DESIGNING A NEW MATHEMATICAL MODEL BASED ON ABC ANALYSIS FOR INVENTORY CONTROL PROBLEM: A REAL CASE STUDY

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Abstract. In modern business today, organizations that hold large numbers of inventory items, do not find it economical to make policies for the management of individual inventory items. Managers, thus, need to classify these items according to their importance and fit each item to a certain asset class. The method of grouping and inventory control available in traditional ABC has several disadvantages. These shortcomings have led to the development of an optimization model in the present study to improve the grouping and inventory control decisions in ABC. Moreover, it simultaneously optimizes the existing business relationships among revenue, investment in inventory and customer satisfaction (through service levels) as well as a company's budget for inventory costs. In this paper, a mathematical model is presented to classify inventory items, taking into account significant profit and cost reduction indices. The model has an objective function to maximize the net profit of items in stock. Limitations such as budget even inventory shortages are taken into account too. The mathematical model is solved by the Benders decomposition and the Lagrange relaxation algorithms. Then, the results of the two solutions are compared. The TOPSIS technique and statistical tests are used to evaluate and compare the proposed solutions with one another and to choose the best one. Subsequently, several sensitivity analyses are performed on the model, which helps inventory control managers determine the effect of inventory management costs on optimal decision making and item grouping. Finally, according to the results of evaluating the efficiency of the proposed model and the solution method, a real-world case study is conducted on the ceramic tile industry. Based on the proposed approach, several managerial perspectives are gained on optimal inventory grouping and item control strategies.

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1. INTRODUCTION

In today's industrialized world, given the intense industrial competition, it is crucial to pay attention to inventory control and the proper control of all types of organizations, especially manufacturing ones. In recent years, a large percentage of the total capital of organizations is made up of their inventories. In developed and developing countries, the holdings of capital at all times are very high. In the United States, for example, this amount is $50 billion (World military spending tops $1T, 2004) for defense projects and more than $95 billion for private companies [45]. Accordingly, it can be said that, nowadays, inventory planning, maintenance and control are significant issues for many organizations in every sector of industry and economy. It is not specific to third-world countries; even advanced industrialized countries have to somehow deal with cost problems related to inventory. Lack of proper inventory control can cost an organization a lot. On the one hand, the lack of one type of item may halt the production process; in this case, failure to deliver the product on time would result in a loss of customers and, consequently, a loss of markets. This can impose enormous costs on the organization. On the other hand, excessive inventory maintenance increases holding costs, which will have a negative impact on the company's profitability. As such is the case in the present era, inventories are assigned a crucial role to play in any production or service system. If properly controlled, inventories can help to balance the flow of operations in organizations [4, 41].

In the industrial world today, the number of inventory items is significantly increasing as customer demand pressures for various products increase. So, creating an appropriate inventory control system should be a priority for all organizations in order to maintain their competitive advantage and rapid and effective response to varied demands. As careful and cautious look at the inventory issue enables organizations to make the best use of their capabilities and attain their goals efficiently. So far, different models and methods have been proposed to classify inventories. Among them, the ABC analysis is an extensively used approach for inventory planning and control [11]. It enables organizations to classify their inventories into meaningful categories. In general, this approach is based on the Pareto principle, known as the 80–20 rule. As it suggests, the policy should be that only 20% of the inventory make up 80% of the total annual cost of the inventory system, and the remaining 80% account for only 20% of the costs. The result of such an analysis shows that class A should be very low, class B low, and class C the least controlled [35].

In contrast to traditional methods, the ABC analysis provides a variety of techniques and criteria for inventory classification, such as the classification of annual order of goods. In line with the first study done by Flores and Whybark [18]. In this field, other methods have been presented in recent years to classify ABC multi-criteria inventories. In this regard, one may refer to the analytic hierarchy process (AHP), artificial intelligence techniques, statistical analysis, data envelopment analysis (DEA) [12], weighted Euclidean distance, standard criterion matrix model, cluster analysis model, meta-heuristic algorithms, optimization algorithms, ABC-FUZZY classification approach, and multiple criteria decision aiding (MCDA). As it turns out, none of the decomposition algorithms such as Lagrange relaxation, Benders and Wolf-Dantzig have been put to practice. In the present study, however, they have been used and even compared.

The method of grouping and inventory control available in traditional ABC has several disadvantages. First of all, there are no specific guidelines in the literature with which to specify the service level for each group [35]. Secondly, since grouping decisions are independent and are preceded by service level decisions, their interactions are not applied and neither of the two decisions can be optimal. Thirdly, since the existing budget is not taken into consideration until the last stage, there is no guarantee that decisions will be made about the grouping or the services offered in the first two stages. As a result, it often has to be reconsidered until decisions can be made to assign different service levels to a diverse group of items. This process can be tedious when there are too many items in stock, which may even lead to infeasible solutions. These shortcomings have led to the development of an optimization model in the present study to improve the grouping and inventory control decisions in ABC. This model also helps inventory and operations managers optimize several items simultaneously, including (a) the number of inventory groups, (b) their service levels, and (c) the allocation of each item to each group under a limited budget.

This paper is structured as follows: The second section reviews the literature existing in the field, which is performed in three streams. Section three describes the proposed model and explains the mathematical model, which includes sets, parameters, decision variables, and so on. Section four presents' methods for solving the mathematical model and provides some explanations. Section five compares the proposed solutions, introduces the best method and offers a sensitivity analysis of the mathematical model. Section six provides an overview of the case study and the implementation of the mathematical model in this industry, including the results of the model implementation. Next, some managerial insights are presented in section seven. Finally, section eight recounts the conclusions of the study and provides some suggestions for future research.

2. Literature review

Based on the topic discussed in the present study, the literature is reviewed in three streams.

2.1. ABC analysis for the inventory control of stock

In general, the inventory of goods is an essential subject to discuss and involves both theoretical and practical aspects to consider. It has been the subject of discussions for many years now. There is still no complete agreement on certain inventory issues, and newer comments are being made every day. Among the different definitions of inventory, the most inclusive seems to be the one given by the American Institute of Certified Public Accountants (AICPA). It postulates that an inventory of goods comprises tangible assets that are (a) held for sale in the ordinary course of business, (b) currently used in the process of producing such goods, and (c) already used in the production of products and services. According to this definition, an inventory of assets includes raw materials, semi-finished products, manufactured goods, and supplies. Because the inventory of goods is of great importance as the current assets of a business, the warehouseman must invariably identify the inventory items, classify them and enumerate them in the warehouse. It is necessary to balance the need for the product and its inventory. Due to the variety of products in a company or store and considering the number of times a product is delivered over one or more days, it is so hard to practically control the goods through numerous cards and records. So, it is necessary to strike a balance between the required goods and the available goods. The most important materials and goods stored in a warehouse, including the products and goods constantly provided to meet the needs of customers, must be clearly accounted for in a precise inventory list. This is why some stores have to invest in inventory and incur costs for holding their goods. There are, however, certain factors that may create disturbances in the regularity of inventories, such as untimely supply of goods, discounts on bulk orders, precautionary storage, and reduction in the number of purchasing operations. Consequently, in order to create a perfect inventory control system, different inventory items should be classified into meaningful categories based on appropriate indicators and criteria. So far, different models and methods have been presented in this regard. Among them, ABC analysis is the most widely used to plan and control inventories [11].

