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# Theory of Rotational Processes in Perpendicular Media and Application to Anisotropy Measurement

Hong J. Zhou, Ganping Ju, Roy W. Chantrell, and Dieter Weller

**Abstract**—Numerical and analytical calculations of rotational process in perpendicular recording media are presented. The work supports recent experimental studies that suggest that the measurement of rotational magnetization processes can be used to determine the value of the anisotropy constant. An expression for the rotational magnetization for a noninteracting system is derived taking into account the dispersion of  $K$  and the easy-axis orientation. The calculations show that the experiments determine the mean value of  $H_K$ , essentially independent of the angular dispersion. A numerical (Monte-Carlo based) micromagnetic model is used to study the effects of magnetostatic and exchange interactions at nonzero temperatures. It is shown that for small values of  $KV/kT$ , irreversible magnetization processes take place, which precludes the use of the rotational magnetization method to determine  $K$  values. This effect is enhanced by the presence of the magnetostatic interaction. However, the presence of exchange interactions is found to enforce coherent rotation in small fields, reducing the irreversible processes and allowing the determination of  $H_K$ . Under these circumstances, it is shown that the exchange does not significantly affect the value of  $H_K$ , and that a well-defined demagnetization correction of  $4\pi M$  is appropriate. Finally, a comparison with experimental data gives good agreement for multilayer and granular media and shows the role of domain formation on the rotational magnetization process.

**Index Terms**—Anisotropy, micromagnetics, perpendicular media, rotational magnetization.

## I. INTRODUCTION

PERPENDICULAR recording is now a main candidate to extend magnetic recording to densities of 1 Tb. In order to achieve these densities, careful control over material properties is required. One of the most important parameters to determine is the anisotropy field of the medium. However, in perpendicular media the presence of the demagnetizing field is a considerable complication. Hard-axis hysteresis loop measured with longitudinal (in-plane) Kerr effect has typically been applied to determine the anisotropy field. However, the demagnetization correction is unknown due to the complicated and undefined domain patterns that form due to irreversible switching at higher fields. Assuming a demagnetization field of  $4\pi M_s$  in the hard-axis method overestimates the anisotropy field. The same concern also exists in full torque magnetometer method, where the demagnetization factor may vary at different field angles. Even though a single domain state is maintained at a  $45^\circ$  method, it still relies on the extrapolation to infinite field. In addition, it can be only applied to characterize samples

without a soft underlayer (SUL). A concern that arises from the overestimation of  $H_K$  is that this leads to incorrectly low ratios of short-time coercivity  $H_0$  over the anisotropy field  $H_K$ . These are normally ascribed to the presence of incoherent rotation mechanisms.

In a previous paper [1], we have shown that it is possible to use the response of a perpendicular medium to an in-plane field to determine the anisotropy field with a consistent correction of  $4\pi M$  for demagnetization. The rotational method was originally proposed and applied to measure the anisotropy field of rare-earth/transition metal magneto-optical alloys [2], and was later applied to Co-Pd (111) wedges [3] and Co-Pd (Pt) multilayer structures [4]. In [1], a magneto-optical rotational method was used to determine the anisotropy field of perpendicular media. The dependence of the perpendicular magnetization component  $M_{\text{perp}}$  on the in-plane field  $H_{\text{in-plane}}$  was measured. When the in-plane field was sufficiently small, the medium was found to remain in a single domain state, with a well-defined demagnetization field of  $4\pi M$ . Moreover, the presence of an SUL does not affect the measurement of perpendicular magnetization even when the optical penetration depth is larger than the total thickness of recording layer and seedlayer, because there is no out-of-plane magnetization from the SUL when the external field is applied in the plane. Measurements showed that it was possible to determine the anisotropy field of perpendicular media with well-defined demagnetization correction. The  $H_0/H_k$  ratio was also measured and found to decrease with increasing intergranular exchange, in good agreement with micromagnetic results. It was found to drop from about 0.8 for weakly coupled media to 0.5 for strongly coupled media. However, the success of the technique is predicated upon the absence of irreversible magnetization processes, and the theoretical analysis takes no account of interactions and also does not consider dispersions of the easy-axis orientation or of  $H_K$ .

