

Dark radiation from particle decays during big bang nucleosynthesis

by

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Abstract

Cosmic microwave background (CMB) observations suggest the possibility of an extra dark radiation component, while the current evidence from big bang nucleosynthesis (BBN) is more ambiguous. Dark radiation from a decaying particle can affect these two processes differently. Early decays add an additional radiation component to both the CMB and BBN, while late decays can alter the radiation content seen in the CMB while having a negligible effect on BBN. Here we quantify this difference and explore the intermediate regime by examining particles decaying during BBN, i.e., particle lifetimes τ_X satisfying $0.1 \text{ sec} < \tau_X < 1000 \text{ sec}$. We calculate the change in the effective number of neutrino species, N_{eff} , as measured by the CMB, ΔN_{CMB} , and the change in the effective number of neutrino species as measured by BBN, ΔN_{BBN} , as a function of the decaying particle initial energy density and lifetime, where DNBBN is defined in terms of the number of additional two-component neutrinos needed to produce the same change in the primordial 4He abundance as our decaying particle. As expected, for short lifetimes ($\tau_X < 0.1 \text{ sec}$), the particles decay before the onset of BBN, and $\text{DNCMB} = \text{DNBBN}$, while for long lifetimes ($\tau_X > 1000 \text{ sec}$), ΔN_{BBN} is dominated by the energy density of the nonrelativistic particles before they decay, so that ΔN_{BBN} remains nonzero and becomes independent of the particle lifetime. By varying both the particle energy density and lifetime, one can obtain any desired combination of ΔN_{BBN} and ΔN_{CMB} , subject to the constraint that $\text{DNCMB} = \Delta N_{\text{BBN}}$. We present limits on the decaying particle parameters derived from observational constraints on ΔN_{CMB} , and ΔN_{BBN} .

Introduction

In the standard cosmological model, the density of the Universe is at present dominated by a cosmological constant, called dark energy, and cold dark matter which make up 70% and 25% of the universe respectively, with the remaining 5% in baryons. While the present day radiation content of the universe is negligible, it was the dominant component at early times. This model has emerged in the past decade based largely upon precision measurements of cosmic microwave background radiation (CMB) fluctuations [1] [2] observations of type Ia supernovae [3] [4] and big bang nucleosynthesis (BBN) [5].

While the cosmological observations are generally consistent with this standard model, there remain a few unresolved problems. The effective number of light neutrinos predicted by the standard model is $N_{eff} = 3.046$. However, precision measurements from WMAP7, combined with baryon acoustic oscillations (BAO) are fit by a higher value of $N_{eff} = 4.34_{-0.88}^{+0.86}$ [8]. Observations by the Atacama Cosmology Telescope, combined with BAO, gives $N_{eff} = 4.56 \pm 0.75$ [6]. In both cases, the standard model value N_{eff} differs by two sigma from observation. This discrepancy hints to new physics beyond the standard model and the extra radiation has been dubbed “dark radiation”.

Big Bang Nucleosynthesis (BBN) is also sensitive to the total radiation content of the Universe but the evidence is more ambiguous. Calculations of the relic helium abundance by Izotov and Thuan [7] and by Aver, Olive, and Skillman [8] have reached opposite conclusions with the former arguing for the additional dark radiations while the latter concluding the standard number of neutrinos suffices with a measurement of $N_{eff} = 3.14_{-0.65}^{+0.70}$. While not conclusive, observations resulting in the case of $N_{BBN} \neq N_{CMB}$ offer a significant challenge to the standard model because for $N_{eff} = 3.046$, both BBN and the CMB should “see” the same N_{eff} .

Thus, our motivation is two-fold. We seek to address both the discrepancy between the standard model’s predicted value of N_{eff} and the observed value of N_{eff} while offering a mechanism for $N_{BBN} \neq N_{CMB}$. Several models have been proposed to account for these discrepancies. The simplest way is to add an additional relativistic relic particle. [9] In this case, we always have $N_{BBN} = N_{CMB}$. Another model which has been studied is the addition of a

massive particle decaying after BBN, resulting in an unchanged N_{BBN} but a higher value of N_{CMB} [10].

We fill the gap between these two regimes by examining a decaying particle scenario which decays during BBN. Such a model allows a range of N_{eff} from $N_{BBN} = N_{CMB}$ to $N_{BBN} < N_{CMB}$, which addresses both discrepancies between observed and predicted N_{eff} .

Background and Theoretical Discussion

The Friedmann Equation

In a spatially homogenous and isotropic universe, the relation among the energy density $\rho(t)$, the pressure $P(t)$, and the scale factor $R(t)$ is given by the Friedmann equation (Λ is the cosmological constant and k is the curvature of space and the dot represents a time derivative),

$$\dot{R}^2 = \frac{8\pi G}{3} \rho R^2 + \frac{1}{3} \Lambda c^2 R^2 - c^2 k,$$

and the fluid equation,

$$\dot{\rho} + \left(\rho + \frac{P}{c^2} \right) \frac{3\dot{R}}{R} = 0.$$

We can rearrange the fluid equation to the useful form

$$\frac{d(\rho R^3)}{dt} = -\frac{3P\dot{R}R^2}{c^2}$$

and can use the relation $\frac{d}{dR} \frac{dR}{dt} = \frac{d}{dR} \dot{R}$ to obtain the more useful relation

$$\frac{d(\rho R^3)}{dR} = -\frac{3PR^2}{c^2}.$$

Finally, we note that the pressure and density of a fluid are related to each other using an equation of state. In cosmology, it is usual to assume that each component of the cosmological fluid has an equation of state given by

$$P = w_i \rho c^2,$$

where the equation-of-state parameter w is a constant depending on the type of matter in question. Particularly, $w=0$ for a pressure-less dust, $w=\frac{1}{3}$ for radiation, and $w=-1$ for the vacuum. Using the equation of state, we can write

$$\frac{d(\rho R^3)}{dR} = -3w_i \rho R^2$$

and thus the immediate solution is

$$\rho_i \propto R^{-3(1+w_i)}.$$

Components of the Cosmological Fluid

In a general cosmological model, the universe is assumed to contain both matter and radiation, and in addition, the cosmological constant Λ is assumed to be non-zero. Thus, the cosmological fluid consists of three components each with a different density relation to the scale factor R which is determined by each component's equation of state parameter as follows:

$$\text{Matter} \leftrightarrow w = 0 \leftrightarrow R^{-3}$$

$$\text{Radiation} \leftrightarrow w = 1/3 \leftrightarrow R^{-4}$$

$$\Lambda \leftrightarrow w = -1 \leftrightarrow \text{constant}$$

The total equivalent energy density is simply the sum of each individual contributor,

$$\rho(t) = \sum_i \rho_i = \rho_m(t) + \rho_r(t) + \rho_\Lambda(t).$$

When we combine the above expressions, we find the total energy density of the universe as a function of time to be:

$$\rho(t) = \rho_{m,0} \left[\frac{R_0}{R(t)} \right]^3 + \rho_{r,0} \left[\frac{R_0}{R(t)} \right]^4 + \rho_{\Lambda,0}.$$

