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REGULAR ARTICLE

Models for the schedule optimization problem at a public transit terminal

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Abstract This work deals with the proposal of some models for the schedule optimization problem for public transit networks. In particular, we consider the case of a transit terminal where passengers are supposed to split among different lines of a service, or even change mode of transportation in case of intermodal systems. Starting from a given schedule for the transit lines arriving at the terminal, the aim is to decide the optimal schedule for the output lines, in such a way to balance the operative costs of the service and the passenger waiting time at the transit terminal. We propose two different models for this problem, which present strong similarities with some well known combinatorial optimization models. Computational results are also presented, showing the suitability of the models to solve real case studies.

Keywords Transportation · Transit network · Timetable/scheduling · Combinatorial optimization

1 Introduction

Intermodal transportation is based on the combined use of several modes and/or lines of transport to move passengers from their own origins to destinations. The integration of multiple transport resources can produce various benefits such as the reduction

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of traffic congestion, pollutant emissions, and fuel consumptions. In this context a crucial role in the efficiency of an intermodal system is played by transfer or transit terminals, i.e., locations where users can split line and/or modes of transport during their trips. Decisions about transit terminals can depend on various time horizons. The general transportation network design problem aims at the definition of the components of transportation networks of various modes and includes decisions about the positioning and the features of terminals which can promote the efficient usage of public transit through a reduction of transfer times. Methodologies for public transit network design problems have, in general, separate characteristics due to the specific peculiarities of each transportation mode. With regard to the bus transit, Ceder and Wilson (1986) define and present a hierarchical model for the whole planning process, distinguishing among problems of network design, headway or frequency setting and timetable development. During an origin-destination trip, it is often necessary to split lines, and excessive waiting time for transfers can discourage transit use (see Stern 1996). For this reason, waiting time reduction can play a crucial role in increasing the attractiveness of public transportation. Various models and techniques have been proposed in literature to simulate the behaviour of transit users in the route choice (see for instance Fernandez et al. 1994; Nguyen and Pallottino 1988; Spiess and Florian 1989; Wu et al. 1994),

Most of these models introduce appropriate network representations in order to explicitly take into account wasted time at a transfer node as sum of costs associated to walk, wait and alight arcs. In the field of transit network design problem, i.e., the determination of a configuration consisting of a set of transit routes and associated frequencies to achieve some desired objectives, a vast literature is available, focused either on single tasks or on simultaneous solution of more problems. A rich survey on existing models and methods is provided by Fan and Machemehl (2004), while Zhao (2006) presents a summary of the main features of some solution approaches. In the following we provide a synthetic overview of the most significant contributions in the literature, with a particular attention to the transit line synchronization problem: given a set of transit lines crossing each other at some points of the network, the aim is to maximize the synchronization of the schedule, in such a way to minimize the waiting time of the passengers in the terminals. Domschke (1989) considers the schedule synchronization problem (SSP) for public transit networks with the objective of minimizing the sum of users transfer waiting time assuming given operation hours. He formulates the problem as a relaxation of the quadratic assignment problem providing solutions for real transit networks. Keudel (1988) proposes an interactive optimization tool aiming at reducing the total users' waiting time and the number of users exceeding a given time limit. Baaj and Mahmassani (1990, 1991, 1995) present a framework of an artificial intelligence/operations research hybrid approach whose main components are given by an algorithm which generates different sets of routes, together with various descriptors associated to the generated routes and an algorithm which produces an improved solution. Bookbinder and Desilets (2002) combine a simulation procedure with an optimization model whose decision variables are the offset times. The model turns out to be a quadratic assignment problem in which the objective function can be represented by various measures of overall user disutility. Desilets and Rousseau (1992) present some results for the application of a variant of the SSP problem on the network of Montreal. Voss (1992) formulates the SSP as a multicommodity network design problem, exploiting the quadratic semi-assignment problem, and proposes a tabu search algorithm to solve the problem. Daduna and Voss (1995) provide the application of the tabu search algorithm for the SSP on some case studies of practical interest. Adamski (1995) formulates and solves the problem of synchronizing different transit lines with shared route segments. Ceder et al. (2001) and Ceder and Tal (1999) address the problem of generating the timetable for a given network of buses so as to minimize their synchronization. In particular, they propose a mixed integer linear programming model to maximize the number of the simultaneous bus arrivals at the connection nodes of the network. Chakroborty (2003) and Ngamchai and Lovell (2003) show how genetic algorithms, through an appropriate representation of the solutions, can be used to solve the transit routing design problem. Wong and Leung (2004) propose a mixed integer programming formulation aiming at the reduction of the waiting time for the passengers of a railway system, also presenting a heuristic approach. Their model requires some particular assumptions such as an unlimited capacity of the trains and a uniform distribution of the passenger flows. Zhao and Ubaka (2004) develop two heuristics for a problem whose goal is to obtain a transit network structure with minimum transfers, while optimizing service coverage. Fleurent et al. (2004) present a more elaborate measure of synchronization including parameters such as minimum, maximum and ideal waiting time, as well as weight factors specifying the relative importance of synchronization with respect to different times, places, routes and directions: an optimization approach is presented, minimizing other vehicles scheduling costs. Schroder and Solchenbach (2006) study the possibility to improve the efficiency of an intermodal transportation system through the modification of the timetable of the lines. They propose a quadratic semi-assignment model based on offset decision variables and a system of assignment of qualitative penalties to the solutions. Fan and Machemehl (2006) formulate a multi-objective nonlinear mixed integer model for the bus network design problem. They develop a solution approach based on genetic algorithms which combines a generation algorithm of candidate solutions and a network analysis procedure that determines service frequencies and other performance measures. Zhao and Zeng (2008) present a hybrid metaheuristic approach for optimizing transit networks in order to minimize the summation of individual transit users travel times. The methodology integrates typical aspects of tabu search and simulated annealing. As several authors underline (Newell 1979; Baaj and Mahmassani 1991; Chakroborty 2003), the transit network design problem is very complex not only due to its combinatorial nature but also due to the difficulty in representing the constraints.

