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Improving multiplication fact fluency by choosing between competing answers

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Developing fluency in arithmetic facts is instrumental to mathematics learning. This study compares the effects of two practice conditions on children's fluency in simple multiplication facts. Third and fourth graders in the Netherlands ($N = 282$) practised in either a conventional *recall* condition where they produced answers to problems, or a *choice* condition where they had to choose between competing answers that included common multiplication errors. Practice in the choice condition was faster and as accurate as recall practice but was not more beneficial to performance on speed tests of practised facts. For more experienced students, recall practice led to greater improvement on a conventional recall fluency test. Differential effects of practice conditions on test performance are explained in terms of practice-to-test transfer demands. The relative merits of recall and choice tasks in multiplication fact learning are discussed.

Keywords: multiplication fact fluency; practice; recall tasks; choice tasks

Introduction

It is commonly accepted that mathematics learning requires three types of knowledge: conceptual, procedural and factual (Delazer, 2003; NCTM, 2000; NMAP, 2008). Much debate in recent years has focused on whether children should first learn core concepts and their interrelationships (i.e. conceptual knowledge or 'knowing why') or the sequences of steps required to solve common problems (i.e. procedural knowledge or 'knowing how'), or whether both should be taught together (Kilpatrick, Swafford, & Findell, 2001; Rittle-Johnson, Siegler, & Alibali, 2001; Schoenfeld, 2004). There is greater consensus about the necessity of having factual knowledge (i.e. answers to basic problems such as $2 + 3 = 5$ or $4 \times 6 = 24$) readily available in memory.

Achieving fluency in arithmetic facts is an important prerequisite for more complex mathematics, such as problem-solving and understanding higher-order concepts and procedures (Cumming & Elkins, 1999; Kilpatrick et al., 2001). Complex tasks place considerable demands on information processing capacity, including working memory (LeFevre, DeStefano, Coleman, & Shanahan, 2005). However, when arithmetic facts can be retrieved fluently – that is, quickly and accurately – from long-term memory, cognitive resources are not consumed by simple operations (Tronsky, 2005). In effect, more

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capacity becomes available for non-routine aspects of the task and for more complex learning (LeFevre et al., 2005). Solving arithmetic word problems, for example, which requires children to carry out, integrate and monitor several cognitive processes, is highly taxing, and fluency with basic facts frees up processing capacity to address these demands (Jitendra, Griffin, Deatline-Buchman, & Sczesniak, 2007).

Poor fact fluency thus has direct consequences for mathematical learning and performance. As well as affecting problem-solving, research has shown that inefficient solving of basic facts depletes resources that are needed for acquiring more advanced computational skills, such as those required for multi-digit sums (Cumming & Elkins, 1999). Weak ability to retrieve arithmetic facts can also impede children's ability to follow mathematical discourse and consequently to understand concepts that are introduced in class, such as equivalence, the commutative property and other concepts that form the basis of more advanced mathematics, such as algebra (Gersten, Jordan, & Flojo, 2005; NMAP, 2008; Woodward, 2006).

Unfortunately, many children have difficulty achieving arithmetic fact fluency. Deficiencies in this area are a defining feature of persistent mathematics difficulties (Geary, Hoard, & Bailey, 2012; Jordan, Hanich, & Kaplan, 2003; Mabbott & Bisanz, 2008). In international studies with nationally representative populations also, Grade 4 students' knowledge of basic arithmetic facts, concepts and procedures lags behind their application and reasoning skills in a third of participating countries, and this situation does not improve greatly by Grade 8 (Mullis, Martin, Foy, & Arora, 2012).

Previous research has established that fluency in arithmetic facts can be improved through systematic practice (e.g. Ruijssenaars, Van Vliet, & Willemse, 2002; Van Galen & Reitsma, 2010; Wong & Evans, 2007; Woodward, 2006). However, there is no clear evidence as to which practice methods and materials are most effective. Most practice materials employ the conventional recall format (e.g. $7 + 3 = ?$ or $7 \times 3 = ?$) in which the answer has to be produced from memory. To date, there has been little research comparing the effectiveness of this format to other formats. In the present study, the effects of practising with the conventional recall format on simple multiplication fact fluency (i.e. single-digit multiplication and multiples of 10) are compared to the effects of practising with a format in which the correct answer has to be selected from competing answers that include common multiplication errors.

Learning multiplication facts

It is commonly assumed that arithmetic facts are organised in associative networks comprising connections between problems and answers in long-term memory (Campbell, 1995; Verguts & Fias, 2005). As the relationship between a problem and its answer is learned, a connection becomes established whose strength is determined by the frequency with which it is activated: frequent activation strengthens the connection. Associative-network models explain why practising arithmetic facts is effective. If a problem–answer pair is repeatedly activated, the connection becomes stronger. This increases the likelihood of that answer being retrieved from memory on presentation of the problem, particularly if its associative strength comes to sufficiently exceed that of competing answers (Campbell, 1995; Siegler, 1988).

Associative-network models also predict specific learning effects. Of particular relevance to the present study is the Identical Elements (IE) model of arithmetic fact representation (Rickard, 2005), by which a single memory node connects each number triplet of an arithmetic problem irrespective of operand order. Thus, the problems 6 × 4 → 24 and 4 × 6 → 24 share the same memory node and strengthening one of these connections should transfer to the other. This means that learning should transfer associatively to commutative counterparts of practised facts.

While the associative-network properties of long-term memory enable arithmetic facts to be learned, they also have some disadvantages that make fast, error-free fact retrieval problematic for some children. In the process of learning a fact, connections are indiscriminately established between the problem and any answers that are associated with it (Siegler, 1988). For example, while learning to multiply 4 by 7, a child could produce answers such as 21 or 24 that could also become associated with the problem. S/he could represent the problem as groups in an array and mistake the number of groups or group-size, or make an error using an addition or shortcut strategy, etcetera. Consequently, there could be several answers associated with any particular problem and incorrect answers compete and thus interfere with the correct answer. A second source of interference occurs when activation in one part of the network spreads to adjacent nodes (Campbell, 1995; Galfano, Rusconi, & Umiltà, 2003; Verguts & Fias, 2005). For example, presentation of the problem 7 × 4 activates the close operand-related problems 7 × 3, 7 × 5, 6 × 4 and 8 × 4 and the close operand-related answers 21, 35, 24 and 32. These then interfere with the correct answer, resulting in errors or longer response times. Common multiplication errors are known to reflect this kind of interference (Lemaire & Siegler, 1995; Siegler, 1988; Verguts & Fias, 2005).