From this expression, we see that the relative contributions of matter, radiation, and dark energy vary as the universe evolves. Thus, the universe goes through phases where specific

types of matter dominate the energy density. Once expects radiation to dominate the energy of the universe for small scale factors, or early times. As the universe expands, the radiation energy dies away most quickly and matter becomes the dominant component. Finally, as the universe continues to expand the matter also dies away and the universe ultimately becomes dominated by the vacuum energy.

Cosmological Parameters

Therefore, in the Friedman model, the entire history of the universe is determined by only a handful of cosmological parameters. As specified in the previous section, if we know the individual energy densities at the present time, we can determine the individual densities, and hence the total energy density, at all previous times t . Indeed, specifying these values and the Hubble parameter is sufficient to determine the scale factor $R(t)$ for all time. Thus, the cosmological model is entirely fixed by specifying four quantities:

$$H_0, \quad \rho_{m,0}, \quad \rho_{r,0}, \quad \rho_{\Lambda,0}.$$

It is common practice in cosmology to define the dimensionless quantities called the density parameters as follows:

$$\Omega_i(t) = \frac{8\pi G}{3H^2(t)} \rho_i(t),$$

where $H(t)$ is the Hubble parameter and the label denotes $m, r,$ or Λ . If we then plug this expression back into the Friedmann equation and divide the result by R^2 , we can write the Friedmann equation in terms of the density parameters as follows

$$1 = \Omega_m + \Omega_r + \Omega_\Lambda - \frac{c^2 k}{H^2 R^2},$$

where k is the curvature of space and can take on three values. From this we see that the three density parameters determine the spatial curvature of the universe. We have three cases:

$$\Omega_m + \Omega_r + \Omega_\Lambda < 1 \leftrightarrow \text{negative spatial curvature } (k = -1) \leftrightarrow \text{open space}$$

$$\Omega_m + \Omega_r + \Omega_\Lambda = 1 \leftrightarrow \text{zero spatial curvature } (k = 0) \leftrightarrow \text{flat space}$$

$$\Omega_m + \Omega_r + \Omega_\Lambda > 1 \leftrightarrow \text{positive spatial curvature } (k = 1) \leftrightarrow \text{closed space}$$

Therefore, the goal of cosmological observations is to measure these quantities because if we know their full time expressions, we can reconstruct the entire history of the universe.

Big-Bang Nucleosynthesis (BBN)

Equilibrium Thermodynamics

The early universe, to good approximation, can be modeled as a hot bath of particles in thermal equilibrium. Because radiation scales as $R(t)^{-4}$, the early universe was dominated by the radiation energy density allowing the matter and vacuum energy density to be ignored. From thermal physics, the energy density ρ can be written as

$$\rho = \frac{g}{2\pi^2} \int_m^\infty \frac{(E^2 - m^2)^{1/2}}{e^{E/T} \pm 1} E^2 dE$$

where g is the number of internal degrees of freedom and T is the temperature. Thus, the total energy density of all species in thermal equilibrium can be expressed in terms of the photon temperature as

$$\rho_r = T^4 \sum_{i=\text{all species}} \left(\frac{T}{T_i}\right)^4 \frac{g_i}{2\pi^2} \int_{x_i}^\infty \frac{(u^2 - x_i^2)^{1/2}}{e^u \pm 1} u^2 dE$$

where $x_i \equiv \frac{m_i}{T}$ and $u \equiv \frac{E}{T}$. Since the energy density of a non-relativistic species is exponentially smaller than that of a relativistic one, it is a good approximation to only include the relativistic species. In that limit,

$$\rho_r = \frac{\pi^2}{30} g_* T^4$$

where g_* represents the total number of effective degrees of freedom:

$$g_* = \sum_{i=\text{bosons}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{i=\text{fermions}} g_i \left(\frac{T_i}{T}\right)^4.$$

Note that g_* is a function of photon temperature and thus its value changes during different eras of the universe as different particles decouple and become non-relativistic.

During the early radiation dominated era, $\rho_r \cong \rho$. If we assume what current observations suggest, that $\Omega_{Total} = 1$, and that all the energy in the early universe was in the form of radiation, it follows from

$$H^2(t) = \frac{8\pi G}{3} \rho_i(t)$$

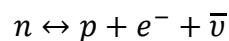
that the expansion rate (the Hubble constant H) is related to g_* by

$$H = 1.66 \sqrt{g_*} \frac{T^2}{m_{pl}}$$

in Planck units.

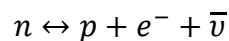
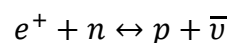
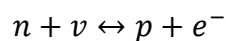
Initial Conditions ($T \gg 1 \text{ MeV}, t \ll 1 \text{ sec}$)

The basic building blocks for nucleosynthesis are the neutrons and protons and thus the abundance of elements depends heavily upon the ratio between neutrons and protons. It is important to note that the neutron is unstable according to the reaction



with a lifetime $\tau_n = 885$ seconds. Thus, if neutrons are left free, they would decay as $\exp(-\frac{t}{\tau_n})$. Thus, neutrons are still around today because of BBN as neutrons bound up in nuclei are stable against decay.

