head with elevation beneath the river. River bed and aquifer saturated hydraulic conductivity: 1 m day^{-1}

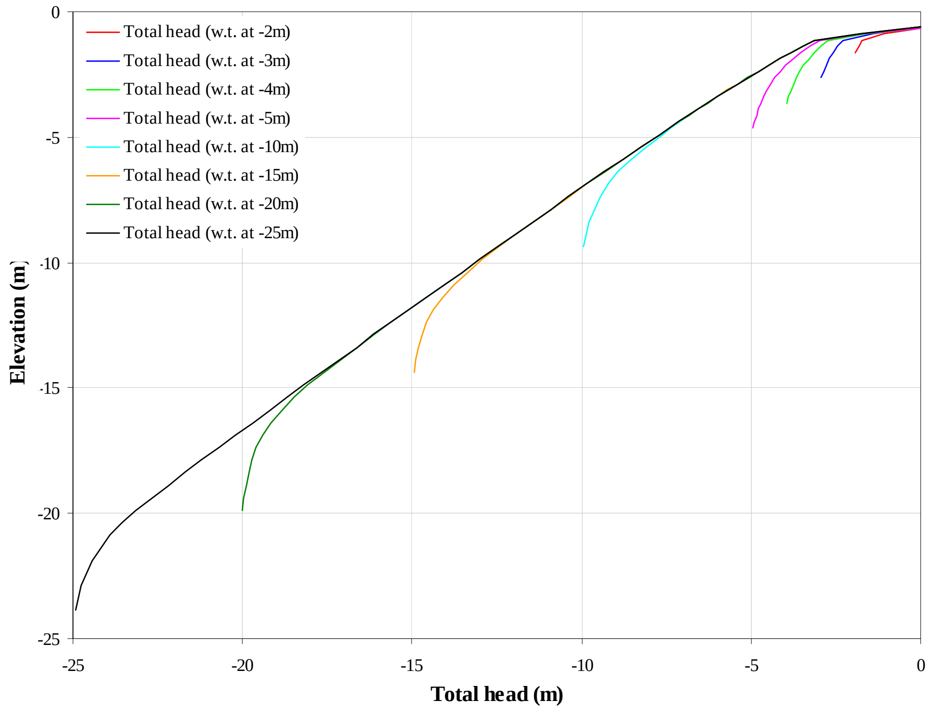


Figure 30 Vertical variation of total head with elevation beneath the river. River bed saturated hydraulic conductivity: 0.1 m day^{-1} (one-tenth of aquifer's conductivity)

4.3.3 Vertical variation of total head gradient

In Figure 31 and 32 the total head gradient is plotted at each of the nodes located on the vertical line directly beneath the centre of the river. The curves are based on the total heads at the end of the 10-day simulation period when steady conditions have been reached as shown in Figures 29 and 30.

In Figure 31 the total head gradient is plotted for the each of the eight models in which the river bed hydraulic conductivity is the same as that of the aquifer. In Figure 32 the total head gradient is plotted for the each of the eight models in which the river bed hydraulic conductivity is one-tenth of that of the aquifer material. In both the figures the curve is plotted for each of the simulations with different water table (w.t.) elevations. The following observations can be made from these two figures:

- Towards the top of each profile the total head gradient is greater than one. This indicates that pressure head decreases (becomes more negative) and moisture content reduces just below the river bed.
- Towards the bottom of each profile the total head gradient is less than one. This indicates that pressure head increases (becomes less negative) and moisture content rises near the water table.
- The head gradients are significantly higher just beneath the river in the models in which the river bed hydraulic conductivity is one-tenth of that of the aquifer; head gradients of up to 7 m per metre are simulated.
- The head gradients are constant and equal to one over only small sections of the profile in the models with lower water table elevations. Over these sections moisture content and pressure head are also constant and the total head varies because of the change in elevation head only.

