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SPEED AND POSITION CONTROL OF A DC MOTOR USING FRACTIONAL ORDER PI-PD CONTROL

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Abstract — **In this paper, fractional order PI and PD controllers are designed for the speed control, respectively, position control of a DC motor. Both fractional order controllers were designed based on time domain specifications. The closed loop performances of the PI and PD fractional order controllers are compared with integer order PI and PD controllers. The experimental results show that the fractional order controllers outperform the classical controllers.**

Keywords: **Fractional calculus, PI controller, PD controller, DC motor**

I – Introduction

The mechatronic systems represent one of the most challenging control applications due to their interdisciplinary nature [1, 2, 3]. Numerous control algorithms have been proposed to deal with nonlinear dynamics of the mechatronic systems. For linear mechatronic systems, the proportional-integral-derivative (PID) controller is often used owing to its simple structure and robustness [4]. Another approach in dealing with mechatronic systems challenges is the fractional-order (FO) control strategies. One of the most common applications in all mechatronic domains is the control of DC motors.

The control of DC motors has been the interest of many researchers, due to the wide variety of applications that require the use of different types of DC motors [5, 6, 7]. The controllers designed for these DC motors range from simple traditional PIDs to advanced control algorithms, among which fractional order control has been gaining more and more popularity [8, 9, 10].

Fractional calculus has been used relatively recently in modeling and control applications [11, 12]. The attractiveness of the fractional order PID controllers resides in their potential to increase the closed loop performance and robustness of the closed loop system, due to the extra tuning parameters available, as compared to the conventional controller. With fractional order controllers, the order of differentiation and integration may be used as supplementary tuning parameters and thus more specifications can be fulfilled at the same time, including the robustness to plant uncertainties, such as gain and time constant changes [12, 13, 14].

In general, frequency domain tuning of the fractional order controllers is preferred using optimization routines to yield the final solutions. The performance criteria are frequently specified in terms of gain crossover frequency, phase crossover frequency, phase margin, gain margin, robustness to open loop gain variations [15, 16].

In this paper we aim to illustrate a laboratory approach for control education. The envisaged students are bachelor level which received a broad training before specialization. The control design method and the application are kept simple, yet effective to illustrate basic time domain and frequency domain concepts.

The paper is structured as follows. Section II presents the tuning procedure for a fractional order PI controller, as well as a tuning example for the control of the speed of a DC motor. Section III presents the tuning methods for the fractional order PD controller, with the position control for a DC motor. Section IV presents the results of the experimental tests, while the last section includes the final conclusions.

II – DC Motor Speed Control

A general model of the DC motor is shown in Figure 1. The applied voltage V_a, which is the manipulated variable, will control the position $\theta(t)$, which is the controlled variable. For the speed control, the controlled variable is the angular velocity $\omega(t)$ and the transfer function has the form in [17]:

$$P_{DC\_motor}(s) = \frac{\omega(s)}{V_a(s)} = \frac{K_m}{(L_a s + R_a)(Js + b) + K_b K_m} \quad (1)$$

However, for many DC motors the time constant of the armature $\tau_a = \frac{L_a}{R_a}$ is negligible and therefore the model can be simplified to:

$$\begin{aligned} P_{DC\_motor}(s) &= \frac{K_m}{R_a(Js+b)+K_b K_m} = \frac{\frac{K_m}{R_a b + K_b K_m}}{\tau s+1} = \\ &= \frac{K_{DC\_motor}}{\tau s+1} \end{aligned} \quad (2)$$

where $\tau = \frac{R_a J}{R_a b + K_b K_m}$ and $K_{DC\_motor} = \frac{K_m}{R_a b + K_b K_m}$.

The transfer function from position $\theta(t)$ as output (controlled variable) to armature voltage V_a as input (manipulated variable) will be:

$$P_{DC\_motor}(s) = \frac{\theta(s)}{V_a(s)} = \frac{K_{DC\_motor}}{s(\tau s+1)} \quad (3)$$

The first experimental set-up consists in the control of the speed of a DC motor using a fractional order