Here, we consider the effect of dispersions of angle and  $K$  on the estimate of  $K$ . The conclusion is that the rotation measurement gives the mean value of  $H_K$  and that for the degree of angular dispersion normally involved in perpendicular media, the effect of the angular dispersion is small. Typically, for a material with a  $5^\circ$  angular dispersion, the value of  $H_K$  will be overestimated by roughly 100 Oe. However, further corrections must be applied for other degrees of texture, for example, a system with a three-dimensional (3-D) random texture is described by a quadratic relationship with a different factor. We also give the results of a computational model that gives quantitative estimates of the effect of intergranular magnetostatic and exchange interactions, and allows a close comparison between theory and experiment.

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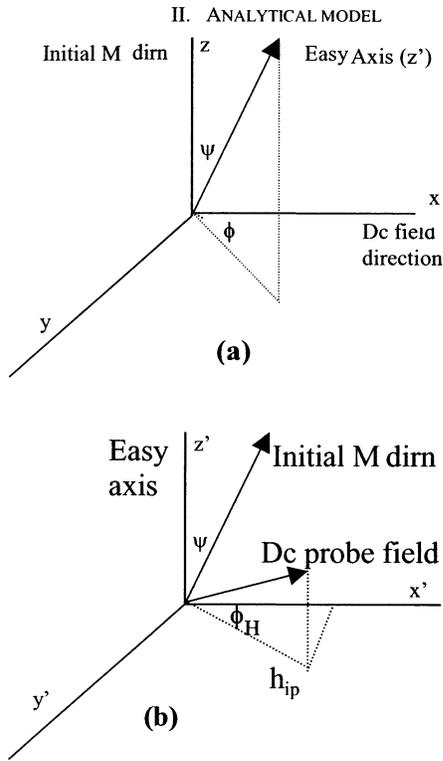


Fig. 1. Coordinate system for the calculation. (a) shows the global coordinate system indicating the initial direction of magnetization and the in-plane dc field direction. (b) shows the rotated coordinate system in which the calculation is carried out. Not shown in (b) for clarity is the direction of the magnetic moment, which has polar coordinates  $(\theta_m, \phi_m)$ .

## II. ANALYTICAL MODEL

The global coordinate system is shown in Fig. 1(a). The system is initially in the saturation remanence state along the  $z$  axis, with the in-plane dc probe field being applied along the  $x$  axis. Each grain has an easy-axis direction that is given by polar coordinates  $(\psi, \phi)$ . We assume that the orientational texture is symmetric about the  $z$  axis with a Gaussian distribution of  $\psi$  and a uniform distribution of  $\phi$ . It is easiest to carry out the calculation in a coordinate system based on the easy-axis direction, which is the equilibrium direction in the absence of an applied field. Thus, the new coordinate system has  $z'$  along the easy axis, and we take the  $x'$  axis to be in the plane defined by the easy axis and the initial magnetization direction. In this coordinate system, the in-plane dc field (normalized with respect to  $H_K$ ) can be characterized by a  $z$  component  $h_z$  and an in-plane component  $h_{ip}$ , which makes an angle  $\phi_H$  with the  $x$  axis. The energy can then be written as

$$E = \sin^2 \theta - 2h_z \cos \theta - 2h_{ip} \cos \phi_H \sin \theta \cos \phi_m - 2h_{ip} \sin \phi_H \sin \theta \sin \phi_m \quad (1)$$

where  $(\theta, \phi_m)$  are the polar coordinates of the magnetic moment. Differentiation with respect to  $\phi_m$  and setting the result to zero leads to the condition  $\phi_m = \phi_H$ . Differentiation with respect to  $\theta$  and using the condition  $\phi_m = \phi_H$  gives the following equation to be solved for the equilibrium orientation  $\theta_m$ :

$$\sin \theta_m \cos \theta_m + h_z \sin \theta_m - h_{ip} \cos \theta_m = 0. \quad (2)$$