These mechanisms result in interference on arithmetic fact retrieval tasks, impacting both speed and accuracy. Performance on these tasks thus depends to a degree on the ability to inhibit such interference (Barrouillet, Fayol, & Lathulière, 1997; Passolunghi, Cornoldi, & De Liberto, 1999). Unfortunately, many children have difficulty with this, and persistent fact retrieval deficits in low-achieving students and children with mathematical learning disabilities are characterised by poor inhibition of irrelevant information during retrieval (Geary et al., 2012). Consequently, there could be benefit in developing practice materials that could improve fluency by training children to inhibit competing answers. Specifically, if children can learn to reject distractors that represent common multiplication errors, speed and accuracy of performance should improve.

Choice tasks

Training children to reject competing answers can be implemented in so-called choice tasks, in which the correct answer has to be selected from several alternatives. Training on a combination of correct and competing answers, particularly if these represent commonly-made errors, could simultaneously strengthen correct associations and suppress interfering associations, increasing the discrepancy between them and thereby the likelihood of the correct answer being retrieved from memory (Campbell, 1995; Siegler, 1988).

Such tasks clearly carry a risk, however: if children are unsuccessful in learning to inhibit competing answers, their associations could be strengthened, resulting in increased interference and a deterioration in performance. Nevertheless, there are reasons to suppose that this method could be successful. Even preschoolers can be trained to improve inhibitory control (Diamond, Barnett, Thomas, & Munro, 2007), while research shows that a large developmental advance in inhibition is made during middle-primary school age (Brocki & Bohlin, 2004; Huizinga & Smidts, 2010), the age at which children typically learn multiplication in schools. Children at this age may therefore be particularly susceptible to stimuli that hone their ability to inhibit undesired associations.

Furthermore, two studies that have investigated the effects of practising with choice tasks indicate that these tasks can be used successfully to improve children's arithmetic fact fluency. In Van Galen and Reitsma (2010)'s study on learning addition facts and Ruijssenaars et al.'s (2002) small-scale study on learning multiplication facts ($N = 49$, of whom 16 in the choice condition), the authors argued that choice tasks produce learning because they can be solved by quickly judging the familiarity of a presented item rather than retrieving exact details from memory, which enables problem–answer connections to be frequently repeated. According to associative-network models, connections would thereby be strengthened and learning improved.

Choice tasks do not necessarily work in this way, however. Romero, Rickard, and Bourne (2006) report that adults use a variety of strategies to solve tasks in which multiplication facts have to be verified (e.g. "$7 \times 3 = 28$, true or false?"); such tasks are similar to choice tasks, which also require targets to be accepted or rejected. Solution strategies include retrieval of the correct answer from memory followed by comparison with the presented answer, calculation followed by comparison, pattern-matching (i.e. the combination of problem and answer "looked right or wrong") and magnitude-estimation (i.e. the answer appeared too large or too small). While the last two strategies are relatively fast, the first two involve rather slower processing, consisting of recall or calculation processes plus additional comparison and decision-making processes. Moreover, it is important to note that Van Galen and Reitsma's (2010) and Ruijssenaars et al.'s (2002) studies deliberately avoided presenting alternative answers that are known to interfere with correct answers, due to the risks noted above. The associative-network perspective of the present study suggests that choice tasks presenting these kinds of distractors could be particularly taxing for persons with poor ability to inhibit irrelevant information, and could therefore be both time-consuming and error-prone for these individuals.

Taken together, this suggests that fast and frequent repetition of problem–answer connections may not necessarily explain how choice tasks work, particularly if they present choices that actively interfere with the correct answer. Thus, the mechanisms by which these tasks could affect learning of multiplication facts in children need to be determined. In short, the potential benefits and risks of using choice tasks, as well as unresolved issues about how these tasks could produce learning, motivate the need to establish whether practice with these tasks is beneficial or detrimental to multiplication fact fluency and what the implications of this could be for educational practice.

The present study

The present study investigates the effects of practice with choice tasks presenting competing answers (i.e. commonly-made errors) on multiplication fact fluency within the context of Realistic Mathematics Education (RME) in the Netherlands. RME is the dominant instructional approach in Dutch primary mathematics education and almost all primary schools – including those involved in this study – use mathematics textbooks based on RME principles (Hop, 2012; Scheltens, Hemker, & Vermeulen, 2013). RME is an investigative and socially-constructed activity within meaningful contexts in which children are encouraged to develop handy strategies for acquiring mathematical knowledge and solving problems (Freudenthal, 1991; Treffers, 1993). RME emphasises 'learning with understanding'. Multiplication instruction is characterised by the use of multiple representations, attention to the relationships between operations, and an emphasis on shortcut strategies such as the commutative rule, doubling, halving, 'one-more' and 'one-less' (Van Zanten, Barth, Faarts, Van Gool, & Keijzer, 2009). Consequently, the focus of the present study is on increasing fact fluency as a supplement to the deeper mathematical learning that is stimulated by RME. As a point of departure, it is assumed that children are familiar with grade-appropriate conceptual and procedural knowledge and are able to construct answers to the problems on their own.

Experiments were carried out in Grades 3 and 4, which allows the influence of different curriculum content and lengths of exposure to multiplication fact learning to be examined. The key issue to be resolved is whether children who practise with choice tasks where they have to reject competing answers representing common multiplication errors produce faster, more accurate performance than children who practise with conventional recall tasks. If the choice method is successful in helping children to suppress interference from competing answers and strengthen correct associations, this should be the case. If, however, the choice method leads to strengthening of incorrect associations and thereby increased interference, performance should be slower and less accurate. Given that low-achieving students often have difficulty in inhibiting interference from irrelevant information (Geary et al., 2012), it is possible that effects differ for higher and lower ability students. To investigate this possibility, maths ability is also taken into account.