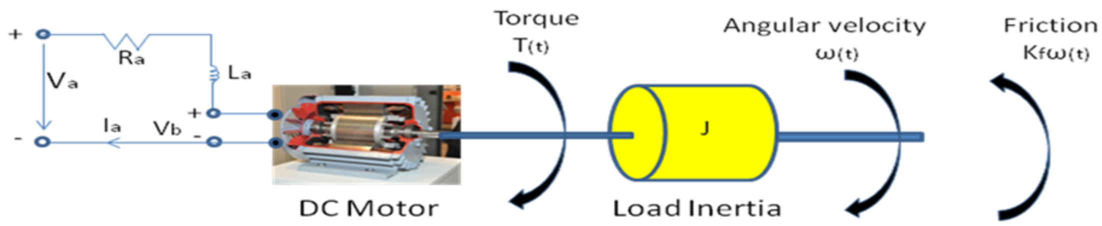

Figure 1: *General model of a DC motor. Photo from Wikimedia Com*

PI controller. The design of the controller is based on a phase margin and a gain crossover condition, to which we add a criteria regarding the robustness to gain variations. The transfer function of the FO-PI controller is given as:

$$H_{FO-PI}(s) = k_p(1 + \frac{k_i}{s^{\mu}}) \tag{4}$$

The tuning of the FO-PI controller in (4) implies the computation of the three parameters k_p, k_i and μ according to three performance specifications imposed:

- an imposed gain crossover frequency of the open loop system

$$\left|H_{open-loop}(j\omega_{gc})\right| = 1 \tag{5}$$

- an imposed phase margin of the open loop system

$$\angle H_{open-loop}(j\omega_{gc}) = -\pi + \varphi_m \tag{6}$$

- a condition for robustness to gain variations

$$\left.\frac{d\left(\angle H_{open-loop}(j\omega)\right)}{d\omega}\right|_{\omega=\omega_{gc}} = 0 \tag{7}$$

where ω_{gc} and φ_m are the imposed gain crossover frequency and the phase margin. For a process, described by the transfer function $H_P(s)$, the performance specifications in (5)-(7) may be rewritten as:

$$\left|H_{FO-PI}(j\omega_{gc})\right| = \frac{1}{\left|H_P(j\omega_{gc})\right|} \tag{8}$$

$$\angle H_{FO-PI}(j\omega_{gc}) = -\pi + \varphi_m - \angle H_P(j\omega_{gc}) \tag{9}$$

$$\frac{d\left(\angle H_{FO\_PI}(j\omega_{gc})\right)}{d\omega_{gc}} = -\frac{d\left(\angle H_P(j\omega_{gc})\right)}{d\omega_{gc}} \tag{10}$$

Replacing in (8)-(10), the transfer function of the FO-PI controller in (4), at the gain crossover frequency yields:

$$\left|k_p\left[1 + k_i\omega_{gc}^{-\mu}\left(\cos\frac{\pi\mu}{2} - j\sin\frac{\pi\mu}{2}\right)\right]\right| = \frac{1}{\left|H_p(j\omega_{gc})\right|} \tag{11}$$

$$\begin{aligned}&\angle\left[1 + k_i\omega_{gc}^{-\mu}\left(\cos\tfrac{\pi\mu}{2} - j\sin\tfrac{\pi\mu}{2}\right)\right] = \\ &= -\pi + \varphi_m - \angle H_P(j\omega_{gc})\end{aligned} \tag{12}$$

$$\begin{aligned}&\frac{d\left(\angle\left[1+k_i\omega_{gc}^{-\mu}\left(\cos\frac{\pi\mu}{2} - j\sin\frac{\pi\mu}{2}\right)\right]\right)}{d\omega_{gc}} = \\ &= -\frac{d\left(\angle H_P(j\omega_{gc})\right)}{d\omega_{gc}}\end{aligned} \tag{13}$$

The identification of the system was done based on a PRBS (pseudo random binary signal) signal. To generate this PRBS signal, the following command is used in Matlab:

- idinput(127, 'PRBS', [0 1/1], [-1 1]).

By using the Prediction Error Method (PEM) for identification [18], the system's model is defined. The transfer function of the DC motor voltage-speed, with 25% break, was identified to be:

$$H_P(s) = \frac{0.25}{(1.45s + 1)} \tag{14}$$

while the imposed performance specifications are: $\omega_{cg} = 1.5$, $\varphi_m = 60^{\circ}$ and robustness to gain uncertainties. Using graphical methods, two curves for the k_i parameter as a function of the fractional order λ are plotted as indicated in Figure 2. The intersection of the two curves yields the solution for k_i and μ as resulting from (12) and (13). The final values, $k_i = 2.28$ and $\mu = 0.89$, are then used to compute the value for the third parameter k_p using (11): $k_p = 1.37$.