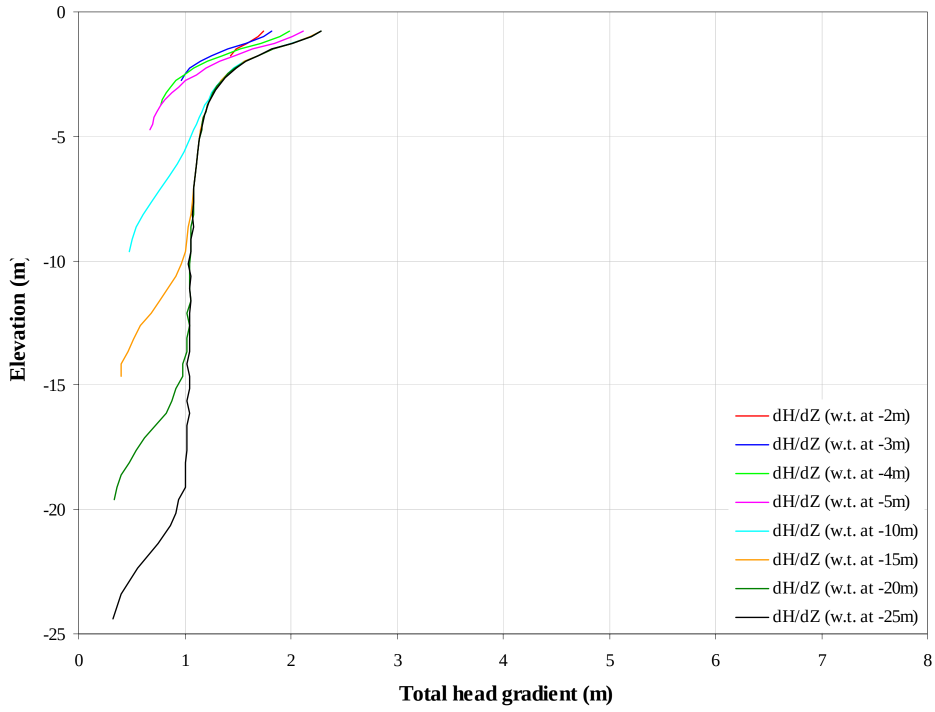


Figure 31 Vertical variation of total head gradient with elevation beneath the river. River bed and aquifer saturated hydraulic conductivity: 1 m day^{-1}

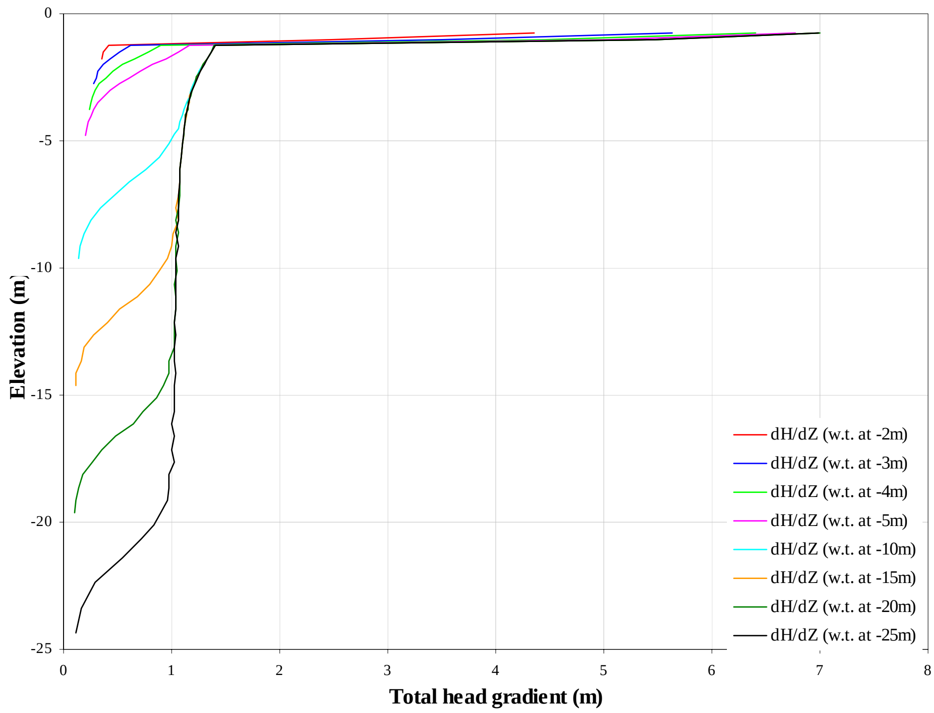


Figure 32 Vertical variation of total head gradient with elevation beneath the river. River bed saturated hydraulic conductivity: 0.1 m day^{-1} (one-tenth of aquifer's conductivity)

4.3.4 Vertical variation of saturation

In Figure 33 and 34 the saturation is plotted at each of the nodes located on the vertical line directly beneath the centre of the river. The curves are based on the saturations at the end of the 10-day simulation period when steady conditions have been reached.

In Figure 33 the saturation is plotted for the each of the eight models in which the river bed hydraulic conductivity is the same as that of the aquifer. In Figure 34 the saturation is plotted for the each of the eight models in which the river bed hydraulic conductivity is one-tenth of that of the aquifer material. In both the figures the curve is plotted for each of the simulations with different water table (w.t.) elevations. The following observations can be made from these two figures:

- The saturations are higher in the models with the higher river bed hydraulic conductivity. In these models more water can leak from the river.
- The saturation reduces to a minimum below the river before rising to a value of one at the water table.
- The lowest saturations are simulated in the models with the lowest water table elevations.
- The saturation is constant over a relatively short section of the vertical profiles. Only over these sections is the pressure head constant and consequently, the total head gradient equal to one.
- In the models with a saturated river bed hydraulic conductivity of 1 m day^{-1} the moisture content is near saturation over the whole depth profile for water table elevations higher than –5 m.