To first order in small quantities, this leads to the result

$$\sin \theta_m \approx h_{ip}. \quad (3)$$

The magnetization relative to the original direction of the magnetization is

$$M = \cos \vartheta_m \cos \psi + \sin \vartheta_m \cos \phi_H \sin \psi. \quad (4)$$

The second term on the right-hand side of (4) vanishes by symmetry on integration over  $\phi_H$ , and using (3) gives

$$M = \cos \psi (1 - h_{ip}^2/2). \quad (5)$$

In order to determine the dc field components in the rotated coordinate system, we use the Euler rotation matrix for a rotation of the polar coordinate, which leads to the following expression for  $h_{ip}$ :

$$h_{ip}^2 = h_{dc}^2 (\cos^2 \psi \cos^2 \phi + \sin^2 \phi). \quad (6)$$

On substituting (6) into (5) and integrating over  $\phi$  with the appropriate weight function of  $1/2\pi$  for the uniform angular distribution, we have that

$$M = \cos \psi (1 - h_{dc}^2 (\cos^2 \psi + 1) / 4). \quad (7)$$

The change of magnetization produced by the rotation of the magnetization from the initial equilibrium direction is

$$\Delta M_\psi = -\cos \psi h_{dc}^2 (\cos^2 \psi + 1) / 4. \quad (8)$$

Taking into account the dispersion in  $K$  and  $\psi$ , we have that

$$M_z = 1 - \frac{H^2}{H_k^2} \int_0^\infty dy y^{-2} f(y) \sqrt{\frac{2}{\pi}} \sigma_\psi^{-1} \times \int_0^{\pi/2} d\psi (\cos^3 \psi + \cos \psi) \exp(-\psi^2/2\sigma_\psi^2) \quad (9)$$

where  $f(y)$  is the distribution of  $H_k$  with  $y = H_k/H_{km}$ , with  $H_{km}$  the median value. Taking the distribution as lognormal having a standard deviation  $\sigma_K$

$$f(y) = \frac{1}{\sqrt{2\pi}\sigma_K} \exp(-(\ln y)^2/2\sigma_K^2)$$

gives

$$\int_0^\infty dy y^{-2} f(y) = e^{2\sigma_K^2}$$

and, therefore, (9) becomes

$$M_z = 1 - \frac{H^2}{H_k^2} e^{2\sigma_K^2} \sqrt{\frac{2}{\pi}} \sigma_\psi^{-1} \int_0^{\pi/2} d\psi (\cos^3 \psi + \cos \psi) \exp(-\psi^2/2\sigma_\psi^2). \quad (10)$$

The integral over  $\psi$  can be written in terms of the error function, which is useful for numerical evaluation, but we can also use

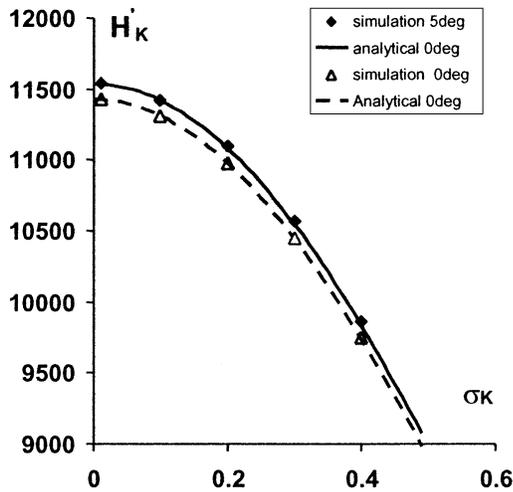


Fig. 2. Effect of the parameter dispersion on the apparent value of  $H_K$ . Symbols: numerical calculations. Solid lines: analytical expressions.

