UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

MODULE **MAT1A01**
CALCULUS OF ONE-VARIABLE FUNCTIONS

CAMPUS **APK**

EXAM **JUNE EXAM**

DATE **14/06/2014** **SESSION: 12:30 – 14:30**

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DURATION **2 HOURS** **MARKS** **70**

SURNAME AND INITIALS: ____________________________________

STUDENT NUMBER: ____________________________________

CONTACT NR: ____________________________________

NUMBER OF PAGES: 1 + 13 PAGES

INSTRUCTIONS:
1. **ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.**
2. **PLEASE ANSWER ALL QUESTIONS IN THE SPACE PROVIDED. IF YOU REQUIRE MORE SPACE, PLEASE CONTINUE ON THE ADJACENT BLANK PAGE AND INDICATE THIS CLEARLY.**
3. **NO CALCULATORS ARE ALLOWED.**
4. **SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.**

Question 1 [10 marks]

For questions 1.1 - 1.10, choose **one** correct answer, and make a cross (X) in the correct block.

Question	**a**	**b**	**c**	**d**	**e**
1.1					
1.2					
1.3					
1.4					
1.5					
1.6					
1.7					
1.8					
1.9					
1.10					

1.1 The contrapositive of $q \to p$ is:

a) $p \to q$

b) $\neg p \to \neg q$

c) $\neg q \to \neg p$

d) $\neg q \wedge \neg p$

e) None of these

1.2 If $f(x) = \dfrac{1}{x-3}$ and $g(x) = 2x+4$, then $g \circ f =$

a) $\frac{1}{2x+1}$

b) $x - 3$

c) $\frac{2x+4}{x-3}$

d) $\frac{2}{x-3} + 4$

e) None of these

1.3 If $y = 3.2^{4x-1}$, then $\dfrac{dy}{dx}$ is equal to:

a) $(12 \ln 2).2^{4x-1}$

b) 12.2^{4x-1}

c) $(12x-3).2^{4x-1}$

d) 3.2^{4x-1}

e) None of these

1.4 Let $\int_c^e f(x)\ dx = 5$, $\int_e^f f(x)\ dx = -3$ and $\int_d^e f(x)\ dx = 1$. Then $\int_c^d f(x)\ dx =$

a) 6

b) 4

c) -4

d) 2

e) None of these

1.5 In order to determine $\int \tan^4 x\ \sec^2 x\ dx$ using u-substitution, let $u =$

a) $\tan x$

b) $\tan^4 x$

c) $\sec x$

d) $\sec^2 x$

e) None of these

1.6 The domain of the function $y = \arctan x$ is:

a) $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

b) $x \in \mathbb{R}$

c) $x \in [0, \pi]$

d) $x \in [-1, 1]$

e) None of these

1.7 If $y = \ln(\sec x + \tan x)$, then $\frac{dy}{dx}$ is equal to:

a) $y \sec^2 x$

b) $2 \sec x$

c) $\tan x + \csc x . \sec x$

d) $\sec x$

e) None of these

1.8 The Intermediate Value Theorem states that if f is continuous on the closed interval $[a, b]$ and 0 is between $f(a)$ and $f(b)$, $f(a) \neq f(b)$, then there exist a number c in (a, b) such that:

a) $f(a) = f(b)$

b) $f(c) = 0$

c) $c = 0$

d) $f(0) = c$

e) None of these

1.9 The point $P(-3, -8)$ is on the graph of $y = f(x)$. Which point will be on the graph of $y = -f(x - 5)$?

a) $(-8, -8)$

b) $(-8, 8)$

c) $(2, 8)$

d) $(8, -8)$

e) None of these

1.10 For what value of k does $\lim_{x \to 4} \left(\frac{x^2 - x + k}{x - 4} \right)$ exist?

a) -12

b) -4

c) 3

d) 7

e) None of these

Question 2 [1 mark]

Write in sigma notation: $\frac{2}{1 \cdot 2} + \frac{2^2}{2 \cdot 3} + \frac{2^3}{3 \cdot 4} + \frac{2^4}{4 \cdot 5} + \frac{2^5}{5 \cdot 6}$ [1]

Question 3 [2 marks]

Solve the following inequality: $\dfrac{2}{|x+1|} \geq 4$ [2]

Question 4 [2 marks]

Prove the identity: $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$ [2]

Question 5 [3 marks]

Given the complex number $z = \sqrt{3} + i$, find $(\sqrt{3} + i)^6$. Write your answer in the form $a + bi$. [3]

Question 6 [2 marks]

Use proof by contraposition to prove that: "For any integer n, if $7n + 9$ is even, then n is odd."