First, the study examines how children's fluency in multiplication facts develops during practice with choice tasks presenting competing answers, compared to practice in the conventional recall format (RQ 1). Most multiplication practice studies focus only on end-results, yet examining the practice process itself could provide valuable information about the learning that takes place. Second, the effect of these practice conditions on improving performance on speed tests of practised facts is compared (RQ 2). Third, the study investigates whether practice conditions differ in the extent of transfer of learning to unpractised commutative counterparts of practised facts (RQ 3). Transfer is predicted by the IE-model (Rickard, 2005), but results of the two previous studies that used choice tasks in arithmetic learning were mixed. In Van Galen and Reitsma's study (2010), choice and recall practice improved equally on unpractised facts, but children in Ruijssenaars et al.'s (2002) choice condition did not improve significantly on these facts. It is therefore possible that choice and recall practice have different effects on memory representations that may be clarified by comparing transfer effects.

Methods

Participants

The study took place in the Netherlands. In the Dutch curriculum, multiples of 1, 2, 5 and 10 are learned in Grade 2. Children in Grade 3 learn multiples of 3, 4 and 6 in the first semester and multiples of 7, 8 and 9 in the second. Children in Grade 4 are expected to achieve full fluency in single-digit multiplication and multiples of 10. The present study was carried out following the first semester. At that point, Grade 3 had been exposed to multiplication learning for a year and Grade 4 for two years.

Six mainstream primary schools participated, involving 282 children from 12 classes in Grades 3 and 4. Of these, 146 (51.8%) were in Grade 3 and 136 (48.2%) were in Grade 4; 128 (45.4%) were boys and 154 (54.6%) were girls. In Grade 3, the average age was nine years and two months (SD = 5.7 months); in Grade 4, average age was 10 years and two months (SD = 6.4 months). Maths ability was determined by a standardised, norm-referenced maths test developed by the Dutch Central Institute for Test Development (CITO, www.cito.nl). According to these norms, level A corresponds to the highest 25% of the norm-referenced population, level B to the above-average 25%, level C to the below-average 25%, and levels D and E together to the lowest 25%. In the present sample, 26.2% scored at level A, 37.3% at level B, 22.2% at level C, 9.3% at level D and 5.0% at level E. To have groups of comparable size, levels C to E were combined. Thus, maths ability was designated in one of three categories: high (26.2%), above-average (37.3%) and below-average (36.5%).

Per class, participants were randomly assigned to one of two practice conditions: *recall* or *choice*. The *recall* condition contained 150 (53.2%) participants and the *choice* condition 132 (46.8%). Sample composition in both conditions was similar for maths ability (Grade 3: $\chi^2(2) = 0.23$, $p = .89$; Grade 4: $\chi^2(2) = 0.09$, $p = .96$), sex (Grade 3: $\chi^2(1) = 0.16$, $p = .69$; Grade 4: $\chi^2(1) = 0.35$, $p = .55$) and age (Grade 3: $t(141) = 0.91$, $p = .36$; Grade 4: $t(133) = 1.53$, $p = .13$).

For the analyses, one Grade 4 class (23 children) and one Grade 3 participant were removed from the dataset because they had worked in both types of practice booklet (see section *Practice procedure and materials*); 258 participants then remained (145 in Grade 3, 113 in Grade 4). For the analysis of practice sessions, one more Grade 3 participant was removed due to an abnormally high percentage of errors (34% errors compared to an average of 2% for the other Grade 3 participants); for this analysis, sample size was 257 (144 in Grade 3, 113 in Grade 4). For the analyses of test scores, 16 participants (seven from Grade 3, nine from Grade 4) had not taken one or both tests, giving a sample of 242 (138 in Grade 3, 104 in Grade 4). The resulting sample composition in both practice conditions in terms of maths ability, sex and age remained similar.

Practice procedure and materials

All children in each class participated. On each school day during two weeks, they practised for 10 minutes between regular lessons with their own teachers, giving a total of 100 minutes of distributed practice. Schools provided absence lists so that actual practice time could be estimated.

Practice materials were curriculum-appropriate; thus, Grade 3 practised multiples of 1 to 6 and multiples of 10 and Grade 4 practised multiples of 1 to 10. Children practised in personal booklets where each pair of facing pages contained all facts to be practised (70 for Grade 3; 100 for Grade 4), presented in three columns per page. Each booklet contained 40 pages: Grade 3 booklets thus contained 1400 items and Grade 4 booklets 2000 items. Presentation order was determined by placing all items in a matrix and applying random number generation to the matrix cell numbers. The booklets used in the second week used a different presentation order. Each practice condition had its own booklet version while maintaining the same presentation order and layout. In the *recall* condition, children had to produce answers, while in the *choice* condition they had to choose between two competing answers. The correct answer was the left-hand alternative for half of the items and the right-hand alternative for the other half. The pattern of correct answers never contained more than three consecutive ‘lefts’ or ‘rights’. Examples of both item formats are shown in Figure 1.

For the *choice* condition, one of four competing answers (i.e. distractors) was presented along with the correct answer for each problem. Distractors represented

8 x 3 = ______ **8 x 3 = 27 24**

(a) Recall format **(b) Choice format**

Figure 1. Item formats.

common multiplication errors known to interfere with correct answers (Lemaire & Siegler, 1995; Siegler, 1988; Verguts & Fias, 2005). Distractors could be close multiplicand-related (i.e. a close neighbour from the multiplication table of one of the multiplicands, e.g. 7 × 3 = 24), close table-related (i.e. a close neighbour from another multiplication table, e.g. 8 × 4 = 30), a 'close miss' (i.e. small errors within a distance of 10% from the correct answer, e.g. 9 × 9 = 84), or could reflect so-called 'operation confusion' (i.e. the sum of the multiplicands rather than the product, e.g. 6 × 3 = 9).

