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The Response Surface Methodology for rapid prototyping of real-time control systems.

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Abstract— **This paper investigates the use of a production control technique known as the Response Surface methodology as a rapid prototyping tool for real time control design. The problem under consideration here is the response of automotive drivetrains to aggressive driver input. The first torsional mode (otherwise known as "shuffle" mode) of automotive drivelines is excited by torque transients and is typically around 2-5Hz. The effect is particularly severe during step changes from the throttle pedal ("tip in" or "tip out"), manifesting itself as an undesirable low frequency longitudinal acceleration oscillation, leading to driver discomfort. The control of this aspect of "driveability" (the error between expected vehicle response and actual vehicle response to an arbitrary control input) is examined, using feedforward control. The overriding principle to be obtained in this examination is the assessment of electronic throttle control in the context of rapid prototyping. The response surface methodology is adopted to achieve this goal. The potential of the electronic throttle for launch control is analyzed and experimentally verified.**

Keywords— **Shuffle, Driveline, Predictive, Driveability, Response Surface.**

I. INTRODUCTION

This paper documents the investigation of the performance potential of a low cost electronic throttle actuator and microcontroller development to control vehicle oscillation. Aside from cost, rapid prototyping was an important factor in the feasibility study.

Drive by wire applications for the replacement of the conventional cable link between the throttle pedal and the throttle body are now the focus of development by many major automotive manufacturers. By fitting a stepper or permanent magnet servo motor [1] to the throttle body, and an electronic throttle pedal with potentiometer, a "drive-by-wire" system can be implemented with no more than a simple linear amplifier. If a microcontroller or DSP is added to the system, then sophisticated control algorithms can be added to the operation of the throttle [2]. Control systems have been designed [3], which allow fast and accurate response to changes in pedal demand, and have been shown to possess robust operating characteristics. The control inputs available to the designer are typically: throttle position, fuel injection timing, exhaust gas recirculation and ignition timing. Current trends indicate that electronic throttle control and variable valve timing are the focus of intense development [4], [5]. A torque controller is designed and implemented in this paper to shape the vehicle response to the first torsional mode of the driveline. The initial requirement is to damp the oscillations generated by throttle tip-in. This dynamic mapping is constrained by the requirement to maximize the vehicle acceleration response available to the throttle. A further requirement might be to accommodate other aspects of driveability , such as maximizing the acceleration overshoot, while minimizing subsequent oscillations during "tip in". Control analysis and design for this automotive system is complicated by a number of factors. There are a number of nonlinearities present, such as backlash in the gearbox, a tyre model which varies nonlinearly with road speed, and nonlinear clutch response. Added to which is a process lag between throttle actuation and torque production and nonlinear engine torque-speed mapping. Experimental data was available from a test car which was fitted with a data acquisition system including three axis accelerometers. A V6 engined saloon vehicle was loaned for the purpose of analysis, design and testing. A systematic excitation of the driveline was made experimentally on the vehicle by performing step demands in all gears at discrete points throughout the effective engine speed range of the vehicle. The generated experimental data (road speed, acceleration, engine speed etc.) was analyzed using the "Response Surface Methodology" (RSM) [6]. The method allows the exploration and optimization of response surfaces, where the response variable of interest (for example vehicle acceleration) is related to a set of predictor variables (for example road speed, selected gear). In the development of a model and control system constrained by computational considerations, and the requirement of rapid prototyping, the RSM allows a low order approximation to be derived from empirical data [7]. A reduced order representation can then be employed in the controller. Application of the RSM analysis to the vehicle response data allows a system model to be developed which lends itself to the design of a scheduled controller structure which is shown to control the first torsional mode of the driveline. Dynamic models of the driveline and engine are widely available, ranging from simple linear models to highly complex nonlinear models. To illustrate the complexity of the control problem, a nonlinear model is presented here which addresses the driveshaft flexibility and adds certain nonlinear elements. Even neglecting certain elements, a high order nonlinear model is obtained which in tandem with other factors