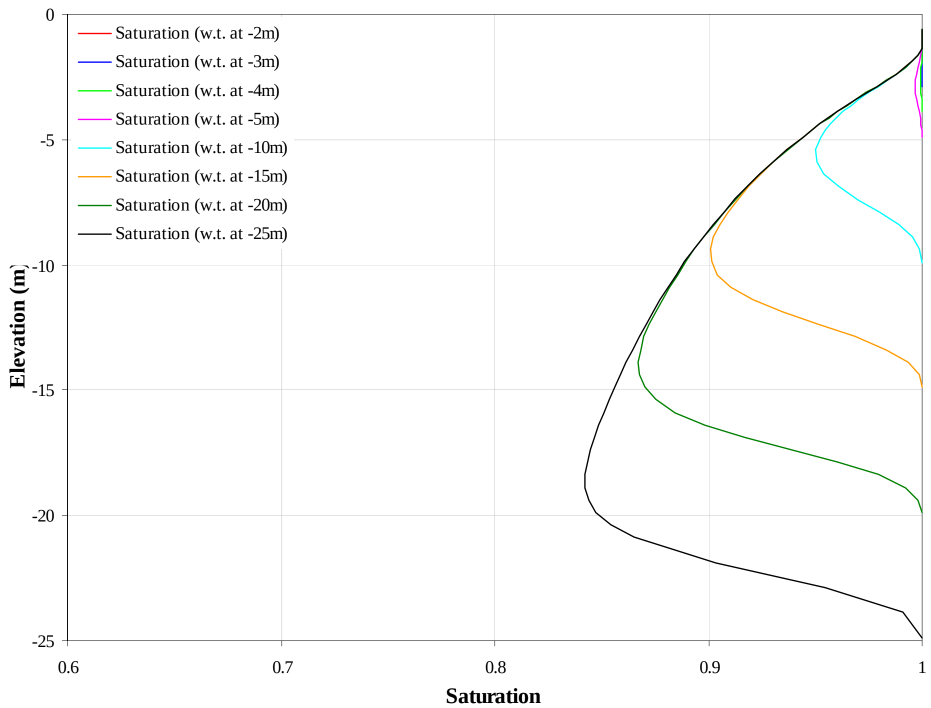


Figure 33 Vertical variation of saturation with elevation beneath the river. River bed and aquifer saturated hydraulic conductivity: 1 m day^{-1}

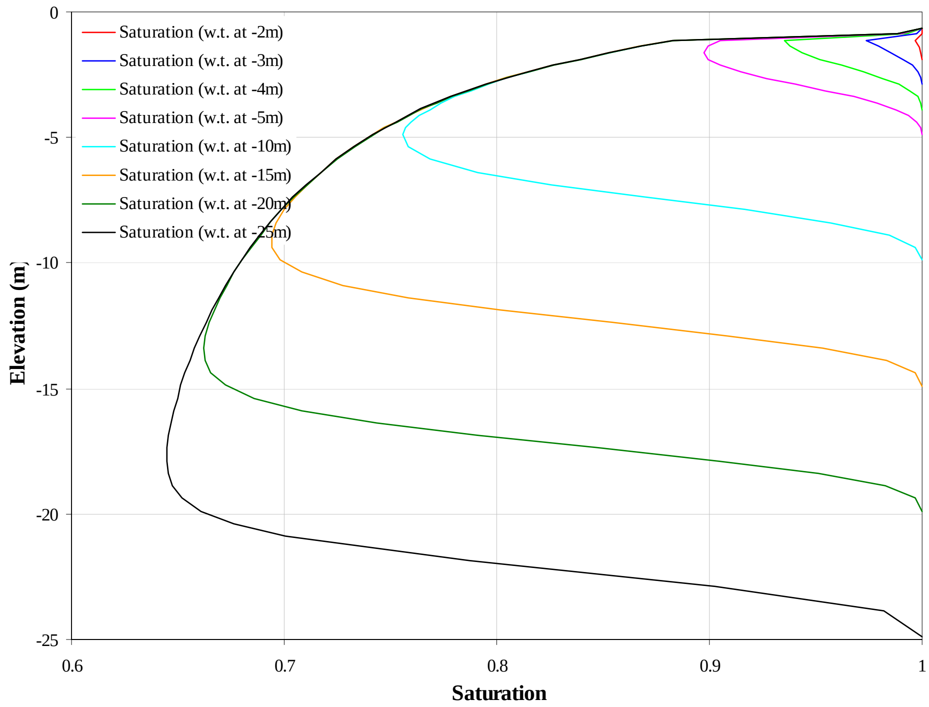


Figure 34 Vertical variation of saturation with elevation beneath the river. River bed saturated hydraulic conductivity: 0.1 m day^{-1} (one-tenth of aquifer's conductivity)

4.3.5 Vertical variation of pressure head

In Figure 35 and 36 the pressure head is plotted at each of the nodes located on the vertical line directly beneath the centre of the river. The curves are based on the pressure heads at the end of the 10-day simulation period when steady conditions have been reached.

In Figure 35 the pressure head is plotted for the each of the eight models in which the river bed hydraulic conductivity is the same as that of the aquifer. In Figure 36 the pressure head is plotted for the each of the eight models in which the river bed hydraulic conductivity is one-tenth of that of the aquifer material. In both the figures the curve is plotted for each of the simulations with different water table (w.t.) elevations. The curves showing the vertical variation of pressure head are similar to those for moisture content. The following observations can be made from these two figures:

- The pressure heads are highest (least negative) in the models with the higher river bed hydraulic conductivity. In these models more water can leak from the river.
- The pressure head reduces with depth below the river to a minimum before rising to a value of zero at the water table.
- The highest suctions are simulated in the models with the lowest water table elevations.
- The pressure head is constant over a relatively short section of the vertical profiles. Only over these sections is the moisture content constant and consequently, the total head gradient equal to one.