Children were instructed to work quickly and accurately from left to right without missing out any problems. When they finished a booklet, they continued in a second one. At the end of each practice session, children drew a line under the last problem they had solved and started from that point in the next session. Teachers checked the booklets daily and highlighted errors. Children were given additional time to correct their errors before the next practice session commenced. In the *choice* condition, the correct answer was immediately evident (being the alternative to the incorrectly given answer), while in the *recall* condition, children had to produce another answer and check it with the teacher. A questionnaire verified whether teachers had adhered to this protocol.

Test procedure and materials

Arithmetic fact fluency was measured by a validated, norm-referenced pencil-and-paper speed test (De Vos, 2010), widely used in schools and in research in the Netherlands. The test comprises four sections in recall format: *addition*, *subtraction*, *division* and *multiplication*. Participants were also given a comparable multiplication test in the choice format. Tests were administered by class teachers 1 week before and 1 week after the practice sessions according to the standard test protocol. The test was administered in its entirety to Grade 4; the section *division* was not administered to Grade 3 as they had not been instructed in division at the time of study. For each section, participants had two minutes to answer as many items as possible. They were instructed to work quickly and accurately and to solve items in the presented order.

Each section contained 50 items, presented in sets of five in increasing order of difficulty within and between sets. Thus, the first set of items contained the easiest items (e.g. 4 × 1) and the last the most difficult ones (e.g. 9 × 7). Multiplication items were single-digit multiples of 1 to 10. Of these, 32 were multiples of 1 to 6 and 10 (i.e. facts practised by Grade 3), and 18 were multiples of 7, 8 and 9 (i.e. facts not practised by Grade 3). The items not practised by Grade 3 included a number of commutative counterparts of practised items: 12 in the recall format and 10 in the choice format. As Grade 4 practised all facts up to 10 × 10, it was not tested on unpractised facts. Pearson correlations between the different sections at pre-test and post-test were all large and highly significant for both Grades (all $p \leq .001$).

Analysis

Analyses were performed in IBM SPSS® Statistics 20 (α = .05). The analysis of the practice sessions used the *number of items answered correctly per week* as the fluency measure. Analysis of test performance used the *number of items answered correctly in two minutes* as the fluency measure. These measures combine both speed and accuracy. A combined fluency measure is relevant to and common in classroom settings as well as in research comparable to this (e.g. Ruijssenaars et al., 2002; Van Galen & Reitsma, 2010; Wong & Evans, 2007; Woodward, 2006).

To investigate how children's fluency in multiplication facts develops during practice (RQ 1), a Linear Mixed Model with post hoc comparisons (Bonferroni correction[1]) was analysed with participants as subjects and practice week as the repeated measure. Fixed effects were *week*, *practice condition*, *maths ability* and *grade*, and *practice time* per week was a time-varying covariate. The outcome variable was the *number of items answered correctly per week*. A subsidiary analysis was performed with percentage *accuracy* (i.e. number of correct answers × 100/number of items answered) per week as the outcome variable, to verify whether accuracy differed across conditions.

To compare the effects of practice condition on improving fluency in practised facts (RQ 2), a repeated measures ANCOVA with post hoc comparisons (Bonferroni correction) was performed on the *number of practised facts answered correctly in two minutes*. *Test moment* (pre-test, post-test) and *test format* (recall, choice) were within-subjects factors. *Grade*, *maths ability* and *practice condition* were between-subjects factors. To control for the amount of practice that children had had, the *total number of practice items answered* was included as a covariate. The comparison of practice conditions for improving fluency in unpractised facts (RQ 3) was performed for Grade 3 only, as Grade 4 had practised all facts presented on the test. A repeated measures ANCOVA with post hoc comparisons (Bonferroni correction) was performed on the *number of unpractised commutative counterparts answered correctly in two minutes*. *Test moment* (pre-test, post-test) and *test format* (recall, choice) were within-subjects factors. *Maths ability* and *practice condition* were between-subjects factors. Amount of practice (i.e. *total number of practice items answered*) was again a covariate. As the focus of both of these research questions was on between-subjects comparisons (i.e. practice condition, Grade, maths ability) in relation to pre-test to post-test improvement, within-subjects comparisons of individuals' relative performance in each test format were not investigated.

Results

RQ 1: development of fluency during practice

Table 1 shows the number of items answered correctly and actual practice time during the practice sessions. There were large effects of *practice time* ($F(1,273) = 147.73$, $p < .001$, $d = 1.47$) and *week* ($F(1,237) = 56.86$, $p < .001$, $d = .98$), with more items answered correctly in the second week, and a medium effect of *practice condition* ($F(1,249) = 25.37$, $p < .001$, $d = .64$), with more items answered correctly in the choice condition. This is in context of a *week* × *practice condition* interaction ($F(1,238) = 13.16$, $p < .001$, $d = .47$), with the increase in correct items in the second week being greater in the choice condition. For *maths ability* ($F(2,249) = 8.23$, $p < .001$, $d = .51$), high-ability students answered more items correctly than below-average students ($p_{\text{Bonf}} < .001$) and showed a

Table 1. Practice sessions: number of items answered correctly and practice time per Grade, practice condition and maths ability level.

