On the fractal nature of the magnetic field energy density in the solar wind

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[1] The solar wind exhibits scaling typical of intermittent turbulence in the statistics of in situ fluctuations in both the magnetic and velocity fields. Intriguingly, quantities not directly accessed by theories of ideal, incompressible, MHD turbulence, such as magnetic energy density, B^2, nevertheless show evidence of simple fractal (self-affine) statistical scaling. We apply a novel statistical technique which is a sensitive discriminator of fractality to the B^2 timeseries from WIND and ACE. We show that robust fractal behaviour occurs at solar maximum and determine the scaling exponents. The probability density function (PDF) of fluctuations at solar maximum and minimum are distinct. Power law tails are seen at maximum, and the PDF is reminiscent of a Lévy flight. **Citation:** Hnat, B., S. C. Chapman, K. Kiyani, G. Rowlands, and N. W. Watkins (2007), On the fractal nature of the magnetic field energy density in the solar wind, *Geophys. Res. Lett.*, *34*, L15108, doi:10.1029/2007GL029531.

1. Introduction

[2] Solar wind fluctuations are characterized by a clear inertial range, with power law power spectrum of exponent ∼5/3 (that of Kolmogorov scaling) over a broad range of temporal scales from minutes to several hours [*Burlaga*, 2001; *Tu and Marsch*, 1995; *Goldstein and Roberts*, 1999]. This scaling is consistent with an intermittent turbulent flow [*Bruno and Carbone*, 2005] at high magnetic Reynolds number [*Matthaeus et al.*, 2005]. Solar wind fluctuations thus may result from local turbulent transport, but may also be generated in the solar corona, then passively advecting downstream [*Matthaeus and Goldstein*, 1986]. One approach is to quantify the statistical properties of the plasma fluctuations in the inertial range, such as the non-Gaussian Probability Density Function (PDF), and the scaling of its moments. These allow quantitative comparison with both the predictions of Magnetohydrodynamic (MHD) turbulence models, and stochastic models for solar wind generation and evolution.

[3] The statistical scaling of solar wind fluctuations have been studied extensively by means of structure functions, which follow the scaling of the moments of the fluctuation PDF with timescale (here, timescale is a proxy for spatial scale via the Taylor hypothesis [*Taylor*, 1938]). Under this measure, the intermittent solar wind velocity v [*Tu and Marsch*, 1995] and the IMF magnitude B [*Burlaga*, 2001] exhibit multifractal scaling. Intermittency relates to the non uniform spatial distribution of energy dissipating structures [*Frisch*, 1995] implying a scale dependent energy transfer rate. The technical aspects of quantifying scaling exponents are critical to experimental studies of turbulence. Importantly, given that the PDF of the fluctuations are "heavy tailed", one has, in a given sample, a few large outliers that are poorly sampled statistically. The challenge is then to differentiate the effect of poor statistics of these outliers on the computed moments, from the *bona fide* signature of intermittent turbulence which is manifest in the heavy tails of the PDF [*Frisch*, 1995; *Horbury and Balogh*, 1997; *Chapman et al.*, 2005; *Kiyani et al.*, 2006].

[4] Recent work has suggested that quantities other than v and B show simpler, close to fractal, scaling [*Hnat et al.*, 2003]. The method of PDF collapse checks for fractality in the time series by testing whether the same functional form of the fluctuation PDF is recovered on different temporal scales, subject to a self-similar rescaling [e.g., *Hnat et al.*, 2003]. For magnetic field energy density fluctuations, $\delta(B^2)$, in the solar wind at 1 AU, this method suggests that the PDF is close to fractal [*Hnat et al.*, 2002, 2003]. The studies of *Hnat et al.* [2002] and *Hnat et al.* [2003] did not, however, differentiate between phases of the solar cycle or varying solar wind conditions. More recently, analysis of Akasofu's parameter [*Perreault and Akasofu*, 1978] which incorporates B^2, have shown variations with the solar cycle [*Hnat et al.*, 2005]. The question then immediately arises as to whether there is a solar cycle dependence in the scaling of B^2. The PDF rescaling technique, which relies on overplotting curves of the PDF, can not precisely differentiate weak departures from fractality, so we will explore the scaling of the moments, which is more sensitive to such differences [*Chapman et al.*, 2005]. Here, we report the first application of a novel generic technique [*Kiyani et al.*, 2006], which is sensitive to departures from fractality, to solar wind in situ observations. We focus on the scaling behaviour of B^2 observations from the WIND and ACE spacecraft with solar cycle. We find that fluctuations in B^2 are scaling and, indeed, fractal to good precision during solar maximum and determine the scaling exponent, while at minimum there is a weak departure from fractality.

