An Application of the Level-set Method to Fire Front Propagation

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Abstract

Wildland fire models and simulators developed in the last two decades are increasingly applied in different ecosystems and countries of the world to predict fire behavior and effects. Fire models range from empirical formulas, such as the ones defined by Rothermel and applied in spatially and temporally explicit fire simulators (i.e. Farsite), to complex three-dimensional CFD approaches solving the partial differential equation of continuity, momentum and energy.

The wide range of length and time scales governing wildland fire (from the millimeter scale of combustion processes to the hundreds of meters scale of synoptic wind flow) complicates the use of a full-3D numerical approach, at least for operational forecasting purposes. At the same time, there is an important two-way influence between weather and fire: wind determines fire propagation and, conversely, the buoyancy effects generated by fire heat modify the local wind field, "creating their own weather".

A possible solution to this problem is the use of a simpler (and thus computationally cheaper) model to describe fire propagation, while maintaining a CFD approach to model wind behavior and, more importantly, the two-way interaction due to fire heat release. A number of studies applied this approach in the last few years [1].

This work describes the initial steps in the development of a model for fire-front propagation based on a level-set methodology and its integration into a CFD model.

Keywords: fire propagation, level-set, CFD

1. INTRODUCTION

The majority of wildfire simulation codes make use of empirical formulas to predict fire propagation. With the relentless increase in computational resources, there is a need for improved reliability by capturing more of the physics of the fire propagation process. While a full 3D approach is still too heavy (at least for operational purposes), a number of studies have demonstrated improved accuracy by means of coupled atmosphere-fire propagation modeling, which combines a CFD approach for the wind behavior with empirical models to predict the displacement of the fire front. The objective of this work is to develop a coupled atmosphere-fire propagation model by expanding the capabilities of available CFD software.

2. METHODOLOGY

2.1 A LEVEL-SET METHOD FOR FIRE PROPAGATION

Level-set methods are Eulerian schemes for tracking fronts that propagate with a given speed function (which can depend on position, time and other local properties such as normal direction and local curvature [2]). The basic idea is to use an implicit definition of the front $\Gamma(t)$ by means of a function $\psi : \Re^n \times [0, T_f] \to \Re$ such that:

$$\forall t \in [0, T_f] \qquad \Gamma(t) = \{x \in \Re^n \mid \psi(x,t) = 0\} \tag{1}$$

The partial differential equation defining the evolution of the front can be obtained by differentiating the equation for the fire front with respect to time:

$$\frac{\partial \psi}{\partial t} + R \cdot \nabla \psi = 0 \tag{2}$$

where R is the front propagation speed, generally assumed perpendicular to the fire front [3]. With this assumption, equation (2) becomes:

$$\frac{\partial \psi}{\partial t} + \|R\| \|\nabla \psi\| = 0 \tag{3}$$

where $\| . \|$ represents the Euclidean norm operator.

On Cartesian grids, equation (3) can be solved using a Finite Difference approach. To preserve stability, special care needs to be placed in the approximation of spatial derivatives. The simplest stable scheme is a first-order upwind. For the x-derivative it reads:

$$\frac{\partial \psi}{\partial x} = \begin{cases} \dfrac{\psi_{i,j} - \psi_{i-1,j}}{\Delta x} & \textit{if} \quad R_x > 0 \\ \dfrac{\psi_{i,j} - \psi_{i-1,j}}{\Delta x} & \textit{otherwise} \end{cases} \tag{4}$$

Similarly, the time derivative in equation (3) can be approximated with a first-order explicit scheme (Euler's method):

$$\frac{\partial \psi}{\partial t} = \frac{\psi_{i,j}^{n+1} - \psi_{i,j}^{n}}{\Delta t} \tag{5}$$