The FO-PI controller was implemented using 9^{th} order Tustin recursive method with a sampling period of 0.2 seconds [19]. In order to test the robustness of the designed FO-PI controller, a case study with a 25% break is considered, the experimental results being given in Figure 3. The simulation results are also included for comparison.

III – DC Motor Position Control

The second experimental set-up consists in the control of the position of a DC motor using a fractional order PD controller. To design the FO-PD controller, a similar approach to the tuning of the FO-PI controller

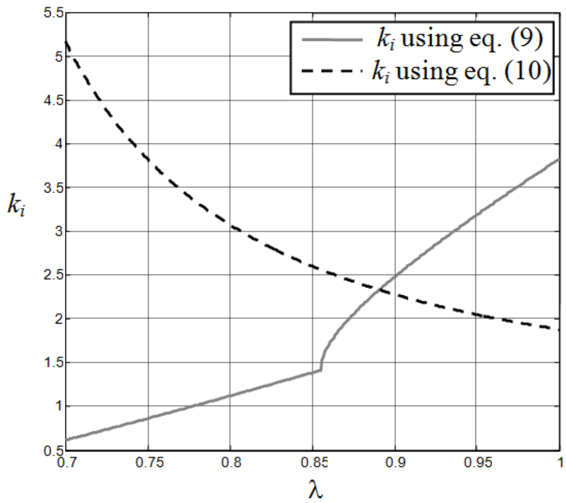

Figure 2: *Graphical selection of the fractional order PI parameters k_i and λ*

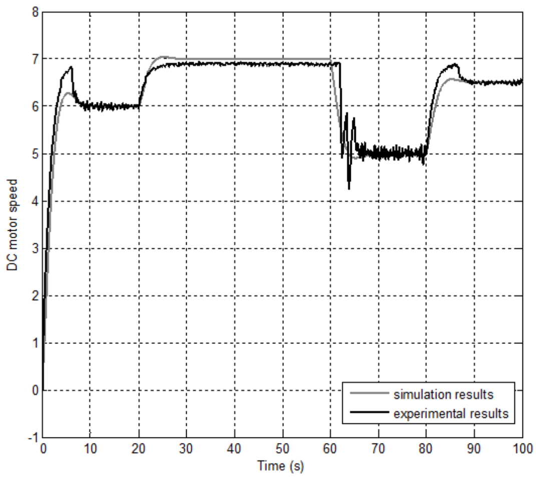

Figure 3: *Closed loop simulation and experimental results comparison considering a 25% break*

in the previous section is taken. The transfer function of the FO-PD controller is given as:

$$H_{FO-PD}(s) = k_p(1 + k_d s^{\lambda}) \tag{15}$$

The transfer function of the DC motor for position control, with 25% break, was identified to be:

$$H_P(s) = \frac{0.25}{s(1.45s+1)} \tag{16}$$

and the imposed performance specifications are the same, $\omega_{cg} = 1.5$ and $\varphi_m = 60^\circ$, as for the speed control.

The tuning of the FO-PD controller in (15) implies the computation of the three parameters k_p, k_d and λ according to three performance specifications imposed (5), (6) and (7). For a process, described by the transfer function $H_P(s)$, the performance specifications in (5)-(7), considering now a FO-PD controller, may be rewritten as:

$$\left|H_{FO-PD}(j\omega_{gc})\right| = \frac{1}{\left|H_p(j\omega_{gc})\right|} \tag{17}$$

$$\angle H_{FO-PD}(j\omega_{gc}) = -\pi + \varphi_m - \angle H_P(j\omega_{gc}) \tag{18}$$

$$\frac{d\left(\angle H_{FO\_PD}(j\omega_{gc})\right)}{d\omega_{gc}} = -\frac{d\left(\angle H_P(j\omega_{gc})\right)}{d\omega_{gc}} \tag{19}$$

