Concrete with triphasic conductive materials for self-monitoring of cracking development subjected to flexure

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Abstract: In this study, the carbon fiber (CF), steel fiber (SF) and nano carbon black (NCB) as triphasic conductive materials were added into concrete, in order to improve the conductivity and ductility of concrete. The influence of NCB, SF and CF on the mechanical behavior and conductivity of concrete was explored. The effect of the triphasic conductive materials on the self-diagnosing ability to the load-deformation property and crack widening of conductive concrete member subjected to bending were investigated. The relationship between the fractional change in surface impedance (FCI) and the crack opening displacement (COD) of concrete beams with conductive materials has been established. The results illustrated that there is a linear relationship between COD and FCI.

Key words: Conductive concrete, Fractional change in resistance, Crack opening displacement, Self-monitoring of cracking, Fibers, Nano carbon black, Hybrid effect

1. Introduction

The crack diagnosing is relevant for crack control in serviceability limit state and for durability requirements of concrete member. The monitoring of load-bearing capacity regarding the whole load-deflection curve can be evaluated as one of the significant points of the safety of structure member. The real-time structural health monitoring systems may able to assist engineers using non-destructive testing and early damage detection, so that proper maintenance can be applied. Electric conductive concrete could be capable of sensing its own strain and damage by the electrical resistance measurement [1-11]. The self-sensing ability of strain and damage of electrically conductive concrete can be one of the useful methods for the non-destructive evaluation of concrete members in practice. Furthermore, electrically conductive concrete also provide wide prospect in specialist applications, such as vibration control, electromagnetic shielding, traffic monitoring and de-icing [12-14].

In order to obtain electrical conductive concrete, conductive substance like NCB, CF or SF, can be incorporated into a cementitious matrix [1-11]. Electric conductive fibers, such as carbon fibers and steel fibers, are effective as admixtures for improving electric conductivity due to the formation of a continuous conducting path [13]. The NCB, with its electric conductivity, low cost and fine filler effect can also be used as ideal admixture for conductive concrete [6-10]. Prior work in the combined use of NCB and CF in cement-based materials has been reported [6-16]. The advantage of combined use of NCB and CF is the synergistic

effect, which refers to the filling (i.e. conductive filler) of the microscopic space between adjacent fibers by NCB, thereby resulting in enhancing the electric conductivity of composites [14-17]. Some previous studies on the self-sensing concrete are focused on the using of mono-phase or diphasic conductive materials, the relationship between the fractional change in resistance and the tension strain before concrete cracking has been suggested. The investigations on the strain and damage of micro carbon fiber and micro steel fiber reinforced cement specimens under compression and tension have been carried out [1-9]; but, there is a shortage of using micro fibers or NCB as self-sensing materials because they cannot keep the conductive path after concrete cracking. Until now, the investigations are mainly concentrated on the self-sensing of concrete damage before cracking [1-17]. In fact, it is important to realize that the concrete bending member works usually with cracks under service load. The concrete member may lose its serviceability by excessive cracking. The crack control is one of the key points of the serviceability, and any cracking should be limited to hairline cracks for reasons of both serviceability and durability. After all, how to self-monitor the crack open displacement and the whole load-deflection process of concrete beam is still an open problem. This work can be considered as a pioneering tentative and paved a new path for the self-diagnosing of the crack opening displacement.

One of the innovation ideas of this work is to use macro SF in the triphasic conductive materials. There are two reasons to provide macro-SF, NCB and CF as triphasic conductive materials:

- Macro SF can enhance the mechanical properties before cracking, avoid the brittle failure of concrete member during the concrete cracking and bridging the cracks;
- 2) In the whole process after cracking of the concrete beam, only the macro SF is capable to restrict the crack widening (Fig. 1b and Fig. 1c), to improve the cracking resistance and toughness, to demonstrate a stable load-deflection curve and to maintain the conductive path of the cracked concrete matrix while the NCB and micro CF are incapable to cross the crack surfaces [7, 18-19].

Based on the previous investigations on the self-diagnosing of damage before cracking by means of the relationship between FCR and strain [8], this study is mainly aimed at investigating of the self-diagnosing ability to cracking property and to



Fig. 1 Conductive network of triphasic materials in the concrete matrix (a) before cracking, b) after cracking, (c) Macro SF cross concrete crack

load-deflection behavior of concrete beam. The relationship between FCR and COD of triphasic electric conductive concrete beams under bending has been established using a tentative data return's method. The results show that the relationship between the FCR and crack opening displacement of concrete beams can be well linearly fitted.