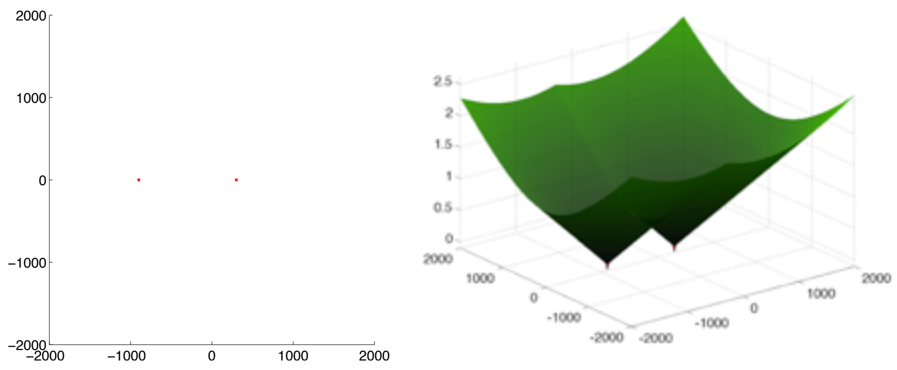
where the superscripts represent the time step. This approach is first-order accurate in both time and space: higher-order discretization can be used, but are not considered in this study due to the uncertainties in the estimation of the fire propagation rate. For an explicit scheme, the maximum stable time step is related to the grid spacing by the Courant-Friedrichs-Lewy (CFL) condition:

$$\max\left(\frac{R \Delta t}{\Delta x}\right) \leq 1 \tag{5}$$

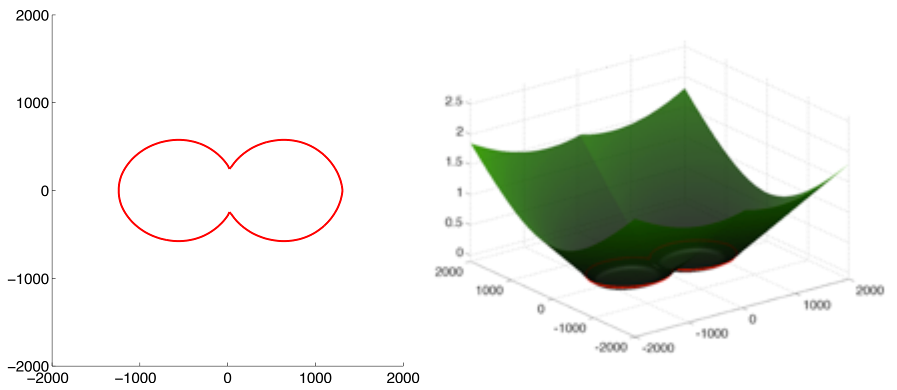
Von Neumann boundary conditions are used at the boundaries of the physical domain (i.e. $\frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial y} = 0$). To approximate the fire front at time $t = 0$, the initial value of the level-set function can be chosen as the signed distance from the fire-line:

$$\psi(x,t) = \begin{cases} d_{\Gamma(t)}(x) & if \quad x \quad lies \quad outside \quad \Gamma(x) \\ -d_{\Gamma(t)}(x) & if \quad x \quad lies \quad inside \quad \Gamma(x) \end{cases} \tag{6}$$

Figure 1 shows the evolution of a fire from two separate ignition points. The level-set function is shown on the right, the fire perimeter on the left, in the presence a low intensity wind (corresponding to an ellipse eccentricity of 0.5). Fire-spread rate has been calculated using Rothermel's formulation. The level-set approach is able to deal with fire-front merging without additional complexities.



(a) Initial fire



(b) Final fire

Figure 1 – Evolution of fire perimeter (left) and corresponding level-set function (right)

2.2 WIND-FIELD PREDICTIONS

To compute the atmospheric flow, we make use of the CFD solver Fluent from Ansys, which provides a wide range of well-validated solvers and models. A body-fitted numerical grid is generated efficiently via a free-form-deformation of a Cartesian grid (an example of the deformed ground-mesh is shown in Figure 2). The grid in this case is structured (evenly- spaced in x- and y-directions), but any other type of grids could be used without any additional requirements.

