Textile Antennas

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***Abstract*—A novel framework to accurately quantify the effect of stochastic variations of design parameters on the performance of textile antennas is developed and tested. First, a sensitivity analysis is applied to get a rough idea about the effect of these random variations on the textile antenna's performance. Next, a more detailed view is obtained by a Generalized Polynomial Chaos technique that accurately quantifies the statistical distribution of the textile antenna's figures of merit, for a given range over which geometry and material parameters vary statistically according to a given distribution. The method is validated both for a simple inset-fed patch textile microstrip antenna and for a dual-polarized textile antenna. For the latter, the probability density function corresponding to its most sensitive design parameter, being the width, is experimentally estimated by means of measurements performed on 100 patches. A Kolmogorov-Smirnoff test proves that, for all considered examples, the results are as accurate as those obtained via Monte Carlo analysis, while the new technique is much more efficient. Indeed, speedups up to a factor 1667 are demonstrated.**

***Keywords*—Textile antenna, polynomial chaos, variability analysis, input impedance, stochastic collocation.**

I. Introduction

DURING the past years, wearable textile antennas have acquired a lot of interest as suitable devices for deployment in critical operations such as rescue missions, military interventions and e-health applications. Although they have the potential to provide high gain and large radiation efficiency [1]–[6], even in proximity of the human body [7], their antenna characteristics are typically more subject to variations than observed in conventional rigid planar-circuit-board antennas [8], [9]. Two causes may be identified that lie at the origin of these deviations from the nominal antenna characteristics. First, the production process of the applied materials and of the final antenna assembly is less accurate compared to antennas defined on high-frequency laminates. Second, the operating conditions combined with the flexibility and compressibility of textile antennas may modify the textile antennas' geometry as well as material characteristics, thereby changing their radiation characteristics [2].

Quite recently, a novel approach [10], based on the combination of a dedicated cavity model with the Stochastic Collocation Method (SCM), was proposed to obtain a generalized Polynomial Chaos (gPC) expansion [11], [12] that relates the textile antenna's resonance frequency to its bending radius. This method enables the textile antenna designer to account for the effect of random variations in bending radius on the antenna performance. Yet, this technique is dedicated to one specific adverse operating condition only, being bending. Currently, all other random effects the wearable antenna is subjected to in its production and operating phases, are accounted for during ments. In addition, a simple sensitivity analysis is typically performed during the computer-aided design phase to verify whether variations due to production tolerances are acceptable or not. A posteriori measurements on prototypes deployed in a variety of adverse conditions are then carried out to verify whether antenna performance remains satisfactory in real-life conditions [13].

It is clear that the above-described design process is suboptimal and may turn out to be uneconomical. In addition, the textile antenna may need redesign after the prototyping phase to ensure the required performance in the actual application. Thus, a more precise stochastic framework to support textile antenna designers is outlined in this work. The conventional sensitivity analysis is taken as a starting point to get a rough idea of the effect of variations on the textile antenna's performance. Yet, to obtain a more precise statistical characterization during the computer-aided design process, we introduce a non-intrusive Stochastic Collocation Method (SCM) based on generalized Polynomial Chaos (gPC) expansions, which we apply as a more efficient method than conventional Monte Carlo analysis to quantify random variations in the textile antenna's figures of merit. In contrast to a posteriori experiments, which only demonstrate the aggregate effect of variations in *all* design parameters, this stochastic design framework enables the efficient analysis of performance variations due to uncertainty in a *single* design parameter.

The polynomial chaos expansion was introduced in a SPICE simulation environment to quantify variability in lumped circuits and in distributed interconnects [14]–[17], as well as in multiport systems [18], [19]. The method was also used to model the statistics of composite media [20] as well as for quantifying the effect of geometric and material variations in scattering problems [21]–[24]. Up to now, however, its application to antenna design remains limited [25], although the importance of taking into account variations in the figures of merit of textile antennas due to adverse deployment conditions and fabrication tolerances has been stressed before [1], [2], [4], [8], [9], [26]. As already mentioned, a stochastic framework combining the polynomial chaos expansion with a dedicated cavity model has recently been presented to study the influence of the substrate bending on the antenna resonance frequency [10]. However, the approach only aims to quantify statistical variations in one particular application scenario, being antenna bending, and the method is not suited to study the effects due to variations of material characteristics, substrate compression and uncertainties in the production process. Therefore, the development of a rigorous and more general approach to characterize the performance of textile patch antennas is of paramount interest.