The objective function often rises additional difficulties either because it is nonconvex or simply because there are several objectives. For these reasons the attempts to formulate the problem as a mathematical programming problem do not represent a realistic and efficient approach to describe the problem. In addition, most of the methods and of the models are oriented to represent large-scale urban contexts where the results provided by the application of the various proposed heuristics are not easy to evaluate, as the main ingredients (demand, flows) are stochastic and not easy to be calculated. These critical aspects suggest that mathematical programming formulations can be used to represent more limited and simple scenarios.

In this paper our interest lies on the optimization of the transit line schedule at a single transfer node, which coincides with a transfer terminal within a transit network. In the following, we refer to this as the Schedule Optimization Problem (SOP). The problem consists in deciding concurrently how many output lines should be activated, and the best time for their departure, given certain levels of demand, in such a way to balance operative costs and user costs. From the practical point of view, this problem arises in several traditional route networks, especially on regional scale, targeted to serve centralized core-oriented land user patterns, as in the case study illustrated in Sect. 5.1. In this case users are in large part commuters who use the network during their home-work trips. In this context, especially in presence of weak and/or dispersed demand, transit frequencies are quite low so the schedule synchronization is particularly crucial in the definition of the users (real or perceived) costs.

We propose mathematical models based on a time-space representation of the network to describe the SOP. The proposed models are applied to a real case study and optimally solved, in order to show the suitability of the SOP to provide results of practical interest. The remainder of this paper is organized as follows: in Sect. 2, a description of the considered optimization problem is provided in detail. In Sect. 3, a time-expanded graph for the representation of the SOP is introduced, and two different mathematical models exploiting this representation are formulated. A description of possible variants and extensions of interest is proposed in Sect. 4. In Sect. 5, the application of the model to a real case study is shown and the results obtained are discussed. The results of computational experiments on test problems are also described in order to show that the proposed formulations can cope efficiently with instances of significant size. Finally, concluding remarks and a discussion on possible future lines of research are given.

2 The SOP for a transit terminal

A transfer or transit terminal is a location, within an intermodal transportation system, where users can change lines of transportation during their own origin-destination trips. We consider in particular the case of trips starting from a set of possible origins and all sharing a common destination, within a given time horizon T. This scheme, focused on a single transfer, could appear quite restrictive. However, this context is typical of many practical cases where the transportation scenario, during the morning peak period is centralized, core-oriented with the presence of a central city where commuters are directed from dispersed origins on a regional scale. As in this situation, in general, trips are characterized by significant travel times; it is infrequent that passengers use more than one line to reach the city, considering that, in order to achieve the final destination within the city they may use further lines of the urban public transportation network. In these circumstances, it is reasonable that users perform a single transfer during their origin-destination trips on the regional transit network, also according to the survey conducted by Stern (1996) on various transit agencies in US. This is due to the fact that users tend to overestimate wasted time for transfers with respect to the other components of the generalized trip costs.

In order to formally define the problem, we introduce the following notation. Let there be a given set I of *input lines*, i.e., transit lines arriving at the transit terminal. We associate to each of these lines:

- an arrival time $a_i \in T$, $i \in I$, at the transit terminal;
- a certain level of demand d_i, $i \in I$, equal to the number of passengers carried by this line to the terminal.

We are interested in defining the set O of *output lines*, that is transit lines that start from the terminal towards the destination; each of these lines must be characterized by:

- an activation cost f_j, $j \in O$, paid by the service operator to activate an output line;
- the departure time $b_j \in T$, $j \in O$, from the transit terminal;
- a capacity Q_j, $j \in O$, i.e., the maximum number of passengers that can be simultaneously transported on the line.