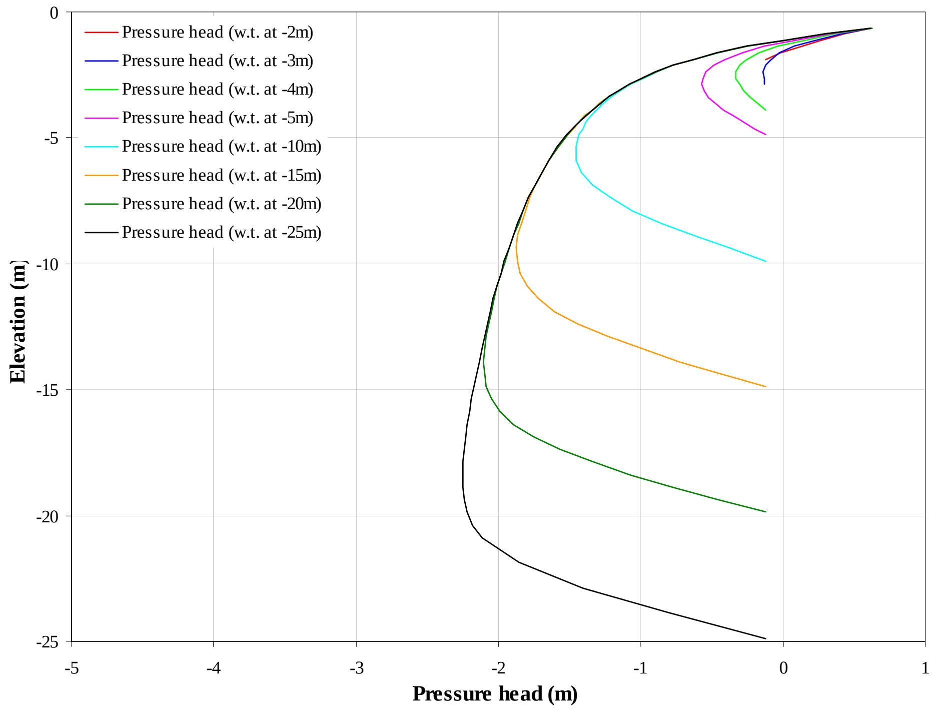


Figure 35 Vertical variation of pressure head with elevation beneath the river. River bed and aquifer saturated hydraulic conductivity: 1 m day^{-1}

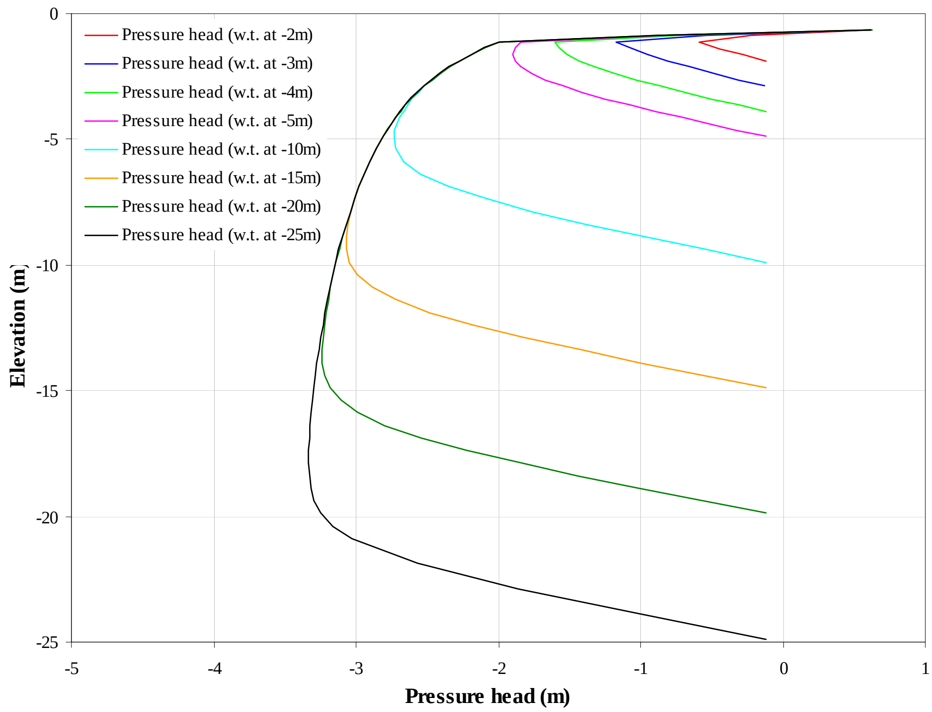


Figure 36 Vertical variation of pressure head with elevation beneath the river. River bed saturated hydraulic conductivity: 0.1 m day^{-1} (one-tenth of aquifer's conductivity)

4.3.6 Water table elevation versus river leakage

One of the main objectives of the modelling was to use VS2DTI to construct graphs showing the relationship between water table elevation and river leakage for two rivers with different bed hydraulic conductivities. These modelled curves could then be compared with that presented in Figure 5 based on the assumptions described in Section 2 i.e. that a unit hydraulic gradient exists beneath the river under perched conditions and that the width of the column of water beneath the river (which is assumed to be saturated) can be assumed to be the same as the width of the channel. The curves could be constructed using VS2DTI because the code provides information on the flow across the model boundaries i.e. from the constant total head cells representing the river.

