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Assessment, diagnosis and strengthening of Outeiro Church, Portugal

P.B. Lourenço *

Department of Civil Engineering, School of Engineering, University of Minho, Azurém, P – 4800-058 Guimarães, Portugal

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Abstract

Historical structures are particularly difficult to analyze due to the lack of mechanical data. Here, a case study is fully detailed, including the aspects of historical, damage and geometric investigations, of advanced numerical analysis, of justification of remedial measures and of detailing the adopted strengthening. The paper also advocates that significant information can be obtained from numerical analysis, namely with respect to the understanding of existing damage and to the minimum and adequate design of strengthening. A clear understanding of the structural behavior and reliable strengthening, based on sophisticated tools of structural analysis, can therefore reduce the extent of the remedial measures in the restoration of ancient structures.

Keywords: Masonry; Historical structures; Structural analysis; Modeling; Strengthening

1. Introduction

The analysis of ancient constructions poses important challenges because of the complexity of their geometry, the variability of the properties of traditional materials, the different building techniques, the absence of knowledge on the existing damage from the actions which affected the constructions throughout their life, and the lack of codes. In addition, restrictions in the inspection and the removal of specimens in buildings of historical value, as well as the high costs involved in inspection and diagnosis, often result in limited information about the internal constructive system or the properties of existing materials.

Nevertheless, significant advances occurred in the last decade concerning the development of adequate tools for the numerical analyses of historical structures [1] and international recommendations have been recently made available [2]. These recommendations are intended to be useful to all those involved in conservation and restoration problems of structures, and contain basic concepts of conservation, together with the rules and methodology that a designer should follow. More comprehensive information on techniques and specific knowledge can be found, e.g. [3–6]. In addition, normative and pre-normative are gradually becoming available, e.g. [7–9], at least with respect to seismic rehabilitation, which is a major concern.

But structures of architectural heritage, by their very nature and history (material and assembly), present significant challenges in conservation, diagnosis, analysis, monitoring and strengthening. These aspects call for qualified analysts that combine advanced knowledge in the area and engineering reasoning, as well as a careful, humble and time-consuming approach. This paper provides a relevant case study that demonstrates the careful use of inspection tools and numerical analyses to tackle practical engineering problems in the field of historical constructions.

* Tel.: + 351 253 510 200; fax: +351 253 510 217.
E-mail address: pbl@civil.uminho.pt.

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2. Investigation and field survey

The church of Saint Christ in Outeiro (Bragança), in the North of Portugal, was built in 1698–1738. The structure is mostly made of local shale stone and thick lime mortar joints (rubble masonry), even if regular masonry (granite ashlars and dry/thin joints) is used in doors/windows frames and the façade, see Fig. 1. For a complete report on this structure please refer to [10].

The structure is of moderate size (38×22 m^2 in plan, with a height of 13 m for the nave and of 22 m for the towers), see Fig. 2. The façade is a piece of significant architectural value due to the false twin arch and the cladding in granite, with pilasters and cushion windows. It possesses a large circular window and a balustrade that connects the bell-towers. The bell towers have a quadrilateral shape in plan, with small openings; the right tower has a staircase in the shape of a snail, while the left tower is hollow with the exception of the top part, where a staircase and pavement, connected to the choir, allow the access to the bells. The side view exhibits framed galleries or galilees, forming three chapels that communicate between themselves by small door openings.

The interior of the church has a single nave (Latin cross) with crossed vaults, split in three parts by two diaphragm arches. The arches are supported by the side walls and by the transversal walls that divide the chapels, which act as buttresses. The choir consists of a granite portico, formed by three arches arranged perpendicularly to the nave, supported by two slender Doric columns. The columns bear also granite arches parallel to the diaphragm arches, which support clay masonry crossed vaults. These vaults have been built in the last years of construction (1726–1738), replacing an existing choir in timber floor. No connection is present between the vaults and the confining walls, with the exception of the stone brackets that support the arches parallel to the diaphragm arches.

2.1. Observed damage

Damage of the structure is localized in the main façade and in the choir. Additional minor cracking at very few locations, of no concern for stability purposes, is visible in other parts of the structure. The façade features large displacement, downwards vertically, in the vicinity of the twin arch, and horizontally, towards the outside, around the central opening and at the top central part of the façade, see Fig. 3. The interior face of the wall is straight and, with the help of a boroscopic camera (HSW Henke-Sass Wolf – PRZ06-0550-VAR-50, equipped with a rod of 0.50 m, a vision angle between 60° and 120°, and a zooming range of 10–20 times), it was possible to check that the wall is made of two leaves,

Fig. 1. Aspects of the irregular masonry in non-plastered areas, including ashlars around window openings.