2 Experimental Investigations

2.1 Materials and mixture design

In this test program, the base mix design of concrete beams without conductive admixture (NCB, CF and SF) was as follows: cement CEM I 42.5R 390 kg/m³, fly ash 155 kg/m³; fine aggregate 848 kg/m³ (0-5mm), coarse aggregate 822 kg/m³ (5-10mm); water 272.5 kg/m³; water binder ratio 0.5, Superplasticizer (SP) 7.63 kg/m³ (1.4% of the binder).

The NCB content with particle size ca. 30 nm – 90 nm (Fig. 2(a)) was between 0.1% and 0.3% by mass of binder (0.55 – 1.64 kg/m3), the density of NCB was about 0.5 g/cm³ and the volume resistivity was $2.3\Omega \cdot \text{cm}$. The carbon fiber content with diameter of 12-15 µm and length of 6 mm (Fig.2(b)) was between 0.4% and 1.2% by mass of binder (2.18 – 6.54 kg/m³), the density of CF was about 1.6 g/cm³ and the volume resistivity of CF was between 3 and 7 m $\Omega \cdot \text{m}$. The macro steel fiber content with diameter of 0.55 mm and length of 35 mm (Fig.1(c)) was between 4% and 8% by mass of binder (22– 44 kg/m³), the density of SF was about 7.85 g/cm³ and the volume resistivity of SF was 10⁻⁶ m $\Omega \cdot \text{m}$. A methylcellulose was used in the amount of 0.4% by mass of cement and a defoamer was used in the amount of 0.19% of sample volume. The different dosages of the conductive admixtures SF and triphasic conductive materials (BCS: NCB + CF + SF) by mass of binder in various concrete samples are compared and listed in Table 1.



Fig. 2. (a) Particle size of nano carbon black using High Resolution Field Emission SEM and (b) Carbon fiber and (c) Macro steel fiber

Serial number		NCB mass/binder%	CF mass/binder%	SF mass/binder%
Concrete containing SF	SF04	0	0	4.00%
	SF08	0	0	8.00%
Concrete containing BCS (NCB + CF + SF)	BCS 01048	0.10%(0.55kg/m ³)	0.40%(2.18kg/m ³)	8.00%(44kg/m ³)
	BCS 02084	0.20%(1.09kg/m ³)	0.80%(4.36kg/m ³)	$4.00\%(22 \text{kg/m}^3)$
	BCS 01128	0.10%(0.55kg/m ³)	1.20%(6.54kg/m ³)	8.00%(44kg/m ³)
	BCS 03124	0.30%(1.64kg/m ³)	1.20%(6.54kg/m ³)	4.00%(22kg/m ³)

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2.2 Samples and set-up description

The specimens prepared for testing were beams with the size of 100mm×100mm×400mm, demolded after 1 day and cured at room temperature in air (relative humidity = 100%) for 28 days. Then, four electrical contacts were prepared in the form of conductive adhesive tape, which was adhered on the tension side of the specimen. Based on the four probe method of electric resistance measurement, contacts A and D were for passing current while contacts B and C were for measuring of voltage [1-2, 8-9]. The relationship between FCR and flexural load-bearing capacity has been studied. We also



analyzed the relationship between FCR and COD. The FCR measured is the fractional change in surface resistance on



the tension side under bending. The dimensions and electrical contact details of all beams are shown in Fig. 3a.

2.3 Test methods

Specimens were tested under flexure over a span of 300mm in third-point loading using a hydraulic servo testing machine (MTS 810). The close-loop test was controlled by displacement, and the deformation rate of mid-span was 0.2 ± 0.02 mm/min until the specified end point deflection of 3.5 mm is reached. Two strain gages were applied at bottom of the beam for measuring the longitudinal strain on the tension side before cracking. Two LVDTs were applied on the two opposite sides for measuring deflection of mid-span. An extensometer was attached at the mid-span to measure the crack opening displacement during the test (Fig. 3b). Other experimental instruments include A.C. stabilized voltage supply, IMC Intelligence Data Collecting System, fixed resistor and AC/DC converter.