The curves of water table elevation against river leakage are plotted in Figure 37 for the model in which the saturated hydraulic conductivity is uniform throughout the system and the model in which the hydraulic conductivity of the river bed is one-tenth of that of the aquifer. The curves are of a similar shape to that presented in Figure 5. As the water table falls below the river bed the leakage from the river approaches a limiting value. However, the maximum leakage does not occur when the water table is just below the bed of the river, which in this case is below –1 m, but at lower water table elevations of approximately –5 to –10 m.

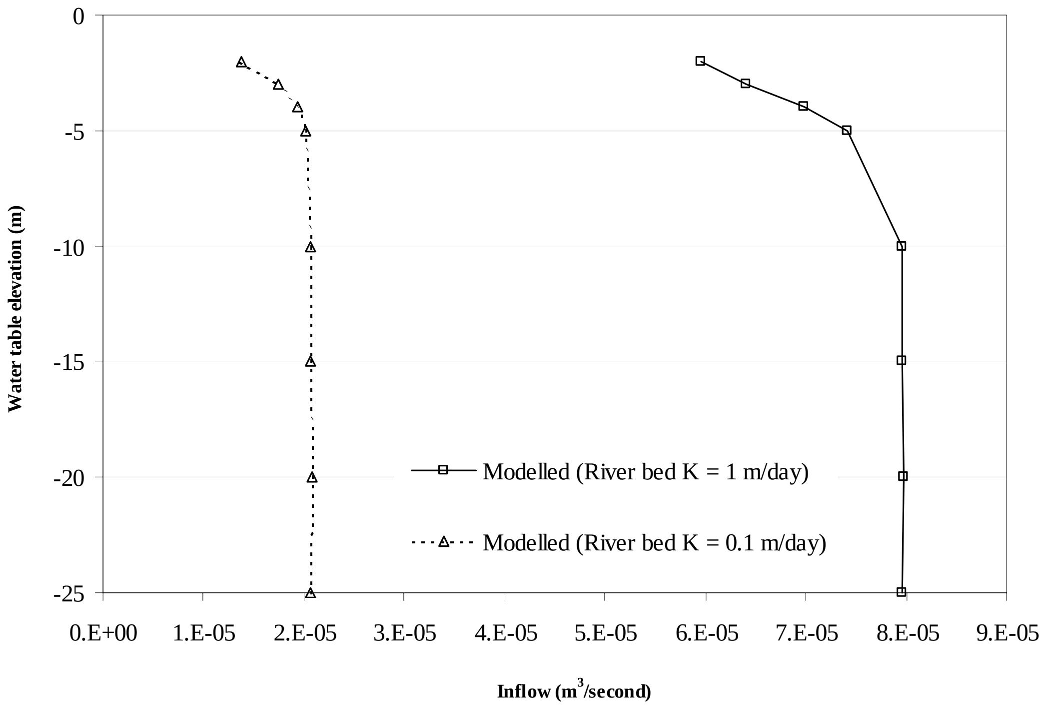


Figure 37 Simulated water table elevation versus river leakage (inflow) for two model scenarios

Whilst these curves correspond to those described in the literature (MacDonald and Harbaugh, 1988; Rushton, 2003), predicting the their exact shape is difficult because of the complex pattern of unsaturated flow beneath the river which incorporates the following features:

- The total head gradient across the river is greater than one for all water table elevations in these two models.
- The total head gradient is equal to one at some point along the vertical profile beneath the river bed. For lower water table elevations the total head gradient is equal to one along a section of the vertical profile beneath the river.

- A continuous vertical total head gradient equal to one indicates that the pressure head and moisture content are constant.
- The width of the flowing column of water from the river to the water table is not constant. It is approximately equal to the channel width (where the vertical total hydraulic gradient is greater than one) just below the river but widens with depth.

Given these features, the question arises of whether it is possible to estimate the maximum leakage rate from the two rivers represented in this modelling investigation prior to running the simulations. If the hydraulic properties and geometry of the river are known estimates can be made based on simplified assumptions. In the next section such estimates are made and compared to those obtained using the numerical models.

5 Estimating leakage rates from perched rivers

In the two models described above the saturated river bed hydraulic conductivity is either 1 m day^{-1} or 0.1 m day^{-1}. In the model with the higher river bed hydraulic conductivity the maximum river leakage is computed to be 8.0×10^{-5} m^3 sec^{-1}. In the model with the lower river bed hydraulic conductivity the maximum river leakage is computed to be 2.1×10^{-5} m^3 sec^{-1}. In this section two methods are used to estimate the maximum river bed leakage assuming that only the saturated hydraulic properties of the aquifer and the geometry of the river are known. In the first method the maximum leakage is estimated by assuming that the vertical total hydraulic gradient in the aquifer is equal to one and that the width of the column of vertically flowing water is equal to the river channel width. This is the approach described in Section 2. In the second method it is assumed that the total head is equal to the elevation head just beneath the river and the maximum leakage is calculated by considering the flow through the river bed only.