This paper is organized as follows. First, Section II outlines the stochastic framework that extends sensitivity analysis with an SCM to accurately quantify the statistics. Next, in Section III the computer-aided design process based on this stochastic framework is illustrated for a representative textile antenna. It is shown that the gPC technique describes the variations in the figures of merit of the antenna with excellent accuracy, also providing a high efficiency in terms of CPU time. Finally, in Section IV the applicability of the new stochastic framework is demonstrated by studying the ISM band textile antenna presented in [27], for which the distribution of geometrical variations corresponding to the antenna's production process is experimentally determined. Conclusions

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are summarized in Section V.

II. Stochastic analysis framework for textile antennas

Consider a textile microstrip inset-fed patch antenna, as shown in Fig. 1. This antenna is designed to exhibit a sharp resonance peak and a specified antenna input impedance Z_{in} at the resonance frequency f_r. Note that this choice of antenna topology and the focus on Z_{in} at f_r as antenna figure of merit is by no means restrictive and the subsequent analysis may be extended to any kind of topology and to any figure of merit. Our aim is to test the effect of stochastic variations of different design parameters on the input impedance Z_{in} of the antenna at the nominal operating frequency f_r. More specifically, in the present analysis we consider both the real part R_{in} and the imaginary part X_{in} of Z_{in} as two distinct parameters. The conventional approach consists of performing a sensitivity analysis where R_{in} and X_{in} are evaluated for the nominal design and at the two extremities of the range over which the random variable is varying [2]. This method, however, does not provide all relevant statistical data to the antenna designer. Therefore, following the Wiener-Askey scheme [12], we introduce a Stochastic Collocation Method (SCM) to carefully investigate the relation between the input and the output distributions. More specifically, we relate the real and the imaginary part of Z_{in} to the random variable y, being the design parameter under study, by means of polynomial expansions:

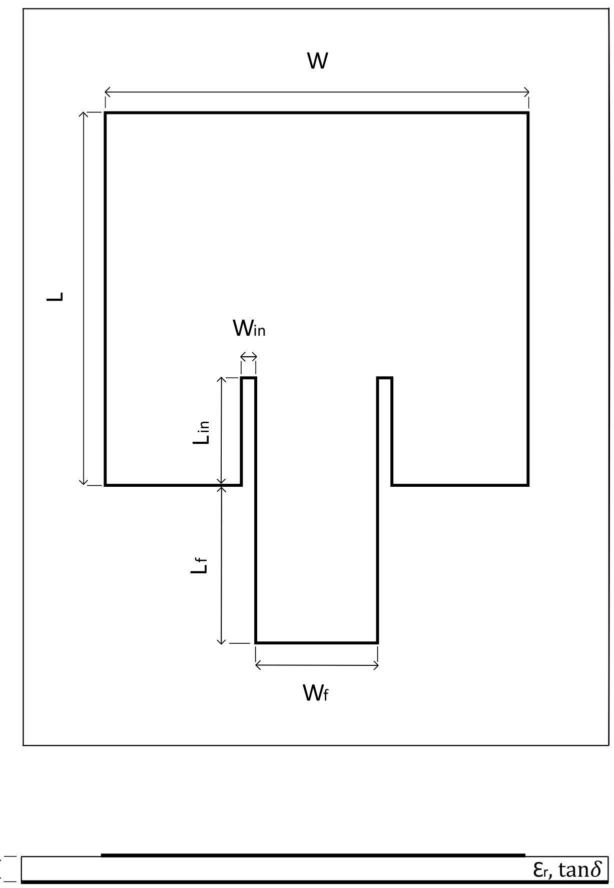

Fig. 1. Representative textile antenna exhibiting a microstrip inset-fed patch antenna topology for which the stochastic framework is developed.