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such as engine lag and uni-directional torque production, shows the difficulties of designing a controller for this system. The driveline can be modelled [8] as a system comprising engine (flywheel), clutch, transmission, propshaft (in the case of rear wheel drive vehicles), final drive, driveshafts, wheels and tyres, and vehicle mass, giving:

$$J_e\ddot{\theta}_c = T_e - T_{fe} - M_{kc}\left(\theta_c - \theta_t i_t\right) - d_c\left(\dot{\theta}_c - \dot{\theta}_t i_t\right) \quad (1)$$

$$\left(J_t + \frac{J_f}{i_f^2}\right)\ddot{\theta}_t = i_t\left(M_{kc}\left(\theta_c - \theta_t i_t\right) + d_c\left(\dot{\theta}_c - \dot{\theta}_t i_t\right)\right) - \left(d_t + \frac{d_f}{i_f^2}\right)\dot{\theta}_t - \frac{1}{i_f}\left(k_d\left(\frac{\theta_t}{i_f} - \theta_w\right) + d_d\left(\frac{\dot{\theta}_t}{i_f} - \dot{\theta}_w\right)\right)$$

$$\left(J_w + mr_e^2\right)\ddot{\theta}_w = k_d\left(\frac{\theta_t}{i_f} - \theta_w\right) + d_d\left(\frac{\dot{\theta}_t}{i_f} - \dot{\theta}_w\right) - \left(d_w + mc_{r2}r_e\right)\dot{\theta}_w - \frac{1}{2}c_a A_v \rho_a r_e^3 \dot{\theta}_w^2 - r_e m\left(c_{r1} + G\sin\left(\tau_r\right)\right) \quad (2)$$

This high order representation of the system gives a reasonable response when implemented as a simulation and compared to experimental responses. It does differ from the real system in significant respects, however. The first difference is the presence of backlash in the geared elements such as the gearbox and differential. This effect is manifest as a deadzone at tip-in while the transmission angular slack is taken up during the application of engine torque. This will also contribute non-linear effects during transmission oscillations after the initial tip-in torque transient. Secondly, the tyre model in reality is far more complex than the description in the given model, in terms of wheel slip and friction coefficient [9].

II. RESPONSE SURFACE METHODOLOGY

The response surface methodology is a technique designed to optimise process control and production by the application of designed experiments in order to characterize a system [6]. The relationship between the response variable of interest (y), and the predictor variables ($\xi_1, \xi_2, ..., \xi_k$) may be known exactly allowing a description of the system of the form

$$y = g\left(\xi_1, \xi_2, ..., \xi_k\right) + \epsilon \quad (3)$$

where ϵ represents the model error, and includes measurement error, and other variability such as background noise. The error will be assumed to have a normal distribution with zero mean and variance σ^2 In general, the experimenter approximates the system function g with an empirical model of the form

$$y = f\left(\xi_1, \xi_2, ..., \xi_k\right) + \epsilon \quad (4)$$

where f is a first or second order polynomial. This is the empirical or response surface model. The variables are known as *natural variables* since they are expressed in physical units of measurement. In the response surface methodology (RSM), the natural variables are transformed into *coded variables* $x_1, x_2, ...x_k$ which are dimensionless, zero mean, and the same standard deviation. The successful application of RSM relies on the identification of a suitable approximation for f. this can for example be a second order model of the form

$$\eta = \beta_0 + \sum_{j=1}^{k}\beta_j x_j + \sum_{j=1}^{k}\beta_{jj}x_j^2 + \sum_{i<j}\sum\beta_{ij}x_i x_j. \quad (5)$$

Response surface methodology is intimately connected to *regression analysis*. For example when considering the first-order model, the β terms comprise the unknown parameter set which can be estimated by collecting experimental system data. This data can either be sourced from physical experiments, or from previously designed dynamic computer models. The parameter set can be estimated by regression analysis based upon the experimental data. The method of least squares is typically used to estimate the regression coefficients. With $n > k$ on the response variable available, giving $y_1, y_2, ..., y_n$, each observed response will have an observation on each regressor variable, with x_{ij} denoting the ith observation of variable x_j. Assuming that the error term ϵ has $E(\epsilon) = 0$ and $Var(\epsilon) = \sigma^2$ and the (ϵ_i) are uncorrelated random variables. The model can now be expressed in terms of the observations

$$\begin{aligned} y_i &= \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + ... + \beta_k x_{ik} + \epsilon_i \\ &= \beta_0 + \sum_{j=1}^{k}\beta_j x_{ij} + \epsilon_i, \\ i &= 1, 2, ..., n \end{aligned} \quad (6)$$

The β coefficients in equation (6) are chosen such that the sum of the squares of the errors (ϵ_i) are minimized via the least squares function

$$\begin{aligned} L &= \sum_{i=1}^{n}\epsilon_i^2 \\ &= \sum_{i=1}^{n}\left(y_i - \beta_0 - \sum_{j=1}^{n}\beta_j x_{ij}\right)^2 \end{aligned} \quad (7)$$