5.1 CONSIDERING FLOW THROUGH THE AQUIFER

As described in Section 2 the maximum leakage from a perched river, Q_{Limit}, could be estimated by assuming that assuming that the vertical total hydraulic gradient in the aquifer is equal to one and that the width of the column of vertically flowing water is equal to that of the river channel. In this case, the simple Darcian calculation reduces to:

$$Q_{Limit} = K_{Aquifer} \times W \times L$$

where:

$K_{Aquifer}$ is the saturated hydraulic conductivity of the aquifer (1 m day^{-1} or 1/86400 m sec^{-1}),

W is the width of the river (2 m) and,

L is the length of the river reach (1 m as the models represent cross-sections through the channel).

This gives an estimate for the maximum river leakage of:

$$Q_{Limit} = \frac{1}{86400} \times 2 \times 1 = 2.3\times10^{-5}\ \mathrm{m^3/sec}$$

The relationship between water table elevation and river leakage based on this value is plotted in Figure 38 in addition to the two modelled curves. Because this method of estimating the maximum leakage from the river does not take into consideration the hydraulic properties of the river it is the same for all river bed hydraulic conductivity values. Compared to the case in which the river bed hydraulic conductivity is the same as that of the aquifer medium, the estimated value is lower than that simulated by the numerical model. This is because the total head gradient just beneath the river, as shown by the modelling, is always slightly greater than unity and the aquifer medium remains near to saturation.

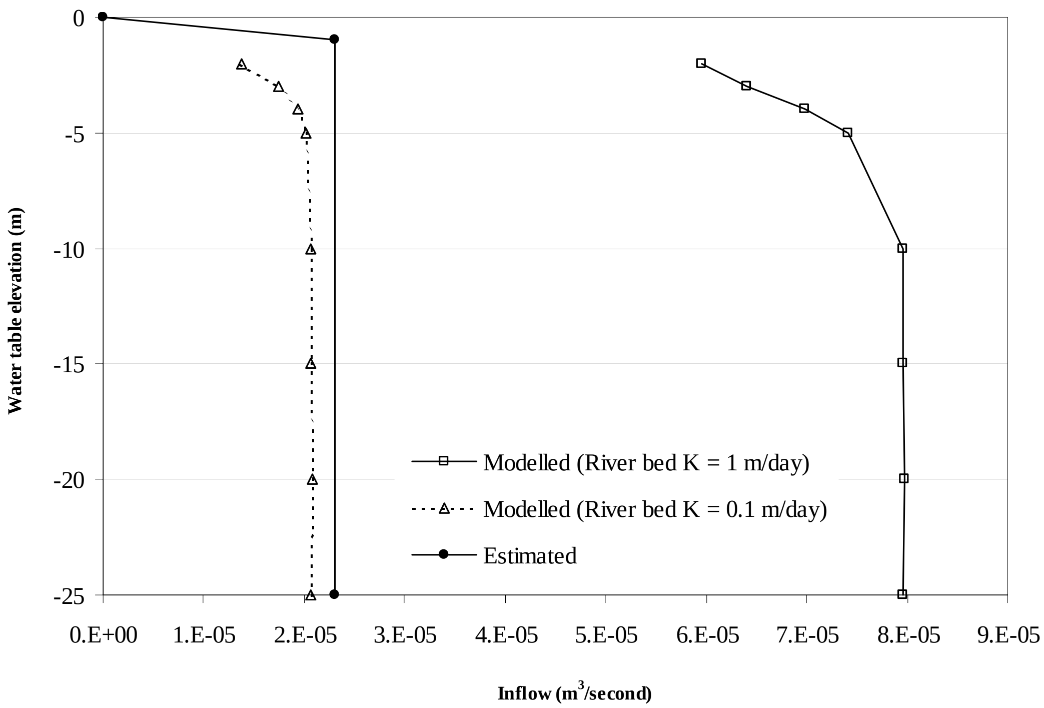


Figure 38 Comparison of the simulated water table elevation versus river leakage curves with the estimated curve based on the consideration of flow through the aquifer under an assumed unit hydraulic gradient

5.2 CONSIDERING FLOW THROUGH THE RIVER-BED

The previous method of estimating the maximum leakage from a river does not depend on the properties of the river bed. However, the numerical modelling has shown that the river bed hydraulic conductivity does affect the leakage to the aquifer under perched conditions in these example river-aquifer systems. A second method by which the maximum rate of leakage from the river could be estimated is through a consideration of the river bed only. It could be assumed that the moisture content just beneath the river bed is very near to saturation and consequently, that the total head is equal to the elevation head at this point. If this is the case, then a simple Darcian calculation can be made to estimate the flow rate based on the head difference across the river bed, the width of the river and its conductivity. The values used to calculate the flow rate are shown in Figure 39 and are:

Total head gradient: 1.125/0.5 = 2.25

River bed hydraulic conductivity: 1 m day^{-1} or 0.1 m day^{-1}

River width: 2 m

River channel length: 1 m

The values for maximum leakage rate calculating using these parameter values are listed in Table 1. The relationships between water table elevation and river leakage based on these estimates are plotted in Figure 40 in addition to those calculated using the numerical models. The values obtained using this method are both underestimates of those simulated by the two numerical models. For the case where the river bed hydraulic conductivity is the same as that of the aquifer material, the difference between the estimated and simulated values is not significantly large. However, for the case in which the river bed hydraulic conductivity is one-tenth of that of the aquifer, the estimated value of maximum leakage is approximately four times less than that simulated.

