UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE
FAKULTEIT NATUURWETENSKAPPE

DEPARTMENT OF MATHEMATICS

MODULE	**MAT2A10** SEQUENCES, SERIES AND VECTOR CALCULUS (Main Stream)
CAMPUS	**APK**
EXAM	**JUNE 2014**

EXAMINER(S)	MRS C DUNCAN MR F SCHULZ
INTERNAL MODERATOR	DR E JOUBERT
DURATION	2 HOURS
MARKS	50

SURNAME AND INITIALS ______________________________

STUDENT NUMBER ______________________________

CONTACT NUMBER ______________________________

NUMBER OF PAGES: 1 + 13

INSTRUCTIONS:
1. ANSWER ALL QUESTIONS ON THE PAPER IN PEN
2. CALCULATORS ARE ALLOWED
3. INDICATE **CLEARLY** ANY ADDITIONAL WORKING OUT

Question 1 [5]

Find all real numbers k for which the sequence

$$\left\{ (-1)^n \, \frac{1 \cdot 3 \cdots (2n+1)}{(2n)!} \, \pi^{3n} \, k^n \right\}_{n=1}^{\infty}$$

converges.

Question 2 [4]

State the Integral Test for series.

Question 3 [10]

By using an appropriate method, determine whether the following series converge or diverge:

(3.1) $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan n}{n^{3/2}}$ (4)

(3.2) $\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n-1}}{n}$ (3)

(3.3) $\sum_{n=1}^{\infty} \frac{(-5)^{2n}}{n^2\, 9^n}$ (3)

Question 4

[4]

Find the radius and interval of convergence of the series:

$$\sum_{n=0}^{\infty} \frac{2^n (x-3)^n}{\sqrt{n+3}}$$

Question 5 [3]

Find the sum of the series:

$$-\pi + \frac{\pi^2}{2} - \frac{\pi^3}{3!} + \frac{\pi^4}{4!} - \cdots$$

Question 6 [4]

Use series to approximate

$$\int_0^1 \cos\left(x^4\right)\, dx$$

correct to three decimal places.

Question 7 [3]

If υ is the speed of a particle along a curve C, $\mathbf{T}$ and $\mathbf{N}$ the unit tangent and unit normal vectors respectively of the particle's position vector $\mathbf{r}$, and κ is the curvature of C, then show that the acceleration $\mathbf{a}$ of the particle is given by

$$\mathbf{a} = \upsilon' \, \mathbf{T} + \kappa \upsilon^2 \, \mathbf{N}$$

Question 8

[3]

Calculate the curvature of the curve $f(x) = e^x$ in the plane.

Question 9 [5]

Find the velocity, acceleration and speed of a particle with the given position function. Sketch the path of the particle and draw the velocity and acceleration vectors for the specified value of t.

$$r(t) = 3\cos t\ \mathbf{i} + 2\sin t\ \mathbf{j}\ ; \quad t = \frac{\pi}{3}$$

Question 10 [5]

If it is given that

$$\mathbf{u}(t) = \mathbf{r}(t) \cdot [\mathbf{r}'(t) \times \mathbf{r}''(t)]$$

then show

$$\mathbf{u}'(t) = \mathbf{r}(t) \cdot [\mathbf{r}'(t) \times \mathbf{r}'''(t)]$$

for $\mathbf{r}$ and $\mathbf{u}$ arbitrary position vectors.

Question 11 [4]

Determine the domain of $\mathbf{r}$ if $\mathbf{r}(t) = \mathbf{F}(t) \times \mathbf{G}(t)$, where

$$\mathbf{F}(t) = t^3\ \mathbf{i} - t\ \mathbf{j} + t\ \mathbf{k} \quad \text{and} \quad \mathbf{G}(t) = \sqrt[3]{t}\ \mathbf{i} + \left(\frac{1}{t^2 - 1}\right)\mathbf{j} + (t+2)\ \mathbf{k}$$