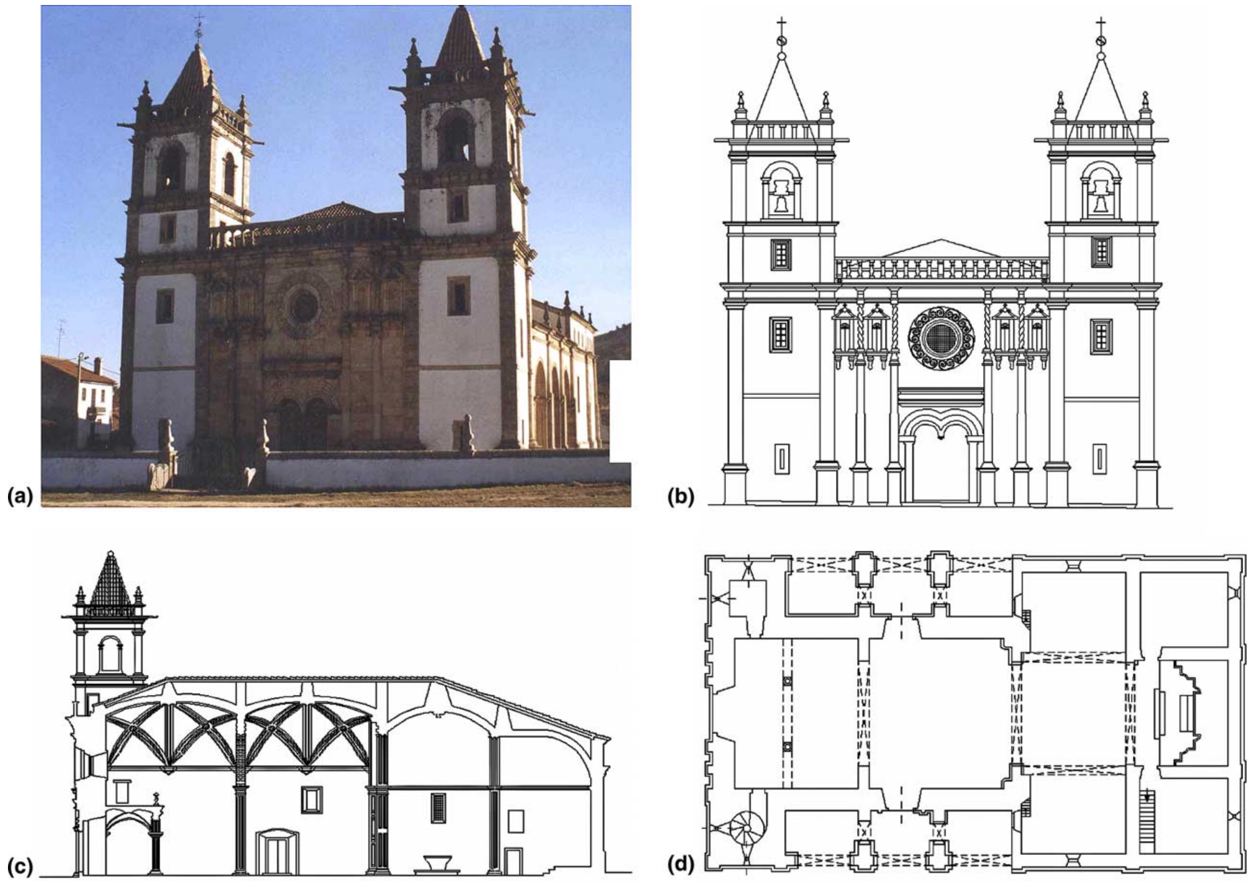


Fig. 2. Geometry of the church: (a) perspective; (b) front view; (c) longitudinal section; (d) plan section.

separated by a crack that reaches approximately 0.10 m width in the vicinity of the opening, see Fig. 4. Out-of-plumb movements are also visible in the granite balustrade that connects the bell-towers, as two of the stone blocks of the top rail have relative horizontal movements of around 0.02 m. It is noted that the stone ashlars of the rail could have been connected with iron dowels in the past (cavities are present but dowels are absent).

The choir features extensive cracking of the arches and vaults, see Fig. 5, being supported by temporary propping at the time of the first visit. The choir is separated from the façade, except in the brackets, and the columns are out of plumb 2.6% (around 0.08 m). The balustrade of the choir is straight, even if the diaphragm arches perpendicular to the nave exhibit significant curvature in plan, see Fig. 6, and the joints of the portico are fully filled with no sign of damage.

All movements do not seem to have occurred recently.

2.2. Masonry characterization

As detailed above, the construction is mostly made of three different masonry types, namely: (a) irregular shale masonry with thick lime mortar joints, used in general; (b) regular granite masonry with thin lime mortar joints, used as cladding of the façade, as window and door frames, as columns and as vault ribs; (c) brick masonry for the vault ribs. Several different techniques can be used for the mechanical characterization of these masonry types, e.g. [11], with particular reference to flat-jack testing and coring.

In the present case of very localized damaged, a complete characterization of these materials was not aimed at due to budgetary reasons and the cost benefit analysis carried out. The benefit to be obtained by opening up the structure in terms of reduced structural intervention must always be weighted against the loss of culturally significant material. Information was already available from similar materials [12,13] and the following conservative values could be estimated from this experience: (a) $E = 1$ GPa, $\nu = 0.2$, $f_c = 1.0$ N/mm^2, for irregular masonry; (b) $E = 6$ GPa, $\nu = 0.2$, $f_c = 6.0$ N/mm^2, for regular masonry; (c) $E = 3$ GPa, $\nu = 0.2$, $f_c = 3.0$ N/mm^2, for brick masonry, where E indicates Young's modulus, ν indicates the Poisson ratio and f_c indicates the compressive strength. In all cases, and as usual for historical structures, tensile strength can be considered equal to zero.