The estimated values of flow are less than those simulated because the vertical total hydraulic gradient across and just beneath the river bed is greater than the assumed value. Even though the aquifer is unsaturated just beneath the river, the hydraulic conductivity does not reduce significantly and the moisture content remains near to saturation in both model scenarios.

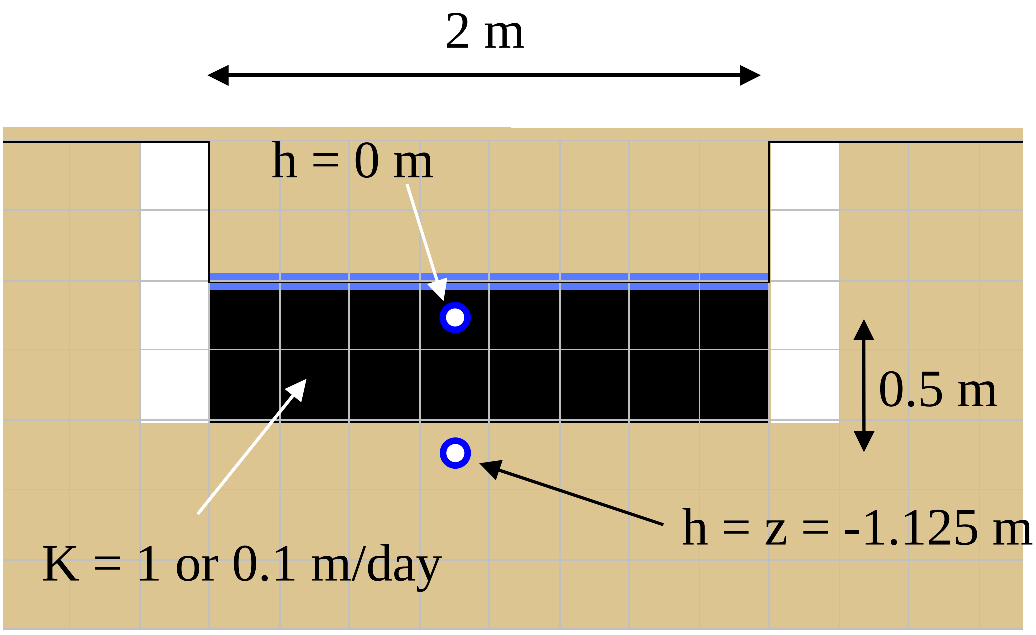


Figure 39 River channel structure and hydraulic properties showing the assumption that the pressure head is equal to zero just below the river bed

Table 1 Comparison of the simulated and estimated maximum leakage rates from the river based on the consideration of the flow through the river bed

	Modelled flow m^3/sec	**Estimated flow m^3/sec**
River bed hydraulic conductivity: 1 m day^{-1}	8.0×10^{-5}	5.2×10^{-5}
River bed hydraulic conductivity: 0.1 m day^{-1}	2.1×10^{-5}	5.2×10^{-6}

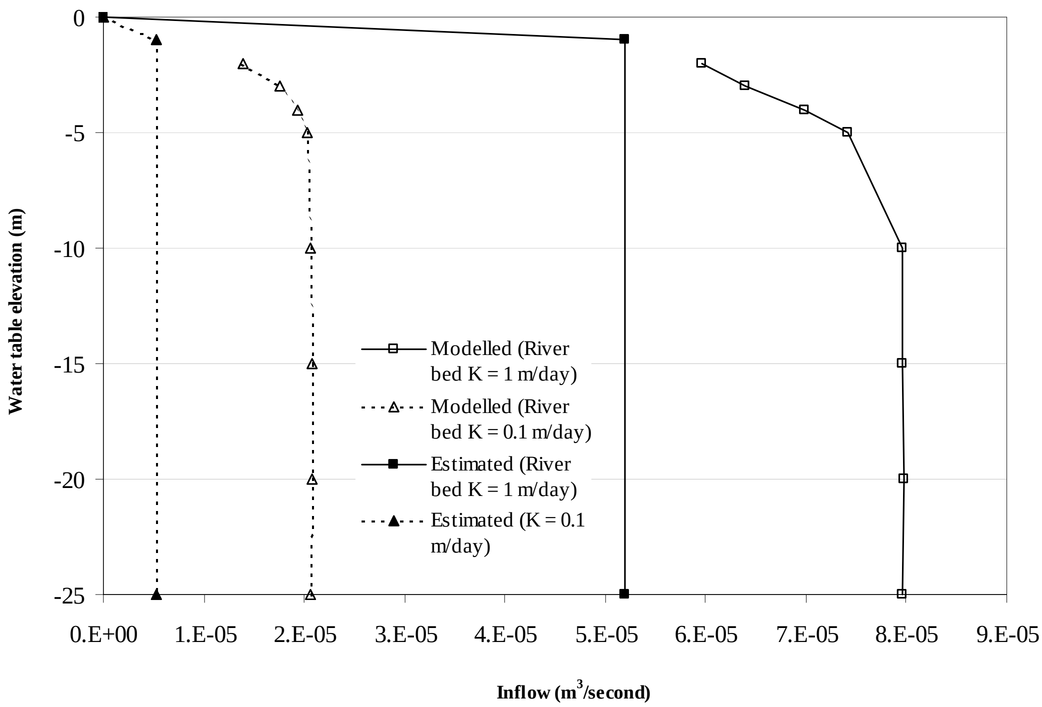


Figure 40 Comparison of the simulated water table elevation versus river leakage curves with the estimated curves based on the consideration of flow through the river bed