III. RSM APPLICATION

In the analysis of the acceleration response of the vehicle, there are three variables of interest, namely vehicle loading, road speed and selected gear ratio. For the purpose of clarity in the description, vehicle loading will be assumed to be a fixed standard two person loading of $160kg$. In order to reduce the overall number of data sets required to construct the response surface of the system, a factorial approach is adopted [10]. The combinations of factorial experiments at $5ms^{-1}$ in each gear requires 25 experiments to be performed. Each proposed factorial combination was assigned a serial number and performed in random order via output from a random number generator. Each experiment consists of coasting the vehicle in the appropriate gear at the appropriate roadspeed, and performing a 100% tip-in with the accelerator pedal. This corresponds to exciting the open loop system with a unit step torque input. On the day of testing, the one mile asphalt test road was dry, with overcast sky and ambient temperature of 60^oF. The effect of the energy storage components in the driveline can be clearly seen in an examination of the vehicle at $10ms^{-1}$ in second gear in Figure 1 Examination of the vehicle response to tip-in reveals a system which can be approximated as a delay and second order dynamic response with an overshoot and settling time which varies with road speed and selected gear. This approximation to describe the entire vehicle response can be formulated by application of RSM to the experimental data. Of note is also the lag between actuation and response. The torque lag is combined with a high level of lash for this high mileage vehicle

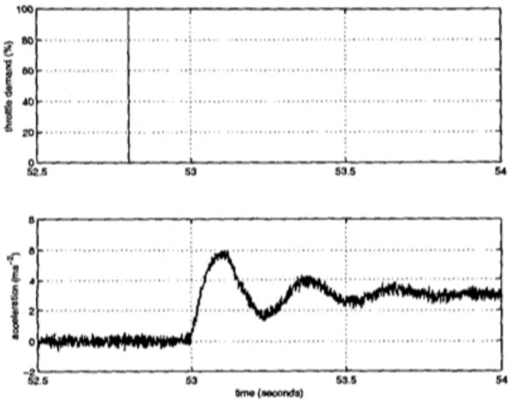

Fig. 1. Vehicle acceleration step response in second gear at $10ms^{-1}$.

The experimental data which has been gathered can be synthesized into a response map for the vehicle in order to design an oscillation control system. For the definition of driveability under consideration here, the vehicle response can be characterized as the damping ratio of the second order approximation map with variables road speed and selected gear. The individual experimental responses may be expressed in terms of overshoot and settling time. The transfer function describing the open loop system may be described as

$$\frac{C(s)}{R(s)} = k\frac{1}{as^2 + bs + c} \tag{8}$$

from which the damping ratio of the system can be calculated as the ratio of the actual damping b to the critical damping $b_c = 2\sqrt{ac}$ [11]. Thus the damping ratio ζ can be calculated from $\zeta = \frac{b}{b_c}$. The roots of the characteristic equation 8 are $s_1, s_2 = -b_c \pm jc\sqrt{1-b_c^2}$. This forms a complex conjugate pair from which the damping ratio and natural frequency can be computed. The natural units ξ_1 and ξ_2 of the experimental data (road speed and selected gear) is first transformed into the corresponding coded variables x_1 and x_2, such that

$$x_{i1} = \frac{\xi_{i1} - \left[max\left(\xi_{i1}\right) + min\left(\xi_{i1}\right)\right]/2}{\left[max\left(\xi_{i1}\right) - min\left(\xi_{i1}\right)\right]/2} \tag{9}$$

and

$$x_{i2} = \frac{\xi_{i2} - \left[max\left(\xi_{i2}\right) + min\left(\xi_{i2}\right)\right]/2}{\left[max\left(\xi_{i2}\right) - min\left(\xi_{i2}\right)\right]/2} \tag{10}$$

thus the response surface in terms of the coded variables is obtained.

$$\begin{aligned} \hat{y} &= 0.4079 - 0.804x_1 + 0.3809x_2 + 0.0519x_1^2 \\ &- 0.0429x_2^2 - 0.0121x_1x_2 \end{aligned} \tag{11}$$

Comparing the computed response surface against a second experimental data set gave an average residual error of 1.65%. The surface in terms of damping ratio, selected gear and road speed is shown in Figure 2. The design of the driveability compensator will be considered in the next section.