		Number of items answered correctly									**Practice time (minutes)**								
		Week 1			**Week 2**			**Total**			**Week 1**			**Week 2**			**Total**		
Practice Condition	**Maths ability**	***N*[b]**	***M***	**SD**	***N*[c]**	***M***	**SD**	***N***	***M***	**SD**	***N*[b]**	***M***	**SD**	***N*[c]**	***M***	**SD**	***N***	***M***	**SD**
										Grade 3									
Recall	High	17	978	332	18	914	435	18	1838	779	17	48	4	18	47	10	18	92	14
	Above-average	27	957	343	29	1031	399	29	1922	762	27	47	6	29	48	5	29	94	13
	Below-average	25	701	327	26	725	332	26	1399	624	25	49	4	26	47	9	26	94	14
	Total[a]	71	858	361	75	891	404	75	1703	755	71	48	6	75	47	8	75	93	14
Choice	High	14	1124	316	17	1298	166	17	2223	572	14	49	5	17	49	3	17	89	22
	Above-average	24	1038	415	24	1109	461	25	2062	818	24	48	6	24	47	7	25	91	18
	Below-average	23	914	393	27	1026	453	27	1804	856	23	50	2	27	48	5	27	91	18
	Total	61	1011	389	68	1123	413	69	2001	789	61	49	5	68	48	5	69	90	19
										Grade 4									
Recall	High	18	1034	354	18	991	420	18	2025	708	18	50	0	18	45	6	18	95	6
	Above-average	21	804	318	21	839	347	21	1643	636	21	50	2	21	48	5	21	98	5
	Below-average	21	735	385	20	749	353	21	1448	711	21	48	8	20	46	7	21	91	16
	Total[a]	61	853	368	60	858	377	61	1696	710	61	49	5	60	46	6	61	94	11
Choice	High	15	1327	582	15	1489	557	15	2816	1109	15	47	5	15	45	5	15	93	7
	Above-average	17	977	590	17	1021	673	17	1998	1241	17	49	3	17	46	8	17	95	9
	Below-average	20	1041	524	19	1174	628	20	2156	1137	20	47	6	19	46	6	20	91	12
	Total	52	1103	571	51	1215	640	52	2295	1191	52	48	5	51	46	6	52	93	10

[a]Maths ability was not available for two Grade 3 and one Grade 4 students; their data are included in the Totals.
[b]Excluding 12 absentees in Week 1.
[c]Excluding 3 absentees in Week 2.

strong tendency to answer more items correctly than the above-average students (p_{Bonf} = .05). There was no effect of *grade* and no other interaction effects.

There were no main effects of *practice time*, *week*, *grade* or *practice condition* on *accuracy* during practice. Mean accuracy in Grade 3 was 97.96% (*SD* = 3.07) in the recall condition and 98.22% (*SD* = 2.73) in the choice condition. Mean accuracy in Grade 4 was 97.67% (*SD* = 3.13) in the recall condition and 98.37% (*SD* = 1.96) in the choice condition. For *maths ability* ($F(2,246) = 19.13$, $p < .001$, $d = .79$), below-average students were less accurate than above-average and high ability students (both $p_{\mathrm{Bonf}} < .001$).There was a small *week* × *practice condition* interaction ($F(1,234) = 4.83$, $p = .03$, $d = .29$), with a slight increase in accuracy (0.3%) in the recall condition and a slight decrease in accuracy (0.3%) in the choice condition in the second week. There were no other interaction effects.

RQ 2: test performance on practised facts

Table 2 shows the number of practised facts answered correctly at pre-test and post-test. There was a medium effect of *test moment* (Wilks' $\lambda = .91$, $F(1,226) = 22.17$, $p < .001$, $\eta_p^2 = .09$), showing that performance increased from pre-test to post-test, and a large effect of *grade* ($F(1,226) = 138.73$, $p < .001$, $\eta_p^2 = .38$), with Grade 4 having higher scores than Grade 3. A *test moment* × *grade* interaction (Wilks' $\lambda = .92$, $F(1,226) = 20.40$, $p < .001$, $\eta_p^2 = .08$) revealed that pre-test to post-test improvement was also greater in Grade 4. There was a large effect of *amount of practice* ($F(1,226) = 255.15$, $p < .001$, $\eta_p^2 = .53$) and a smaller effect of *maths ability* ($F(2,226) = 4.82$, $p = .009$, $\eta_p^2 = .04$), with high ability students having higher scores than above-average students ($p_{\mathrm{Bonf}} = .008$).

There was a small main effect of *practice condition* ($F(1,226) = 9.37$, $p = .002$, $\eta_p^2 = .04$) in favour of the recall condition, but this was in context of a three-way *test moment* × *test format* × *practice condition* interaction (Wilks' $\lambda = .95$, $F(1,226) = 10.78$, $p = .001$, $\eta_p^2 = .05$) and a four-way *test moment* × *test format* × *practice condition* × *grade* interaction (Wilks' $\lambda = .98$, $F(1,226) = 4.11$, $p = .04$, $\eta_p^2 = .02$). The Grade 4 recall condition improved more than the choice condition on the recall test. Improvement was similar in both conditions otherwise, though there was a slight tendency for the Grade 3 choice condition to improve more than the recall condition on the choice test (Figure 2). The absence of an interaction with *maths ability* indicates that these effects held for all maths ability levels.

RQ 3: test performance on unpractised facts (Grade 3 only)

Table 3 gives the number of unpractised commutative counterparts answered correctly at pre-test and post-test. There was again a large effect of *amount of practice* ($F(1,129)$ = 123.32, $p < .001$, $\eta_p^2 = .49$) and a small main effect of *practice condition* ($F(1,129)$ = 3.98, $p = .048$, $\eta_p^2 = .03$) in favour of the recall condition, but this was in context of a two-way *test format* × *practice condition* interaction (Wilks' $\lambda = .93$, $F(1,129) = 10.16$, $p = .002$, $\eta_p^2 = .07$) and a three-way *test moment* × *test format* × *practice condition* interaction (Wilks' $\lambda = .86$, $F(1,129) = 20.81$, $p < .001$, $\eta_p^2 = .14$). This revealed that the recall condition improved more than the choice condition on the recall test, while the choice condition improved more than the recall condition on the choice test (Figure 3). There was no effect of or any interaction with *maths ability*; thus, these effects held for all maths ability levels.

Table 2. Number of practised multiplication facts answered correctly at pre-test and post-test per Grade, practice condition, maths ability level and test format.