For the first time in 1915, Ford Harris of the Westinghouse Institute developed a simple formula for inventory accumulation [16]. Since then, this seemingly independent formula has been developed by many people. The formula is also known as the Wilson formula because it was R.H. Wilson who raised it as an integral part of an inventory control scheme. An article by Arrow *et al.* [7] provided an in-depth analysis of a simple inventory model, which was later followed by mathematicians such as Massart [33]. In recent years, massive research has been done on inventory classification based on the ABC multi-attribute approach to improve inventory control. The study by López-Soto *et al.* [31] is a recent attempt in this case. Chakravarty [10] proposed a dynamic model for the classification of inventories. In this model, inventories were categorized according to the increase of unit demand and the cost per unit. A main strength of this model was the minimization of the overall cost, but its significant weakness was the long runtime. Flores and Whybark [18] developed a two-character matrix approach to classify inventories and provide comprehensive control over them. Their approach classified the inventory items according to the ABC standard classification and based on individual indexes; two single-item groups were combined by a conjugate matrix. A major disadvantage of this approach was that several indices

were too complicated and impractical. Cohen and Ernst [13] presented a model to classify inventories. They used only one annual dollar usage index for this purpose. Guvenir and Erel [19] proposed a multi-characteristic genetic algorithm to classify inventories. In essence, this model determined the weights of the indexes when classifying inventories, which showed better results than the analytical hierarchy process (AHP) method. Van Eijs *et al.* [46] researched the classification of inventory items. They referred to two types of demand-side classification strategies including direct and indirect groupings. The indirect grouping strategy assumed one-base order intervals and multiple-order intervals that were linear to them. Partovi and Burton [38] used the AHP to classify inventories. This classification took into account both qualitative and quantitative criteria.

In 2008, Tsai and Yeh [44] proposed the flexible particle swarm optimization (PSO) algorithm to classify inventories. They proposed an optimization approach to inventory classification problems when inventory items were categorized for multiple purposes, such as maximizing inventory turnover rates and minimizing costs. Their model challenged the other models of inventory classification in that they required identifying the number of groups before classifying the inventories. The advantage of that algorithm over other models is that it can specify the optimal number of classification groups while completing the inventory classification. Hadi-Vencheh [20] presented a wide-ranging form of the NG model for the classification of inventories based on several indicators. This was a nonlinear programming model to determine a standard set of weights for all inventory items. Yu [50] conducted a study to compare classification techniques based on artificial intelligence (AI) and traditional classification techniques. The results showed that AI-based techniques would have greater accuracy in the classification of inventories. Through statistical analysis, it was also suggested that the SVM technique was more accurate for classification than the other two solutions (*i.e.*, BPN and K-NN).

Inventory managers often classify inventories to control them better. The well-known ABC approach classifies inventory items to A, B, and C based on their usage and sales. Millstein *et al.* [35] presented an optimization formulation to improve the quality of inventory grouping under a limited budget. In the research by Kaabi *et al.* [25], TOPSIS was used to calculate the score of each inventory item, and a continuous-valued number system (CVNS) algorithm was used to create the weighting criterion. Finally, the inventory was classified according to the item weighting criterion.

In the literature, most existing classification models use the ABC inventory classification as a ranking problem. For example, a set of inventory items that are weighted based on their performance are listed in a descending order of scores. Douissa and Jabeur [14] examined the problem of classifying the ABC inventory list as an allocation problem. For example, an inventory item was classified in a group of items with the most similar properties. Multiple criteria ABC analysis has also been extensively used in inventory control to support organizations allocate inventory items to several classes according to different evaluation criteria. Many solution methods have been suggested in the literature to address this problem. However, most of them are quite compensatory in multi-criteria aggregation, which means that an item with a bad grade on one or more main criteria can be given good grades because the other criteria can compensate corrupt practices. It is, thus, necessary to consider the ABC analysis of multiple criteria. Liu *et al.* [30] proposed a new efficient classification method according to the outranking model to tackle such a problem. They combined the simulated annealing algorithm and the clustering analysis to search for an optimal classification. Recently, Mehdizadeh [34] has introduced a new criterion for the ABC analysis of car spare parts classification. The rough set theory distinguishes strong knowledge from ABC analysis. Using the rough set theory, the researcher has derived rules and patterns from stochastic information already obtained by the ABC analysis. These rules are extracted to predict future retailers' demands. The order is then determined based on the periodic review approach. Moreover, Jesujoba and Adenike [24] investigated ABC analysis inventory management practice and product quality of manufacturing companies in De United Foods Industries Limited. a populace of 385 people was considered. They entailed all the staff of inventory-related departments, and a sample size of 196 was selected. The quantitative method utilizing a questionnaire to gather data from the respondents was applied in this study. Research data were examined by means of regression analysis. The results depicted that a strong positive and significant relationship exists between ABC analysis on product quality of De United Foods Industries Limited. In the paper by Abdolazimi *et al.* [5], a bi-objective mathematical model was suggested to enhance the inventory grouping based on ABC

analysis. The proposed model concurrently optimized the service level, the number of inventory groups, and the number of assigned items. To solve the model in small and large dimensions, two exact methods (LP-Metric and ε-constraint) and two meta-heuristic methods (NSGA-II and MOPSO) are used.

2.2. Lagrange and benders algorithms in comparison

Optimization problems have different matrix structures that are based on the arrangement of matrix blocks and their relationship [1]. The methods that utilize this particular problem matrix structure are usually more efficient and find the right answer to the problem at the right time. In general, the structure of optimization problems often includes complex constraints or complex variables. These constraints and variables usually reflect the shared use of problem blocks by one or more scarce sources. Lagrange relaxation algorithm, Bander's decomposition, Dantzig–Wolfe decomposition, and some other formats can be used to solve such problems. In fact, in order to solve a problem using the decomposition methods, it is necessary first to identify the problem structure. Next, the methods of decomposing the set of blocks are put to practice separately to achieve an overall solution to the problem. Many researchers have used these analytical algorithms, some of whose works are presented below.

Multipurpose assembly lines usually exist in industries producing large-scale products (*e.g.*, the car industry), where multiple workers are assigned to the same station to perform several tasks simultaneously on the same product. The efficient mathematical formulas presented are only able to solve a few small samples, while larger ones are solved by heuristic or meta-heuristic methods that do not guarantee optimality. The article by Wang *et al.* [47] presents a new short-interest LP formulation with strong symmetry break constraints and breaks down the original problem in a new Benders decomposition algorithm to solve great samples optimally. Li *et al.* [29] introduced cluster supply chains to prevent this potential operational risk through chain collaboration. Due to the large amounts of data from actual operations and the complexity of the cluster supply chain structure, a parallel Lagrange heuristic was presented to solve the problem of the nonlinear mixed program (MINLP). The Benders algorithm was also used for performance evaluation through comparisons.