A passenger, during the trip from his own origin to the destination node, will choose an input line $i \in I$ and arrive at time a_i at the terminal; afterwards, he will choose the first available output line $j \in O$ to reach the destination. Being $b_j \in T$ the departure time of such line from the terminal, each passenger using this trip will have a waiting time at the terminal equal to $b_j - a_i$. A sketch of this operation context is depicted in Fig. 1. In the proposed scheme some simplifications have been adopted. We assume that the arrival times a_i are deterministic and that passengers arriving at time a_i can use any output line with departure time $b_j \geq a_i$. In real situations, b_j must be greater than $a_i + l_i$ where l_i represents the total time spent in the transfer from the arriving location to the departure location. This aspect can be easily considered including the transfer times in the arrival time. Furthermore, the demand d_i has been defined as the number of users arriving at time a_i through the input line i. In this way, the model seems to consider only passengers who transfer at the terminal. However, through the introduction of a discrete time-expanded model (which will be discussed in the next section), the scheme is defined in terms of total demand d_i arriving at time i directed to the common destination. This way, it is possible to include in this value the component of the demand generated by passengers directly arriving at the terminal without using any of the input lines, such as those passengers arriving either by private car

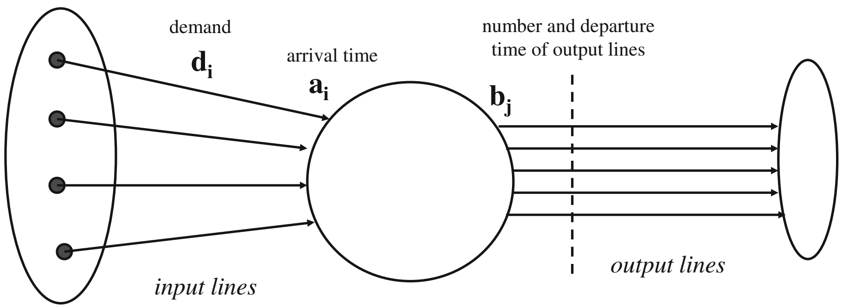

Fig. 1 A scheme of the considered problem. Each line $i \in I$ arrives at time a_i at the transit node. A decision must be taken concerning how many output lines to activate, and the departure times b_j

or by walking. We also assume that the level of service defined by the number of the activated lines does not influence the user choices: in practice, we suppose that users have no alternatives to the transfer; this is reasonable in a transportation scenario on regional scale with reference to a given time horizon.

On the base of the described representation, we consider the SOP that consists in determining the number of lines to be activated, and the departure time for each of these lines, in order to optimize a certain performance index, satisfying at the same time all the demand present at the transit terminal. We consider, as a performance index to be minimized, an opportunely weighted sum of the total waiting time for the users (user cost) and of the total activation costs of the lines (operator cost).

3 Time-expanded models for the SOP

The mathematical models we propose in this section to describe the SOP are based on a discrete time expansion of the terminal node. The time horizon T is divided into n time periods, obtained by dividing T by the length of a time unit τ. Therefore, the time expansion of the terminal node over the time horizon T is given by the graph $G_T = (N_T; A_T)$, being:

- $N_T = \{1, \ldots, n\}$
- $A_T = \{(i, j) : i \in \{1, \ldots, n\}, j \in \{1, \ldots, n\}, i \leq j\}$.

Each node i in the time-expanded terminal node corresponds to a time instant of the time horizon T, that can coincide with the arrival or departure time of some lines. Each arc (i, j) links an instant node i with an instant node j, such that $j \geq i$. We refer to these as the *holdover arcs*. A weight d_i is associated to each node $i \in N_T$ representing the total number of passengers that are supposed to arrive at the terminal at time i, all of them being directed to the common destination. A waiting time $w_{ij} = j - i$ (between instants i and j) is associated to each holdover arc $(i, j) \in A_T$. The waiting costs c_{ij} can be defined as $k_{ij} \cdot w_{ij}$, being k_{ij} chosen in a set of possible weights which can be associated to the waiting time (otherwise, k_{ij} can be equal to 1 for any pair i, j). For instance, k_{ij} can be defined as an increasing function of the waiting time. Figure 2 depicts the discrete time expansion of the terminal node. In the following, we present two mathematical models for the SOP, both using the time-expanded scheme just introduced to represent the transit terminal node.

3.1 A model with variables on the holdover arcs

The SOP can be mathematically formulated introducing the following decision variables:

- $x_{ij} \geq 0$, $(i, j) \in A_T$, are continuous variables associated to the holdover arcs, which represent the number of passengers arriving at time i at the terminal and leaving at time j (see Fig. 3);
- y_j, $j \in N_T$, are binary variables and assume value 1 if an output line is activated at time j, and 0 otherwise.