			Pre-test				Post-test			
			Recall		Choice		Recall		Choice	
Practice condition	Maths ability	*N*	*M*	SD	*M*	SD	*M*	SD	*M*	SD
					Grade 3					
Recall	High	17	27.53	5.61	31.00	2.29	29.00	5.60	31.35	1.50
	Above-average	28	26.18	5.02	28.89	4.52	30.18	4.00	31.32	1.96
	Below-average	26	20.46	8.79	26.04	7.69	23.81	8.21	27.23	6.65
	Total[a]	73	24.32	7.49	28.22	5.98	27.38	7.01	29.66	4.92
Choice	High	15	27.00	5.39	30.40	2.41	29.33	3.66	32.00	0.00
	Above-average	24	24.04	5.74	29.13	3.72	26.42	6.55	30.42	4.73
	Below-average	26	21.42	6.25	27.85	4.64	26.00	7.30	30.85	4.53
	Total	65	23.68	6.18	28.91	3.96	26.92	6.40	30.95	4.05
					Grade 4					
Recall	High	17	33.35	8.06	39.35	9.60	39.59	11.27	44.47	8.19
	Above-average	20	27.05	8.77	33.45	7.87	34.80	9.97	37.80	8.48
	Below-average	21	27.00	10.33	32.81	8.82	33.95	13.11	38.86	10.45
	Total[a]	59	29.10	9.55	34.98	8.98	36.14	11.66	40.31	9.45
Choice	High	13	38.92	11.54	42.85	10.02	41.38	10.81	45.15	7.88
	Above-average	16	28.00	12.25	32.63	11.79	33.88	14.00	39.88	12.79
	Below-average	16	31.44	13.05	36.75	12.21	33.87	12.21	42.06	7.13
	Total	45	32.38	12.86	37.04	11.94	36.04	12.71	42.18	9.73

[a]Maths ability was not available for two Grade 3 and one Grade 4 students; their data are included in the Totals.

Discussion

This study investigated the comparative effects of practising with choice tasks versus conventional recall tasks on simple multiplication fact fluency within the context of RME in the Netherlands. Children in Grades 3 and 4 (N = 282) practised 10 minutes a day for two weeks on curriculum-appropriate multiplication problems. Fluency in practised and unpractised facts was measured before and after the practice period using a validated, norm-referenced speed test in the recall format and an equivalent test in the choice format. The issue to be resolved is whether practice with problems where children have to choose between answers that include distractors known to interfere and compete with correct answers is more beneficial or detrimental to improving fact fluency than practice in the conventional recall format where children have to produce answers to problems.

Practice with choice tasks

Children answered more items correctly during choice practice than recall practice. Given that accuracy in both conditions was comparable and high (i.e. children were not simply guessing more successfully in the choice condition), the choice method thus appears to produce more frequent repetition of correct problem–answer connections than recall practice. Importantly, the presence of competing answers representing common multiplication errors does not have a negative effect on learning. Moreover, as more problems were

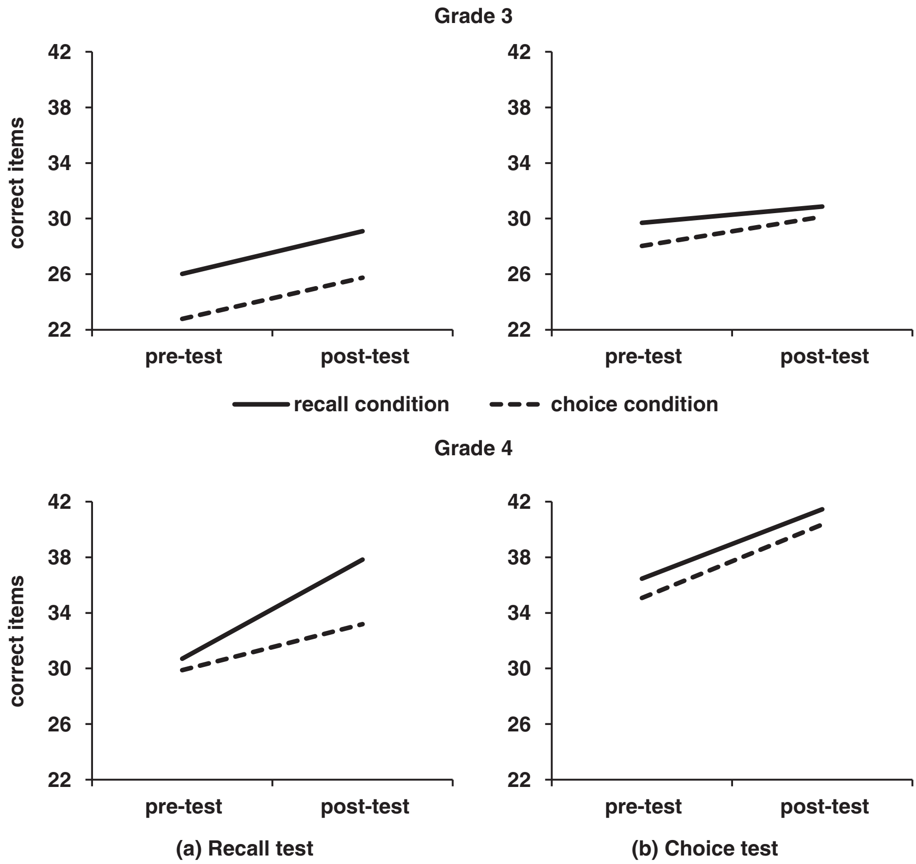

Figure 2. Interaction between Grade, test moment, test format and practice condition on practised facts controlling for amount of practice.
Note: Comparison of pre-test to post-test improvement between practice conditions is based on the line gradients.

Table 3. Number of unpractised commutative counterparts answered correctly at pre-test and post-test per practice condition, maths ability level and test format (Grade 3).

			Pre-test				Post-test			
			Recall		Choice		Recall		Choice	
Practice condition	**Maths ability**	***N***	***M***	**SD**	***M***	**SD**	***M***	**SD**	***M***	**SD**
Recall	High	17	2.71	4.19	4.47	4.00	5.00	5.15	6.71	3.74
	Above-average	28	1.21	3.19	3.04	3.93	4.79	5.04	6.25	4.28
	Below-average	26	0.50	1.24	1.85	3.15	2.15	4.09	3.88	4.27
	Total[a]	73	1.32	3.00	2.97	3.77	3.82	4.81	5.48	4.30
Choice	High	15	2.80	4.93	4.33	4.43	4.20	4.71	8.73	2.89
	Above-average	24	1.00	3.39	3.21	3.97	2.83	4.28	7.38	3.74
	Below-average	26	0.81	2.59	2.27	3.78	1.85	3.94	6.88	3.66
	Total	65	1.34	3.57	3.09	4.02	2.75	4.28	7.49	3.55

[a]Maths ability was not available for two students; their data are included in the Total.