In a study by Kovački *et al.* [27], the Lagrange relaxation approach was used to propose a new method for the dynamic reconfiguration of a distribution network (DRDN). The purpose of the DRDN was to determine the optimal topology (configuration) of a distribution network over a specified period. First, the "switch-to-switch path" method was used to model distribution networks and to formulate DRDN as a mixed linear integer programming (MILP) problem. Then, the problem was solved using the two-step Lagrange method. At the first step, the Lagrange related dual problem was also solved. It was created by relaxing the switching performance constraints that separated the Lagrange dual problem. This problem was much easier to solve than the original one. At the second step, the Lagrange dual problem was solved to perform the heuristic search. Then a suboptimal solution, though possible for the main problem, was presented. Finally, the proposed DRDN model was expanded to a multi-purpose formula which accounted for the impacts of network reliability and switching costs on the DRDN process. In a study by Rohaninejad *et al.* [40], an accelerated Benders decomposition algorithm was suggested to address the location of the multi-echelon reliable capacitated facility location problem (ME-RCFLP). The goal was to offer a trade-off between total investment and system reliability on one hand and operating costs on the other while making facilities more costly (increasing their capacity and reducing the likelihood of complete or partial failure).

In the real world, regulated electricity market operations, electricity price forecasting, profit-based unit commitment (PBUC) and optimal bidding strategy are significant issues. Among them, PBUC is a problem of hybrid optimization. Sudhakar *et al.* [42] proposed a hybrid approach of Lagrange Relaxation (LR) and Differential Evolution (DE) to solve the PBUC problem. In this approach, LR was used to solve a single-commitment problem, and the DE algorithm was applied to update Lagrange multipliers. Recently, Mohebifard and Hajbabaie [36] have formulated the problem of optimizing the network traffic signal scheduling as a mixed integer non-linear program (MINLP). They have also devised a proprietary method to solve the problem with a tight optimality gap. In another recent study, Mardan *et al.* [32] have presented a comprehensive formulation for multi-product, multi-period, multi-modal, and two-objective green closed-loop supply chains. Minimizing the total cost and

the emissions of greenhouse gases is the purpose of this model, which is met by deciding on the location of facilities, the amount of transportation and the balance of inventory. The results show that the proposed solution approach can reduce the total cost by more than 13%.

In the research by Zetina *et al.* [51], two detailed Benders decomposition-based algorithms are presented to solve a multi-commodity incapacitated fixed-charge network design problem. In addition, Li and Jia [28] have studied the issue of order fulfillment in an electronic environment where the e-tailer assigns orders to its delivery centers and sets routes for shipments from delivery centers to delivery stations. In their research, a mix integer program is proposed for the problem, its computational complexity is analyzed, and the Benders decomposition algorithm is considered to solve it. The computational performance of the proposed algorithm is also evaluated on problem instances created based on the JD.com logistics network in Shanghai. It is necessary to equip the network with monitoring sensors to avoid the severe risks of contaminants entering the water distribution network. Hooshmand *et al.* [21] have investigated the problem of sensor location with identification criteria, assuming that a limited budget is available for sensor placement. The aim of the study is to minimize the number of vulnerable nodes with the same alarm pattern. First, the problem is considered as a MIP formulation, assuming that the objective functions were ordered on the basis of specific prioritization. Then, using the basic problem structure, the exact logic-based Benders decomposition algorithm is developed. The challenge is to provide a solution to multi-objective problems with nonlinear constraints on large networks.

Networks are made up of many nodes that have millions of streams of dynamic nature. Jaglarz *et al.* [23] have addressed this issue. A novel linear programming problem of energy awareness, multi-objective, complex and integer formulation is modeled and solved using the Lagrange decomposition algorithm. This algorithm has been improved with new ergodic sequences to enhance the quality of primal solutions recovered from dual problems. Finally, In the paper by Abdolazimi *et al.* [2], a bi-objective mixed integer linear programming model was developed to minimize the overall cost and maximize the use of eco-friendly materials and clean technology. The paper evaluates the exact (LP-metric, modified ε-constraint, and TH), heuristic (Lagrange relaxation algorithm), and meta-heuristic (MOPSO, NSGA-II, SPEA-II, and MOEA/D) approaches in solving the proposed model in both small and large sizes. This study provided an all-inclusive view regarding the importance of selecting an appropriate solution methodology based on the problem dimension to ensure obtaining the optimum and accurate solution within the reasonable processing time.

2.3. Research gap

In the industrial world today inventories serve as a vital element for all organizations. In recent decades, organizations have encountered thousands of different types of inventories, and inventory management and creation has been the subject of discussions in this regard. Proper inventory control systems have become a significant challenge for all organizations, and this has brought a need for research in this area. Lack of appropriate inventory control systems creates many problems for organizations. First of all, they face inventory-related costs, as for holding, ordering and shortages. In some organizations, the shortage of an inventory may hinder the production process, resulting in the problem of not delivering the product to customers on time, thereby increasing the cost of scarcity. In some cases, the organization may also face excessive inventory increases, which would also increase inventory holding costs. Secondly, because the number of inventory items is increasing rapidly along with the increasing customer demand for various products, organizations must maintain a rapid and effective response to their customers' demand. This is essential if they are to survive and continue their competitive advantage. Finally, as the case is nowadays, the total capital of a large percentage of organizations is made up of their assets. In developed and developing countries, the capital held in inventories at any given time is enormous. Therefore, research on appropriate inventory control systems is necessary for all organizations.

Given the above, the necessity of research on inventory control systems is undeniable. In addition to the problems mentioned, another problem with most organizations is that their inventory control system uses only the same control policy for all inventories, which is not economically viable. Also, the resources of all organizations are limited, and the application of a common control policy to all inventories cannot ensure the useful management of resources, which results in additional costs for the organization. Various studies have

shown that, to overcome this problem, organizations must classify their inventories into meaningful categories. A commonly used approach in this area is ABC analysis. In most organizations, inventory classification is based on only one index; therefore, this classification is not able to meet all the needs of inventory control systems, and research is essential to identify different indicators that can be of use in inventory classification.

For the present research, several articles were reviewed in the field of grouping different inventories, especially warehouses. Unlike other inventory grouping models, this study focuses on optimizing the relationships among income, inventory stock, service level, and management costs to maximize profit. Table 1 presents the literature review in three sections. In the first one, the reviewed articles are labeled for their critical formulating features including performance criteria (single or multi-criteria), objective function type (single or multi-objective) and budget constraints. In the second section, the studies that have used decomposition algorithms (Lagrange and Benders) are reviewed to solve their models and compare them with each other, and even with other methods or algorithms. The last section deals with reviewing articles that report case studies. The model in this paper contributes to the existing literature by presenting a combination of new formulating features such as simultaneous identification of inventory groups, their service levels, allocation of items to groups, maximizing the profits of a company or industry and optimal allocation of limited funds to inventory items. It also enables purchasing and inventory managers to cope with inventory grouping and service levels with a limited budget. Moreover, it simultaneously optimizes the existing business relationships among revenue, investment in inventory and customer satisfaction (through service levels) as well as a company's budget for inventory costs. Considering the products in different packages, the model proposed in this study takes into account inventory control policies such as inventory shortages and holding costs. It is, thus, brought closer to reality.