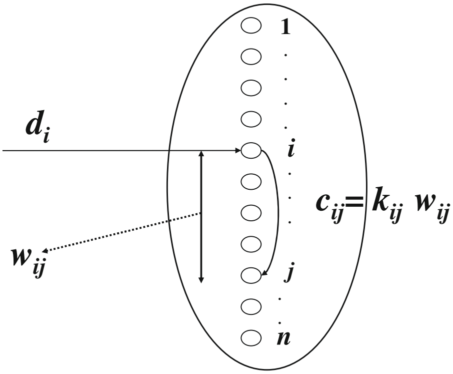

Fig. 2 Sketch of the time-expanded transit terminal node: each holdover arc (i, j) is associated with a waiting time w_{ij} and a waiting cost c_{ij}

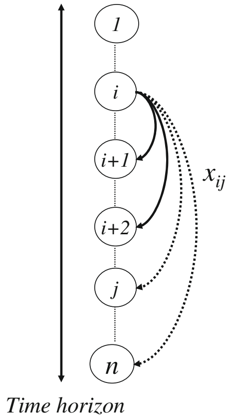

Fig. 3 A scheme of the FLP formulation for the SOP: variables x_{ij} refer to passengers arriving at time i and leaving at time j

This way the model is given by

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} \cdot x_{ij} + \sum_{j=1}^{n} f_j \cdot y_j \tag{1}$$

$$\text{s.t.} \sum_{j=i}^{n} x_{ij} = d_i \quad \forall i \in \{1, \ldots, n\} \tag{2}$$

$$\sum_{i=1}^{j} x_{ij} \leq Q_j \cdot y_j \quad \forall j \in \{1, \ldots, n\} \tag{3}$$

$$y_j \in \{0, 1\} \quad \forall j \in \{1, \ldots, n\} \tag{4}$$

$$x_{ij} \geq 0 \quad \forall (i, j) \in A_T. \tag{5}$$

The objective function (1) is the sum of the total users' waiting costs and the total activation costs. Constraints (2) ensure the satisfaction of the demand arriving at time i while constraints (3) indicate that variable x_{ij} must be equal to 0 unless a line starts from the terminal at time j. In that case, x_{ij} must be less than or equal to the capacity of the line Q_j. Model (1)–(5) presents $O(n^2)$ continuous variables and n binary variables. This model emphasizes that the problem can be seen as a Capacitated Facility Location Problem (CFLP) (see for instance Sridharan 1995). In fact, the set of nodes N_T can be viewed as a set of potential locations for a facility with a fixed cost f_j for installing the facility at j. Values c_{ij} can be seen as the transportation cost for assigning the demand of customer i to facility j and Q_j as the maximum amount of demand that can be assigned to facility j.

It is therefore possible to solve instances of the SOP exploiting the approaches developed and proposed in literature for the CFLP problem, concerning in particular lagrangian relaxation and column generation methods. The CFLP belongs to the class of NP-hard problems. Hereafter we refer to model (1)–(5) as the CFLP formulation for the SOP.

In a situation in which we can assume that each activated transit line is large enough to contain all passengers that at the time of the line departure are waiting in the transit terminal we face an *uncapacitated* SOP. Relations (3) can be modified as follows:

$$\sum_{i=1}^{j} x_{ij} \leq M \cdot y_j \qquad \forall j \in \{1, \ldots, n\} \tag{6}$$

being M a constant higher than the sum of all the demands. Model (1), (2), (6), (4) and (5) can be seen as a simple plant location problem or uncapacitated facility location problem (UFLP) (Krarup and Pruzan 1983; ReVelle and Laporte 1996). We refer to (1), (2), (6), (4) and (5) as the UFLP formulation for the uncapacitated SOP. Being a generalization of the well-known set covering problem, the UFLP is NP-hard in the strong sense. It follows that both the capacitated and the uncapacitated variants of the SOP are hard to solve when it is formulated associating the variables with the holdover arcs. In the following, we also refer to the CFLP and UFLP formulations together as the facility location problem (FLP) formulation for the SOP.

3.2 A model based on the conservation of passenger flows at time nodes

It is possible to formulate the SOP in a different way, reducing the number of continuous variables. This is accomplished by defining new sets of variables as follows (see also Fig. 4):

- x_j, $j \in N_T \cup \{0\}$, are continuous variables indicating the number of passengers which remain in the terminal during the time interval $[j, j+1]$, waiting for the next departure;
- q_j, $j \in N_T$, represent the number of passengers leaving the terminal at time j using an output line activated at time j;
- y_j, $j \in N_T$, defined as in the previous model.