Consider the era when the universe has $T \gg 1 \text{ MeV}$. In this era the neutrons and protons are in equilibrium via the following weak interactions:



As long as they are kept in equilibrium by the reactions above, their number densities are related by Boltzmann statistics:

$$\frac{n}{p} = \exp\left(-\frac{Q}{T}\right).$$

where $Q = m_n - m_p = 1.293 \text{ MeV}$. If the neutrons and protons were to remain in equilibrium, then the number of protons would begin to rapidly outnumber neutrons and no nuclei could be formed. However, equilibrium does not last long enough for the number of neutrons to become negligible. The interactions that mediate between neutrons and protons involve the interaction of a baryon with a neutrino. Thus, when the reaction rate of the neutrino reactions is on rate with the expansion rate (i.e. $\Gamma_\nu = H$), the neutrinos decouple from the neutrons and protons and the ratio of neutrons and protons is “frozen.”

By comparing the reaction rate to the expansion rate of the universe, $H = 1.66\sqrt{g_*} \frac{T^2}{m_{pl}}$, we find

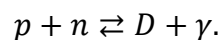
$$\frac{\Gamma}{H} = \left(\frac{T}{0.8 \text{ MeV}} \right)^3.$$

Thus the freeze-out temperature turns out to be $T = 0.8 \text{ MeV}$. The universe reaches this temperature when the age is $t_{freeze} \approx 1 \text{ second}$. The neutron-to-proton ratio at this freeze-out value is thus

$$\frac{n}{p} = \exp\left(-\frac{Q}{T_{freeze}}\right) \approx \exp\left(-\frac{1.29 \text{ MeV}}{0.8 \text{ MeV}}\right) \approx 0.2.$$

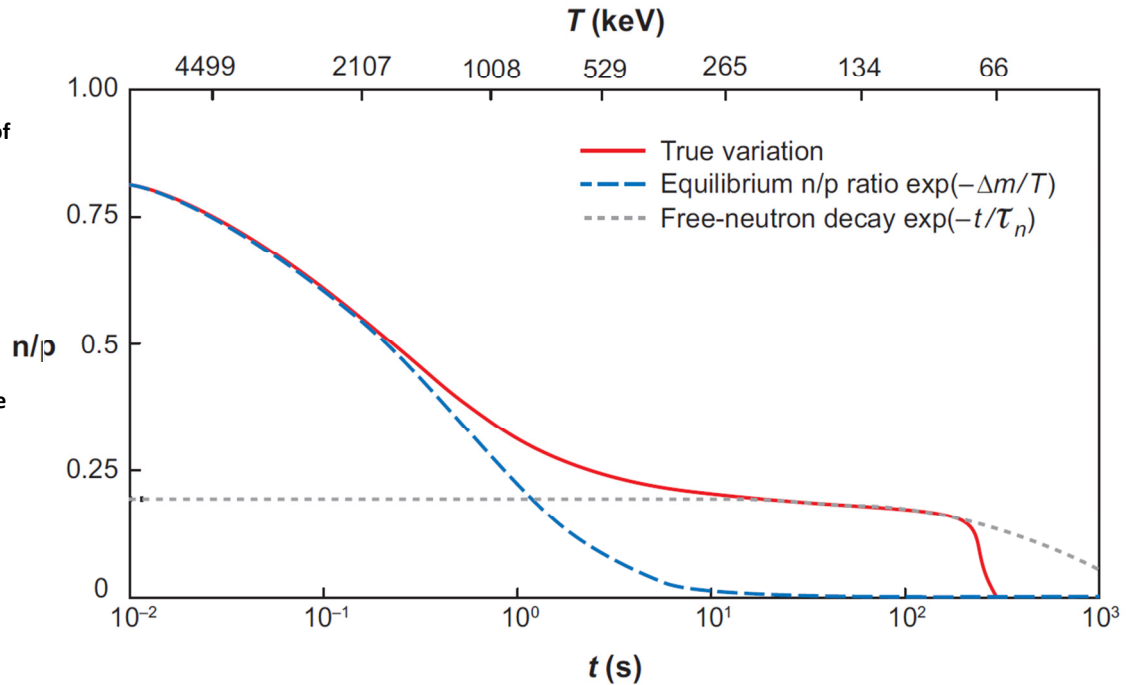
Deuterium Synthesis

Moving forward, when universe is about two seconds old, the neutron-proton ratio is $n_n/n_p = 0.2$. The neutrinos have decoupled from the rest of the universe but photons are still strongly coupled to neutrons and protons through the fusion reaction for deuterium. BBN takes place through a series of two body reactions and builds up nuclei up step by step. The first step in BBN is the formation of deuterium by the reaction



Following from the Saha equation, we find the ratio of deuterium to neutrons as a function of temperature is

Figure 1: The time-temperature evolution of the n/p ratio. The solid red curve indicates true variation. The steep decline after a few 100 s indicates the onset of BBN. The dashed blue curve indicates the equilibrium ratio and the dotted grey curve represents free-neutron decay. [5]



$$\frac{n_D}{n_n} = 6.5\eta \left(\frac{T}{m_n}\right)^2 \exp\left(\frac{B_D}{T}\right).$$

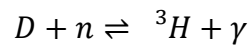
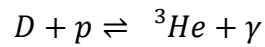
If we define T_{nuc} as the temperature when $\frac{n_D}{n_n} = 1$, then $T_{nuc} \approx 7.6 \times 10^8 K$ and $t_{nuc} \approx 200 s$.

Even with the rough approximation, we see that the time of the “beginning” of nucleosynthesis is not negligible compared to that of the neutron’s half-life and thus means that once deuterium production comes into full swing the ratio of neutrons to protons is less than the freeze-out value due to neutron decay.

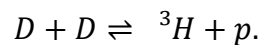
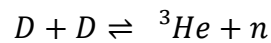
It is important to note that the freeze-out ratio has significant consequences on the deuterium synthesis, and thus the formation of other light elements. The formation of deuterium is dominated by the proton-neutron fusion reaction and thus heavily depends on the ratio of protons to neutrons. Once neutrons run out, there can be no more deuterium formation. Therefore, higher freeze-out numbers will result in a higher neutron number when deuterium synthesis starts.