6 Conclusions and recommendations

This numerical modelling has illustrated that the shape of the curve describing the relationship between water table elevation and river leakage, which is included in MODFLOW and other saturated groundwater flow models, is similar to that which is predicted by the unsaturated flow simulation code, VS2DTI. Under perched conditions the leakage from a river does tend towards a maximum value. However, this maximum leakage rate cannot be assumed to occur immediately as the water table falls below the bed of the river. The assumptions that it is solely the aquifer that governs the rate of flow to the water table and that this occurs through a saturated column with a width that is equivalent to that of the river is simplistic. The approach to modelling perched rivers that is incorporated into saturated groundwater flow models based on these assumptions is likely to be adequate for regional scale models. In this case the detailed flow pattern beneath the river is not of interest and the river bed conductance is essentially only a "calibration parameter" but with some basis on the experience of other similar catchments. However, this will not be the case for the simulation of channel scale river-aquifer interaction processes.

Whilst the shape of the water table elevation versus river leakage curve is adequately described by two straight lines (as in Figure 5), it is difficult to estimate the maximum leakage from a river and, thus assign a river bed conductance, prior to running a saturated flow model. The rate of leakage from a perched river is complex and governed by the following features:

- The river stage and water table elevation.
- The vertical hydraulic conductivity of the river bed *and* the aquifer.
- The pressure head-moisture content and hydraulic conductivity-moisture content relationships for the porous media.
- The horizontal hydraulic conductivity of the aquifer, which will affect the width of the column of vertically flowing water.
- The width and length of the river channel.
- The thickness of the river bed deposits.
- The shape of the river channel.

The influence of the shape of the river channel has not been addressed in this modelling work but is worthy of investigation. The simulation of a river channel in which flow occurs through its banks will result in different moisture content distributions and may reduce vertical hydraulic gradients beneath the river. The nature of the relationship between water table elevation and river leakage would again be of interest.

Models of only two different river-aquifer systems have been developed in this investigation: (i) a homogeneous system represented by a single porous medium and (ii) the same system containing a lower permeability river bed. The number of different types of river-aquifer systems that could be simulated is infinite. However, the construction of models to simulate typical hydrogeological settings found in UK river catchments would be worthwhile. These models could include the representation of heterogeneous or anisotropic aquifers with different river channel geometries and varying river stages. Layered aquifers may be of particular interest for example, to investigate the influence of the vertical variation of horizontal hydraulic conductivity in Chalk aquifers.

It has only been possible to review the literature briefly during this work. A thorough search of the literature would be beneficial.

Finally, the installation and use of VS2DTI for the simulation of unsaturated flow systems and, the interpretation of its results, has proved to be straightforward. Though it could be argued that unsaturated zone models are difficult to parameterise and thus verify, their application to simple systems is of major benefit in terms of gaining a better understanding of the processes operating.

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Appendix 1 Steps involved in constructing an unsaturated flow model using VS2DTI

The process of constructing an unsaturated flow model using VS2DTI is described by the following procedure. The simulation of solute transport is not considered.

1. Define the basic model parameters. This involves specifying the following:
   a. Model units used for length, time and mass.
   b. Co-ordinate system to be used: radial or Cartesian.
   c. Whether simulating transport or not.
2. Specify parameters relating to fluid flow
   a. Specify the type of initial hydraulic condition: equilibrium profile, pressure head, or moisture content distribution.
   b. Specify the method for calculating the inter-cell relative hydraulic conductivity.
   c. Specify which hydraulic characteristic function is being used to define the relationship between pressure head and, moisture content and hydraulic conductivity: van Genuchten, Brooks-Corey, Haverkamp or Tabular Data.
3. Define the parameters relating to the numerical solution process.
4. Specify parameters controlling the writing of model output.
5. Specify the textural classes. A textural class defines the hydraulic and transport properties of a porous medium in the model domain.
6. Set up the model drawing area and draw the model domain using the polygon drawing tools.
7. Draw polygons relating to the regions of different textural class.
8. Specify the initial hydraulic condition i.e. the equilibrium profile, pressure head or moisture content distribution.
9. Specify the recharge periods which are time periods when the boundary conditions and stresses remain unchanged.
10. Specify the boundary conditions. These may be either no-flow, specified pressure head, specified total head, specified flux (L/T), specified volumetric flow rate (L^3/T), evaporation/transpiration or a seepage face. If a seepage face is defined, the condition will not be known a priori, but the boundary will either be impermeable or the pressure head will be set to zero along it. The condition along a seepage face can switch between these two types during a simulation.
11. Specify the location of observation points where total head, pressure head, moisture content, saturation and concentration (if transport is simulated) are recorded over time.
12. Construct the finite difference mesh.
13. Run the model and analyse the results using the post-processor or text output files.