the fact that  $\sigma_\psi$  is very small for a perpendicular medium. This means that we can replace the upper limit of the integration by infinity, leading to the closed-form solution

$$\sqrt{\frac{2}{\pi}} \sigma_\psi^{-1} \int_0^{\pi/2} d\psi \cos^2 \psi \exp(-\psi^2/2\sigma_\psi^2) = \frac{1}{4} e^{-9\sigma_\psi^2/2} (1 + 7e^{4\sigma_\psi^2}) \quad (11)$$

and using the relationship between the mean ( $\overline{H_K}$ ) and the median for the longnormal distribution (LND)

$$M_z = 1 - \frac{H^2}{H_k^2} \cdot \frac{1}{16} e^{-9\sigma_\psi^2/2} (1 + 7e^{4\sigma_\psi^2}). \quad (12)$$

Finally, given that  $\sigma_\psi$  is small

$$M_z^2 = 1 - \frac{H^2}{H_k^2} \cdot (1 - \sigma_\psi^2). \quad (13)$$

For an angular dispersion of  $5^\circ$ ,  $\sigma_\psi = 0.09$  leading to a correction to  $H_K$  of less than 1%. However, the texture is otherwise important.

Numerical calculations support the weak dependence on  $\sigma_\psi$ . The introduction of a  $5^\circ$  dispersion of orientation produced an increase in the apparent  $H_K$  of 109 Oe for a material with a real  $H_K$  of 11 429 Oe, an increase of 0.95%.

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Fig. 2 demonstrates the effect of the parameter dispersion. This shows the apparent value ( $H'_K$ ) determined from the rotation measurement as a function of the dispersion of the  $H_K$  value. The median value is  $H_{km} = 11 429$  Oe, and the two sets of data refer to a fully aligned system and one with an angular dispersion of  $5^\circ$ . Solid lines are a fit to the expression  $H'_K = H_K^0 \exp(-\sigma_K^2)$ , which relates the mean to the median for the LND. For the case of a fully aligned system,  $H_K^0 = 11 426$  Oe, which agrees well with the expected value of

$H_K$ . For the system with  $\sigma_\psi = 5^\circ$ ,  $H_K^0 = 11 536$  Oe, which is essentially in exact agreement with (7).

### III. COMPUTATIONAL MODEL

The model uses Monte-Carlo (M-C) techniques to simulate the long-term slow dynamic behavior, and is described in detail by Chantrell *et al.* [5]. The physical microstructure of the model is generated using a Voronoi construction, which produces a physically realistic picture of the film, including grain size dispersion and some microstructural disorder. A lognormal distribution for diameters and anisotropy fields was used. In order to avoid problems concerning demagnetizing fields, a periodic boundary condition was used.

Interactions are taken into account by calculating the total field acting on a particle as the total of the external field  $H_{ext}$  and the dipolar and exchange field produced by the neighboring particles. The magnetostatic term is evaluated by a direct summation and the exchange term is represented by a factor  $C^*$ , which characterizes the exchange field relative to the anisotropy field.

The equilibrium orientations of the magnetic moment are found using the Stoner–Wohlfarth [6] model. However, this model may give two equilibrium positions if the total field acting on the particle is smaller than the critical field. In this case, the actual equilibrium position is chosen to be the one that is closer to the previous equilibrium position of the grain, but thermally activated transitions between the two positions are allowed. To do that, one calculates the transition probability

$$p_r = 1 - e^{-t_m/\tau} \quad (14)$$

where  $t_m$  is the measuring time and  $\tau$  is the relaxation time that has the expression

$$\tau^{-1} = f_0 \exp(-\Delta E/k_B T) \quad (15)$$

where  $\Delta E$  is the height of the total energy barrier for reversal, and  $f_0 = 9 \times 10^9$  Hz is the frequency factor. The expression for the free energy of a particle is