2. Data and Methods

[5] The calibrated magnetic field magnitude observations from WIND and ACE analysed here were obtained directly from the CDAWeb site (http://cdaweb.gsfc.nasa.gov/). Two WIND data sets, for the years 1996 and 2000, provide observations at solar minimum and maximum respectively. These 60 second averaged data sets comprised $\sim 4.5 \times 10^5$

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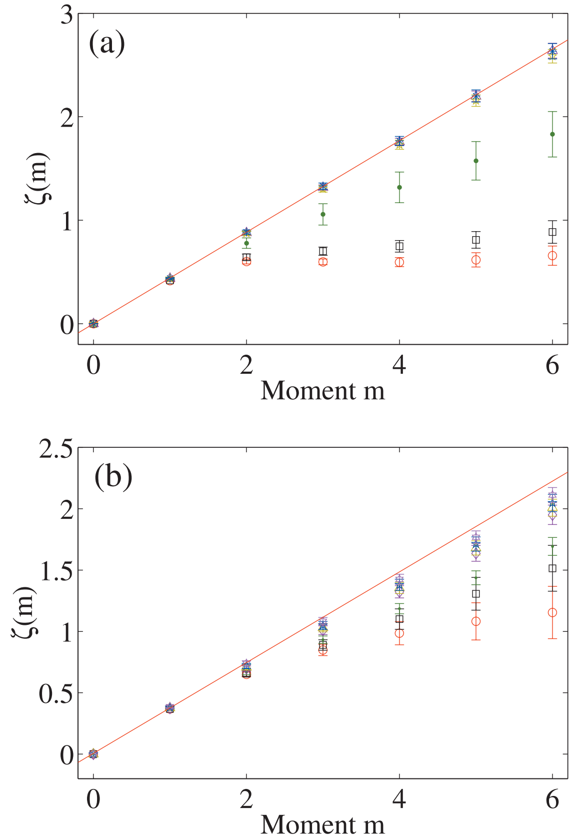

Figure 1. Scaling exponents $\zeta(m)$ as functions of the order m for (a) solar maximum and (b) solar minimum. Different symbols correspond to values of K (fraction of excluded points) used in conditioning: open circle, K = 0; square, K = 0.005%; closed circle, K = 0.05%; diamond, K = 0.5%; triangle, K = 1%; asterisk, K = 2%; star, K = 5%. Straight lines have been fitted to pass through $\zeta(m) = 0 - 2$.

samples each. Contemporaneous ACE data is available for the year 2000, i.e. solar maximum. The ACE 96 second averaged data set used here comprised $\sim 4 \times 10^5$ samples.