3 Influence of conductive admixtures on the mechanical and electrical properties of concrete beam

3.1 Influence of conductive admixture on the compression and flexural behavior

3.1.1 Effect of conductive materials on the compressive and flexural strength

The average values of the compressive strength (f_{cu}) of three specimens at the age of 28 d can

be found in Table 2. The increment of the compression strength ranges between 1.6% and 7.5%. The addition of NCB, CF and macro SF does not show significant trend of improving compressive strength. The flexural strength and toughness parameters (the equivalent flexural strength and energy absorption capacity) of beams with



Fig.4 Comparison of load-deflection curves of conductive concrete beams a) Curves of PC beam and beams with 22kg/m³ SF; b) Curves of PC beam and beams with 44 kg/m³ SF

different conductive admixture contents have been investigated based on German Guideline [19]. The flexural strength (σ_u) and toughness parameters are shown in Table 2. Fig. 4a shows the comparison of the load-deflection curves of plain concrete (PC) beam and beams with different NCB, CF and 22 kg/m³ SF, and the comparisons of the load-deflection relationships of PC beam and beams with different NCB, CF and 44 kg/m³ SF are illustrated in Fig. 3b. From Fig.4 and Table 2, it can be seen that:

- Compared with plain concrete (PC) beam without any conductive admixtures(Fig. 4a), the flexural strength σ_u of SF04 (beams with 22kg/m³ SF), BCS02084(beams with 1.1kg/m³ NCB, 4.36 kg/m³ CF and 22kg/m³ SF) and BCS03124 (beams with 1.65kg/m³ NCB, 6.54kg/m³ CF and 22kg/m³ SF) increased by 12.5%, 27.5% and 37.5%, respectively.
- Compared with PC beam (Fig. 4b), the flexural strength σ_u of SF08(beams with 44kg/m³ SF), BCS01048 (beams with 0.55kg/m³ NCB, 2.18 kg/m³ CF and 44 kg/m³ SF) and BCS01128 (beams with 0.55 kg/m³ NCB, 6.54 kg/m³ CF and 44 kg/m³ SF) increased by 45%, 70% and 97%, respectively. It means that the flexural strength can be improved clearly by addition of conductive materials (NCB, CF and macro SF).
- Compared with beam SF04 with 22kg/m³ SF only (Fig. 4a), the σ_u of BCS02084 and BCS03124 increased by 13% and 22%, respectively.
- Compared with beam SF08 with 44kg/m³ SF only (Fig. 4b), the σ_u of BCS01048 and BCS01128 increased by 17% and 36%, respectively. It means that the combined use of conductive admixtures (NCB, CF and SF) showed positive hybrid effect on the flexural strength.
- Compared to post-crack behavior of beams with only SF (Fig.4a and Fig. 4b), the triphasic conductive beams behave much better over the entire deflection range. The hybrid use of NCB, CF and macro SF has positive hybrid effect on the flexural toughness.

3.1.2 Effect of conductive materials on the flexural toughness

The experiment and evaluation of the flexural strength and toughness parameters (the

equivalent flexural strength/Equ. β_{BZ} and energy absorption capacity/ D_{BZ}^{f}) are carried out based on RILEM and German Guideline [19-20]. The energy absorption of conductive concrete is determined in accordance with the following expression:

$$D_{BZ} = \int_0^{\delta_i} F(x) dx$$

The post crack parameters for SF04, BCS02084, BCS03124, SF08, BCS01048, BCS01128 after 28 days are demonstrated in Table 2. From Table 2, the increase rate of the flexural strength and toughness parameters (the energy absorption and the equivalent flexural strength) with different conductive admixture contents are analyzed (Fig.5).



Table 2 Comparison of the flexural and compressive strength, the equivalent flexural strength and energy absorption of beams with different conductive admixtures

Fig. 5 Diagram of bending toughness

Samples	f_{cu}	F_u	$\sigma_{\rm u}$	$D^{\rm f}_{BZ2}$	Equ. β_{BZ2}	$D^{\rm f}_{\ BZ3}$	Equ. β_{BZ3}	Total diphasic materials
	N/mm ²	kN	N/mm ²	kN∙mm	N/mm ²	kN∙mm	N/mm ²	kg/m ³
PC	35.6	13.23	4					0
SF04	36.2	14.97	4.5	1.48	0.89	13.05	1.3	22
SF08	37.8	19.46	5.8	7.49	4.49	38.13	3.81	44
BCS02084	36.9	17.08	5.1	1.96	1.18	17.21	1.72	27.45
BCS03124	37.4	18.22	5.5	2.5	1.5	18.27	1.83	30.18
BCS01048	38.3	22.77	6.8	9.02	5.41	51.55	5.16	46.73
BCS01128	35.9	26.29	7.9	7.48	4.49	50.04	5	51.09