Beyond Deuterium

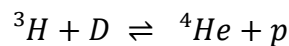
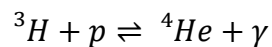
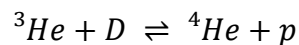
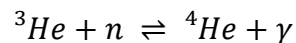
Once a significant amount of deuterium forms, many possible nuclear reactions can occur. For instance, a deuterium nucleus can fuse with a proton to form Helium-3 or a neutron to form tritium:



Deuterium can also fuse with itself in two ways to form tritium and helium 3:



However, a large amount of helium 3 and tritium are never present during nucleosynthesis because both are efficiently converted into helium 4 by the following process:



It is important to note that none of the preceding reactions involve neutrinos. Therefore all these reactions have large cross-sections and fast reaction rates. Thus, once nucleosynthesis

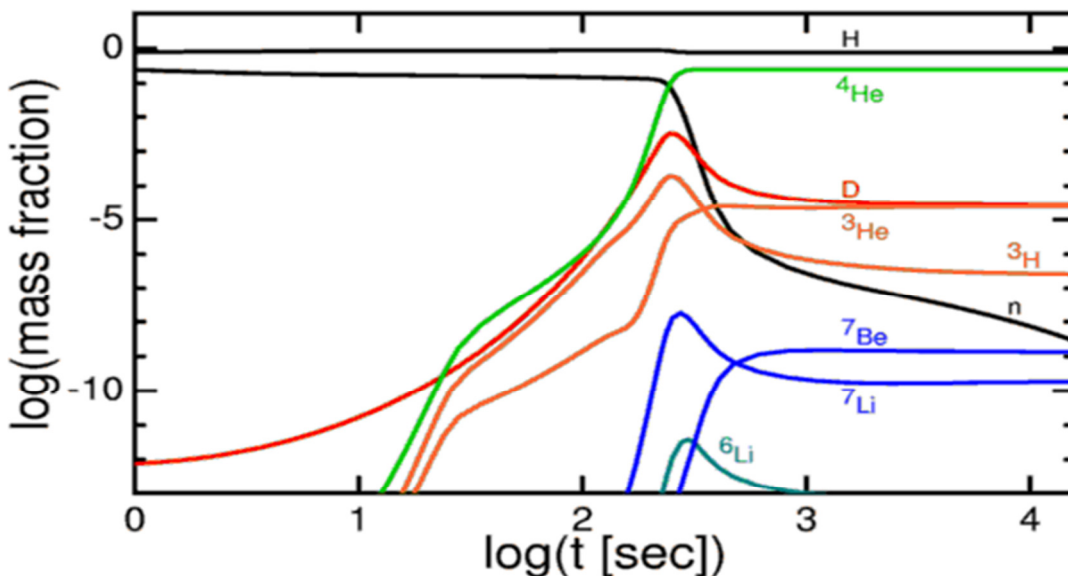


Figure 2: Helium-4 and hydrogen dominate the nuclei created during BBN with heavier elements created in small but non-zero abundances. [19]

begins, deuterium, tritium, and Helium-3 are all effectively converted into Helium-4.

Higher atomic number nuclei are much less in relative abundance. Atoms with atomic number $A=5$ are unstable, thus helium-4 will not fuse with either a proton or neutron. Small amounts of stable lithium and beryllium are created but in very small quantities due to the time taken to reach this stage. Once such atoms can begin to occur, the universe has cooled so that the coulomb barrier becomes a significant and BBN effectively ends. Therefore, once deuterium forms, the reactions up to helium-4 happen very rapidly and by the time BBN ends, nearly all the baryons are in the form of helium-4 or free protons.

Primordial Nucleosynthesis as a Cosmological Probe

In light of our research, it is important to look at the effects of additional species of neutrinos. If we look back to our definition of g_* ,

$$g_* = \sum_{i=bosons} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{i=fermions} g_i \left(\frac{T_i}{T}\right)^4,$$

any addition of a new particle species outside of those contained in the standard model would result in a higher value of g_* . In the case of extra neutrinos, we have additional relativistic fermions to take into account. Since $H \propto \sqrt{g_*} T^2$, an increase in the value of g_* leads to a faster expansion rate which in turn leads to an earlier freeze out of the neutron-proton ratio. With more neutrons, we obtain more deuterium and as a result more Helium-4.

Methods

Addition of a decaying particle

We assume an expansion rate given by

$$H = \frac{\dot{R}}{R} = \sqrt{\left(\frac{8\pi G}{3}\rho\right)}.$$

To the standard cosmological model we add a nonrelativistic particle X which is unstable with a lifetime τ_X which decays into an invisible (“dark”) radiation component. The equations governing the evolution of ρ_X and the decay component ρ_{dec} are

$$\frac{d\rho_X}{dt} = -3H\rho_X - \frac{\rho_X}{\tau_X}$$

$$\frac{d\rho_{dec}}{dt} = -4H\rho_{dec} + \frac{\rho_X}{\tau_X}.$$

In each equation, the first term represents the dilution of the energy density as the universe expands with the matter particle diluting like R^{-3} and the radiation component like R^{-4} , while the second term represents the decay itself. The density of the particle can be integrated analytically to give

$$\rho_X = \rho_{X0} \left(\frac{R}{R_0} \right)^{-3} \exp\left(-\frac{t}{\tau_x}\right)$$

while the energy density of the decay radiation must be integrated numerically.

Parameterization in terms of Entropy

We first follow Scherrer and Turner [4] and parameterize the energy density in terms of

$$r \equiv n_X/n_\gamma.$$

To use a constant value for r , we take the ratio at a temperature T_0 to be immediately prior to nucleosynthesis, chosen to be $T_0 \sim 10^{12} K$,

$$r \equiv n_X/n_\gamma |_{T_0=10^{12}K}.$$

However, in addition we follow [11] and parameterize the density of the decaying particle in terms of its number density relative to the entropy density, s , prior to decay

$$Y_X = \frac{n_X}{s}$$

where s is given by

$$s = \frac{2\pi^2}{45} g_{*S} T_\gamma^3.$$

and $g_{*S} = g_* \sum_{i=bosons} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{i=fermions} g_i \left(\frac{T_i}{T}\right)^3$.

This approximation is useful because Y_X remains constant through the epoch of e^+e^- annihilation.

To change between $Y_X m_X$ and rm_X , we must find the constant that relates $n_X/n_\gamma = aY_X$.