3. Description of the mathematical model

Consider N items. Each item has an average monthly demand (d) and a standard deviation (σ). The demand for each stock keeping unit (SKU) follows the normal distribution $N(d, \sigma)$. The delivery time of each item is h_{it}, and π is the gross income of each item of goods whose selling price is minus the purchase cost. To reduce overhead costs and simplify the inventory management process, the inventory manager has the task of classifying the goods into different groups (j groups) and then setting a service level for each group. Thus, each stock group in the stock has a service level β_j. The cost of the inventory maintenance for each SKU is e_{it}. It is clear that the 99.99% service level for all SKUs has the highest revenue but is practically not feasible, as it also involves a high cost because the model has a total fixed budget and limited D to store products in stock. The inventory decision maker should optimally allocate D to the SKUs to optimize the total net profit. When the demand is distributed normally, the inventory level of each SKU in the warehouse to reach β_j (in group j) can be calculated by the standard method through equation (3.1) as follows:

$$\delta_{ijt} = d_{it}h_{it} + Z_j\sigma_i\sqrt{h_{it}} \tag{3.1}$$

where Z_j is the value of Z corresponding to β_j in the standard normal distribution. Generally, the level of inventory in equation (3.1) can be negative in the case of (a) negative Z_j (*i.e.*, β_j is less than 50%), (b) significant standard deviation (many changes in demand) and (c) long delivery time.

From a cost management point of view, θ_j is for the maintenance and management of each inventory group. This cost may also include purchase costs and executive costs for each group. This paper assumes that θ_j is a variable and a function of the number of inventory groups. It is also assumed that the warehouse faces a shortage, so there is a cost (CL_{ijt}).

In the inventory optimization problem of this paper, the manager simultaneously decides about two topics; (a) select the number of inventory groups (with the corresponding service levels) and (b) assigning each SKU to a suitable group. These decisions should be made in such a way that the total holding and the overhead costs of the inventory management are not more than the limited budget.

According to the previous description, the presented mathematical formulation is a kind of MILP. The sets, parameters and variables of this mathematical formulation are as follows:

TABLE 1. Summary of the literature review.

Reference	Section A ABC analysis					Section B Decomposition algorithms			Section C Case study
	Objective function		Criteria		Solution methods	Lagrange	Benders	Comparison	
	Single	Multi	Single	Multi					
Guvenir and Erel [19]	✓	–	–	✓	Genetic algorithm	–	–	–	–
Partovi and Burton [38]	✓	–	–	✓	AHP method	–	–	–	✓
Hadi-Vencheh [20]	✓	–	–	✓	Extended version of NG-model	–	–	–	–
Tsai and Yeh [44]	✓	–	–	✓	PSO algorithm	–	–	–	✓
Massart [33]	✓	–	✓	–	–	–	–	–	–
Chakravarty [10]	✓	–	✓	–	IBM 370/140	–	–	–	–
López-Soto *et al.* [31]	✓	–	✓	–	Multi-start constructive algorithm	–	–	–	✓
Arrow *et al.* [7]	✓	–	✓	–	Markov chain	–	–	–	✓
Chen *et al.* [11]	✓	–	–	✓	Quadratic optimization program	–	–	–	✓
Flores and Whybark [18]	–	–	–	✓	A simple mechanical procedure	–	–	–	✓
Van Eijs *et al.* [46]	✓	–	✓	–	Optimal and heuristic solutions	–	–	–	–
Yu [50]	✓	–	–	✓	Artificial intelligence	–	–	–	–
Millstein *et al.* [35]	✓	–	✓	✓	Branch-and-cut method	–	–	–	✓
Kaabi *et al.* [25]	–	–	–	✓	Automatic learning method	–	–	–	✓
Douissa & Jabeur [14]	✓	–	✓	–	PROAFTN method	–	–	–	✓
Liu *et al.* [30]	✓	–	–	✓	Clustering analysis and simulated annealing algorithm	–	–	–	✓
Massart [33]	–	✓	✓	–	Periodic review approach	–	–	–	✓
Kovački *et al.* [27]	–	–	–	–	–	✓	–	–	✓
Mohebifard and Hajbabaie [36]	–	–	–	–	–	–	✓	✓	✓
Hooshmand *et al.* [21]	–	–	–	–	–	–	✓	–	–
Mardan *et al.* [32]	–	–	–	–	–	–	✓	–	✓
Jaglarz *et al.* [23]	–	–	–	–	–	✓	–	–	–
Li *et al.* [29]	–	–	–	–	–	✓	✓	✓	–
Zetina *et al.* [51]	–	–	–	–	–	–	✓	–	–
Rohaninejad *et al.* [40]	–	–	–	–	–	–	✓	–	–
Sudhakar *et al.* [42]	–	–	–	–	–	–	✓	–	–
Wang *et al.* [47]	–	–	–	–	–	✓	–	✓	✓
Li and Jia [28]	–	–	–	–	–	–	✓	–	✓
Current study	✓	–	–	✓	Decomposition algorithms	✓	✓	✓	Ceramic tile industry

Sets

i	Set of inventory items (SKUs)
j	Set of inventory groups
t	Set of periods

Parameters

d_{it}	Mean of monthly demand for SKU i in period t
σ_i	Standard deviation of monthly demand of SKU i
h_{it}	Lead time of SKU i
π_{it}	Gross profit per unit of SKU i in period t
e_{it}	Inventory holding cost per unit of SKU i in period t
θ_j	Variable overhead control cost for inventory group j
M	Number of inventory items (SKUs)
D	Available budget
β_j	Service level associated with inventory group j
Z_j	Z-value associated with the service level β of inventory group j
O_{it}	Fixed order cost for SKU i from the central stock to supplier s in period t
CL_{ijt}	Fixed cost of shortage for SKU i at inventory group j in period t

Decision variables

V_{it} The inventory level of SKU i in the central stock in period t

La_{ijt} The amount of shortage for SKU i at inventory group j in period t

X_{ijt} If SKU i is assigned to group j in period t, 1 and otherwise 0

Y_{jt} If inventory group j is selected in period t, 1 and otherwise 0

$$\text{Max } Z = \sum_i \sum_j \sum_t \pi_{it} d_{it} \beta_j X_{ijt} - \sum_j \sum_t \theta_j Y_{jt} - \sum_i \sum_j \sum_t \text{CL}_{ijt} \text{La}_{ijt} \tag{3.2}$$