Where, F_u : maximum value of the load in the interval of 0.1 mm (kN); δ_1 : deflection corresponds to the F_u (mm); D^b_{BZ} : energy absorption of the unbroken concrete (kN mm); $D^f_{BZ2} = D_{BZ2} - D^b_{BZ}$ (kN mm) by $\delta_2 = \delta_1 + 0.65$ (mm); $D^f_{BZ3} = D_{BZ3} - D^b_{BZ}$ (kN mm) by $\delta_3 = \delta_1 + 3.15$ (mm); Equ. β_{BZ2} : equivalent flexural tensile strength (MPa) by δ_2 ; Equ. β_{BZ3} : equivalent flexural tensile strength (MPa) by δ_3 [19 - 20].

Based on the analyzing of Table 2, it can be observed that:

- (1) Plain concrete (PC) beam without conductive materials does not show any toughness or post crack energy absorption because PC beam shows strong brittle behavior and is broken down after cracking. Compared with PC beam, the addition of conductive admixture could enhance both the flexural strength and the post crack behaviour or flexural toughness greatly.
- (2) Compared to SF04 with 22kg/m^3 SF,
 - a) The toughness parameter (the energy absorption D_{BZ2}^{f} , D_{BZ3}^{f} , and the equivalent flexural strength Equ. β_{BZ2} , Equ. β_{BZ3}) of BCS02084 increased about 32%, when

the dosage of total conductive materials (NCB, CF and SF) in BCS02084 increased 20% only.

- b) The D_{BZ2}^{f} and D_{BZ3}^{f} , Equ. β_{BZ2} and Equ. β_{BZ3} of BCS03124 increased 68% and 40%, respectively, when the dosage of total conductive materials in BCS03124 increased by 30% only.
- (3) Compared to SF08,
 - a) The D_{BZ2}^{f} and D_{BZ3}^{f} , Equ. β_{BZ2} and Equ. β_{BZ3} of BCS01048 increased 20% and 35%, respectively, although the dosage of total conductive materials in BCS01048 increased by 5% only.
 - b) Although the energy absorption D_{BZ2}^{f} and the equivalent flexural strength Equ. β_{BZ2} of BCS01128 did not increase, the D_{BZ3}^{f} and Equ. β_{BZ3} increased by 31%, when the dosage of total conductive materials in BCS03124 increased by 15% only.

The hybrid use of NCB, CF and macro SF has positive hybrid effect on the flexural behavior. Comparing SF04 with SF08, it can be seen that the steel fiber plays a dominant role in improving the flexural behavior, especially in flexural toughness.

3.2 Influence of conductive admixture on relationship between FCR and flexural load-bearing capacity

Fig.6(a) – Fig. 6(f) illustrate the variation of the load with deflection, and of the fractional change in surface resistance (FCR) on tension side with deflection for various conductive admixture contents under bending. Compared to SF04/SF08 with mono fiber (Fig.6(a), Fig.6(b)), the signal to noise ratios [5] of FCR-deflection curves of triphsic conductive concrete are much lower which indicates that the combined use of NCB, CF and macro SF demonstrates the positive hybrid effect on the improving of signal-to-noise ratio of FCR (Fig. 6(c) – Fig. 6(f)).



Fig. 6 Relationships between load and deflection, FCR and deflection of (a) SF04, (b) SF08, (c) BCS02084, (d) BCS01128, (e) BCS03124, (f) BCS01048

A significant increment of FCR can be observed (Fig. 6(c) - Fig. 6(f)) during the first concrete cracking. For instance, the FCR of BCS01048 in Fig.6(f) increased strongly during the first concrete cracking. After cracking, the FCR increased with decreasing of load-bearing capacity, especially for a clear perturbation over the deflection range between 0.60 mm and 0.74 mm, where the load-bearing capacity dropped. Similar phenomenon can be observed for BCS01128 (Fig.6(d)) over the deflection range between 0.46 mm and 1.20 mm. This reflects that the triphsic electric conductive concrete can be very suitable for self-sensing of change of load-bearing capacity.