Dividing through by n_γ we get

$$\frac{Y_X}{n_\gamma} = \frac{n_X}{n_\gamma s} \rightarrow \frac{Y_X s}{n_\gamma} = \frac{n_X}{n_\gamma}.$$

Setting the constants $k_B = \hbar = c = 1$, we get $n_\gamma = 60.42 \left(\frac{T_\gamma}{2\pi}\right)^3$. Thus

$$\frac{s}{n_\gamma} = \frac{\frac{2\pi^2}{45} g_{*S} T_\gamma^3}{60.42 \left(\frac{T_\gamma}{2\pi}\right)^3}$$

During BBN, $g_{*S} = 43/4$ and we get the relation $n_X/n_\gamma = 19.36Y_X$.

Deriving ΔN_{CMB}

We look at the limit where neither the decaying particle nor its radiation ever dominates the energy density of the universe. In this limit, we may use Scherrer and Turner's [12] relation between ρ_{dec} and ρ_ν ,

$$\frac{\rho_{dec}}{\rho_\nu} = 0.43 \left(\frac{rm_X}{MeV}\right) \left(\frac{\tau_X}{s}\right)^{1/2},$$

where the proportionality constant is determined from numerical fit. Using our parameterization, we get the relation

$$\Delta N_{CMB} = 8.3 \left(\frac{Y_X m_X}{MeV}\right) \left(\frac{\tau_X}{s}\right)^{1/2}$$

This relation is true for the limit where all the particles have decayed and thus the ratio $\frac{\rho_{dec}}{\rho_\nu}$ has become constant. Because ρ_{dec} varies during BBN when the particles are decaying, we determine this ratio by looking at limits when $t \gg \tau_X$. In our program, this entails measuring the ratio near the end of time program long after BBN ends. Since the program measures the

age of the universe in temperature, we set a threshold temperature inequality which says that when T is less than this threshold, the program saves our ratio $\frac{\rho_{dec}}{\rho_\nu}$. The specific value of this threshold is not important as long as it is in the limit of $t \gg \tau_x$, or in our case $T \ll 0.1 \text{ MeV}$. In our program we choose the threshold temperature to be 10^6 Kelvin, or 8.6 KeV.

Deriving ΔN_{BBN}

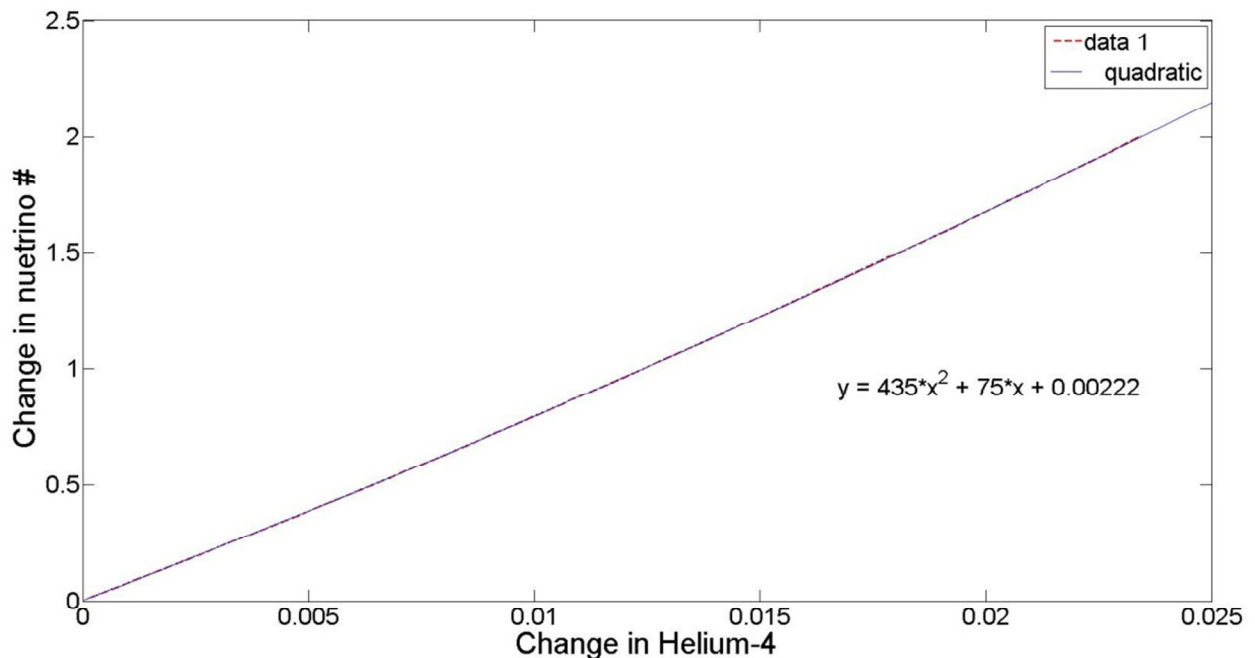


Figure 3: Fit to determine the relationship between the changes in Helium-4 to the change in the effective neutrino number.

The change in N_{eff} from BBN is related to the difference in helium-4 production with and without the addition of extra decaying particles. Because the decaying particle is effectively extra energy, we can relate the change in neutrino numbers to a change in helium-4 using a basic fit. To determine this relation, we first use a version of the program [13] [14] without any additional particles to determine the unperturbed amount of helium-4 produced. Next, we run the same program but alter the number of neutrino flavors (by default, the value is set to the standard models prediction of $N=3.0$) in increments of 0.01 up to a final value of $N=5.0$. We subtract the unperturbed amount of helium-4 from the perturbed amount of Helium-4 resulting for additional neutrinos and plot this against the matching change in N from its

baseline value of 3.0 (see figure above). Using a quadratic numerical fit, we gain the relation between the change in helium-4 and ΔN_{BBN} as

$$\Delta N_{BBN} = 435(\Delta {}^4He)^2 + 75(\Delta {}^4He) + 0.00222.$$

Thus, for each pair of values for $Y_X m_X$ and τ_X (and hence a change in the Helium-4 abundances) we have a corresponding change ΔN_{BBN} that procures the same effect on Helium-4 production.

Simulation

Using the above relations and limits for determining the change in both ΔN_{BBN} and N_{CMB} , we ran simulations using a modified version of Kawano's [15] version of the Wagoner [14] [13] big-bang nucleosynthesis code with the addition of ρ_{dec} and ρ_X . We take the baryon-photon ratio to be $\eta = 6.1 \times 10^{-10}$ and the neutron lifetime to be 881.5 seconds.