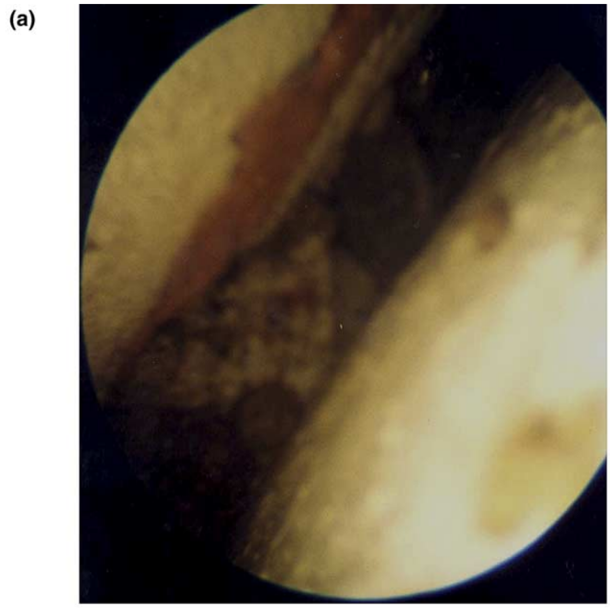
(a)

(b)

Fig. 3. Damage in the façade: (a) twin arch and (b) opening.

2.3. Geotechnical characterization

The geotechnical survey included pits for foundation inspection of the towers and main façade, and plate indentation tests plus SPT tests for the evaluation of the mechanical properties of the soil. The pits allowed to verify that the foundations are built with irregular granite blocks joined by lime mortar joints, with a depth ranging from 0.35 to 0.70 m. The plate indentation tests were carried out up to a stress level of 500 kN/m^2. The SPT tests indicated a layer of organic soil, mostly of silty–sandy nature, with a thickness up to 0.40 m. Below this layer, the substrate is constituted by deteriorated shale, varying between 35



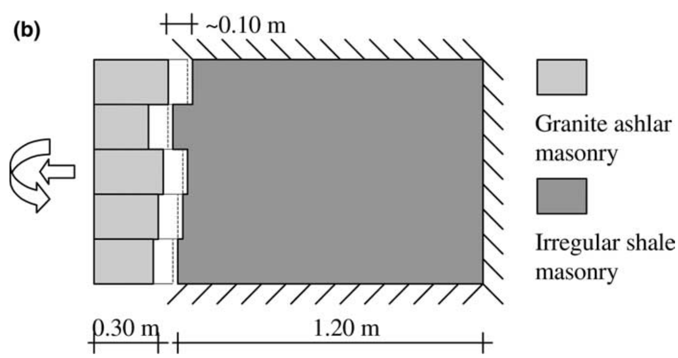


Fig. 4. Wall of the main façade: (a) inspection with a boroscopic camera and (b) observed movements.

and over 60 strokes, being the last value obtained frequently.

The obtained geotechnical parameters for the foundation soil were: specific weight $\gamma_G = 20$ kN/m^3, internal friction angle $\phi' = 36°$, cohesion $c' = 0.03$ N/mm^2 and vertical elasticity modulus $E = 45$ N/mm^2. In short, the soil possesses high strength and low deformability mechanical characteristics, meaning that no correlation exists between the geotechnical behavior of the foundation materials and the existing damage.

2.4. Historic seismology

The information related to the historic seismology must be considered carefully, because data from the past is usually scarce and no strong earthquakes hit the region in the last century, providing reliable seismic records. Nevertheless, there are no doubts about past seismic activity in the area and an active fault is located close to Outeiro. Documented earthquakes of intensity VI (modified Mercalli scale) occurred in Miranda do Douro and Izeda (only 20 km away from Outeiro) [14]. In particular, the earthquakes that hit Torre de Moncorvo, a city on the same fault 60 km away, are very well documented. In December 19, 1751 (11:00) and March 19, 1858 (13:30), strong shakes hit the region. The last shake, of intensity VII in this city, provoked the fall of a large stone in the cornice of the church and a full crack, from top to bottom, in the main façade (visible today). The houses moved violently and rendering fell from ceilings and walls. This shake, followed by eight replicas, was felt in all the Northern part of the country, in Coimbra, Aveiro and Viseu (all important towns up to 175 km of Torre de Moncorvo), and provoked several damages in various buildings of this last city (intensity VI).

2.5. Preliminary conclusion

The damage of the structure concentrates in the main façade and choir, which exhibit large out-of-plane displacements. These displacements are ancient and seem not to be progressing. The geotechnical conditions of

Fig. 5. Aspects of the damage in the choir: (a) cracked ribs; (b) temporary support; (c) columns out-of-plumb; (d) no connection between vaults and walls.

the site are not correlated with the damage. The church was probably affected by, at least, two earthquakes of intensity VI (modified Mercalli scale); it is possible that other, non-documented, shakes of higher intensity hit the building because Outeiro is a small village, with low population and low regional importance. Therefore, earthquakes are likely to be responsible for the separation between the leaves of the main façade and the choir damage, which is poorly built (no connection between the vaults and the neighboring walls).