Subject to:

$$\sum_j X_{ijt} \le 1 \qquad\qquad \forall i, t \tag{3.3}$$

$$\sum_i X_{ijt} \le M Y_{jt} \qquad\qquad \forall j, t \tag{3.4}$$

$$V_{it} + \sum_j \text{La}_{ijt} = \sum_j d_{it} h_{it} X_{ijt} + \sum_j Z_j \sigma_i \sqrt{h_{it}} X_{ijt} + V_{it-1} + \sum_j \text{La}_{ijt-1} \qquad \forall i, t \tag{3.5}$$

$$\sum_i e_{it} V_{it} \le D \forall t \tag{3.6}$$

$$V_{it}, \text{La}_{ijt} \ge 0 \tag{3.7}$$

$$X_{ijt}, Y_{jt} \in [0, 1]. \tag{3.8}$$

Objective function (3.2) calculates the maximum total depreciated net profit from which costs are deducted. Herein, the service level is a fixed rate per unit of time to calculate the amount of demand. If item i falls into inventory group j with service level β_j, $d_{it}\beta_j$ is the average warehouse demand that can be completed by any SKU. The first statement in objective function (3.2) is the total gross profit of all the SKUs. The second statement is the variable cost overhead management of the inventory groups in the warehouse, and the third statement is the cost of inventory shortage at the end of each period if the warehouse faces a lack of inventory. Constraint (3.3) assigns one SKU to a maximum of one group. It is not possible to assign any item to any group because, for each SKU$_i$ and group j, if $\delta_{ijt} > 0$, item i cannot be attributed to group j, *i.e.*, $X_{ijt} = 0$ [35]. Constraint (3.4) suggests groups must be selected and each SKU must be assigned to a group; *i.e.*, no SKU can be assigned to group j if group j is not selected. Constraints (3.5) calculate the amount of inventory based on equation (3.1) to which the shortage is added. Constraint (3.6) guarantees that the cost of holding the inventory in a warehouse does not exceed the total fixed budget. Constraint (3.7) indicates that the model variables are positive, and constraint (3.8) specifies the binary variables of the model.

4. Proposed decomposition algorithms

In many mathematical models, with increasing the problem size, the computational complexity of the model also increases exponentially so that the exact solutions cannot be calculated in a logical time [2, 3]. Following this, various methods have been proposed by researchers who use a particular approach to seek approximate and near-optimal solutions. These methods are generally divided into two categories of heuristic and meta-heuristic algorithms. Decomposition algorithms are among the heuristic approaches that seek to simplify complex mathematical models in order to achieve an approximate answer in a logical time. Numerous applications of these algorithms have led to their utilization in various optimization problems. Studies by Yolmeh and Saif [49], Wang *et al.* [48], Naderi *et al.* [37], and Aydin and Taşkin [8] are examples of the application of these algorithms to supply chain problems.

This section presents the solutions to the proposed model. In this regard, references are made to Lagrange and Benders decomposition algorithms. Then, they are compared so as to select the best solution to the model.

As mentioned before, decomposition solutions are intended to simplify the mathematical model proposed in this research. These solutions are introduced in the following subsections.

4.1. Lagrange relaxation algorithm

The Lagrange relaxation algorithm is one of the innovative techniques that utilize the Lagrange theorem to simplify complex mathematical models in order to obtain an approximate response within a reasonable time. This method has been used in various optimization problems. Studies by Diabat *et al.* [15], Kang and Kim [26] and Ahmadi-Javid and Hoseinpour [6] are examples of the application of the Lagrange method to these problems. The mathematical approach of this method is described below. Consider the mathematical model presented in equation (4.1):

$$
\begin{aligned}
&\min c^T x \\
&\text{Subject to} \\
&Ax \le b \\
&x \in X.
\end{aligned}
\tag{4.1}
$$

The general approach of the Lagrange relaxation algorithm is based on releasing the complex constraints of the mathematical model and adding them to the objective function of the problem using Lagrangian multiplier coefficients. The basis of this approach is that, first, the problem constraints are relaxed. Then, considering the effect of the constraints on the problem, if the relaxed constraints are violated, the penalty functions for each constraint are added to the objective. Thus, the relaxed form of model (4.2) is as follows:

$$
\begin{aligned}
&\min c^T x + \mu^T (Ax - b) \\
&\text{Subject to} \\
&x \in X
\end{aligned}
\tag{4.2}
$$

where μ^T is the Lagrangian coefficient of the algorithm. Finally, the Lagrangian function is considered as the equation $L(\mu) = c^T x + \mu^T (Ax - b)$. In most cases, the result of the relaxed mathematical model is unjustified for the original mathematical model. This unreasonable response is considered as a low limit (in problems with the minimization objective function) or a high limit (in problems with the maximization objective function) for the original mathematical model. In the following, the Lagrangian relaxation algorithm justifies the solution by the use of innovative processes. It should also be noted that a justified response is either an upper limit (in problems with the minimization objective function) or a lower limit (in problems with the maximization objective function) for the original mathematical model. Given that the optimal solution of the original mathematical model is obtained between these upper and lower bounds, the algorithm seeks to reduce this boundary and, thereby, achieve a near-optimal solution to the problem.

Correct selection of complex constraints for relaxation is one of the crucial parts of the Lagrange algorithm that has a direct effect on the performance of this algorithm. In this paper, constraint (3.5) is chosen for relaxation due to the complexity that it has created for the mathematical model. By releasing this constraint, the proposed relaxed model is transformed as follows:

$$
\begin{aligned}
\text{Max } Z + u \times &\left(-\sum_i \sum_t V_{it} - \sum_i \sum_j \sum_t \text{La}_{ijt} + \sum_i \sum_j \sum_t d_{it} h_{it} X_{ijt} + \sum_i \sum_j \sum_t Z_j \sigma_i \sqrt{h_{it}} X_{ijt} \right. \\
&\left. + \sum_i \sum_t V_{it-1} + \sum_i \sum_j \sum_t \text{La}_{ijt-1} \right).
\end{aligned}
\tag{4.3}
$$

Constraints (3.3)–(3.8), except for constraint (3.5), are also added to the formulation.

In the formulation delineation above, u is the Lagrange coefficient and a free of sign. The Lagrange relaxation algorithm starts with the constant values of this Lagrange coefficient. However, the coefficient value must be updated in each iteration of the algorithm using a mechanism. Various methods have been proposed to update the values of the Lagrangian coefficients, the most famous of which is the sub-gradient method. It has been used in this paper as a well-known and popular method of solving the Lagrange relaxation problems [17]. Based on the sub-gradient approach, the Lagrangian coefficient in the $C + 1$ iteration of the algorithm is calculated by equation (4.4):

$$u^{c+1} = \max\left[0, \left\{ u^c + \pi^c.\left(-\sum_i\sum_t V_{it} - \sum_i\sum_j\sum_t \mathrm{La}_{ijt} + \sum_i\sum_j\sum_t d_{it}h_{it}X_{ijt} + \sum_i\sum_j\sum_t Z_j\sigma_i\sqrt{h_{it}}X_{ijt} \right.\right.\right.$$

$$\left.\left.\left. + \sum_i\sum_t V_{it-1} + \sum_i\sum_j\sum_t \mathrm{La}_{ijt-1} \right) \right\} \right] \tag{4.4}$$

where π^c is the size of the steps of the algorithm. It is calculated by equation (4.5):

$$\pi^c = \frac{v^c \times (\mathrm{BUB}^c - \mathrm{LB}^c)}{\left(-\sum_i\sum_t V_{it} - \sum_i\sum_j\sum_t \mathrm{La}_{ijt} + \sum_i\sum_j\sum_t d_{it}h_{it}X_{ijt} + \sum_i\sum_j\sum_t Z_j\sigma_i\sqrt{h_{it}}X_{ijt} + \sum_i\sum_t V_{it-1} + \sum_i\sum_j\sum_t \mathrm{La}_{ijt-1} \right)^2}. \tag{4.5}$$

BUB is equivalent to the best upper bound calculated for the c iteration, and LB^c is the lower bound of the problem in the c iteration. Coefficient v^c is usually between 0 and 2. Furthermore, it is worth noting that LB is an initial feasible solution for the problem. This scalar can be gained employing other solution methods like CPLEX, LP-metric, ε-constraint, and the like. In this study, LB has been achieved based on the CPLEX solution in GAMS software.