We separated the simulations into six 100x100 grids, 2 groups based on particle mass and 3 groups based on particle lifetime. The two mass groups are split equally with $Y_X m_X$ ranging $\sim 1 \times 10^{-34}$ to ~ 1 (or 0.01 to 60 MeV). The three lifetime groups are split into eras where the particle decays before BBN, during BBN, and after BBN with group one ranging from 0.1 to 1 seconds, group two from 1 to 100 seconds, and group three from 100 to 1000 seconds.

At each grid point we calculated ΔN_{BBN} and ΔN_{CMB} according to the discussion above. For ΔN_{CMB} , we took the appropriate ratio at the end of the program and generated a matrix corresponding to the correct values of $Y_X m_X$ and τ_X . For ΔN_{BBN} , the simulations generated a matrix of Helium-4 abundances corresponding to the correct values of $Y_X m_X$ and τ_X from which we subtracted a baseline matrix of helium-4 = 0.2477 which was calculated using an unmodified version of the program. Then, using the appropriate relation we calculated ΔN_{BBN} . Using these matrices, we generated contour plots for eight different values of ΔN_{BBN} and ΔN_{CMB} ranging from 0.1 to 2.0.

The results of these simulations are plotted below.

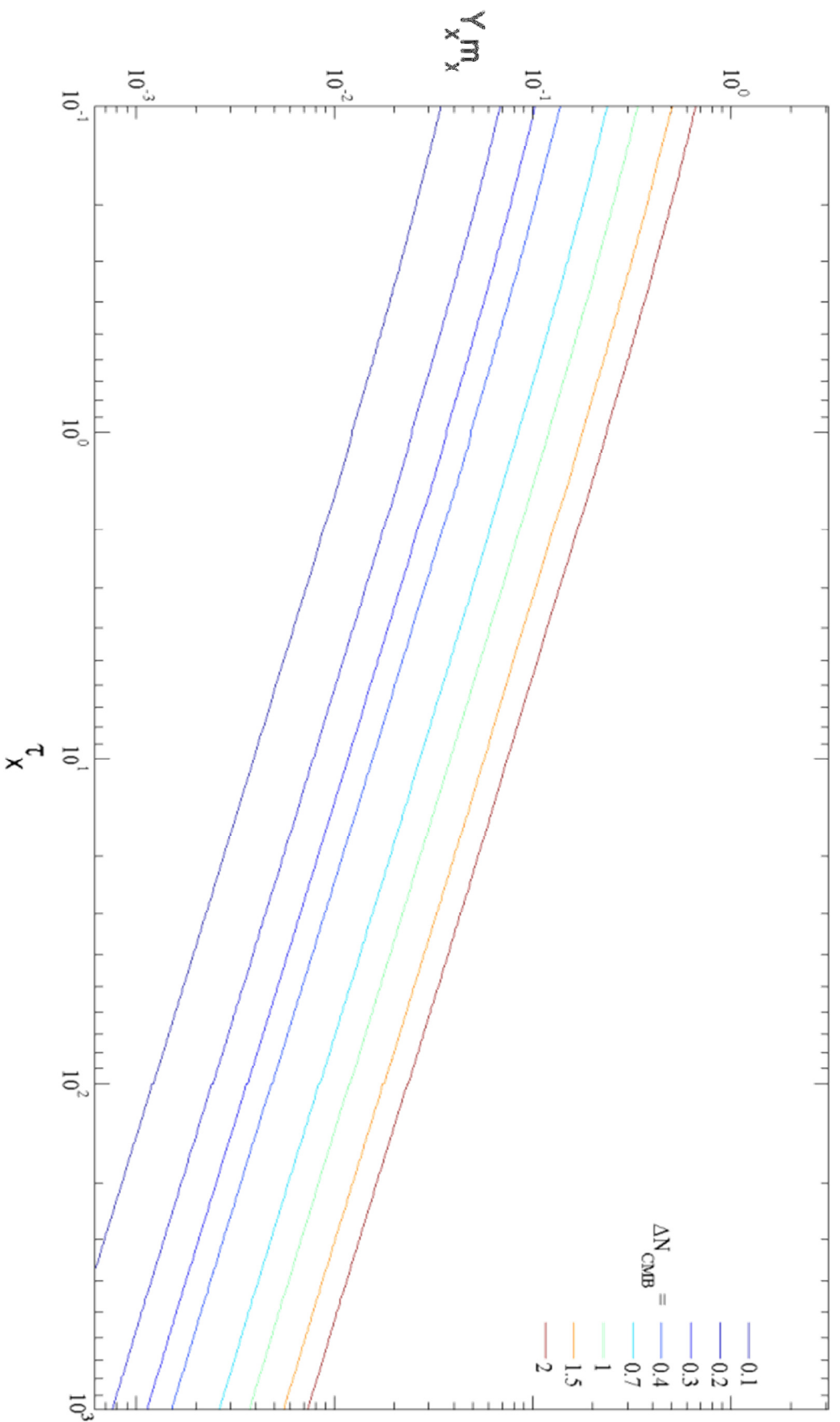


Figure 4. Contour plot of ΔN_{CMB} , the change in the effective number of neutrinos giving the same change in the N effective as measured by the CMB. The short lifetime limit matches with the BBN case because decay occurs before BBN and thus both CMB and BBN see the same effective N . The long lifetime limit continues to depend on particle lifetime.