$$F = K \sin^2 \alpha - \mu \mathbf{H}_T \cdot \mathbf{M}_s \quad (16)$$

where  $K$  is the anisotropy constant,  $\alpha$  is the angle between the particle's easy axis and magnetic moment  $\mu = M_s V$ , and  $M_s$  is the saturation magnetization of the material. This probability is compared to a random number  $x$  between 0 and 1. If  $x > p_r$  the transition is not allowed, otherwise, the transition is allowed. It can be shown using a master equation approach [7] that the probability for the moment to go to one of the two positions is

$$p_i = \frac{\exp(-E_i/k_B T)}{\exp(-E_1/k_B T) + \exp(-E_2/k_B T)} \quad (17)$$

where  $i = 1, 2$ , and  $E_i$  represents the total energy of the grain in the  $i$ th orientation, so that the system will have a Boltzmann distribution of energies. Essentially, the use of (17) ensures that any superparamagnetic particles have zero remanence and coercivity. All these steps are used for every particle several times so that the system reaches the thermal equilibrium state.

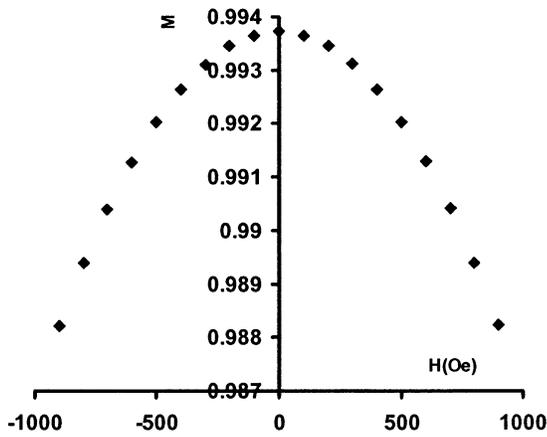


Fig. 3.  $M$  versus in-plane field at a temperature of 5 K for a system with magnetostatic interactions only.

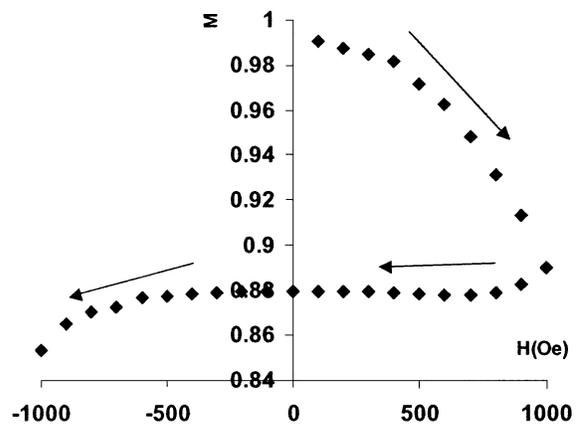


Fig. 4. Magnetization as a function of in-plane field for a medium with magnetostatic interactions only at a temperature of 300 K. Arrows show the direction of field change.

TABLE I  
 $H_K$  VALUES (CORRECTED FOR DEMAGNETIZATION WITH A DEMAGNETIZING FACTOR OF  $4\pi$ ) FOR SYSTEMS WITH VARIABLE EXCHANGE AND MAGNETOSTATIC INTERACTIONS AT  $T = 5$  K AND  $T = 300$  K

	$H_K, T=5K$	$H_K, T=300K$
Non-interacting	12365	12365
Magn. only	13026	-
Magn. + exchange	12942	12998

IV. RESULTS

We have used the computational model to study the effects of interactions on the rotational magnetization process. We are particularly concerned with the onset of irreversible processes since this marks the point at which the rotational magnetization becomes nonquadratic in the field and cannot be used to determine the value of  $H_K$ . For these calculations, we include the shape anisotropy of the particles. In this case ( $M_s = 350$  emu/cm<sup>3</sup> and an aspect ratio of 2 : 1), this increases  $H_{km}$  to 12 365 Oe. Consider first calculations for a low temperature ( $T = 5$  K) and magnetostatic interactions only.