[6] Generalized structure functions (GSF) S_m are widely used to extract the statistical scaling of time series [*Rodríguez-Iturbe and Rinaldo*, 1997; *Hnat et al.*, 2003]. From the observed magnitude of B, the fluctuations in B^2 are given by $\delta B^2(t, \tau) = B^2(t + \tau) - B^2(t)$, and the GSF are then defined as:

$$S_m(\tau) \equiv \langle |\delta B^2|^m \rangle \equiv 1/N \sum_i^N |\delta B_i^2|^m$$
$$= \int_{-L_n}^{L_p} |\delta B^2|^m P(\delta B^2, \tau) d(\delta B^2) \propto \tau^{\zeta(m)} \quad (1)$$

where $P(\delta B^2, \tau)$ is a PDF of fluctuations on temporal scales τ and the proportionality on the r.h.s. of (1) holds if S_m exhibits scaling with respect to the time lag τ for a sufficiently large range $\tau < \tau_{max}$. For a finite length time series, the limits of integration L_p, L_n in (1) are given by the extrema of δB^2 found in the data. The gradients of log-log plots of S_m versus τ then yield the values of the scaling exponents $\zeta(m)$. Generally, the $\zeta(m)$ can be a nonlinear function of order m; for example in the multifractal velocity field of hydrodynamic turbulence they are quadratic and concave with m [*Frisch*, 1995]. However, if $\zeta(m) = m\alpha$ with α constant, then the fluctuations δB^2 are statistically self-similar and the time series is fractal (or more precisely, self-affine). In any given ensemble of the $\delta B^2(t, \tau)$ used to construct the integral (1), the L_n, L_p can differ significantly from their average values, and it is the average values that follow the scaling of the PDF of the ensemble. As a consequence, these large events, or outliers, can drastically affect the values of $\zeta(m)$ obtained from a given ensemble [*Katul et al.*, 1997; *Mangeney et al.*, 2001; *Chapman et al.*, 2005; *Jespersen et al.*, 1999; *Kiyani et al.*, 2006]. A recent technique of *iterative conditioning* systematically tests the effect of outliers on the $\zeta(m)$ [*Kiyani et al.*, 2006]. Essentially, one repeatedly computes the $\zeta(m)$ whilst successively removing outlying points that are poorly represented statistically. In each stage of this iterative method the *kth*-largest fluctuation (where $k = 1, 2, 3, \ldots, K$) is excluded from the set $\delta B^2(t, \tau)$ and the scaling exponent $\zeta(m)$ is recomputed using the new limits L_n, L_p in (1). For the particular case of a fractal time series, a rapid convergence of scaling exponent $\zeta(m) \rightarrow \alpha m$ for small K is expected [*Kiyani et al.*, 2006]. For the case of a Lévy flight, it has been shown [*Kiyani et al.*, 2006] that convergence is recovered when less than 1% of fluctuations are removed. We will apply this conditioning method to explore the scaling of magnetic field energy density.

3. Results and Discussion

[7] The effect of iterative conditioning is shown in Figures 1a and 1b, for magnetic field energy density solar maximum, B^2_{Max} and minimum, B^2_{Min}, respectively. We plot $\zeta(m)$ versus m for a range of K, that is, as we successively remove outlying data. In Figure 1 we see that the $\zeta(m)$ from the raw data move toward a constant level as m increases. This saturation is especially pronounced during solar maximum for the moments of order $m > 2$, reminiscent of the behaviour found in Lévy flights (in both standard and fractional cases, e.g., *Watkins et al.* [2005], see also *Kiyani et al.* [2006]). Different symbols in Figure 1 denote the fraction of outliers $K \in [0\%, 5\%]$ successively excluded. The error bars are regression errors of the fitted $S_m \propto \tau^{\zeta(m)}$ relation on a log-log plot. A straight line has been fitted to pass through the points $(m, \zeta(m))$, for $m = 0, 1, 2$ on both plots. We see that at $K = 0.1\%$ the exponents all approach this line for solar maximum, consistent with a fractal time series. At minimum there is a departure from linear $\zeta(m)$ behaviour which is weak but just resolvable within the errors at higher m, even when K is relatively large. The rate of convergence can also be seen to qualitatively differ and is more rapid for solar maximum.