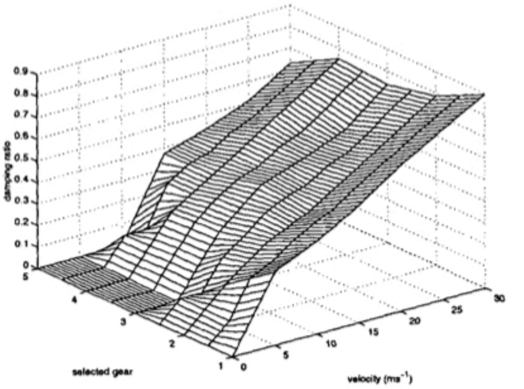

Fig. 2. Vehicle damping ratio response surface.

IV. Controller Design

The control of driveline shuffle has been the subject of controller design, including LQR and generalized optimal control (GOP) [12] noting that the technique is "unlikely to prove beneficial in any practical implementation of driveline control", and pole placement [13],[14], which utilizes a ninth order controller which although damping the decay oscillations, adds an undesirable initial acceleration transient.

As previously noted, it is required to assess the usefulness of RSM for rapid prototyping, and also to provide an initial simple useable controller to identify the potential of the electronically operated throttle body for controlling the driveline. The simple feedforward controller is designed as follows. A response surface has been obtained (Figure 2) which describes accurately the vehicle's damping ratio map. As an initial design target, a damping ratio of 0.7 across the entire operating map would be a desirable response.

The problem as presented to the control designer is nonlinear and time variant (varying with selected gear and roadspeed), a transport delay exists in terms of engine torque production, and the engine torque output varies nonlinearly in terms of engine speed, engine temperature and other factors. The RSM analysis however allows a simple open loop feedforward controller to be immediately designed and implemented to allow a fast appraisal of the actuator potential. The system response surface extracted from the experimental data is a representation of the complex conjugate pole pairs of the approximation in terms of the system's varying damping ratio. The initial approach will be to effect a pole-zero cancellation of these complex conjugate poles by utilizing the damping ratio response map, and pole placement to give a satisfactory response, by utilizing the damping ratio error response map. This method does rely on accurate knowledge of the position of the uncompensated poles which has been ascertained experimentally for the purpose of this development. The parameters of the feedforward controller are derived in real-time from the response surfaces and are a function of selected gear and roadspeed. The feedforward compensator takes the form $\frac{as^2+bs+c}{as^2+ds+c}$ where the coefficient b is calculated from the damping ratio response surface, and performs pole-zero cancellation. Coefficient d produces the desired pole placement and forms the requires damping ratio, and is calculated from the sum of the damping ratio response surface, and the damping ratio response error surface. The demand from the throttle pedal, throttle position and roadspeed were read in via A/D ports, and the selected gear read in via the digital I/O. Output from the controller was sent to a power amplifier via the PWM port. With the controller in place, the experimental set was repeated to asses the potential of the electronic throttle control.

V. EXPERIMENTAL RESULTS

Of particular note in this implementation is the fact that the hardware assembled to achieve the electronic throttle control is both cheap and readily available. The goal of rapid prototyping was achieved in a matter of days between initial experimental testing and final implementation testing. The original set of experiments were repeated with the electronic throttle both compensated and uncompensated. A comparison at the $10ms^{-1}$ in second gear step response is shown in Figure 3. The time axis in both traces was zeroed at the initiation of the step demand for the purpose of clarity. The marked improvement in vehicle oscillation obvious in figure 4 was repeated throughout the operating map of the vehicle. The throttle demand computed by the algorithm at this operating point in Figure 3 is shown in figure 4. The compensated vehicle responses over the operating region of the vehicle were found to have a mean damping ratio of 0.68, with a maximum residual of 0.07. The tip-in driveability of the vehicle was found to be subjectively very improved, in addition to the experimental evidence of the vehicle damping ratio map. Although a small amount of acceleration rate is sacrificed to achieve the suppression of oscillations, subjectively the vehicles performance was not felt to be affected. The controller did endow the vehicle with a "turbine like" feel.

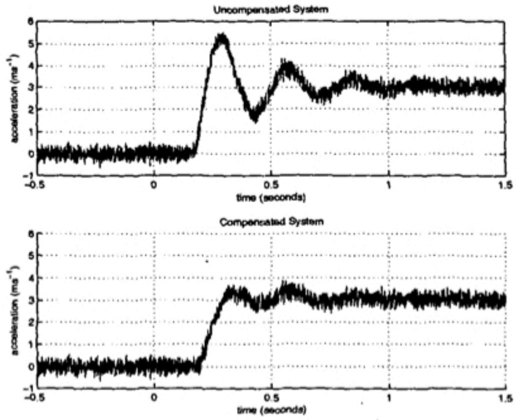

Fig. 3. Vehicle compensated and uncompensated step response.