Fig. 3 shows  $M$  versus in-plane field at a temperature of 5 K for a system with magnetostatic interactions only. It can be seen that the quadratic behavior is preserved in this case. The value of  $H_K$  (corrected for demagnetization with a  $4\pi$  demagnetization factor, giving a field of 4398 Oe) is 13 026 Oe, which is slightly larger than the expected value of 12 365 Oe, suggesting that the rotation method gives a reasonable estimate of  $H_K$  in this case. However, the applicability of the method depends crucially on the absence of an irreversible component of the magnetization, which itself depends on interactions and also the temperature. We have carried out a series of calculations for interacting and noninteracting systems at temperatures of 5 K and 300 K, the results being summarized in Table I.

The results obtained give a consistent value of  $H_K$  except for the exchange decoupled system at room temperature. For the parameters chosen, the mean value of  $KV/kT$  at room temperature was 58, which gives rise to a strong irreversible component of the magnetization as shown in Fig. 4.

Here, the nucleation field is positive because of the reduction in coercivity due to the increase in temperature. This leads to

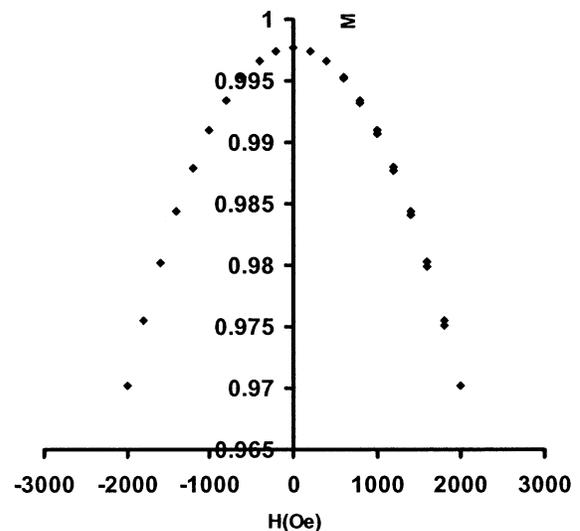


Fig. 5. Rotational magnetization as a function of in-plane field for a medium with strong exchange coupling ( $C^* = 0.1$ ) at  $T = 300$  K.

irreversible behavior and breakdown of the theory of the rotational process; hence, no value of  $H_K$  can be estimated from the data. Interestingly, this tendency is overcome by the introduction of exchange coupling, as shown in Fig. 5.

The effect of the exchange coupling is to remove the irreversible behavior present in the case of the exchange decoupled system, leading to a reasonable value of  $H_K$  after correction with a demagnetization factor of  $4\pi$ . The effect of the exchange coupling on the determined value of  $H_K$  is shown in Fig. 6. It can be seen that for the particular medium considered, there is a slight increase in the effective value of  $H_K$  with exchange coupling, but the magnitude of the change is rather small, and for realistic values of  $C^*$ , the exchange coupling does not apparently have a significant effect on the effective value of  $H_K$ . The effect of interactions can be seen more prominently in the rotational magnetization loop extended to large fields. It is found that the form of the loop depends strongly on the type of material and the strength of the intergranular exchange coupling. As examples, we consider the experimental data given in [1], which