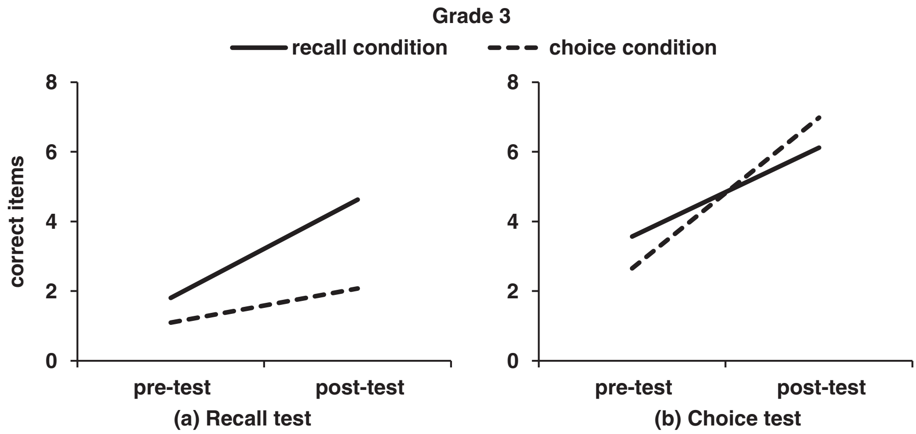

Figure 3. Interaction between test moment, test format and practice condition on unpractised facts controlling for amount of practice (Grade 3).
Note: Comparison of pre-test to post-test improvement between practice conditions is based on the line gradients.

answered correctly in the second week of practice and the difference between conditions also became greater, children appear to become more efficient in ignoring distractors the longer they practise. This was the case for all maths ability levels, including those with lower ability who are more likely to have difficulty in inhibiting competing answers (Geary et al., 2012). In short, the choice method – whereby children have to ignore distractors that represent common multiplication errors – appears to be more efficient than conventional recall tasks in terms of the frequency of correct problem–answer connections made in relation to the amount of time spent during practice.

The finding that the choice method is faster than recall practice can also be seen in light of Romero et al.'s (2006) strategies for verifying multiplication facts discussed earlier: choice tasks may make substantial use of faster strategies – namely pattern-matching or magnitude-estimation – rather than slower recall/calculation-plus-comparison. This interpretation suggests that there may be a developmental aspect to learning with choice tasks. Neuroscientific studies show that, as arithmetic facts become learned, activity shifts from parts of the brain involved in procedural and quantity-based processing to parts involved in retrieval of representations from long-term memory (Ischebeck, Zamarian, Egger, Schocke, & Delazer, 2007). From this perspective, if magnitude-estimation – a quantity-based process – is invoked by choice tasks, this could contribute to early fact learning. This could be particularly relevant for less experienced students. Pattern-matching, on the other hand, would involve accessing representations already present in long-term memory. This could be particularly relevant for more experienced students. Thus, depending on the mix of strategies used, early and/or later stages of learning may be promoted during practice with choice tasks.

Performance on fluency tests

Fluency on practised facts improved in both practice conditions; thus, there was an item-specific effect of practice. Grade 4 benefited more from practice than Grade 3. This is in keeping with findings from previous research that performance improves more when practice is coupled with greater experience (Ericsson, 2006). In this case, fourth-graders had more experience with multiplication and presumably had sufficiently established representations of multiplication facts in memory that allowed them to benefit more from an intervention that aimed to strengthen those representations.

While less experienced students (i.e. Grade 3) in both practice conditions produced comparable performance on the recall test, more experienced students (i.e. Grade 4) performed better on the recall test when they had practised in the same format. On the choice test, practice conditions were equally effective in Grade 4, while the choice condition had a marginal advantage in Grade 3. Thus, although the choice method does improve performance on speed tests of practised facts, it is not more beneficial than conventional recall practice in general. Importantly, when children are more experienced with multiplication, the choice method leads to lower improvement on a conventional recall test compared to children who practise in the conventional way.

It seems likely that children who practise with choice tasks could be disadvantaged by the mismatch between the item formats of choice practice and a recall test, compared to children who practise with recall tasks. This explanation fits with the concept of transfer-appropriate processing (Roediger, Gallo, & Geraci, 2002), where memory performance may benefit more when processes involved in learning overlap with processes required on tests of that learning. If more experienced students primarily used pattern-matching during choice practice – as suggested by the analysis of the practice sessions – they would not have practised freely retrieving memory representations. Thus, they would have been at a disadvantage on the recall test compared to children who did practise retrieval. This does not tell the whole story for less experienced students, however, for whom the choice condition was as effective as the recall condition on the recall test. In support of the study motivation, this could indicate that – at a relatively early stage of multiplication learning – the choice method strengthens correct associations relative to competing answers to an extent that retrieval is likely to be successful (Campbell, 1995; Siegler, 1988), even if not explicitly practised.

From the transfer perspective, it should also be noted that recall condition children tested in the choice format need not necessarily be at a disadvantage compared to children who practised with the choice method, as they could continue to use a recall strategy on the choice test. Indeed, more experienced students improved equally in both conditions on the choice test while less experienced students in the recall condition were at only a slight disadvantage.

The study also investigated the extent to which practice in each condition transfers to unpractised commutative counterparts of practised facts. As Grade 4 practised all simple multiplication facts, this issue was only investigated for Grade 3. The transfer-appropriate processing effect was now clearly apparent: for both test formats, improvement was higher when children had practised in the same format. It is likely that less experienced students have more difficulty coping with a double transfer (i.e. between formats *and* from practised to unpractised facts) than with a single transfer (i.e. between formats when tested on practised facts).