3. Structural analysis

In order to assess the safety of the structure and clearly justify the damage described in the previous

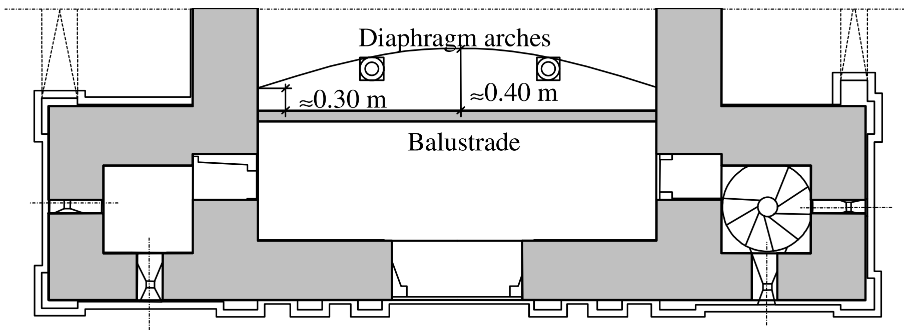


Fig. 6. Detail of choir in plan, including curvature of diaphragm arch versus straight balustrade, and adopted thickness for the plane stress model (shaded).

section, three different models of increasing complexity have been adopted for the façade and one model has been used for the choir. Modeling, which resorts to the finite element method, included linear elastic and non-linear behavior of the material. As stressed in [1], it is essential to verify the adequacy of the models with the existing building. This can be carried out using a number of techniques, namely flat-jack testing or dynamic identification, but also a comparison with the damage survey (in terms of cracks, displacements, tilting, etc.). In the present case, the adopted model validation technique was the comparison with the damage exhibited by the church. Nevertheless, it is stressed that the numerical analyses carried out aim at justifying the damage exhibited by the structure and at understating its behavior, and not at defining its safety or at detailed strengthening.

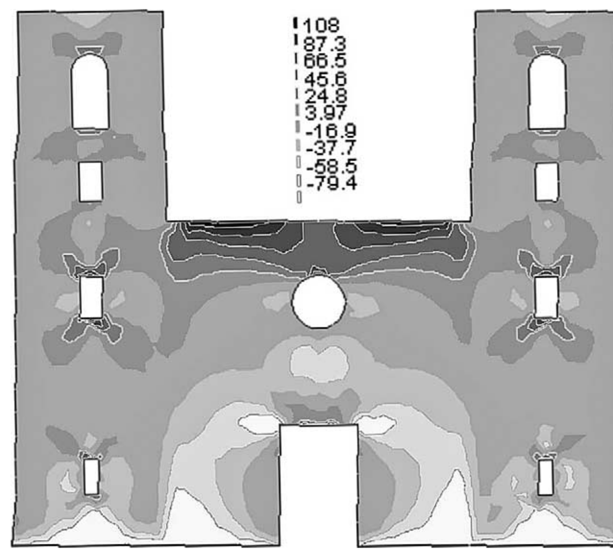


Fig. 7. Plane stress analysis for vertical loading – picture of the maximum (tensile) principal stresses (10–3 N/mm^2) on the deformed structure.

3.1. Analysis of the façade subjected to vertical loading

For the analysis of the façade subjected to vertical loading a plane stress representation has been adopted, assuming a homogeneous material. The thickness of the façade is given in Fig. 6, in order to simulate the three-dimensional shape of the structure.

Eight-noded isoparametric finite elements were adopted in the analysis. As detailed in Section 2, the adopted mechanical properties were: Young's modulus of 1000 N/mm^2 and Poisson's coefficient of 0.2. The vertical loads considered in the analysis included the self-weight of the structure, the weight of the pyramidal roof of the bell towers and the weight of the main nave roof. The soil–structure interaction has been modeled by interface elements, with elastic properties obtained from the in situ testing of the soil [15].

The results of the analysis, assuming linear elastic behavior of the material, are given in Fig. 7, in terms of maximum (tensile) principal stresses. Under the most unfavorable hypothesis that all permanent loads are applied simultaneously and not in agreement with the construction phases, it is observed that the maximum value of the tensile stresses and the compressive stresses are extremely low, respectively, +0.11 (tension) and −0.54 (compression) N/mm^2.

Even if the maximum stress values are questionable in the framework of linear elastic analysis (the values depend on the mesh refinement), this analysis demonstrates that, under the assumption of a homogeneous material for the façade, the structure should not present any damage due to vertical loading. Indeed, the structure is robust and very stiff. The localized damage of the structure, the signs that the movements of the structure have stabilized and the excellent conditions of the foundations/soil, indicate that the damage is due to seismic action. Earthquakes are normally unexpected in the North of Portugal but, in the vicinity of the structure, shakes were reported in 1751 and 1858 (Mercalli intensities VI and VII, respectively). For this reason, the analysis of the structure under combined actions (vertical and horizontal) seemed necessary.

3.2. Analysis of the façade subjected to vertical and horizontal loading

For this analysis, the model of the previous section was adopted but the plane stress elements have been replaced by eight-noded isoparametric shell elements. For the purpose of horizontal loading, it was assumed that the walls normal to the façade would act as shear walls, preventing any horizontal movement in this direction.

The vertical loading is the same as in the previous section. To assess the behavior of the structure under earthquakes, a simplified approach was used, replacing the seismic action by equivalent horizontal loading. According to the Portuguese Code, the horizontal loading is given by 6.6% of the mass, for this particular region in Portugal.