4.2. Benders decomposition algorithm

The Benders decomposition algorithm was proposed by Benders [9] to solving complex integer problems. It is an exact solution method based on the problem decomposition, and it operates in two main parts including (MP) and sub-problem (SP). It, thus, uses the relationship between the sub-problem and it's dual. Once cutting planes are added to make the algorithm optimal and feasible, the convergence of the upper and the lower bounds continues until an optimal solution is reached. In each algorithm's iteration, a corner point is obtained from the sub-problem, and the problem-solving space is cut from that point. The optimal response is obtained when the upper and the lower limits are closely matched or intertwined. Bender's flowchart is shown in Figure 1.

The first step is to standardize the problem. So, the primary problem can be rewritten in the following standard form:

$$\text{Min } Z = -\sum_i\sum_j\sum_t \pi_{it}d_{it}\beta_j X_{ijt} + \sum_j\sum_t \theta_j Y_{jt} + \sum_i\sum_j\sum_t \mathrm{CL}_{ijt}\mathrm{La}_{ijt} \tag{4.6}$$

Subject to:

$$1 \geq \sum_j X_{ijt} \qquad\qquad \forall i, t \tag{4.7}$$

$$0 \geq -MY_{jt} + \sum_i X_{ijt} \qquad\qquad \forall j, t \tag{4.8}$$

$$\sum_j d_{it}h_{it}X_{ijt} + \sum_j Z_j\sigma_i\sqrt{h_{it}}X_{ijt} + V_{it-1} + \sum_j \mathrm{La}_{ijt-1} - V_{it} - \sum_j \mathrm{La}_{ijt} = 0 \qquad\qquad \forall i, t \tag{4.9}$$
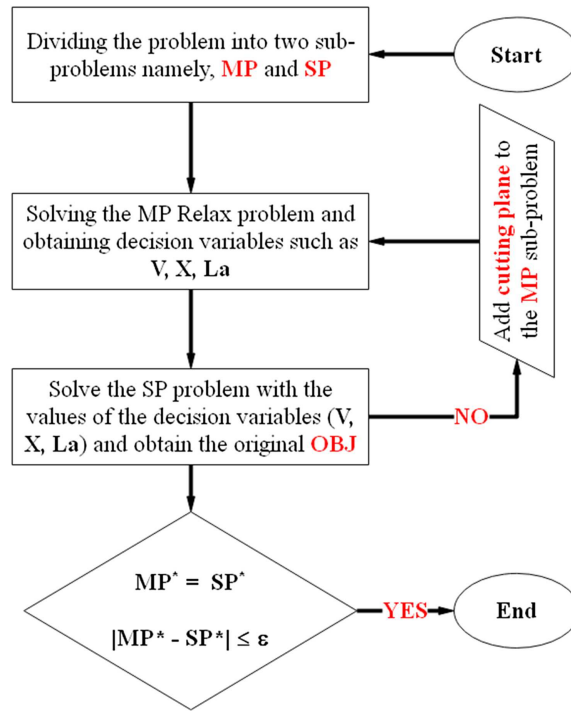
FIGURE 1. The flowchart of Benders' Algorithm.

$$-\sum_i e_{it} V_{it} \geq -D \qquad\qquad \forall t. \qquad (4.10)$$

As can be seen, to conduct standardization, all the integer variables are moved to the right; on the left, there are only continuous variables.

4.2.1. The master problem (MP) and the lower bound of the algorithm

In the proposed model, Y_{jt} is a hard variable because it is binary. To use the Benders algorithm in the master problem, therefore, a constraint to relaxation (defined as a nonnegative variable in the domain of real numbers) is added to the MP. It is defined as equation (4.11):

$$
\begin{aligned}
&\text{Min } Z \\
&\text{Subject to:} \qquad\qquad\qquad\qquad\qquad (4.11)\\
&Y_{jt} = 1 \qquad \forall j, t.
\end{aligned}
$$

4.2.2. Sub-problem (SP) and upper bound of the algorithm

After the Y_{jt} variable is relaxed in the main problem and the MP model is solved, for the sub-problem in the SP problem, the relaxed variable becomes binary and the relaxing constraint (*i.e.* constraint (4.8)) is removed. Considering $V1_{it}$, $V2_{jt}$, $V3_{it}$ and $V4_t$ as the dual variables for constraints (4.7), (4.8), (4.9), and (4.10), respectively, it is possible to solve the dual problem of this sub-linear problem; the upper bound in each iteration generates an algorithm written as follows:

$$\text{Max } Z = -\sum_i \sum_t V h_{it} - \sum_j \sum_t V1_{jt} M \overline{Y}_{jt} - \sum_t V4_t D + \sum_j \sum_t \beta_j \overline{Y}_{jt} \qquad (4.12)$$

Subject to:

$$-V1_{it} - V2_{jt} - \left(D_{it} h_{it} Z_j \sigma_i \sqrt{h_{it}} \right) V3_{it} + \beta_j \overline{Y}_{jt} \leq - (\pi_{it} D_{it} \beta_j) \qquad \forall i, j, t \qquad (4.13)$$

$$V3_{it} - V3_{it-1} \leq \mathrm{CL}_{ijt} \qquad \forall i, j, t \qquad (4.14)$$

$$V3_{it} - e_{it} V4_t \leq 0 \qquad \forall i, t. \qquad (4.15)$$

4.2.3. Cutting planes of the algorithm

As indicated in Figure 1, in each iteration of the Benders algorithm, the upper and the lower bounds of the problem converge as the two main types of feasibility (constraint (4.17)) and optimality (constraint (4.18)) cutting planes are added to the MP. The former remains in the problem-solving space and becomes unjustified, but the latter will work to make the problem function get worse. Given the following dual solution to the linear problem, the finite element problem that creates a lower bound in each iteration of the algorithm is obtained as follows:

$$\underset{y}{\mathrm{Min}}\, Z \qquad (4.16)$$

Subject to:

$$-\sum_i \sum_t V1(i,t,\mathrm{unbcut}) - \sum_j \sum_t V2(j,t,\mathrm{unbcut})M\overline{Y}(j,t) - \sum_t V4(t,\mathrm{unbcut})D \leq 0 \qquad (4.17)$$

$$Z \geq -\sum_i \sum_t V1(i,t,\mathrm{optcut}) - \sum_j \sum_t V2(j,t,\mathrm{optcut})MY(j,t) - \sum_t V4(t,\mathrm{optcut})D + \sum_j \sum_t \beta_j \overline{Y}_{jt} \quad (4.18)$$

where *unbcut* is dynamic optimal cut set and *optcut* is dynamic feasibility cut set. These sets first take the value *no* and based on each iteration, wherever they take the value *yes*, equations (4.17) and (4.18) are activated. They are as follows:

$$\mathrm{optcut(iteration)} = \mathrm{No}$$
$$\mathrm{unbcut(iteration)} = \mathrm{No}.$$