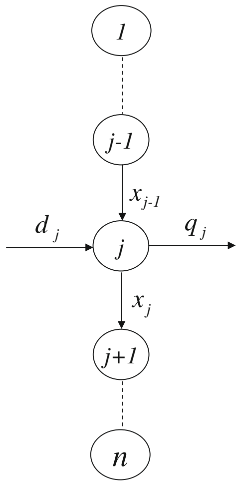

Fig. 4 A scheme of the set of variables in the LSP formulation for the SOP

The model becomes

$$
\begin{aligned}
\min\ & \sum_{j=1}^{n} c_j \cdot x_j + \sum_{j=1}^{n} f_j \cdot y_j && (7)\\
\text{s.t.}\ & x_{j-1} + d_j = x_j + q_j \quad \forall j \in \{1, \ldots, n\} && (8)\\
& q_j \le Q_j \cdot y_j \quad \forall j \in \{1, \ldots, n\} && (9)\\
& x_0 = x_n = 0 && (10)\\
& x_j \ge 0 \quad \forall j \in \{0, \ldots, n\} && (11)\\
& q_j \ge 0 \quad \forall j \in \{1, \ldots, n\} && (12)\\
& y_j \in \{0, 1\} \quad \forall j \in \{1, \ldots, n\} && (13)
\end{aligned}
$$

being:

- f_j the fixed cost paid by the operator for the activation of a line starting at time j from the terminal;
- c_j the waiting cost associated to a passenger waiting at the transit terminal during interval $[j, j+1]$.

If $c_j = 1$ for each j, waiting costs coincide with waiting times; otherwise it is possible to associate to each j a different weight c_j. For instance, waiting time during peak periods is more strongly perceived.

This model has $2 \cdot n$ continuous variables and n binary variables. The objective function (7) has the same meaning as in the model (1)–(5). Constraints (8) express the conservation of passenger flows at each time instant, while conditions (10) ensure that the number of passengers staying in the terminal at the beginning and at the end of the time horizon is equal to 0. Constraints (9) hold the same meaning of constraints (3), ensuring that $q_j = 0$ (no passengers leaving at time j) if there is no output line starting at time j ($y_j = 0$), and that q_j is less than or equal to the capacity Q_j of the line activated in instant j otherwise.

If we modify model (7)–(13) by replacing d_j with $-d'_j$ and q_j with $-q'_j$ we obtain the well-known formulation of the single-item Lot Sizing Problem (LSP) (see for instance Drexl and Kimms 1997; Karimi et al. 2003, and Wolsey 1995). In that problem, d'_j refers to quantity of items, or demand, that must be ready in a manufacturing system at time j and q'_j to the quantity of items to be produced at each time period, while x_j expresses the inventory of items at time j. The LSP was proved by Florian et al. (1980) to be NP-hard by reduction from the Knapsack problem. Hereafter we will refer to the model (7)–(13) as the LSP formulation for the SOP.

In order to consider the uncapacitated variant of the SOP, we can replace constraints (9) by:

$$q_j \leq M \cdot y_j \qquad \forall j \in T \tag{14}$$

assuming that $M \geq \sum_{i=1}^{j} d_i, \forall j \in T$. It is possible to show that model (7), (8), (14), (10)–(13) falls in the class of polynomially solvable problems, which can be proved, for instance, by recalling and adapting the proof for the dynamical programming algorithm proposed by Wagner and Within (1958). Therefore, the uncapacitated variant of the SOP is polynomially solvable when formulated as a LSP, while it is NP-hard when the problem is formulated as a UFLP, associating variables with the holdover arcs. The reason for this difference in the complexity can be understood by observing that the LSP formulation of the SOP implies a loss of detail in the knowledge of the whole transportation system, with respect to the FLP formulation. Indeed, the arrival times of the passengers waiting at the transit terminal at each time interval are explicitly available in the latter, while in the former such information is lost. For instance, in the FLP formulation, constraints on the maximum number of periods that each user can spend inside the terminal can be explicitly considered, as presented in Sect. 4.

The assumption of uncapacitated transit lines seems to be far from the reality for most of the transportation services, but it can be exploited to obtain lower bounds on the optimal solution.

4 Some variants of the models

It is possible to introduce extensions of the proposed mathematical models either in the FLP or in the LSP formulation, in order to take into account further operational constraints.

In the FLP formulation, the presence of an additional constraint on the maximum waiting time allowed can be easily considered by setting either $c_{ij} = \infty$ or $x_{ij} = 0$ for any $j > i + W_i$, where W_i is the maximum allowable waiting time for users arriving at time i.

Constraints on the minimum number of output lines $N_{\min}^{ij}$ to be activated on a given interval $(j - i)$ can be introduced by imposing, for both the formulations, that:

$$\sum_{k=i}^{j} y_k \geq N_{\min}^{ij} \qquad \forall i, j \in T, j > i. \tag{15}$$

Such a constraint can be seen as an alternative way to impose a maximum waiting time constraint, and it can be introduced to represent a possible public intervention oriented to ensure a minimum threshold for the quality of service.