Assuming that the top of the façade is capable of moving freely and adopting linear elastic behavior for the material, the results of the analysis are given in Fig. 8, in terms of maximum (tensile) principal stresses. Again, the maximum value of the tensile stresses and the compressive stresses are extremely low, respectively, +0.12 (tension) and −0.51 (compression) N/mm^2. Moreover, the maximum displacement normal to the façade is only 0.4 mm, which cannot be compared with the value of 0.10 m observed.

From these results it was concluded that the assumption of a homogeneous material is incapable of justifying the damage exhibited by the structure, even with the action of the horizontal forces equivalent to the seismic load. It is, however, stressed that the shape of the deformation shows some agreement with the observed deformation of the real structure.

The previous analyses demonstrated the need of representing the multiple leaves of the façade wall. For the purpose of a non-linear dynamic analysis, the wall was modeled with two different leaves, see Fig. 4: granite ashlars with an average thickness of 0.30 m and shale masonry with and average thickness of 1.20 m, with the elastic properties provided in Section 2. For this purpose, fully three-dimensional twenty-noded elements were considered for the leaves, connected with sixteen-noded plane interface elements. The continuum elements are kept linear elastic (same mechanical properties as above), while the interface elements have zero stiffness in tension and infinite stiffness in compression, in such a way that interpenetration is not allowed and no tensile strength exists between the leaves.

Raleigh damping was assumed [16], in which the damping matrix C is considered a linear combination of the mass M and the stiffness K matrices:

$$C = \alpha M + \beta K, \qquad (1)$$

where the α and β coefficients are given by

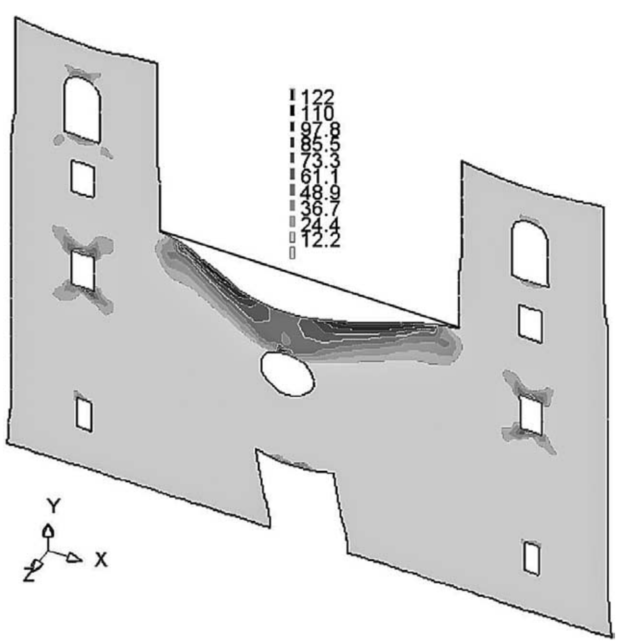


Fig. 8. Shell analysis for vertical and horizontal loading – picture of the maximum (tensile) principal stresses on the upper layer (10–3 N/mm^2) on the deformed structure.

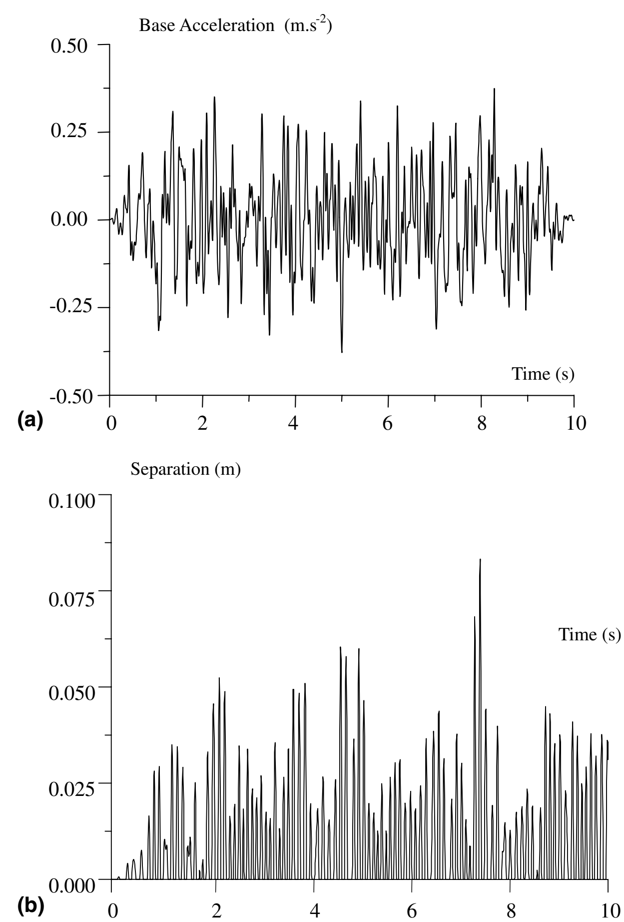


Fig. 9. Dynamic analysis with time-integration: (a) artificially generated base acceleration and (b) separation between the two leaves.