$$R_{in} = f_1(y) = \sum_{k=0}^{P_1} t_{1k} \phi_k^Y(y), \tag{1}$$

$$X_{in} = f_2(y) = \sum_{k=0}^{P_2} t_{2k} \phi_k^Y(y), \tag{2}$$

where $\phi_k^Y(y)$ are suitably chosen polynomial basis functions of degree k, which are normal with respect to the following innner product:

$$< u(y), v(y) >= \int_\Gamma u(y)v(y)p_Y(y)dy, \tag{3}$$

where the input PDF $p_Y(y)$, with support Γ, is taken as a weighting function. Such an orthonormal set of polynomials $\phi_k^Y(y)$ can computed by means of the modified-Chebyshev algorithm [28]. The corresponding expansion coefficients t_{1k}, t_{2k} are as yet unknown.

We now exploit the orthonormality of the basis functions $\phi_k^Y(y)$, to calculate the coefficients t_{1k}, t_{2k}:

$$t_{1k} =< f_1(y), \phi_k^Y >= \int_\Gamma f_1(y)\phi_k^Y(y)p_Y(y)dy, \tag{4}$$

$$t_{2k} =< f_2(y), \phi_k^Y >= \int_\Gamma f_2(y)\phi_k^Y(y)p_Y(y)dy. \tag{5}$$

We compute integrals (4), (5) by means of the following N_1 and N_2-points quadrature rules:

$$t_{1k} = \sum_{i=1}^{N_1} w_i f_1(y_i)\phi_k^Y(y_i), \tag{6}$$

$$t_{2k} = \sum_{j=1}^{N_2} w_j f_2(y_j)\phi_k^Y(y_j), \tag{7}$$

where the quadrature points y_i, y_j and the corresponding weights w_i, w_j are derived by the Golub-Welsch algorithm [29]. Finally, the values $f_1(y_i)$ and $f_2(y_j)$, in (6), (7), are evaluated by means of the ADS Momentum full-wave electromagnetic field solver.

Finally, we test the accuracy of the statistical data resulting from the SCM analysis by generating a representative population of realizations by means of a Monte Carlo simulation, to which we apply the Kolmogorov-Smirnoff test [30]. This test enables us to reject the null hypothesis that the sample set matches the distribution imposed by the SCM analysis, with a significance level α. More specifically, the maximum distance D between the two cumulative distribution functions (CDFs) generated by the Monte Carlo samples and resulting from the SCM analysis, respectively, is compared to a threshold distance D_α. If $D > D_\alpha$, the null hypothesis is rejected with a significance level α, otherwise it is accepted.

III. Validation for a representative textile antenna

A. *Sensitivity analysis*

Consider the design of a representative textile antenna on a flexible closed-cell expanded rubber protective foam substrate with a height h equal to 3.94 mm, permittivity ϵ_r equal to 1.52 and a loss tangent given by 0.012, characterized with the same procedure as in [31]. A computer-aided design procedure was carried out to match the antenna impedance to 50 Ω at

2.45 GHz. The design parameters considered are the permittivity ϵ_r, the thickness of the substrate and the geometrical dimensions of the patch and the inset. Table I provides the optimized dimensions of the antenna (see also Fig. 1). The corresponding input impedance at the frequency of 2.45 GHz is $50.29 + 1.028i\ \Omega$.

TABLE I. NOMINAL VALUES OF THE ANTENNA GEOMETRICAL PARAMETERS (FIG. 1) ON A PROTECTIVE FOAM SUBSTRATE.

parameter	nominal value
patch length L	48.01 mm
patch width W	55.3 mm
inset length L_{in}	14 mm
inset width W_{in}	2.27 mm
feed length L_f	20 mm
feed width W_f	15.43 mm

We now first investigate the statistical variation in Z_{in} due to a variation of a single parameter by means of a sensitivity analysis. Since the closed-cell expanded rubber foam exhibits a compression set of 30%, after being compressed by 50% at 23°C for 72 h, we assume that the substrate height h varies between 70% and 100% of the nominal value, being [2.758 mm, 3.94 mm]. For all other design parameters, we set a variation of $\pm 5\%$ with respect to the nominal value. The corresponding values of Z_{in} are reported in Table II.