5. Comparison of decomposition algorithms

The purpose of this section is to validate the proposed methods for solving the mathematical model presented to determine the optimal number of inventory groups in stock. Two decomposition algorithms, namely Lagrangean and Benders, are presented. Then, the model is solved using these algorithms, and the results are compared. In order to practice these two proposed methods and compare them with each other, the mathematical model for each method is implemented in 10 different numerical examples. Then, using the statistical assumption test, the results of these methods are compared in all the numerical examples using t-tests. The TOPSIS technique is also used to select the best method. It is worth noting that all the numerical examples are implemented to solve the proposed mathematical model in the GAMS software version 24.1.3 and CPLEX solver and in a system with Cori7 6700 HQ CPU and 16 GIG DDR4 RAM. For this part of the research and the comparisons, the research data are generated experimentally.

5.1. Numerical examples

In this section, to evaluate the proposed mathematical formulation and the proposed methods for solving it, the indicators are first defined. Two indices are, thus, determined. They include the value of the objective functions calculated by running the model with each proposed method as well as the time spent by that method. Also, through generating several numerical instances, the proposed methods are compared to each other.

TABLE 2. The results of the implementation of the model by different numerical examples.

Numerical examples	Benders decomposition		Lagrangean relaxation	
	OBJ ($)	Time (s)	OBJ ($)	Time (s)
1	4.58150E+8	3.25	4.579227E+8	10.62
2	3.73781E+8	4.14	3.735936E+8	7.34
3	5.33425E+8	8.78	4.702978E+8	10.75
4	5.52722E+8	58.99	5.524483E+8	13.23
5	2.11601E+8	4.15	2.114914E+8	8.18
6	4.34186E+8	20.78	4.339716E+8	9.64
7	2.42491E+8	5.54	2.423645E+8	5.39
8	3.94054E+8	9.42	3.938539E+8	9.75
9	4.44046E+8	8.43	4.438289E+8	10.07
10	4.52280E+8	10.28	4.520566E+8	92
Average	4.09674E+08	13.38	4.031829E+08	9.42

The other parameter values used in this numerical instance are based on uniform distribution. In the case of θ_j, which is a variable and a function of the number of groups, it is set to get a uniform value $(120, 200)$ if the number of groups is greater than 10; otherwise, it is a uniform value $(50, 100)$.

The results of the implementation of Lagrange as well as Benders are compiled on the numerical examples produced in Table 2 and Figure 2.

As shown in Figure 2 and the last row of Table 2, in the objective functions section, on the average, the Benders algorithm has provided a better response. On the other hand, concerning the solution time, it is clear that the average of Lagrangean is the best of all.

5.2. Statistical analysis of the results

Several *t*-tests are used to analyze the results of the two proposed methods of solving the mathematical model and to compare them with each other. Given the 95% confidence level, the statistical comparison of the means of the results of the two suggested methods is performed for each of the defined evaluation indices. In each comparison, the assumption of zero (H_0) is equal to the mean of the results of the two proposed methods, and the opposite hypothesis (H_1) seeks to refute this hypothesis. This hypothesis test is performed for both specified indices, *e.g.*, the objective function value and the duration of the model implementation. The results of this test using MINITAB version 19 are presented in Tables 3 and 4. Based on the results in the tables, since the *P-value* for the objective function index is higher than the significance level, the null hypothesis for the first index is accepted. This means that at a 95% confidence level, there is no meaningful difference between the responses of the two proposed solutions in terms of the objective function value index. Likewise, the null hypothesis regarding the CPU Time index of the model is accepted because its *P-value* is higher than 0.05, which means there is no significant difference between the responses of the two proposed solution methods in terms of the CPU Time index.

5.3. Determine the best algorithm using the TOPSIS technique

Based on the results of the numerical examples as well as the statistical comparisons, it is not possible to determine a solution method which is superior to the others in terms of both criteria. Therefore, Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) technique is used to select the best method.

The term TOPSIS means preference based on similarity to the ideal solution. The model developed by Hwang and Yoon [22] is an excellent way to rank options. This algorithm evaluates m options by defining ideal positive and negative solutions, using n evaluation criteria. Finally, the alternative that has the shortest distance from the positive ideal solution and the longest distance from the negative ideal solution are known as the superior
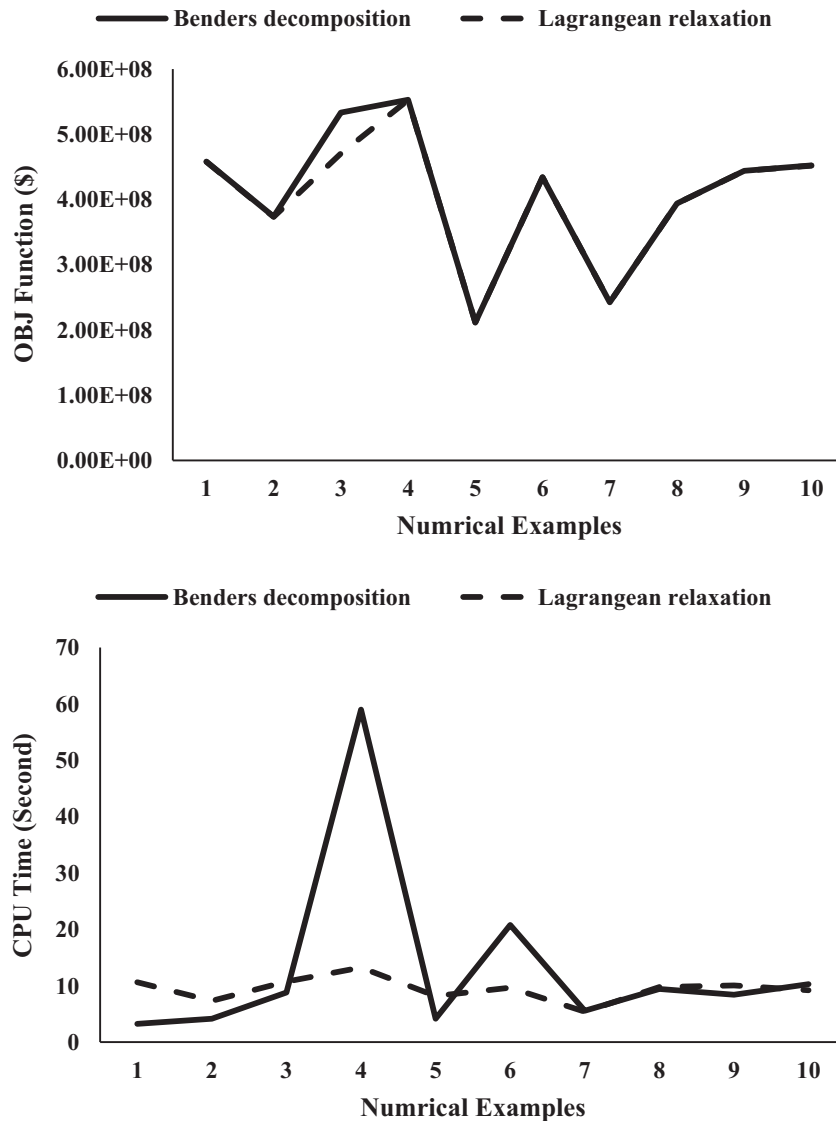
FIGURE 2. The comparison of the model in terms of OBJ function and CPU time.

options. Using the results of the numerical example to implement the TOPSIS technique, the Lagrangean method has been selected as the better one out of the two proposed solution methods.