$$\begin{Bmatrix} \alpha \\ \beta \end{Bmatrix} = 2\frac{\omega_m\omega_n}{\omega_n^2-\omega_m^2}\begin{bmatrix} \omega_n & -\omega_m \\ -1/\omega_n & 1/\omega_m \end{bmatrix}\begin{Bmatrix} \xi_m \\ \xi_n \end{Bmatrix}. \quad (2)$$

Here, ω_m is the first natural frequency and ω_n should be a higher frequency that significantly contributes for the dynamic response of the structure (the second lower frequency was adopted in this case). The natural frequencies of the two leaves of the walls considered independently provide the following α and β coefficients:

$$\begin{Bmatrix} \alpha \\ \beta \end{Bmatrix}_{\text{Granite}} = \begin{Bmatrix} 2.11 \\ 0.00108 \end{Bmatrix} \text{ and } \begin{Bmatrix} \alpha \\ \beta \end{Bmatrix}_{\text{Shale}} = \begin{Bmatrix} 2.99 \\ 0.000444 \end{Bmatrix}. \quad (3)$$

The above equation confirms the fact that the two leaves possess rather different dynamic properties, both with respect to the first natural frequency and to the damping properties. These aspects are essential to explain the separation between the leaves of the masonry wall that constitutes the façade.

Assuming the usual damping coefficient ξ equal to 5%, a base acceleration was generated according to the Portuguese code, see Fig. 9(a). As the dynamic properties of the two leaves differ substantially, separation of the leaves will occur in the adopted dynamic, time-integration non-linear contact analysis. The response of the structure, in terms of separation (i.e., relative displacement in the z direction) of the top edge of the façade is illustrated in Fig. 9(b), where it is confirmed that separation of the two leaves reaches very large values (around 0.09 m), which is comparable to the observed separation.

Fig. 10 presents some examples of the deformed structure through time. It was possible to obtain different configurations of separation between the two leaves (in the top and center of the wall). For example, Fig. 10(a) shows the internal wall practically in the original configuration and the external wall moving out of plane. Fig. 10(b) shows the walls moving in opposite directions. Fig. 10(c) shows the walls moving in the same direction, with small separation at the top and large separation around the opening. This last shape of separation seems to be in good agreement with the damage exhibited by the façade. Therefore, it seems possible to conclude that an earthquake was the main origin of this damage and the insufficient connection between the leaves allowed the separation.

3.3. Analysis of the choir

For the analysis of the choir, a simplified model was chosen in which vaults were represented by shell elements, arches and ribs were represented by beam elements and columns were represented by truss elements. All elements were quadratic isoparametric elements, keeping the elastic properties adopted before. All arches and ribs were supported in stone brackets, with the exception of the arches supported by two columns.

Loading included all permanent loads and static horizontal forces equivalent to the seismic loading, as before. Fig. 11 illustrates the deformation of the choir under these loads, assuming linear elastic material behavior. The deficient original conception of the choir is clear, as it moves apart from the façade and side walls, even for vertical loading. If the vaults of the choir were originally connected to the side walls, or side arches were originally placed around the borders, the behavior would be monolithic. The values obtained for the maximum horizontal displacement is 0.006 m for vertical loading and 0.016 m for the seismic loading, amounting to a total of

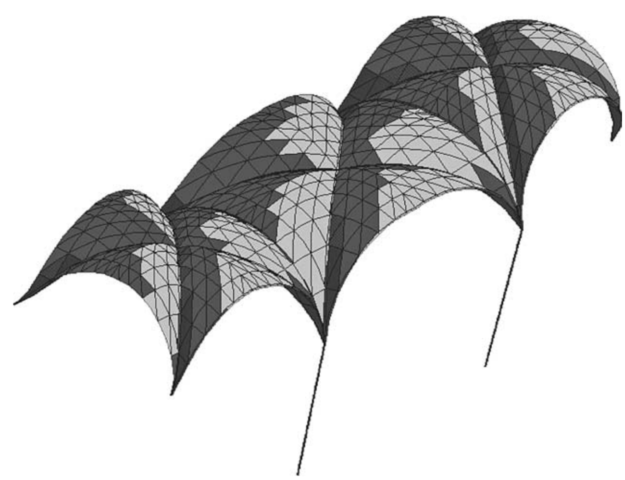


Fig. 11. Deformed mesh of the choir under vertical and horizontal loading.

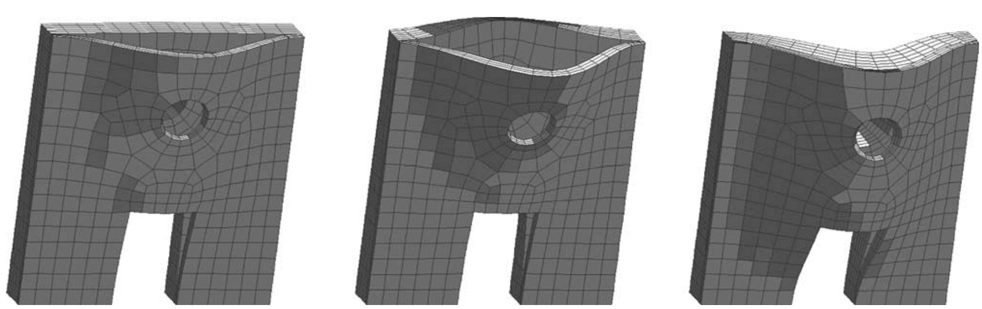


Fig. 10. Examples of deformed meshes through time.