TABLE II. Z_{in} VALUES CORRESPONDING TO ALL GEOMETRICAL PARAMETERS VARIATIONS.

	nominal value - 5%	nominal value + 5%
L	$15.398 - 26.443i\ \Omega$	$150.678 - 68.855i\ \Omega$
W	$38.454 + 3.291i\ \Omega$	$62.998 - 7.474i\ \Omega$
L_{in}	$42.331 + 5.96i\ \Omega$	$58.177 - 9.89i\ \Omega$
W_{in}	$49.462 + 2.025i\ \Omega$	$50.995 + 0.038i\ \Omega$
L_f	$49.822 + 1.128i\ \Omega$	$50.557 + 0.675i\ \Omega$
W_f	$56.699 + 0.491i\ \Omega$	$44.342 + 1.265i\ \Omega$
ϵ_r	$26.506 - 13.446i\ \Omega$	$98.58 - 20.153i\ \Omega$
	70 % nominal value	nominal value
h	$19.917 - 5.933i\ \Omega$	$50.29 + 1.028i\ \Omega$

We notice that Z_{in} is appreciably affected by a variation of all parameters under study, except for the width of the inset and the length of the feed line. Therefore, these parameters may be discarded.

B. SCM-based full statistical analysis

Based on the sensitivity analysis, we now further focus on the patch length L, which has the most profound effect on Z_{in}. Hence, the random variable y introduced in Section II equals the design parameter L. Then, we relate R_{in} and X_{in}, on the one hand, and the input random variable, on the other hand, by means of the gPC expansions (1), (2), constructed using the SCM. Both for a truncated Gaussian distribution and a uniform distribution of the length L, we generate the CDFs of R_{in} and X_{in}, based on sample points computed with the full-wave simulator ADS Momentum. We gradually increase the order of the gPC expansion until the Kolmogorov-Smirnov test confirms that stable CDFs have been found. We then validate these CDFs by comparing them with the ones obtained by a Monte Carlo full-wave simulation with a sample set of 10000 points.

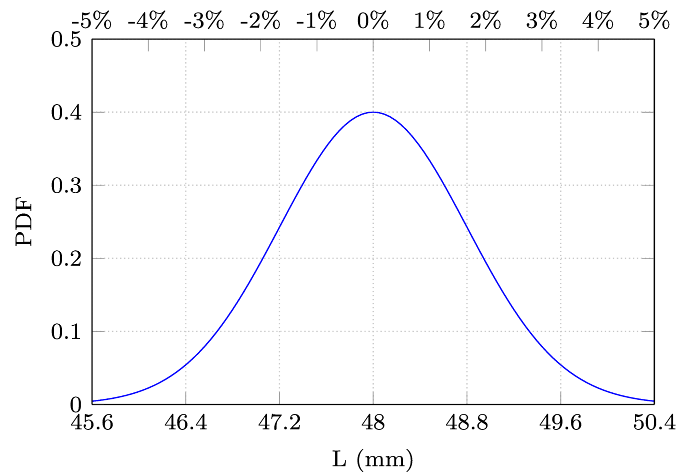

Fig. 2. Truncated Gaussian distribution used as input PDF $p_Y(y)$ for the random variable $y = L$. Bottom axis: absolute variation of L. Top axis: percentage variation of L.

We first assume that the random variable L is distributed following a truncated Gaussian distribution with mean value μ equal to 48.01 mm and standard deviation σ equal to 0.8 mm, i.e. one third of the half of the variation interval, which spans a support Γ of $\pm 5\%$ with respect to its nominal value (see Fig. 2). First, we construct a set of normal polynomials $\phi_k^K(y)$, with respect to the *truncated* Gaussian distribution as a weighting function, by means of the Modified Chebyshev algorithm. More specifically, the algorithm takes the Hermite polynomials, which are conventionally used for the closely related Gaussian distribution *of infinite support*, as a starting point to generate a new set of orthogonal polynomials that have the *truncated* Gaussian distribution as a weighting function. Based on this set of polynomials, we adaptively construct the gPC expansions (1), (2) up to orders of expansion $P_1 = 5$ and $P_2 = 8$, for which we obtain convergence for both $f_1(y)$ and $f_2(y)$. Both R_{in} and X_{in} as functions of L are reported in Fig. 3. Finally, we apply the Kolmogorov-Smirnov test to verify whether the CDFs obtained by the SCM and Monte Carlo techniques correspond to the same distribution. If we set the significance level α to 0.05, we obtain that D_α is equal