Similarly, constraints about a maximum number of output lines $N_{\max}^{ij}$ to be activated on a given interval $(j - i)$ can be defined:

$$\sum_{k=i}^{j} y_k \le N_{\max}^{ij} \qquad \forall i, j \in T, j > i. \tag{16}$$

This way, it is possible to formulate additional constraints on the available fleet size and, if the interval coincides with the time horizon, also the existence of a budget constraint.

Both formulations can accommodate the existence of a given number of lines NL to be activated during the time horizon

$$\sum_{k=1}^{n} y_k = NL. \tag{17}$$

Solving the models for a series of NL values, within a range $[N_{\min}, N_{\max}]$, means defining a trade-off relationship between operative costs and user costs. In fact, in this case the models determine the departure time for each of the NL lines which permit to produce the minimum user cost.

In Sect. 3 we showed the SOP can be reduced to the LSP. The model can be easily extended to consider the case of *multiple destinations*, using the extension of the lot sizing model for multiple items (see for instance Pochet and Wolsey 2006). This extension allows such models to represent also the reverse transportation scenario, when users want to go back in the opposite direction from the common destination towards their respective origins.

5 Application of the models and computational results

Part of the motivation for this work comes from some applications arising in the field of regional transportation services. This yielded to an immediate application of the models on instances based on real cases. In this section a specific case study is illustrated and solved at the optimum, and results are presented studying the system behaviour when some parameters are changed, such as line capacity and fixed costs for lines activations. Furthermore, the results of a computational comparison between the two presented models are presented, studying the way computational time grows with the size of instances.

5.1 Applying the models: the Grottaminarda transit terminal

A first test of the model was performed using the data related to a bus transport company (A.Ir. spa) operating in the south of Italy, in the Province of Avellino.

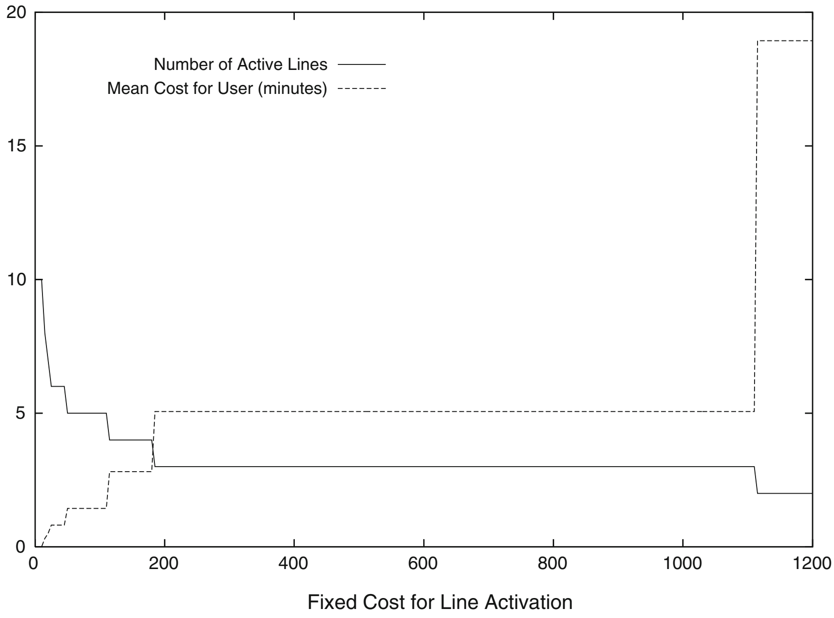

Fig. 5 Number of active output lines versus value of the fixed costs for the Grottaminarda instance. *Dashed line* indicates the corresponding mean cost per user

The aim was to define the optimal number of buses to be used and their departure time during the morning peak period at the transit terminal of Grottaminarda. The transit terminal plays a crucial role in the connection of an internal area characterized by a dense presence of towns of limited population and the city of Avellino.

Starting from a set of historical data collected by the transportation firm, the instance is built as follows: the morning demand peak period (6.00–8.00 a.m.) is discretized using a time unit $\tau = 5$ min, such that the number of nodes of the time-expanded graph is equal to 24. We assume $k_{ij} = 1$ for any (i, j), hence the user costs correspond to the waiting times. The number of input lines arriving at the terminal during the considered time horizon is equal to 18, each of them carrying a different number of passengers. Since some of the arrival times of the input lines fall in the same time interval, the maximum number of lines to be activated in order to set to zero the overall user waiting cost would be equal to 10, assuming the outbound bus capacity Q (60 passengers) to be always high enough to satisfy the current level of demand. Models were implemented in Mosel language, and solved by means of the Xpress-MP Optimizer (Dash optimization, http://www.dashoptimization.com) running on a Windows PC with a Pentium M 1.60 GHz processor and 512 Mb RAM. Each instance of the model was solved at the optimum in <1 s.