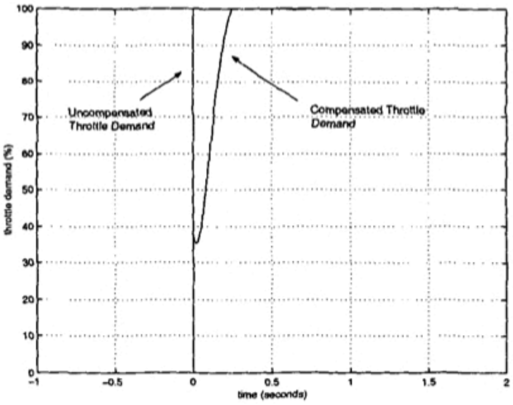

Fig. 4. Uncompensated and compensated throttle demands.

VI. CONCLUSIONS

This project had two hierarchically related objectives, firstly to examine the application of the Response Surface Methodology to rapid control design prototyping, and having successfully achieved this goal, the second objective was to use the RSM to examine the potential of electronic throttle control to shape the acceleration response of an experimental vehicle. A controller based upon non-linear techniques and certain aspects of pole placement is currently being developed, but the timescale for analysis and design available for this project being particularly stringent, proved the usefulness of the RSM technique. At no point is the application of RSM being suggested as a replacement for standard control design procedures, however the benefits of its addition to the controller designer's toolbox become immediately apparent when we consider the design process presented in this paper. The timescale from experimental data capture through RSM analysis to experimental verification and assessment of the electronic throttle's potential for vehicle acceleration shaping was under four days. At this point, a judgement upon the viability and potential of the project could easily be made with confidence. In parallel with the RSM analysis, standard controller designs were being implemented and assessed. However these designs are still in the development stage after many months. In conjunction with the existing mathematical models of the vehicle driveline presented in a previous section, it is necessary to develop a fully dynamic model in a software simulation package such as Simulink. The development of the mathematical models in conjunction with a simulation and also highly complex nonlinear controller design is by its nature an extremely time consuming task. In this context, the use of the RSM becomes extremely attractive. The technique has allowed an examination of the system control potential to be made at the start of the project, benefitting both the confidence of the industrial partners, and giving a realistic benchmark of potential performance. The controller derived by the RSM is immediately useable on the experimental vehicle, providing a demonstration facility at project inception. Some other benefits of this development tool are also significant. A controller is quickly available for verifying the computational, mechanical and electrical components of the control system, giving a stable platform for subsequent controllers as and when they become available. The controller as designed via the RSM is simple and low order by nature, and thus can be installed on a very simple microcontroller. Finally, a quick and cheap assessment of a system's potential can be rapidly made in order to support project development proposals. A controller has been developed to examine the potential of electronic throttle control. The stated requirements were rapid prototyping, algorithm simplicity in both design and implementation, oscillation control, and the ability to work effectively across the entire operating region of the vehicle. All these requirements have been fulfilled, with the conclusion that electronic throttle control is a capable tool in a suite of applications to control the various aspects of "driveability". A secondary achievement has been to implement the control system on extremely cheap widely available components, further adding to its attractiveness to the automotive industry. A design method has been followed, centered around the Response Surface Methodology which has enabled the rapid prototyping process from empirical data. The design of a useable implementation has been achieved on a time varying process with significant time delays and nonlinearities. The feedforward controller has been the subject of further developments, notably the development of parameter estimation and adaptive elements to make the controller not only robust to parameter variations, but also optimal in terms of the vehicle response map. Empirical tests with large driver groups have revealed that in fact the initial overshoot may be desirable to some drivers, as long as the subsequent oscillations are completely damped. A gain scheduled version of the basic feedforward controller is being developed to deliver this response.

Physical Variables
J moment of inertia
θ angular position
T torque
M mass
d damping coefficient
i gearing ratio
k spring stiffness
m mass
r effective rolling radius
G acceleration due to gravity

Subscripts
e engine
c clutch
fe engine friction
t transmission
f friction torque
d differential
w wheel
a aerodynamic drag
ρ air density

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