0.022 m and confirming again the significant contribution of horizontal loading to the observed movements and damage. The value of the total displacement is well under the 0.08 m observed in the top of the columns but is in reasonable agreement with the separation between vaults and façade wall (around 0.015 m). The difference of displacement in the columns can be explained by the severe cracking of arches, ribs and vaults in the structure, which was not considered in this analysis.

This simplified analysis demonstrates that the vertical load thrust is inadequately absorbed by the columns and a connection between the side walls and the vault is missing, meaning that the original conception of the choir is inadequate.

4. Remedial measures

The conclusions of the numerical analyses of interest for the definition of the intervention are that (a) the damage was due to seismic action and (b) there were two misconceptions in the construction of the church (façade and choir). Therefore, it is seems necessary to correct these deficiencies so that further damage to the structure is prevented. The remedial measures must include some new connection in the main façade wall and adequate connection between the choir and the external walls. If the remedial measures would not be taken, it would be likely that the structure would collapse in the event of a future moderate earthquake.

Keeping these conclusions in mind, the remedial measures were designed taking into account the modern principles of architectural heritage protection, namely, minimal repair, unobtrusiveness, removability (as reversibility is obviously impracticable) and respect for the original conception, together with the obvious requirements of stability, durability and compatibility, see Figs. 12 and 13. As shown in Fig. 12(a), a stiffening steel frame was built inside the infill of the choir vault using UNP (channel) and INP (I section) steel profiles, for architectonical and technical reasons. The channel profiles were laid with the web horizontal due the very

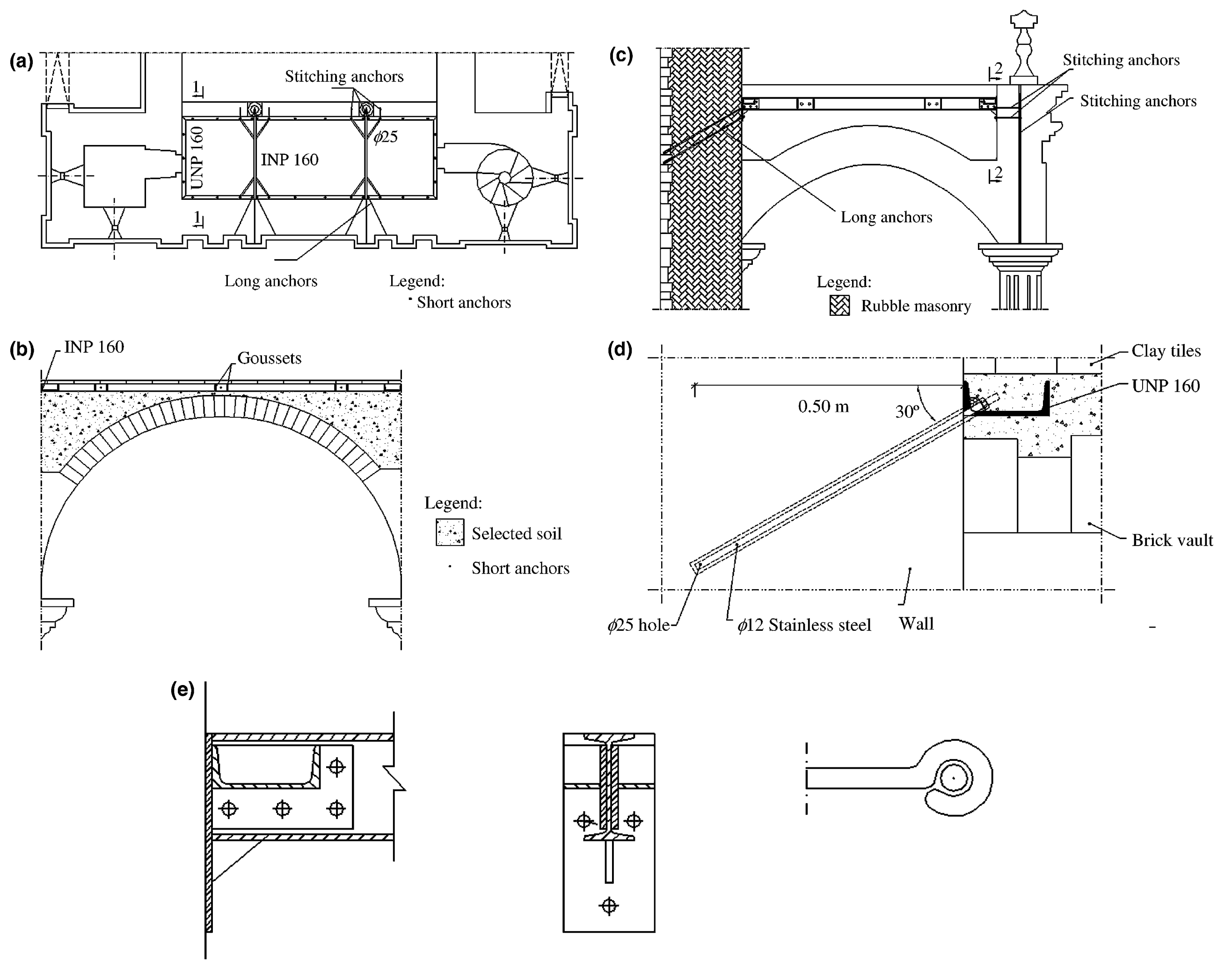


Fig. 12. Remedial measures: (a) general plan; (b) section 11 (channel section UNP 160); (c) section along the ties (I section INP160); (d) detail of a short anchor; (e) section 22 and detail of the connection between vertical and horizontal stitching ties.