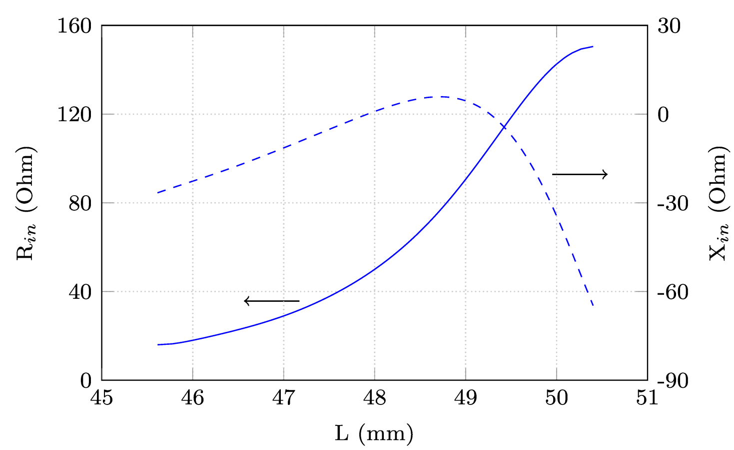

Fig. 3. X_{in} and R_{in} as function of the patch length L according to the Stochastic Collocation Method (SCM).

to 0.019233. We find that the maximum distances $D_{R_{in}}$ and $D_{X_{in}}$ are equal to 0.0052 and 0.0182, respectively, proving that the CDF obtained by the SCM is a good approximation for the output CDF, considering a significance level of 5%. The CDFs resulting from the Monte Carlo full-wave simulation and from the SCM are shown in Fig. 4.

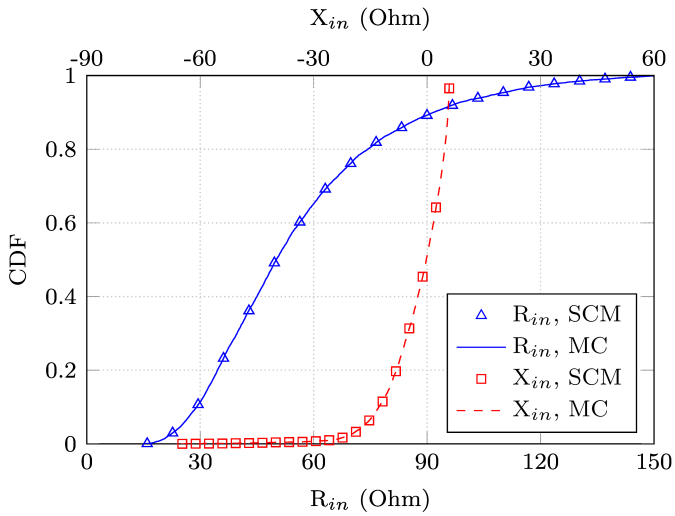

Fig. 4. Comparison between the CDFs constructed with the Stochastic Collocation Method (SCM) and the Monte Carlo (MC) simulations, assuming a truncated Gaussian distribution of L.

Second, we turn our attention to a uniform distribution. We assume that the distribution of the patch length L has a mean value μ equal to 48.01 mm and varies within a support of $\pm 5\%$ with respect to its nominal value. We adaptively construct two generalized Polynomial Chaos expansions of order $P_1 = 6$ and $P_2 = 10$, respectively, to relate the input and output distributions. Then, we again applied the Kolmogorov-Smirnov test to verify if the CDFs obtain by the SCM technique and the Monte Carlo approach correspond to the same distribution. The maximum distances $D_{R_{in}}$ and $D_{X_{in}}$ are now equal to 0.0169 and 0.0093. Hence, even in this case, the SCM allows to accurately approximate the output CDF, considering a significance level of 5%. All the computed CDFs are reported in Fig. 5.