In Fig. 5 we present the results in terms of number of buses activated when varying the value of the fixed costs, showing that the optimal number of output lines decreases as the fixed cost increases. In Fig. 6, the number of activated lines is presented when

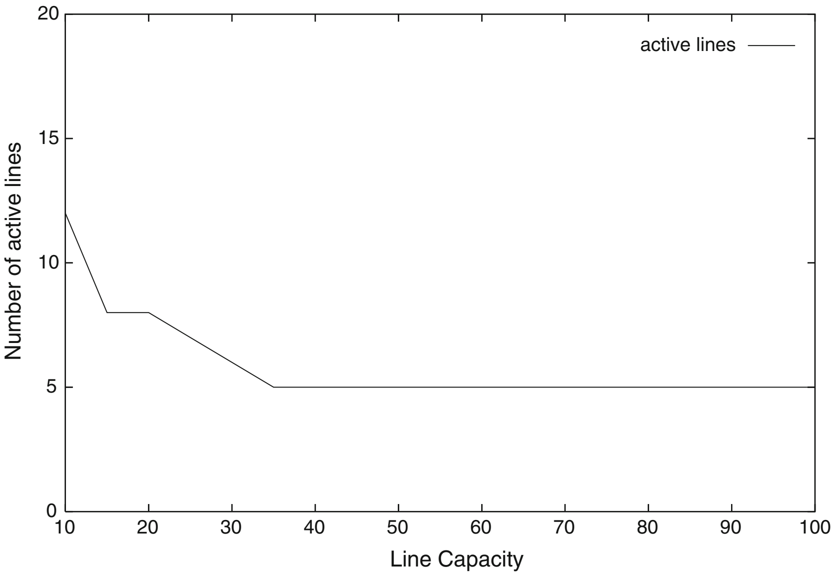

Fig. 6 Number of active output lines versus value of the bus capacity for the Grottaminarda instance

varying the value of the bus capacity Q. The results provided by the models on this instance seem coherent and reasonable, and the computational time needed to solve the problem at the optimum was negligible for both cases. In order to make easier the analysis of the results, we include in Fig. 5 a second line representing the value of the mean cost per user, expressed in minutes of waiting time. It turns out that when activation costs are close to zero, ten lines must be activated in such a way to completely eliminate the waiting time for the users in the transit node. As the bus activation cost increases, the number of lines decreases, assuming all of the integer values in the range from 10 to 2, the latter corresponding to a very high level of activation cost and a very poor solution in terms of the quality of service. We can also observe a symmetrical behaviour for the mean user cost that starts from zero and grows as the number of active lines decreases. Starting from these results, some considerations could be made with respect to the current quality and efficiency of the A.Ir. spa transportation service, that was performed by exploiting four bus lines whose departure times were, respectively 6:30, 7:00, 7:30, and 8:00 a.m., with a mean waiting time close to 15 min for each user. The service could hence be improved, from the quality of service point of view, by fixing new departure times for the buses, according to the result of the schedule obtained from the models for all those values of the fixed costs that induce a number of active lines in the optimal solution equal to 4. For all of them, we can read a mean value for the user costs that is less than 3 min. A second issue to be considered concerns the chance to reduce to three the number of active lines, still achieving a much better service, with a mean user waiting time equal to about 5 min. This would yield to a concurrent increase in the effectiveness and the efficiency of the service.

5.2 A computational experience

In order to test the suitability of the proposed models to solve large instances of the problem, a set of instances was produced ad-hoc for the experiments. A random instances generator was therefore designed and implemented in ANSI C language, and a set of 1,000 instances was created and tested using both the CFLP and the LSP models. More in detail, each instance was created by choosing:

- the number of time intervals n, varying from 1 to 1,000 with a step equal to 1;
- the number of input lines arriving at the terminal, randomly generated with uniform distribution ranging from 0 to $\lceil n/4 \rceil$;
- the number of passengers carried by each input line, randomly generated with uniform distribution ranging from 0 to 60;
- the transit line capacity Q, equal to 50.

The pseudo-random values were generated by means of the $rand()$ C function which, for a range $[0, r]$, provides uniformly distributed values with theoretical mean and variance equal to $r/2$ and $r^2/12$, respectively. The experiments were performed on a Windows PC with Pentium M 1.60 GHz processor and 512 Mb RAM. The Xpress-MP Optimizer (Dash optimization, http://www.dashoptimization.com) was used to solve the models.