[8] We can more clearly see the rate of convergence by plotting the value of a particular exponent $\zeta(m)$ versus the fraction of outlying points removed. This is shown in Figures 2 where we plot $\zeta(2)$ for both solar maximum (main plot) and minimum (inset). We can corroborate the WIND observations on this plot for solar maximum by overplotting the $\zeta(2)$ obtained by the same procedure for ACE observations. From Figure 2 we see that at solar maximum, the exponent rapidly converges (i.e. reaches a constant value that does not vary with K) at $\zeta(2) = 0.88 \pm$

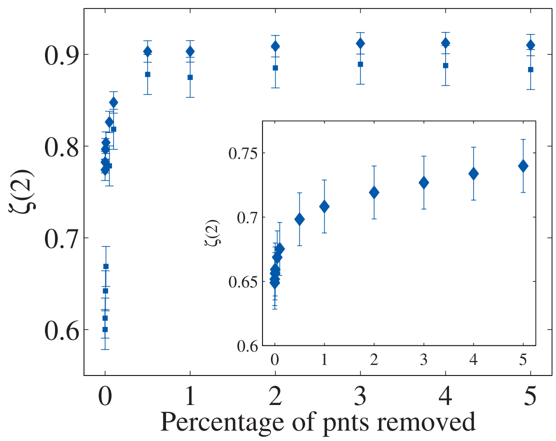

Figure 2. Scaling exponent $\zeta(2)$ versus percentage of points removed during conditioning for solar maximum: circle, ACE calibrated $|B|$; diamond, WIND calibrated $|B|$; and square, WIND $|B|$ from components. (inset) The same format, solar minimum.

0.02, corresponding to a Hurst exponent of $H = \zeta(1) = 0.44$. For solar minimum such clear convergence is not found, rather a weak rising trend continues up to our upper limit of $K = 5\%$ of removed outliers. We have found the same behaviour in the other scaling exponents calculated for B^2_{Min} and B^2_{Max}, as suggested by Figure 1. Rapid convergence to a constant value with increasing K for each individual exponent is a necessary but not sufficient condition for self-similarity (one must also have $\zeta(m) = m\alpha$). These plots of rate of convergence provide a clear discriminator of fractal scaling which is more sensitive than discerning a precise linear trend in plots of $\zeta(m)$ versus m. The former emphasizes the sharp plateau in the rate of convergence while the latter requires distinguishing weak departures from linear (fractal) dependence of $\zeta(m)$ with m. Figure 2 leads us to conclude that the fluctuations in B^2 during solar minimum are not self-similar, whereas at solar maximum, they are fractal to quite high precision. Intriguingly, the behaviour of $\zeta(m)$ versus m seen in the raw data, and the rapid conversion toward a linear relation $\zeta(m) = m\alpha$ again resembles that found for Lévy motion.

[9] We stress that the systematic conditioning of fluctuations, as described here, is not equivalent to an elimination of the large spikes from the field magnitude data set. Recently, *Podesta et al.* [2006] concluded, based on the scaling of the unconditioned absolute moments of fluctuations in B^2 obtained from the *ACE* spacecraft, that approximate self-similarity was not valid. Such self-similarity was later recovered, in the same data, when conditioning was used (J. J. Podesta, private communication, 2006).

[10] From the above results we would anticipate that the PDF of fluctuations in B^2 at solar maximum and minimum will not share the same functional form. We verify this in Figure 3 where we compare the distributions of positive fluctuations in B^2_{Min} and B^2_{Max}, at $\tau \approx 30$ minutes, normalized to their respective standard deviations (this log-log plot emphasizes the PDF tails). We see that the PDFs do differ significantly for the entire range of fluctuations. The fluctuations in B^2 at solar maximum are in many aspects similar to those seen in Lévy flights. A power law tail is evident in Figure 3 at solar maximum over about 1–1.5 decades in fluctuations of B^2. This is consistent with, but not unique to, the limiting form of a Lévy PDF, $P_L(|\delta B^2| \to \infty) \propto |\delta B^2|^{-(1+\mu)}$. The slope of the best fitted line suggests a μ value of $\approx$1.4. At solar minimum, the tails of the PDF suggest an exponential roll-off.