As a final remark, we point out that the time required to process a single point with ADS Momentum is about 6 s (Intel i7 CPU, 16 Gb RAM). Thus, we need about 17 hours to process the complete sample set of 10000 points, whereas only a few seconds are required to obtain the values $f_1(y_i)$ and $f_2(y_j)$, to perform the quadrature rules (6), (7), to construct the basis and the weights in (1), (2), and to complete the gPC expansions. Therefore, it is clear that the SCM is able to correctly reconstruct the CDFs of both R_{in} and X_{in}, in a significantly more efficient manner than the full-wave Monte Carlo simulation, reaching speedup factors up to 1667. Note that, specifically for microstrip inset-fed patch antennas, approximate empirical formulas may be used to compute both R_{in} and X_{in} as a function of the patch length L [32], which would considerably speed up our analysis. However, the procedure outlined in this paper aims to be generally applicable for any antenna topology and for all figures of merit, for which such approximate empirical formulas may not be available. Therefore, our analysis is based on an SCM combined with an accurate full-wave simulator.

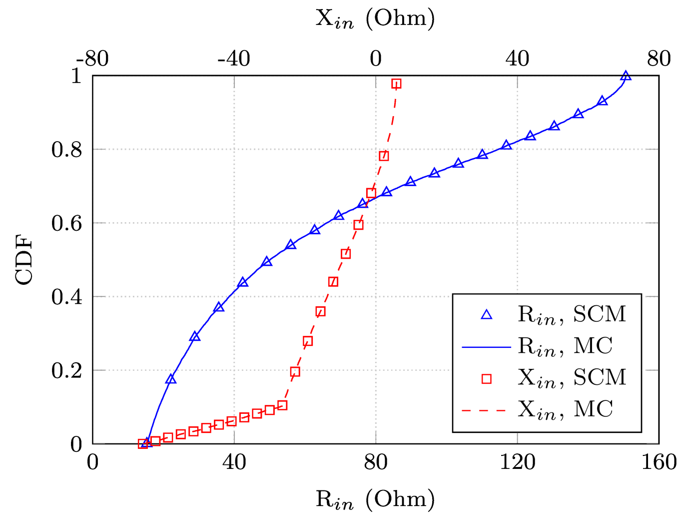

Fig. 5. Comparison between the CDFs constructed with the Stochastic Collocation Method (SCM) and the Monte Carlo (MC) simulations, assuming a uniform distribution of L.

IV. Application to a dual-polarized antenna

A. Sensitivity analysis

We now apply the developed stochastic framework to the textile antenna presented in [27] to demonstrate its applicability to any antenna configuration. More specifically, the antenna considered is a dual-polarized probe-fed patch antenna operating in the 2.4-2.4835 GHz ISM band (see Fig. 6). The antenna's substrate again consists of a flexible closed-cell expanded rubber protective foam with a height h equal to 3.94 mm, permittivity ϵ_r equal to 1.53 and a loss tangent given by 0.012. A computer-aided design procedure was carried out to match the antenna impedance, which is equal for both feed 1 and feed 2, to 50 Ω, and to have an isolation $-20\log|S_{21}|$ better than 15 dB within the entire ISM band. Table III provides the optimized dimensions of the antenna. The corresponding input impedance at the frequency of 2.45 GHz is $49.91 - 1.93i$ Ω.

TABLE III. Nominal values of the antenna geometrical parameters (Fig. 6) on a protective foam substrate.

parameter	nominal value
patch length L	44.46 mm
patch width W	45.32 mm
slot length L_s	14.88 mm
slot width W_s	1 mm
feed points $(\pm x_f, y_f)$	(±5.7,5.7) mm

We repeat the procedure outlined in Section III. Therefore, we carry out a sensitivity analysis on the antenna, in order to investigate the statistical variation of Z_{in} and $-20\log|S_{21}|$ corresponding to the variation of a single parameter. We find that the patch width W has the most profound effect on both Z_{in} and $-20\log|S_{21}|$. Therefore, we decide to proceed with our analysis by considering only W.