All the instances were solved at the optimum within a reasonable cpu time, but the performance of the LSP model always prevails with respect to the CFLP model. In Figs. 7 and 8, we present the results in terms of time needed to solve the instances at the optimum for the two models. The size of the instances considered is large enough

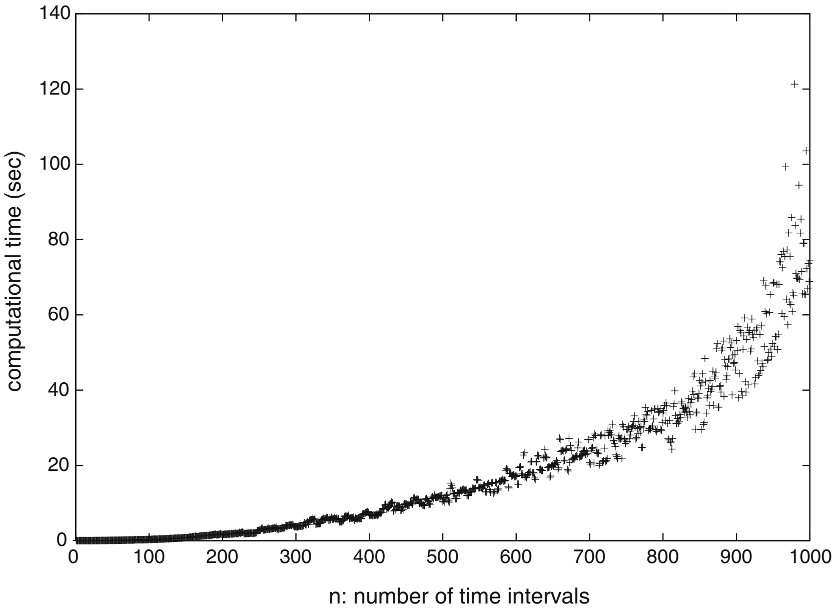

Fig. 7 Computational times for the CFLP formulation of the SOP for the considered set of instances

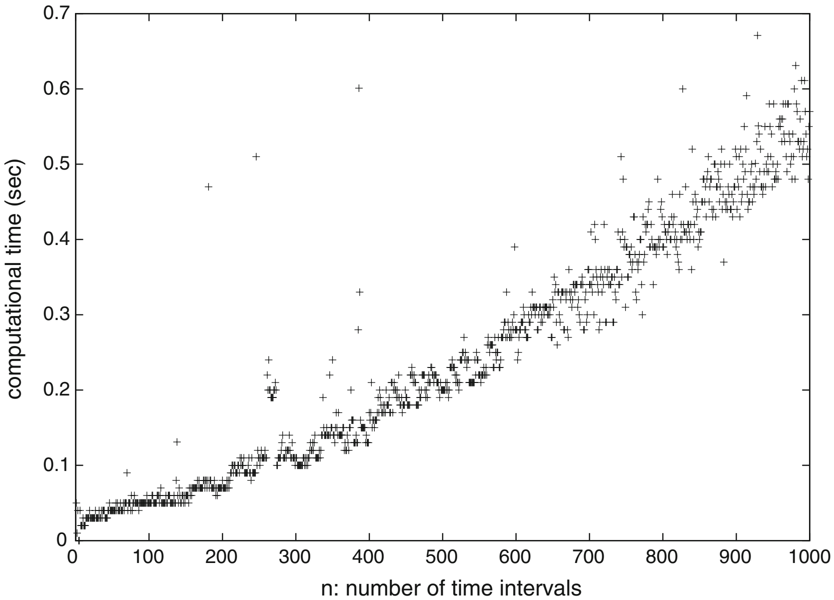

Fig. 8 Computational times for the LSP formulation of the SOP for the considered set of instances. All instances are optimally solved in <1 s

to allow the representation of a whole working day with $\tau \cong 1$ min, hence it turns out that it is possible to optimally solve the SOP on real operational cases within a reasonable computational time by means of both the models, even if the LSP model runs in negligible time with respect to those required by the CFLP model.

6 Conclusions

In this work we considered a public transportation system in which some transit lines arrive at a transfer terminal during a given time horizon carrying passengers proceeding towards a common final destination. This situation frequently arises in regional public transportation services, occurring in particular in case of intermodal transit terminals. The problem we faced regards the decision on the optimal number of transit lines to be activated, starting from the transit terminal and directed to the final shared destination, and the departure time from the terminal of such output lines. Such optimization problem includes in the objective function both the operational costs needed for the transit line activation and the costs of the passenger waiting time at the terminal node, in such a way to find a sustainable trade-off between these two components.

We presented two different mixed-integer programming formulations for the SOP showing that the problem under study can be reduced to the CFLP or to the single-item lot sizing problem. The models proposed for the SOP were tested on a case study extracted from a real transportation network, confirming the consistency and the suitability of the models to be applied for real operational purposes, providing some indications on possible actions to improve quality and efficiency of the transportation service. A set of instances was randomly generated in order to evaluate the computational effort needed for solving optimally large instances. Although results showed an expected computational time growth for the two models, both seem to be suitable for solving real size instances in a reasonable cpu time.

The future developments of this work concern the study of some relevant extensions of the proposed models, to extend the suitability of the SOP to more complex transportation networks. In particular, models with multiple destinations will be studied in depth, together with transportation scenarios with the concurrent presence of more than one transfer station. Also, the case of stochastic parameters (i.e., demand, arrival times) will be dealt with, in such a way to increase the robustness of the solutions provided by the models.

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