Replacing in (17)-(19), the transfer function of the FO-PD controller in (15), at the gain crossover frequency yields:

$$\left|k_p\left[1 + k_d\omega_{gc}^{\lambda}\left(\cos\frac{\pi\lambda}{2} + j\sin\frac{\pi\lambda}{2}\right)\right]\right| = \frac{1}{\left|H_p(j\omega_{gc})\right|} \tag{20}$$

$$\angle\left[1 + k_d\omega_{gc}^{\lambda}\left(\cos\tfrac{\pi\lambda}{2} + j\sin\tfrac{\pi\lambda}{2}\right)\right] = \\ = -\pi + \phi_m - \angle H_P(j\omega_{gc}) \tag{21}$$

$$\frac{d\left(\angle\left[1 + k_d\omega_{gc}^{\lambda}\left(\cos\frac{\pi\lambda}{2} + j\sin\frac{\pi\lambda}{2}\right)\right]\right)}{d\omega_{gc}} = \\ = -\frac{d\left(\angle H_P(j\omega_{gc})\right)}{d\omega_{gc}} \tag{22}$$

Following the same procedure as for speed control, based on (20)-(22), the next tuning parameters for fractional order PD controller were obtained: $k_p = 1.05$, $k_d = 1.51$ and $\lambda = 0.8$.

IV – Results and Discussion

In this section, a part of the simulation and experimental results that were conducted in order to validate the fractional order controllers are presented.

Hence, a discrete time version of the controllers need to be developed for the final implementation. The equivalent discrete-time formulation (for sampling time 0.2s) of the identified model for DC motor voltage-speed is given by:

$$H_P(z) = \frac{0.032}{z - 0.87} \tag{23}$$

and for the position control of the DC motor, the discrete model, is:

$$H_P(z) = \frac{0.0032z + 0.0031}{z^2 - 1.87z + 0.87} \tag{24}$$

As a result, the step response of the DC motor voltage-speed with 25% break is shown in Figure 4, while the BODE diagram is depicted in Figure 5. The BODE diagram of the DC motor position control described by (24) is illustrated in Figure 6.

Before to implement and evaluate the performance of the controllers on the real time setup, a Simulink model has been designed to test and validate the controllers.

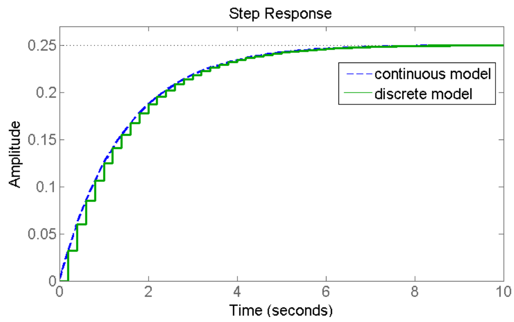

Figure 4: *Validation of the continuous time and discrete time system of the DC motor voltage-speed. The discrete time model is the transfer function from (14) divided by the sampling time (0.2s).*

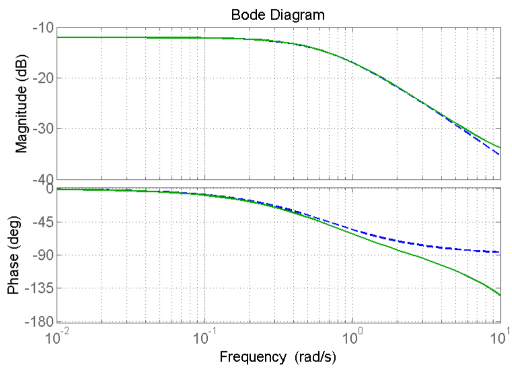

Figure 5: *BODE diagram for the continuous and discrete models of the DC motor voltage-speed.*

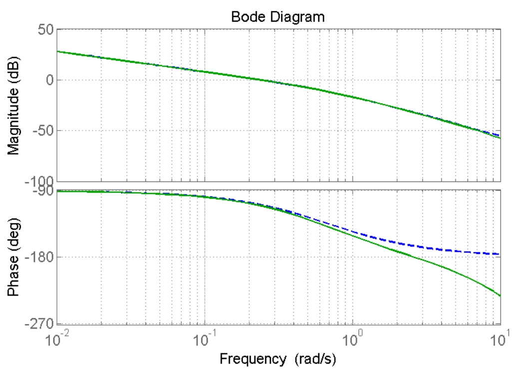

Figure 6: *BODE diagram for the continuous and discrete models of the DC motor for position control.*

In Figure 7, a Simulink diagram of the fractional order PI controller for speed control is illustrated. Block constants were set according to parameters of DC motor and fractional-order controller. The simulation results for fractional order feedback loop by using the Simulink model are depicted in Figure 8. The fractional order PI controller is compared with the integer order PI controller. The integer order PI controller has been designed using the same performance specifications and the same tuning algorithm as for the fractional controller. Hence, the fractional order is $\mu = 1$ and the tuning parameter are: $k_p = 1.23$ and $k_i = 2.41$. As observed from the simulation results, the fractional order controller outperforms the classical controller.