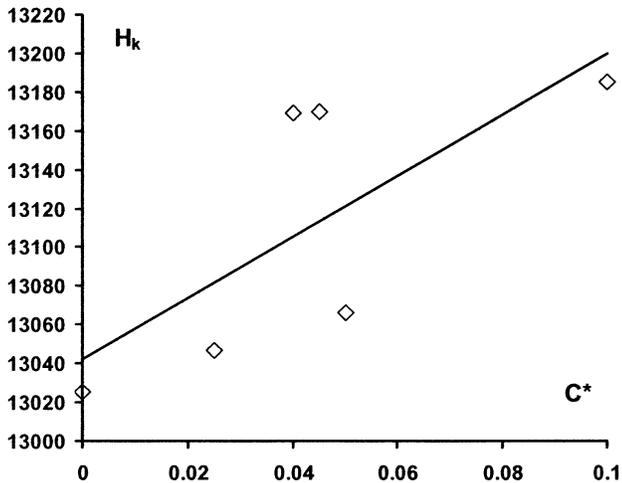


Fig. 6. Effective value of  $H_K$  as a function of  $C^*$  for a perfectly aligned medium with a diameter dispersion  $\sigma_D = 0.03$ , and  $\sigma_K = 0.1$ . The value of  $KV/kT = 95$ .

were representative of the behavior of granular and multilayer (ML) materials. In particular, the ML showed a strong deviation from quadratic behavior at around 9 kOe, which was accompanied by the formation of a pronounced domain structure.

Fig. 7 shows the calculated rotational magnetization curve for a simulated multilayer medium. The parameters for the calculation are grain diameter 12 nm, film thickness 17 nm,  $K = 3.8 \times 10^7$  erg/cm<sup>3</sup>, and  $M_s = 395$  emu/cm<sup>3</sup>. The dispersions of size, angle, and  $K$  are rather narrow ( $\sigma_D = 0.03$ ,  $\sigma_\theta = 1^\circ$ , and  $\sigma_K = 0.1$ , respectively). The grains are also strongly exchange coupled, with the parameter  $C^* = 0.03$ . The calculations shown in Fig. 6 are qualitatively similar to the data for the ML material given in [1] and have a similar value of  $H_K = 20.7$  kOe (corrected for a demagnetization factor of  $4\pi M$ ) in comparison with the experimental value of 19.1 kOe [1]. This value of  $H_K$  is also close to the value of 20.3 kOe expected for a noninteracting system, indicating that again the rotational method gives a reasonable value of  $H_K$  even for a system with magnetostatic and exchange interactions. As before, the effect of exchange is to stabilize the magnetization in small fields, leading to an extended range over which quadratic behavior is observed. This is demonstrated in Fig. 6 by the inclusion of data calculated for an exchange decoupled system. The value of  $H_K$  calculated for the exchange decoupled system is the same as that calculated in the presence of exchange coupling. The effect of exchange coupling, in addition to giving rise to the extended range of quadratic behavior, is also to give a rather more rapid change of the magnetization in large fields. In agreement with experiment, we find this to be associated with the formation of a quasi-domain structure due to the intergranular exchange interactions.

Fig. 8 shows a grayscale representation of the perpendicular magnetization component in a field of 10 kOe, which corresponds to a magnetization  $M_z = 0.5$ .

Fig. 9 shows the domain structure for the ML medium close to zero magnetization in an in-plane field of 15 kOe. There is an extended stripe-domain-like structure, which is in good agreement with the experimental data. We have also carried out a comparison with the rotational magnetization curve measured

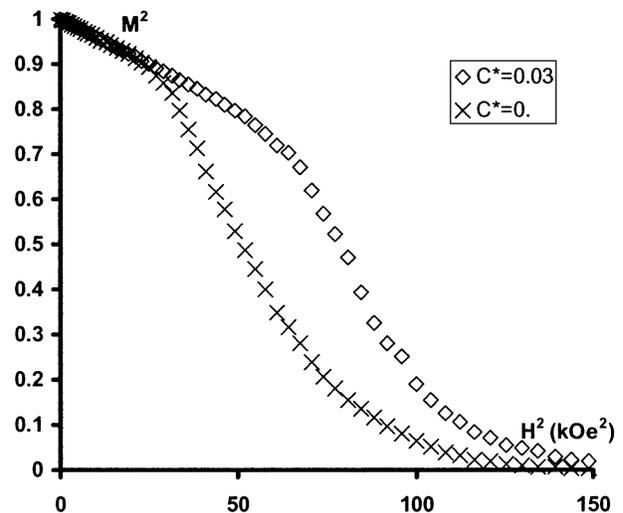


Fig. 7. Rotational magnetization curve for a simulated multilayer medium. Calculations are given for an exchange decoupled system and a medium with  $C^* = 0.03$ .