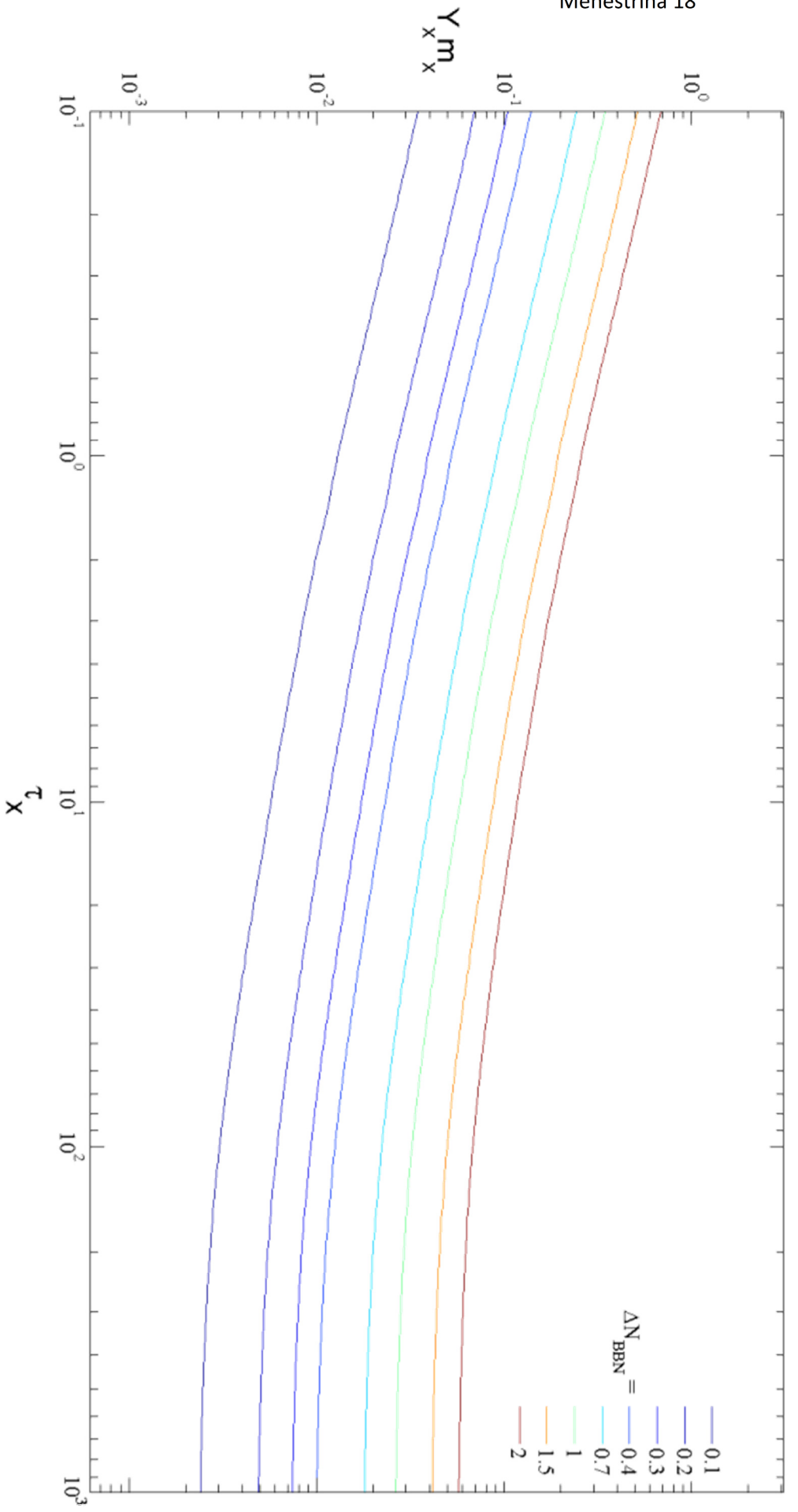


Figure 5. Contour plot of ΔN_{BBN}^N , the change in the effective number of neutrinos giving the same change in the primordial Helium-4 abundance as a decaying particle. The short lifetime limit matches with the CMB case because the particle decays before BBN occurs. The long lifetime limit becomes independent of lifetime because the particle does not decay during BBN and thus the density of the non-relativistic particle is the only contributing factor.

Conclusion

For $\tau_X \leq 0.1 \text{ sec}$, $\Delta N_{BBN} = \Delta N_{CMB}$. In this short-lifetime limit, all of the decaying particle energy density is converted to dark radiation before BBN begins. In this case, the N_{eff} seen by both BBN and the CMB is the “dark radiation” produced by the particle decay. Therefore, for this model, any future observations in which $\Delta N_{BBN} = \Delta N_{CMB}$ would indicate a particle which decays before BBN occurs.

In the opposite limit, $\tau_X \geq 1000 \text{ sec}$, the contours for BBN become horizontal lines. In this long-lifetime limit, all of the particles decay after BBN and then increase in the expansion rate during BBN that alters the helium-4 abundances is due solely to the energy density of the nonrelativistic particles before the decay. However, ΔN_{CMB} would still vary as a function of both $Y_X m_X$ and τ_X because decay would still take place after BBN and thus the relative abundance of particle and “radiation” would have different effects on the last scattering. Thus, in this limit, ΔN_{BBN} becomes a function of only $Y_X m_X$ and is independent of τ_X while $\Delta N_{BBN} \sim \sqrt{\tau_X}$. However, even in this limit, ΔN_{BBN} never goes to 0 precisely because of the continued contribution of the nonrelativistic particles to the expansion rate.

The transitional regime we explored is where $0.1 \leq \tau_X \leq 1000 \text{ sec}$. This is the region of interest if more precise measurements of N_{eff} from the CMB and BBN yield nonzero values for both ΔN_{BBN} and ΔN_{CMB} with $\Delta N_{BBN} \neq \Delta N_{CMB}$. Future observations, such as those expected from Planck, which yielded differing values ΔN_{BBN} and ΔN_{CMB} could be compared to our chart and a potential particle mass and lifetime could be directly read off allowing for an explanation for the discrepancy. Additionally, this model can be falsified by observations because we always have $\Delta N_{CMB} \geq \Delta N_{BBN}$.