3.3 Influence of conductive admixture on relationship between FCR and crack open displacement

Figs.7(a) - 7(f) illustrate the relationship between FCR and crack opening displacement (COD) on the tension side of beam with different conductive admixture contents under bending. They demonstrate a monotone increasing relationship between FCR and COD, and the FCR correlates linearly with COD.



Fig. 7 Relationships between FCR and COD of (a) SF04, (b) SF08, (c) BCS02084, (d) BCS01128, (e) BCS03124, (f) BCS01048

The relationship between FCR (denoted by Y) and COD (denoted by X) can be expressed in Eqn. (1):

$$Y = a + b \cdot X$$

(1)

Where: **a** denotes the intercept and **b** denotes the slope of fitting line, both are constant parameters related to the types and the contents of electric conductive phases. The unit of X is mm. The parameters fitted and the correlation coefficients (C_R^2) are illustrated in Table 3.

Table 3 Fitted parameters of regression equation					
Serial number	Constant a	Constant b	Correlation		
			coefficient C_R^2		
SF04	5.88	2.04	0.87675		
SF08	4.87	5.55	0.88241		
BCS02084	8.37	2.09	0.92510		
BCS03124	7.41	3.44	0.91982		

BCS01048	2.04	2.80	0.98372
BCS01128	1.85	2.15	0.97193

From Table 3, it can be seen that:

- The correlation coefficient C_R^2 of all beams range from 0.88 to 0.98. It means that the relationship between FCR and COD is very strong correlated with the Eqn. (1).
- Compared with SF04 and SF08 (Macro steel fiber as the sole conductive admixture), the correlation coefficient C_R^2 of triphsic conductive concrete beams are higher, which means that the combined use of NCB, CF and SF shows positive hybrid effect on C_R^2 of concrete member.
- The parameter **b** is the slope of Eqn. (1), represents the gauge factor [5] (the fractional change in resistance per unit COD) and can reflect the self-sensing ability to COD. The gauge factors **b** of all beams range from 2.04 to 5.55.
- The parameter **a** of all beams range from 1.85 to 8.37. Generally, the value of **a** decreases with the increasing of steel fiber content. It means that the macro SF may work both as the structural materials for enhancing the post crack behavior of concrete and as the functional materials for improving the conductivity of concrete after cracking.
- The value of **a** of BCS02084 and BCS03124 is larger than the others (Table 3), it can be attributed to the decreasing magnitude of load bearing capacity and increasing of FCR when concrete is cracking (see Fig.6). Thus, the value of the intercept **a** of Eqn. (1) could reflect the reducing of load-bearing capacity at cracking. The conductive concrete is suitable for the self-sensing of crack. The hybrid use of NCB, CF and SF has positive hybrid effect on the crack sensing ability.

4. Conclusion

The purpose of this study was to explore the application of the NCB, CF and macro-SF as triphasic electric conductive materials for self-sensing of crack of concrete beam. A series of experiments and analysis on the mechanical properties like the strength and post crack behaviour, and the electric properties like the FCR of concrete beam with conductive materials have been performed, and the effects of macro SF and triphasic conductive materials on the relationships among the FRC, the load bearing capacity and deflection of concrete beam after cracking have been evaluated. The relationship between FCR and COD has been developed. The experimental and analytical results have led to the following conclusions:

- (1) Compared with plain concrete member, the addition of conductive admixture could improve the flexural strength and toughness.
- (2) The composed use of NCB, CF and macro SF demonstrates the positive hybrid effect on improving of signal-to-noise ratio of FCR.
- (3) As the load-bearing capacity declines rapidly at the first cracking point, the FCR of triphasic conductive concrete beams shows a clear linearly increasing.
- (4) The FCR increases with the decreasing of the load-bearing capacity over the entire post-crack zone of concrete beam. Triphsic electric conductive concrete beams could reflect both the crack opening displacement and the decreasing of load-bearing capacity

under bending.

- (5) The FCR correlates linearly well with crack opening displacement on the tension side of conductive concrete beam.
- (6) Compared with the concrete beam with mono steel fiber, the hybrid use of NCB, CF and macro SF shows clear positive hybrid effect on the flexural behavior and self-sensing ability to crack widening.

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