Fig. 13. Aspects of the execution: (a) general view of the steel frame, tie and injection of the vaults; (b) details of the steelwork; (c) drilling of the holes for the anchors; (d) aspect of the dowels used in the balusters; (e) final aspect of the vault, with the soil infill.

low spacing between the top of the vault and clay tiles used as floor finishing. These profiles were anchored to the neighboring walls using short anchors, see Fig. 12(b) and (d). The I section profiles, which serve as ties, had to be kept straight because the brackets from the old timber floor were found inside the vault infill (see Fig. 12(c) and Fig. 13(a)). This poses additional complexity to the system because the thrust of the longitudinal arches is located at a lower level. For this reason, stitching vertical anchors where also introduced, and connected to the I section ties with horizontal stitching anchors (see also Fig. 12(e)). In the other end of the tie (main façade), the anchors were taken almost until the external part of the wall (see Fig. 12(c)), so that the granite external leaf would be connected to the internal shale leaf, at least at this level. Mortar injection of this crack allowed further bond connection between the two leaves.

Fig. 13(a) shows a general view of the stiffening frame, together with one longitudinal tie and the equipment for the vault injection (using fluid lime mortar). Fig. 13(b) shows details of the steelwork connections and Fig. 13(c) shows the execution of a hole using a drilling machine that works solely on rotation and introduces no vibration to the structure. The balustrade was dismounted and rebuilt using stainless steel dowels, in order to prevent further damage, see Fig. 13(d). Finally, in Fig. 13(e), an image of the vault with the soil infill, just before placing again the clay tiles in the pavement.

It is noted that the selection of materials included stainless steel (AISI 316) and highly galvanized steel for corrosion protection, selected lime mortars and non-intrusive grouting anchors (with a cloth socket that prevents dissemination of the injection throughout the wall).

5. Conclusions

This paper presents a detailed case study of remedial works in a church affected by earthquakes in the past, with a focus on the possibilities and advantages of using advanced numerical analysis in the diagnosis and strengthening of the architectural heritage. For the particular case study, it was possible to conclude that: (a) the structure was already subjected to moderate earthquakes twice; (b) the structure is robust and conveniently tied, and the soil is of good quality, resulting in no need for a global intervention; (c) the main façade is a two-leaf wall, not conveniently tied and featuring partial separation, due to seismic action; (d) the choir structural misconception lead to heavy cracking and separation from the external walls, under seismic action. With the present numerical analysis it was possible to define adequate and minimum remedial measures in the structure. The adopted measures are fully detailed in the text.

References

[1] Lourenço PB. Computations of historical masonry constructions. Prog Struct Eng Mater 2002;4(3):301–19.

[2] ICOMOS. Recommendations for the analysis, conservation and structural restoration of architectural heritage; 2001.

[3] Croci G. The conservation and structural restoration of architectural heritage. Southampton: Computational Mechanics Publications; 1998.

[4] CUR. Structural masonry: an experimental/numerical basis for practical design rules. In: Rots JG, editor. Rotterdam: Balkema; 1997.

[5] Giuffrè A. Safety and conservation of historical centers. Bari: Editori Laterza; 1993. [in Italian].

[6] Meli R. Structural engineering of historical buildings. Mexico-City: Fundación ICA; 1998. [in Spanish].

[7] ISO. Basis for design of structure – assessment of existing structures, ISO 13822; 2003.

[8] CEN. Eurocode 8: design of structures for earthquake resistance. Part 3: strengthening and repair of buildings, prEN 1998-3:2004; 2004.

[9] BSSC. Prestandard and commentary for seismic rehabilitation of buildings, FEMA 356; 2000.

[10] Lourenço PB, Oliveira DV, Mourão SC. Stability study of the Church of Saint Christ in Outeiro [in Portuguese]. Report no. 99-DEC/E-2. Guimaràes: University of Minho; 1999.

[11] Rossi PP. Possibilities of the experimental techniques for the structural analysis of historical constructions. In: Roca P et al., editors. Structural analysis of historical constructions. Barcelona: CIMNE; 1996.

[12] Roque JA. Strengthening and structural rehabilitation of ancient masonry walls [in Portuguese]. MSc Thesis, Guimarães: University of Minho; 2002. Available from: www.civil.uminho.pt/masonry.

[13] Lourenço PB, Vasconcelos G, Ramos LF. Assessment of the stability conditions of a Cistercian cloister. In: Arun G, Seçkin N, editors. Studies in ancient structures. Istambul: Yldiz Technical University; 2001. p. 669–78.

[14] Monteiro VJ. Historic seismicity of continental Portugal. J Natl Inst Meteorol Geophys 1984. [in Portuguese].

[15] Bowles JE. Foundation analysis and design. 4th ed. New York: Mc-Graw Hill; 1988. p. 405–11.

[16] Clough RW, Penzien J. Dynamics of structures. New York: Mc-Graw Hill; 1993.