B. Estimation of the input PDF

In order to estimate the real distributions of the patch width W and length L, and to give a practical validation to the

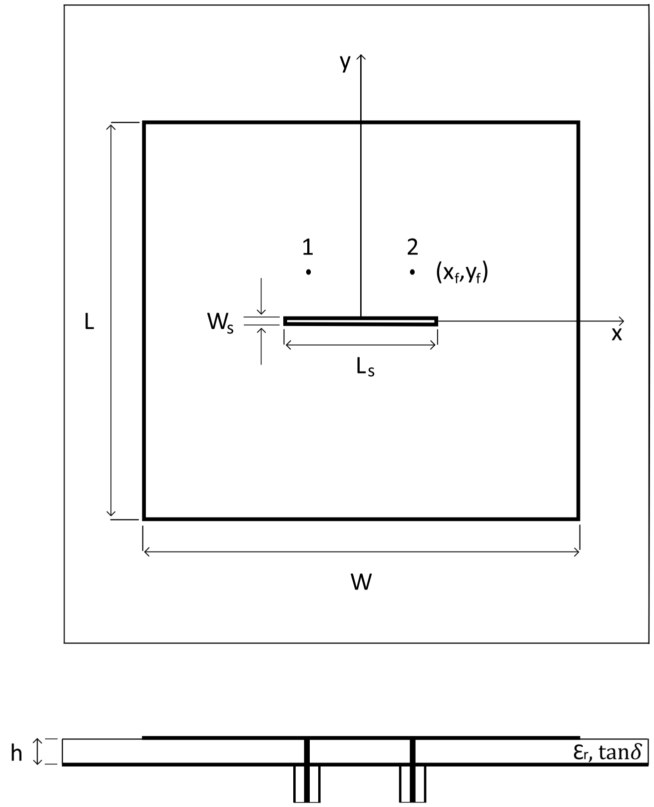

Fig. 6. Schematic of the dual polarized probe-fed ISM band textile antenna under study.

presented framework, 100 patches corresponding to the dual-polarized textile antenna shown in Fig. 6 were hand-cut and measured. To ensure the randomness of the manual cutting process, the patches were carefully prepared with the highest possible accuracy by several people. The textile patches were then measured by means of a Nikon Veritas VM-250V optical setup with an accuracy of 0.4 μm. Measurements yield a marginal distribution for the values of W, which fits a Gaussian distribution with mean value μ equal to 45.385 and standard deviation σ equal to 0.1268 (see Fig. 7). As for the length

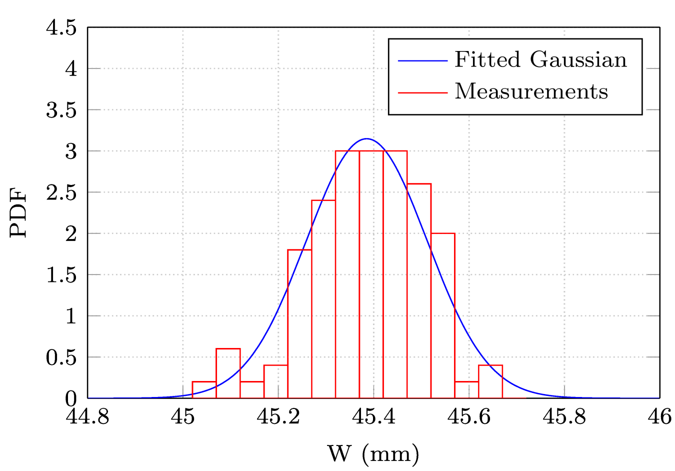

Fig. 7. Results of the measurements of the patch width W and fitted Gaussian distribution.

L, we find it to be fitted by a Gaussian distribution with with mean value μ equal to 44.515 and standard deviation σ equal to 0.1627. The infinite support of this distribution is then truncated to $\pm 4\sigma$ with respect to the mean value. Moreover, as for the joint distribution, the correlation between the measured values of W and L equals -0.0089. Therefore, W and L are not correlated, and we will further focus on W in the next section.