However, a deviation from the standard model’s value of N_{eff} in either BBN or CMB measurements is still inconclusive. Recent precision measurements of the CMB fluctuations are best fit by larger values of N_{eff} , with seven-year data from the Wilkinson Microwave Anisotropy Probe (WMAP), combined with observations of baryon acoustic oscillations (BAO) and measurements of the Hubble parameter, give $N_{eff} = 4.34 + 0.86 - 0.88$ [16]. Observations by the Atacama Cosmology Telescope combined with BAO, gives $N_{eff} = 4.56 \pm 0.75$ [6]. Both differ from the standard model value of N_{eff} by about 2.5 sigma. Measurements of primordial helium are tricky because helium-4 is produced in nuclear reactions inside stars and thus the source of measurements must be chosen to carefully avoid measuring stellar produced helium-4. In addition, theoretical calculations for primordial helium production depend on other variables, such as neutron lifetime, which are not precisely known and present problems when trying to generate an accurate theoretical calculation to compare

to observations. Because of these uncertainties, we can use the upper and lower bounds on

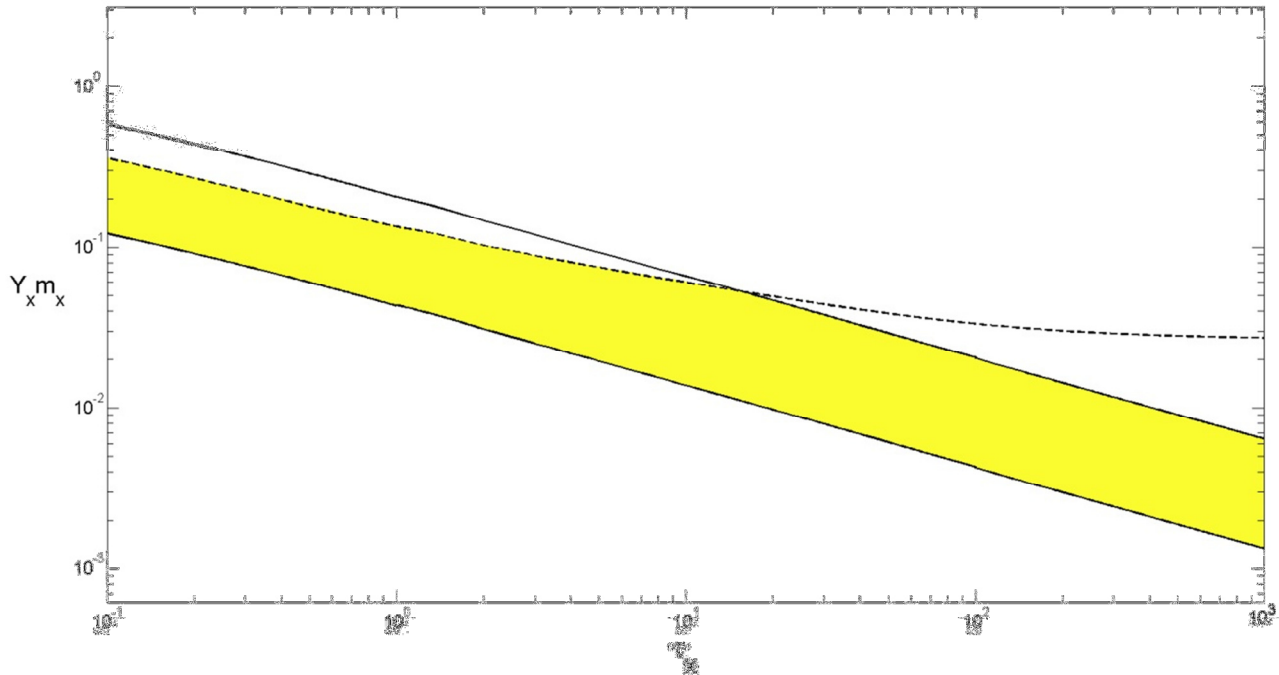


Figure 6: The solid lines give upper and lower bounds on ΔN_{CMB} and the dotted line gives an upper bound on ΔN_{BBN} . Current limits give little distinction between the two.

ΔN_{CMB} from Ref. [17] and the upper bound on ΔN_{BBN} from ref. [18] to set limits on $Y_X m_X$ and τ_X . Current bounds are not restrictive enough for major distinctions between ΔN_{CMB} and ΔN_{BBN} .

Despite the challenges and as of yet inconclusive measurements, the higher fit for N_{CMB} from more than one source strongly hints at new physics beyond the standard model. More accurate determinations of ΔN_{BBN} will require more precision for variables going into theoretical calculations along with refined measurements methods. The future for ΔN_{CMB} will come from Planck as its measurements are expected to reduce the error in N_{eff} by an order of magnitude and thus completely resolve the controversy on whether N_{eff} for the CMB is higher than the standard model's value of 3.046.

Works Cited

- [1] E. K. e. al., *Astrophysical Journal Supplement Series 192, 18*, 2011.
- [2] N. J. e. al., *Astrophysical Journal Supplement Series 192,14*, 2011.
- [3] M. Kowalski et al., *Astrophys. J. 686, 749*, 2008.
- [4] M. Hicken et al., *Astrophys. J. , vol. 700, p. 1097*, 2009.
- [5] G.Steigman, "Annu. Rev. Nucl. Part.," *Science 57*, p. 467, 2007.
- [6] J. Dunkley and e. al, *Astrophysical Journal*, vol. 739, p. 52, 2011.
- [7] Y. Izotov and T. Thuan, *Astrophysical Journal Letters*, vol. 710, p. L68, 2010.
- [8] E. Aver, K. Olive and E. Skillman, *Journal of Cosmology and Astroparticle Physics*, vol. 2010, 2010.
- [9] J. Hamann, S. Hannestad, G. Raffelt, I. Tamborra and Y. Wong, *Physical Review Letters*, vol. 105, 2010.
- [10] K. I. e. al, *Journal of Cosmology and Astroparticle Physics*, vol. 5, 2007.
- [11] E. Kolb and M. Turner, *The Early Universe*, Addison-Wesley, 1990.
- [12] R. Scherrer and M. Turner, *Astrophysical Journal*, vol. 33, p. 331, 1988.
- [13] R. Wagoner, *Astrophysical Journal*, vol. 179, p. 343, 1973.
- [14] R. Wagoner, W. Fowler and F.Hoyle, *Astrophysical Journal*, vol. 3, p. 148, 1967.
- [15] L. Kawano, "Report No. Fermilab-pub-92/04-A," 1992.
- [16] E. K. e. al., *Astrophysical Journal Supplement Series*, vol. 18, p. 192, 2011.
- [17] M. Archidiacono, E. Calabrese and A. Melchiorri, *Physical Review D*, vol. 84, pp. 123-128, 2011.
- [18] G. Mangano and P. Serpico, *Physical Letters B*, vol. 701, p. 296, 2011.
- [19] K. M. N. M. T. Scott Burles. [Online]. Available: arXiv:astro-ph/9903300v1.