[2]

Question 7 [2 marks]

Show, by using the definition, that $f(x) = \dfrac{x-3}{x+2}$ is one-to-one. [2]

Question 8 [2 marks]

Find the inverse function $f^{-1}(x)$ of $f(x) = \ln(\frac{1}{2}x + 7)$. [2]

Question 9 [2 marks]

Graph the function of $y = \frac{\pi}{2} + \arctan\theta$ and state the domain and the range in interval notation. [2]

Question 10 [7 marks]

Evaluate each of the following limits, if they exist, otherwise show why they do not exist. Show all steps.

a) $\lim_{x\to\infty} \frac{\sqrt{9x^6 - x}}{x^3 + 1}$ [2]

b) $\lim_{x\to\infty} \frac{2\cos x + 2}{x}$ [2]

c) $\lim_{x\to 0} \frac{1 + x - e^x}{x(e^x - 1)}$ [3]

Question 11 [6 marks]

Let f be a function defined by

$$f(x) = \begin{cases} \sqrt{x+1}, & \text{if } \ 0 \leq x \leq 3 \\ 5 - x, & \text{if } \ 3 < x \leq 5 \end{cases}$$

Determine:

a) $\lim\limits_{x \to 3^+} f(x)$ [1]

b) $\lim\limits_{x \to 3^-} f(x)$ [1]

c) Is $f(x)$ continuous at $x = 3$? Give a reason for your answer. [2]

d) Is $f(x)$ differentiable at $x = 1$? Give a reason for your answer. [2]

Question 12 [7 marks]

a) Prove that $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$ (Do NOT use L'Hospital's Rule.) [4]

b) **Hence**, use the definition of derivatives and the special limit $\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} = 0$, to prove that

$$\frac{d}{dx}(\sin x) = \cos x$$ [3]

Question 13 [7 marks]

Use the differentiation rules to determine:

a) $\frac{d^2x}{dy^2}$ if $x = \frac{1}{2 - \pi y}$ [2]

b) $\frac{dy}{dx}$ if $y = x.\sinh(x^2)$ [2]

c) y' if $y = \ln\left(\dfrac{\sqrt[3]{1+e^{x^2}}}{\sqrt[3]{x.\cos x}}\right)$ [3]

Question 14 [7 marks]

a) The curve of $y = \dfrac{10}{2x+1} - 2$ intersects the x-axis at A. The tangent to the curve at A intersects the y-axis at C.

(i) Find the coordinates of the points A and C. [2]

(ii) Show that the equation of AC is $5y = -4x + 8$. [2]

b) Find the slope of the tangent to the curve of $x^2 + 4xy + y^2 = 13$ at $(2; 1)$. [3]

Question 15 [4 marks]

Determine $\dfrac{d}{dx}\displaystyle\int_0^{2x} (\cos t - \sin t)\ dt$,

a) without integrating, by using the Fundamental Theorem of Calculus (Part 1). [1]

b) by first integrating the function $f(t) = \cos t - \sin t$ and then differentiating the result. [3]

Question 16 [6 marks]

a) Evaluate $\int xe^{3x^2-5}\ dx$ [2]

b) Evaluate $\int \left(\frac{2}{x} - 3\sqrt{x} - \frac{4}{x^2}\right)\ dx$ [2]

c) If f is continuous and $\int_0^2 f(u)\ du = 6$, use u-substitution to find $\int_0^{\pi/2} f(2\sin\theta)\cos\theta\ d\theta$. [2]