[11] Encouraged by the apparent self-similarity of fluctuations at solar maximum we have tested the applicability of various models for random fractal time series and these are shown in Figure 4 overplotted on the PDF from the data. A Fokker-Planck (F-P) model [*Hnat et al.*, 2003] has been developed to describe self-similar fluctuations in this context, and is shown on the plot as a dashed line. In Figure 4 we overplot as a dotted line a Lévy PDF calculated from the characteristic function as $P_L(x) = \int dk e^{-ikx} e^{-\gamma|k|^{\mu}}$ with $\mu = 1.4$ and $\gamma = 0.3$. We can see that both these functions adequately describe the data.

4. Summary

[12] We have applied a novel technique, that of iterative conditioning of the structure functions, to quantify the

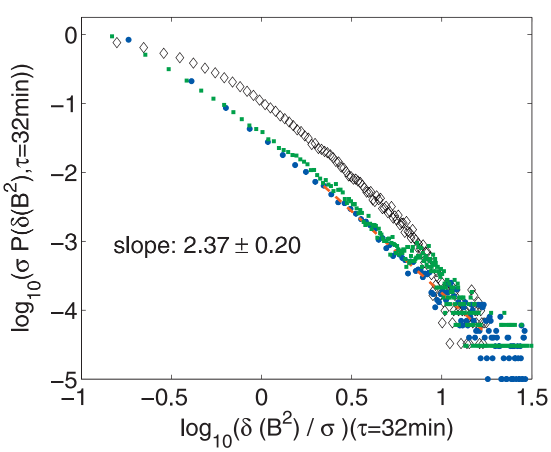

Figure 3. Tails of the distribution for positive fluctuations in B^2: diamond, WIND at solar minimum; circle, WIND at solar maximum; and square, ACE at solar maximum.

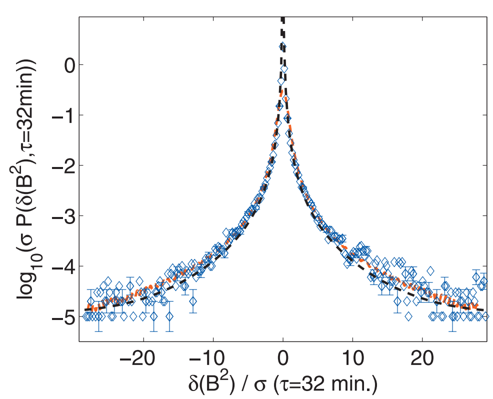

Figure 4. Fitted curves over experimental pdf for τ = 32 minutes. Dashed line indicates a Fokker-Planck solution and a dotted line corresponds to a Lévy PDF with $\mu = 1.4$ and $\gamma = 0.3$. Error bars are proportional to $1/\sqrt{n}$, where n is a number of data points per bin.

scaling of fluctuations in magnetic field energy density observed by WIND and ACE at solar minimum and maximum. This method is a particularly sensitive discriminator of fractality, and where the time series is indeed fractal, quantifies the scaling exponent to good precision. At solar maximum, the timeseries is found to be fractal, whereas there is a weak but clearly discernable departure from fractality at solar minimum. This difference is reflected in the functional form of their PDFs. Intriguingly, at solar maximum, the PDFs show power law tails. This may reflect the increased complexity of the magnetic field structure in the corona at solar maximum. Importantly, the scaling that we have established here is found within the inertial range of turbulence seen in the solar wind - that is, coincident with the signature of approximately $-5/3$ power law power spectra - rather than at the lower frequencies typically associated with the $1/f$ scaling seen in energy containing scales. Thus our results may inform understanding of the interplay between the signature of coronal heating and solar wind acceleration, propagated to 1 AU [*Milovanov and Zelenyi*, 1998], and that of locally evolving turbulence.

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