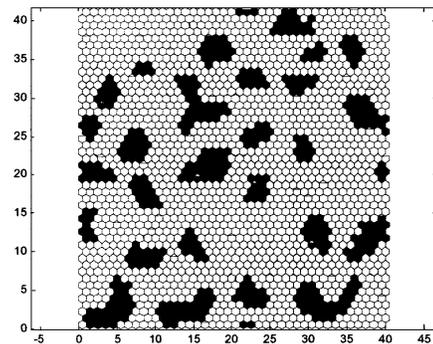


Fig. 8. Domain structure for the exchange coupled ML in a field of 10 kOe corresponding to a magnetization  $M_z = 0.5$ . Dimensions are in units of the median particle diameter.

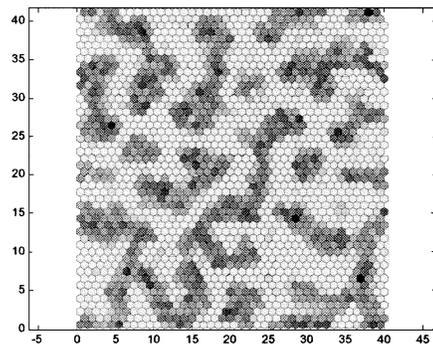


Fig. 9. Domain structure for the exchange coupled ML in a field of 15 kOe corresponding to a magnetization  $M_z = 0.05$ . Dimensions are in units of the median particle diameter.

for a granular medium. A good qualitative fit to the experiments, shown in Fig. 10, was found for  $D = 11$  nm,  $t = 22$  nm,  $K = 2.2e6$  erg/cm<sup>3</sup>, and  $M_s = 350$  emu/cm<sup>3</sup>. The parameter dispersion values are rather larger than for the ML medium, specifically  $\sigma_D = 0.3$ ,  $\sigma_\theta = 5^\circ$ , and  $\sigma_K = 0.3$ . The grains are also less strongly exchange coupled, with a value of  $C^* = 0.015$ .

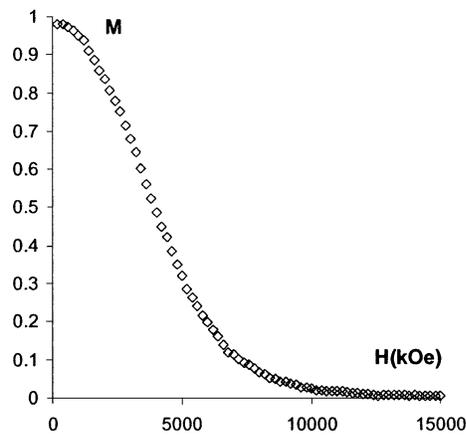


Fig. 10. Rotational magnetization curve for a medium with parameters consistent with a granular medium.

The combination of weak exchange coupling and large parameter dispersion leads to a nucleation dominated reversal, which is consistent with the magnetic force microscopy images given in [1].

## V. CONCLUSION

We have carried out a theoretical and computational analysis of rotation processes in perpendicular media. It is found that thermally activated processes can lead to irreversible magnetization changes in small fields, which can be exacerbated by the presence of magnetostatic interactions. The introduction of exchange coupling forces coherent rotation onto the system and reduces the irreversible magnetic behavior, leading to estimates of  $H_K$  that are weakly dependent on the exchange. The

computational model has been qualitatively fitted to rotational magnetization curves for multilayer and granular media. The fits give reasonable values for the material parameters and also predict domain structures that are consistent with experimental data. The form of the curves are dependent on the dispersion in materials properties in addition to the exchange coupling, and this suggests that the rotational magnetization curve is a useful source of information for the determination of materials parameters.

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