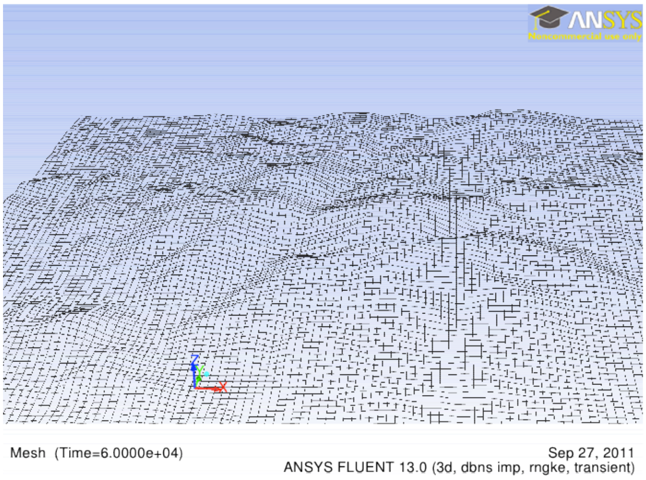


Figure 2: Ground mesh in Fluent

Fluent allows a generic wind field to be predicted, given boundary conditions at the extremes of the computational domain. At vertical boundaries, inlet-velocity, pressure-outlet or symmetry boundary conditions are used (depending on the wind direction), a symmetry boundary condition is used at the top of the computational domain and a no-slip boundary condition at the bottom, with a roughness wall-function to account for the large frictional forces exerted by the vegetation on the atmospheric boundary layer. A renormalization group $k-\varepsilon$ model is used for turbulence closure.

For generality, equation (2) has been rewritten in Finite Volume formulation:

$$\frac{d}{dt}\int_V \psi \, dV + \int_\Omega R \cdot \psi \, dS = 0 \tag{7}$$

where V is a generic control volume and Ω the corresponding boundary surface. The same approximation of Section 2.1 is obtained using the upstream value of ψ to calculate the surface integrals in equation (7). For simplicity, the mesh used to solve the level-set problem corresponds to the ground mesh in the fluid problem.

The fire propagation solver has been linked to Fluent by means of user-defined-functions (UDFs) written in C. To simplify the integration, we have defined a *cell* structure containing a number of information required to solve equation (7), mostly calculated during the initialization process -- e.g. number of neighboring cells and their IDs, cell volume and boundary areas, ID of fluid cell for wind velocity, current and old values of level-set function (if needed), fire propagation parameters (vegetation, slope, etc.) and fire-front rate of spread -- to reduce run times.

2.3 WIND-FIRE INTERACTION

To resolve the interaction between the heat generated in the combustion process and the wind flow, the heat has been introduced at the ground boundary. The reaction intensity from Rothermel's formulation gives the heat generated per unit surface. The surface has been assumed to start burning when the level-set function assumes a negative value and to continue burning until all the fuel is consumed (the burning time can be calculated as the product of the reaction intensity and the inverse of fuel load and fuel heat content).

3. RESULTS

Figure 3 presents a comparison between the evolution of a line fire without (left) and with (right) wind-fire interaction, in the presence of a 5 m/s intensity wind coming from the left boundary. Figure 4 shows the complex vortical structures generated the buoyant flow.

Figure 5 demonstrates the application of the solver in a real case (note the formation of an unburned island due to the combined effect of terrain morphology and wind).