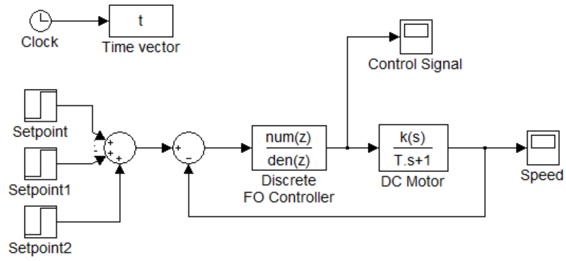

Figure 7: *Simulink diagram for the DC motor speed control.*

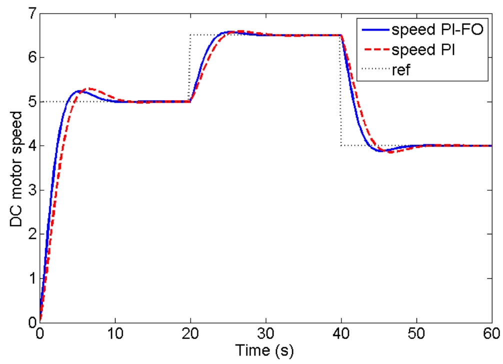

Figure 8: *Step response via Simulink for the DC motor speed control.*

The performance of the PI controllers has also been evaluated in the real time application using the setup depicted in Figure 9. After the system reaches steady state, different changes in setpoint were applied. Again, the experimental results from Figure 10 show that the fractional order controller outperforms the integer order PI controller. If for the simulation results the superiority of the fractional controller is slightly visible, in case of real time the fractional order PI is clearly superior to the classical controller.

In the second experimental set-up, PD controllers were designed in order to control the position of the DC motor. Similarly as for PI controllers, a Simulink model was made to validate the fractional order PD and integer order PD controllers. The integer order PD controller was designed based on the same principle as fractional order PD controller, where $\lambda = 1$. The other PD controller parameters are $k_p = 0.95$ and $k_d = 1.15$. The experimental result are illustrated in Figure 11.

Analyzing the experimental results for the position control it can be concluded that the control effort is similar for both PD controllers. For setpoint

Figure 9: *The real set up configuration for the DC motor*

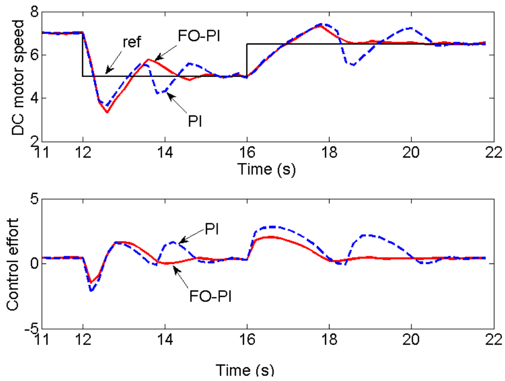

Figure 10: *Setpoint tracking for the DC motor speed control.*

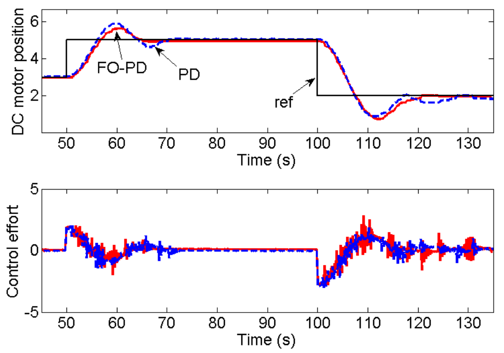

Figure 11: *Setpoint tracking for the DC motor position control.*

tracking, the fractional order PD controller outperforms the classical PD controller. The simulation and experimental results reveal that both fractional order controllers obtain better performances when dealing with this type of process in comparison with classical PI and PD controllers.

V – Conclusion

In this paper, a design of a fractional order PI and PD controllers has been made to control the speed and position of a DC motor. The fractional order controllers were designed based on frequency domain specifications. The experimental results revealed good performances and show a stable and convergent behavior of the system when dealing with fractional order control law. The performances of both fractional order PI and PD controllers are analyzed and compared with integer order PI and PD controllers. The experimental results show that the fractional order controllers outperform the classical integer order controllers.

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