C. SCM-based full statistical analysis

We now apply the SCM analysis to this distribution for the patch width W. On the one hand, for the interval of variation assumed, we find that the isolation $-20\log|S_{21}|$ is always larger than 15 dB. As for R_{in} and X_{in}, on the other hand, the variations remain not negligible. Therefore, we directly relate R_{in} and X_{in} to the patch width W by means of the gPC expansions (1), (2), with orders of expansion $P_1 = 2$ and $P_2 = 2$, where now $y = W$. Following the adaptive fitting procedure, we obtain convergence for both $f_1(y)$ and $f_2(y)$, with orders of expansion $P_1 = 2$ and $P_2 = 2$. Both

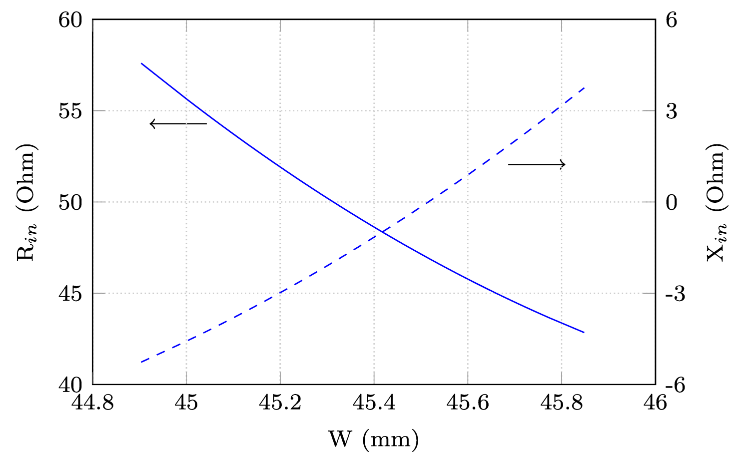

Fig. 8. X_{in} and R_{in} as a function of the patch width W according to the Stochastic Collocation Method (SCM).

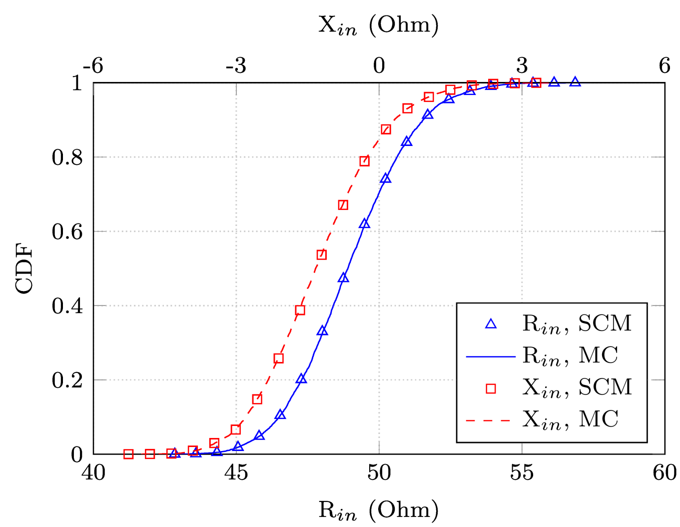

Fig. 9. Comparison between the CDFs constructed with the Stochastic Collocation Method (SCM) and the Monte Carlo (MC) simulations for the measured truncated Gaussian distribution of W.

R_{in} and X_{in}, as functions of W, are reported in Fig. 8. From the comparison between the CDFs obtained by a Monte Carlo analysis based on 10000 samples and those resulting from (1), (2), we derive that the maximum distances $D_{R_{in}}$ and $D_{X_{in}}$ are equal to 0.0083 and 0.0075, respectively. Therefore, the CDF obtained by the SCM is a good approximation for the output CDF, considering a significance level of 5%. The CDFs resulting from the Monte Carlo full-wave simulation and from the SCM are shown in Fig. 9.

V. Conclusions

In this paper, a novel stochastic framework for the variability analysis of a general figure of merit of textile patch antennas was presented. First, a representative antenna is considered and a gPC analysis is performed to accurately reconstruct the stochastic behavior of the figure of merit under study. Then, the analysis is repeated on a textile antenna known from literature to prove its general applicability and reliability, also experimentally quantifying the uncertainty on the antenna's geometrical dimensions. The results show an excellent agreement and superior efficiency with respect to the standard Monte Carlo approach, used to validate the method.

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