UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

MODULE	**ASMA2B1** MULTIVARIABLE AND VECTOR CALCULUS (MAIN STREAM)
CAMPUS	**APK**
EXAM	**JUNE 2014**

DATE	14/06/2014
EXAMINER	MRS C DUNCAN
INTERNAL MODERATOR	DR J SOUTHEY
DURATION	2 HOURS
MARKS	45

SURNAME AND INITIALS ______________________________

STUDENT NUMBER ______________________________

CONTACT NUMBER ______________________________

NUMBER OF PAGES: 1 + 10

INSTRUCTIONS:
1. ANSWER ALL QUESTIONS ON THE PAPER IN PEN
2. CALCULATORS ARE ALLOWED
3. INDICATE **CLEARLY** ANY ADDITIONAL WORKING OUT

Question 1 [5]

(1.1) Define clearly what is meant by saying "$f(x, y)$ is continuous at the point (a, b)". (2)

(1.2) Is the function

$$f(x,y) = \begin{cases} \dfrac{2xy}{x^2 + 2y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

continuous at $(0, 0)$? (3)

Question 2 [3]

Show that a differentiable function f decreases most rapidly at (x, y) in the direction opposite to the gradient vector, that is, in the direction $-\nabla f(x, y)$.

Question 3 [8]

Consider the volume represented by the following triple integral:

$$V = \left[\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} dz\,dy\,dx\right] - \left[\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} dz\,dy\,dx\right]$$

(3.1) Explain, in words, the represented volume. (1)

(3.2) Rewrite the **first term only** in the order $dx\,dz\,dy$. (2)

(3.4) Rewrite V in **spherical** coordinates using only one triple integral. (2)

(3.4) Rewrite V in **cylindrical** coordinates. (3)

Question 4 [5]

A circular cylindrical hole of radius 1 is drilled through the centre of a sphere with radius 2. Sketch the resulting solid in an appropriate orientation, set up a triple integral in **cylindrical** coordinates representing the volume of the solid (using ONLY one triple integral) and calculate this volume.

Question 5

[4]

Rewrite the following triple integral in **spherical** coordinates:

$$\int_0^1 \int_{\sqrt{3}\,x}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} \, dz \, dy \, dx$$

Question 6

[5]

Use an appropriate change of variable to evaluate the double integral

$$\iint_R \cos\left(\frac{x-y}{x+y}\right)\, dA$$

where R is in the first quadrant and bounded by the lines $x+y=1$ and $x+y=3$.

Question 7 [6]

Consider the following vector field:

$$\mathbf{F} = \left\langle y^3 + 1,\ 3xy^2 + 1 \right\rangle.$$

(7.1) Is $\int_C \mathbf{F} \cdot d\mathbf{r}$ path-independent? Justify your answer clearly. (2)

(7.2) Show that $\int_C \mathbf{F} \cdot d\mathbf{r} = 2$, where C is the semi-circular path, in the first quadrant, with starting point $(0, 0)$ and terminal point $(2, 0)$. (4)

Question 8 [5]

The force exerted by an electric charge at the origin on a charged particle at a point (x, y, z) with position vector $\mathbf{r} = \langle x, y, z \rangle$ is $\mathbf{F} = \frac{K\mathbf{r}}{||\mathbf{r}||^3}$, where K is a constant.
Find the work done by this latter force as the particle moves along a straight line from $(2, 0, 0)$ to $(2, 1, 5)$.

Question 9 [4]

Given a vector field $\mathbf{F} = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}$. Show that

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C P\,dx + Q\,dy + R\,dz$$

along a smooth curve C.