Teaching General Problem-Solving Skills is Not a Substitute for, or a Viable Addition to, Teaching Mathematics

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Problem solving is central to mathematics. Yet problem-solving skill is not what it seems. Indeed, the field of problem solving has recently undergone a surge in research interest and insight but many of the results of this research are both counter-intuitive and contrary to many widely held views. For example, many educators assume that general problem-solving strategies are not only learnable and teachable but are a critical adjunct to mathematical knowledge. The best known exposition of this view was provided by Polya (1957). He discussed a range of general problem-solving strategies, such as encouraging mathematics students to think of a related problem and then solve the current problem by analogy or to think of a simpler problem and then extrapolate to the current problem. The examples Polya used to demonstrate his problem-solving strategies are fascinating and his influence probably can be sourced, at least in part, to those examples. Nevertheless, in over a half century, no systematic body of evidence demonstrating the effectiveness of any general problem-solving strategies has emerged. It is possible to teach learners to use general strategies such as those suggested by Polya (Schoenfeld, 1985) but that is insufficient. There is no body of research based on randomized, controlled experiments indicating that such teaching leads to better problem solving.

Recent 'reform' curricula both ignore the absence of supporting data and completely misunderstand the role of problem solving in cognition. If, the argument goes, we are not really teaching people mathematics but rather are teaching them some form of general problem solving, then mathematical content can be reduced in importance. According to this argument, we can teach students how to solve problems in general and that will make them good mathematicians able to discover novel solutions irrespective of the content.

We believe this argument ignores all the empirical evidence about mathematics learning. While some mathematicians, in the absence of adequate instruction, may have learned to solve mathematics problems by discovering solutions without explicit guidance, this approach was never the most effective or efficient way to learn mathematics.

The alternative route to acquiring problem-solving skill in mathematics derives from the work of a Dutch psychologist, De Groot (1965/1946) investigating the source of skill in chess. Researching why chess masters always defeated weekend players, De Groot managed to find only one difference. He showed masters and weekend players a board configuration from a real game, removed it after 5 seconds and asked them to reproduce the board. Masters could do so with an accuracy rate of about 70% compared to 30% for weekend players. Chase and Simon (1973) replicated these results and additionally demonstrated that when the experiment was repeated with random configurations rather than real-game configurations, masters and weekend players had equal accuracy (±30%). Masters were superior only for configurations taken from real games.

Chess is a problem-solving game whose rules can be learned in about 30 minutes. Yet it takes at least 10 years to become a chess master. What occurs during this period? When studying previous games, chess masters learn to recognise tens of thousands of board configurations and the best moves associated with each configuration (Simon & Gilmartin, 1973). The superiority of chess masters comes not from having acquired clever, sophisticated, general, problem-solving strategies but rather from having stored innumerable configurations and the best moves associated with each in long-term memory.

De Groot's results have been replicated in a variety of educationally relevant fields, including mathematics (Sweller & Cooper, 1985). They tell us that long-term memory, a critical component of human cognitive architecture, is not used to store random, isolated facts, but rather to store huge complexes of closely integrated information that results in problem-solving skill. That skill is knowledge domain-specific, not domain-general. An experienced problem solver in any domain has constructed and stored huge numbers of schemas in long-term memory that allow problems in that domain to be categorised according to their solution moves. In short, the research suggests that we can teach aspiring mathematicians to be effective problem solvers only by providing them with a large store of domain-specific schemas. Mathematical problem-solving skill is acquired through a large number of specific mathematical problem-solving strategies relevant to particular problems. There are no separate, general problem-solving strategies that can be learned.

How do people solve problems that they have not previously encountered? Most employ a version of means-ends analysis where differences between a current problem-state and goal-state are identified and problem-solving operators are found to reduce those differences. There is no evidence that this strategy is teachable or learnable because we use it automatically.

But domain-specific mathematical problem-solving skills can be taught. How? One simple answer is by emphasising worked examples of problem solution strategies. There is now a large body of evidence showing that studying worked examples is a more effective and efficient way of learning to solve problems than simply practicing problem-solving without reference to worked examples (Paas & van Gog, 2006). Studying worked examples interleaved with practice solving the type of problem described in the example reduces unnecessary working memory load that prevents the transfer of knowledge to long-term memory. The improvement in subsequent problem-solving performance after studying worked examples rather than solving problems is known as the worked-example effect (Paas & van Gog).

While a lack of empirical evidence supporting the teaching of general problem-solving strategies in mathematics is telling, there is ample empirical evidence of the validity of the worked-example effect. A large number of randomised, controlled experiments demonstrate this effect (e.g. Schwonke et al., 2009; Sweller & Cooper, 1985). For novice mathematics learners, the evidence is overwhelming that studying worked examples rather than solving the equivalent problems facilitates learning. Studying worked examples is a form of direct, explicit instruction that is vital in all curriculum areas, especially areas that many students find difficult and that are critical to modern societies. Mathematics is such a discipline. Minimal instructional guidance in mathematics leads to minimal learning (Kirschner, Sweller, & Clark, 2006).

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