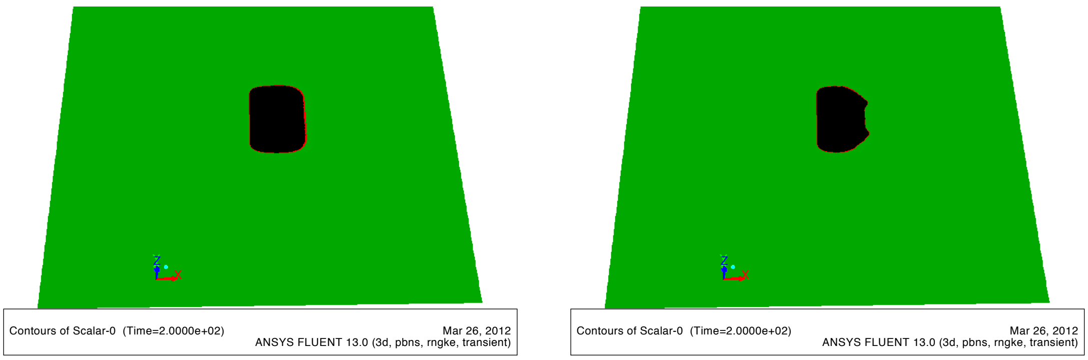


Figure 3 – Effects of buoyancy on the fire perimeter

4. CONCLUSIONS

This work presents the first steps in the development of a tool to predict wildland fire propagation, capable to account for wind-fire interaction. Future plans include extensive testing and further development of available capabilities.

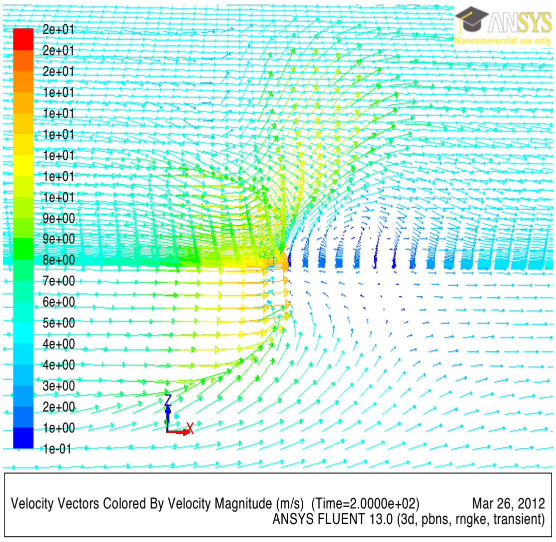


Figure 4 – Wind-fire interaction

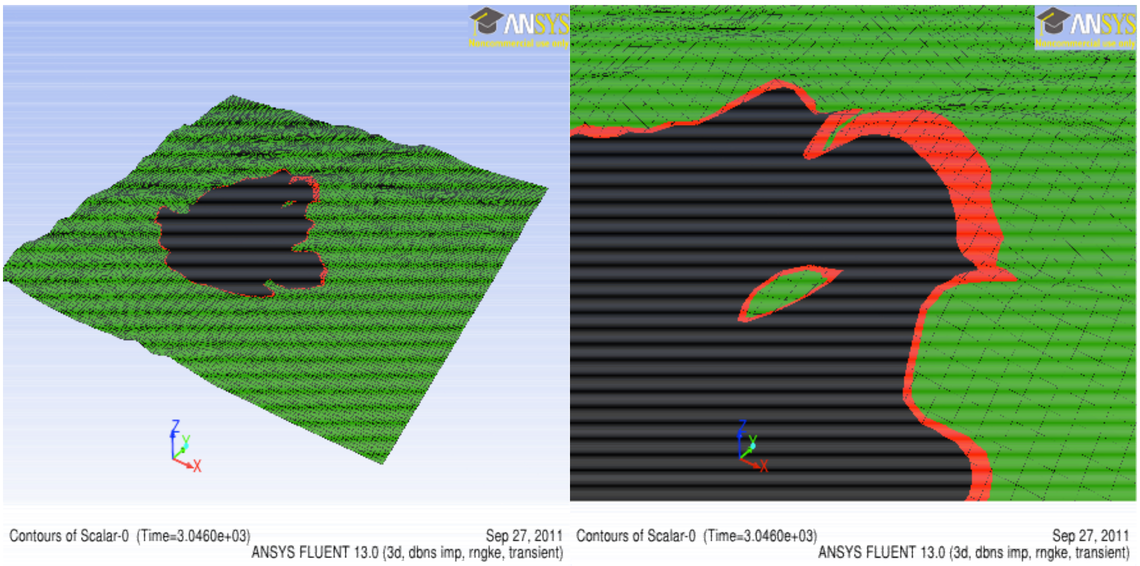


Figure 5 – Application in a real case

5. REFERENCES

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