It is notable that improvement for unpractised facts was similar on both tests for the recall condition, but that improvement on the choice test was much greater than improvement on the recall test for the choice condition (see Figure 3). Possibly, the transfer effect here was enhanced by another mechanism that relates directly to the study rationale: the choice method may help children to ignore competing answers on problems that they have only associatively (and not explicitly) learned. We propose that, although multiplication facts may ultimately come to be represented in a single memory node irrespective of multiplicand order – as posited by the IE-model (Rickard, 2005) – this may not yet be the case during early learning of these facts. If memory representations of commutative pairs overlap without being fully integrated, it is possible that they are not equally strengthened by practising only one pair member. When a competing answer is then presented, the strength of its activation may not be sufficient to interfere with the stronger representation of the practised pair member but may be sufficient to interfere with the weaker representation of the unpractised pair member. However, training to ignore or suppress competing answers may decrease their baseline strength enough that subsequent activation is not enough to produce interference with either pair member. Consequently, the choice method would produce an advantage over recall practice on a choice test of unpractised facts, additional to any transfer effects.

Educational implications

As previously noted, fact fluency is of high importance for problem-solving and conceptual understanding (Cumming & Elkins, 1999; Kilpatrick et al., 2001; NMAP, 2008; Woodward, 2006). Difficulties in achieving fluency in multiplication facts are often reported by students and teachers, however, and findings that may aid this process would be welcome in many classrooms (Kilpatrick et al., 2001; Wallace & Gurganus, 2005). The present study provides educators with insights into the relative merits of choice and recall practice materials for improving multiplication fact fluency in the classroom.

First, the study showed that children in Grades 3 and 4 can improve multiplication fact fluency during a short and compact programme of systematic practice. This is an important finding, as studies targeting multiplication fact fluency in regular classrooms usually focus on Grade 4 and upwards (e.g. Ruijssenaars et al., 2002; Wong & Evans, 2007; Woodward, 2006). While more experienced students did benefit more from practice, children at a relatively early stage of learning about multiplication within the context of RME already seem to possess sufficient knowledge to allow consolidation over a short period of time. Educators should therefore not wait until children are older before focusing on these skills: the earlier fluency can be achieved, the earlier children can direct cognitive resources to more complex learning. Of course, practice can only be effective if children possess sufficient conceptual and procedural understanding to be able to construct answers to problems on their own, as is emphasised with RME. It is unclear what children who lack this understanding would gain from practice.

Choice tasks are not commonly used to support mathematics learning in schools. However, choice tasks with well-chosen distractors that children must learn to ignore may promote early stages of fact learning and enable children to practise faster than and as accurately as conventional recall tasks. Furthermore, children appear to become more efficient in ignoring competing answers the longer they practise, which could be an advantage of using these tasks systematically. The choice method may also help children to reject competing answers when presented with unpractised commutative counterparts of practised facts. Although not investigated here, it is possible that this may contribute to more efficient learning of those counterparts.

The choice method offers no advantage over conventional recall practice for improving fluency test performance for practised facts, however. Indeed, when children are more experienced with multiplication, it is less effective for improving recall fluency, which is arguably the most relevant criterion for actual mathematical problem-solving.

Thus, taken together, the findings of the present study provide a number of pointers as to the relative merits of recall and choice practice for improving multiplication fact fluency. During practice, the choice method appears to be an efficient and effective alternative to recall tasks, and may be useful during early stages of fact learning and when children need to learn to inhibit commonly-made errors. However, if the outcome learning criterion is quick and accurate fact recall, recall practice is to be preferred, particularly as children become more experienced with multiplication. This is in accordance with the general principle – of which teachers need to be aware – that selecting practice methods that correspond to the form in which knowledge is to be tested or used will likely benefit performance (Roediger et al., 2002; Van Galen & Reitsma, 2010).

Given current societal and scientific interest in how best to serve groups of pupils sharing particular characteristics within the regular classroom (George, 2005), it is important to note that the materials used here appear equally effective for all ability levels. This is relevant in light of persistent fact retrieval deficits in low-achieving students and children with mathematical learning disabilities, which are thought to arise from poor inhibition of irrelevant information during retrieval (Geary et al., 2012). Although children with mathematical learning disabilities were not included in this study, it is encouraging that students with below-average maths ability were able to learn from the choice method, indicating that they learned to inhibit interference from competing answers. This result could also improve perceptions of both teachers and lower ability students, who often believe that maths ability is fixed and consequently limit effort put into improving their attainment (Marks, 2011).

Future research

This study provides several directions for future research. First, it is possible that practising on a combination of recall and choice tasks may be more beneficial to learning than practising with only one type of problem, as memory traces may then be more robust under varying retrieval conditions (Van Merriënboer & Kirschner, 2013). This could be investigated in a future study. Second, while the kinds of strategies used were important for interpreting the study results, these could only be inferred from performance measures. Future research could investigate strategy use in more depth, through verbal reports or choice/no-choice methods used in other areas of arithmetic learning (e.g. Torbeyns & Verschaffel, 2013). Third, the study entailed 100 minutes of distributed practice. It would be useful to explore the effects of different practice intensities, so that teachers can make informed choices about how to organise practice. Fourth, children with poor maths ability were underrepresented. Targeting these children in follow-up studies would enable their specific learning trajectories to be examined. Fifth, it was not possible to carry out a meaningful retention measurement, as learning conditions could not be controlled in participating classes for an extended period. Measuring retention effects after a period of weeks to months (for example over the summer break) could be included in a future study. Finally, the study was designed to be carried out as a curricular classroom activity in a regular school setting. Consequently, stimuli were not manipulated to investigate well-known factors that affect multiplication fact learning (e.g. distance, problem-size, five, tie, parity). It would be of interest to investigate these effects within a recall/choice practice paradigm in an experimental setting.

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Note

1. In order to correct for the increased probability of making false discoveries (type I errors) when multiple hypotheses are tested, the Bonferroni correction performs each individual test at a significance level of α/n, where α is the desired overall significance level and n is the number of tests.

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