6. CASE STUDY

In this section, the proposed formulation is applied for the strategic analysis of a ceramic tile factory. Mehseram ceramic tile factory was established in 1997 in a land area of 171 000 sqm located in Yazd, Iran. It currently produces all kinds of granite tiles (*e.g.*, salt pepper, cellubel salt and vin) and porcelain products (*e.g.*, matte glaze, bright glaze and polish glaze).

The ceramic tile industry studied in this research has several warehouses for various types of granite tiles and porcelain products, semi-finished products, chemicals, spare parts, and more. Since the ceramic tile warehouse

TABLE 3. The result of statistical analysis for the OBJ function.

OBJ function	N	Mean	St. Dev	SE Mean	P-Value
Benders decomposition	10	409 673 600	110 774 981	35 030 125	**0.894**
Lagrangean relaxation	10	403 182 930	104 530 932	33 055 583	

TABLE 4. The result of statistical analysis for the CPU time.

CPU time	N	Mean	St Dev	SE Mean	P-Value
Benders decomposition	10	13.4	16.8	5.3	**0.479**
Lagrangean relaxation	10	9.42	2.12	0.67	

of this factory is very important, this research is carried out in this place. For under-examination inventory control and tile ordering, the logistics department of the factory often uses inventory planning models with the expertise and experience of its experts. At present, warehouses use a traditional ABC analysis with different categories, taking into account cost and annual consumption metrics. The main advantages of the traditional ABC analysis are its simplicity and applicability in difficult and significant situations as well as the practical benefits observed in inventory management. However, the method has some disadvantages as well, as mentioned in the previous sections.

Using the information obtained, the proposed ABC analysis model is implemented with regard to the price criteria of the commodity and the annual consumption rate for the factory warehouse items. Table 5 lists the tiles in the ceramic tile warehouse of the factory.

According to the data, there are 140 ceramic tiles in the warehouse that are created in different production lines. It should be noted that the factory also has separate warehouses for semi-finished goods, raw materials, etc. In this research, the largest end-product warehouse is selected for the case study. Next, the ABC analysis model is run by the factory's specialists for these products. Table 6 presents the annual turnover rate calculated on the basis of equation (6.1):

$$\text{Annual Turnover Rate} = \text{Annual Consumption} * \text{Unit Cost}. \tag{6.1}$$

In addition, based on the calculated annual turnover, the warehouse items are ranked in such a way that the higher the annual turnover item is, the higher the item's rank will be. Table 6 presents the values in a descending order and in A, B, and C classes.

As it can be seen in the table, the items in stock are totally 12 in class A, 27 in class B and 103 items in class C. In Table 7, classes A B, and C are compared by the percentage of the items and the final values. Table 6 shows that 103 items in class C, *i.e.*, 72.55% of the total factory warehouse items, are worth 9.85%, while only 12 items in class A, *i.e.*, 8.45% of all the items, are worth 65.82%, or $102 038 493 853.37. Moreover, 19% of all the items (27 items) are in class B, with a value of 24.33% or $37 722 210 667. Table 7 summarizes the corresponding analyses.

The company's annual sales are approximately $12 million, and there are 2650 items in its stock. It is to be noted that only a ceramic tile warehouse with a stock of 140 items is taken into consideration in the present study. The plant has a planned budget of $3.5 million set by the inventory manager annually. Based on the number of the groups, each inventory group is estimated at $500–$700 for the groups of 7 and more than $750–$2000 for the management or overhead costs. The sales and inventory data for 12 months are provided for this article.

As mentioned earlier, the plant is currently implementing the traditional ABC approach to classify SKUs according to its sales volume. After the identification of the groups (A, B, and C) and SKU memberships, an

TABLE 5. Inventory of ceramic tile factory plus their unit prices.

No.	Annual consumption	Unit cost ($)	No.	Annual consumption	Unit cost ($)	No.	Annual consumption	Unit cost ($)	No.	Annual consumption	Unit cost ($)	No.	Annual consumption	Unit cost ($)
1	140 000	29 742.825	31	138	138	61	15 000	15 000	91	2000	38 240.775	121	4300	15 296.31
2	1 518 924	1 518 924	32	1000	28 893.03	62	20	339 918	92	7300	66 284.01	122	15 000	127 469.25
3	456 000	456 000	33	150	28 482.7189	63	30 000	15 721.2075	93	5000	33 991.8	123	94	84 979.5
4	1050	35 224.58	34	11	25 493 850	64	5000	18 508.626	94	27 000	10 197.54	124	160	169 959
5	4140	95 627.901	35	1000	33 991.8	65	3680	47 813.3466	95	5000	62 204.994	125	37 000	4248.975
6	1727	1727	36	50 000	50 000	66	300	300	96	14 000	113 108.27	126	106 000	2124.4875
7	7300	10 514.8	37	85 000	85 000	67	19 000	12 459.504	97	16 000	76 481.55	127	35 000	11 047.335
8	20 000	21 244.875	38	19 000	20 565.14	68	15	25 493.85	98	5000	5000	128	3000	13 596.72
9	1100	514 128.5	39	900	900	69	80	25 493.85	99	65 000	8158.032	129	540	24 644.055
10	5500	5500	40	20 000	25 706.425	70	28	25 493.85	100	16 300	32 292.21	130	86 000	22 689.5265
11	40 000	34 960.738	41	690	690	71	300	16 995.9	101	64 000	14 956.392	131	29 103	29 742.825
12	9000	17 845.695	42	15 000	56 554.135	72	300	16 995.9	102	161 400	20 989.9365	132	50 919	20 395.08
13	30 000	35 988.995	43	360	257 064.25	73	300	16 995.9	103	350 000	4248.975	133	14 550	40 790.16
14	150	150	44	85 000	25 706.425	74	300	16 995.9	104	1340	79 880.73	134	36 366	18 695.49
15	400 000	400 000	45	8000	33 142.005	75	2400	25 706.425	105	66 000	16 146.105	135	201 737	16 995.9
16	150 000	879 159.735	46	12 500	243 891.165	76	120	120	106	40	263 436.45	136	2182	33 991.8
17	6000	264 286.245	47	280	280	77	20 000	21 244.875	107	1750	36 541.185	137	14 541	80 730.525
18	3 886 000	3 886 000	48	4800	280 432.35	78	2400	48 863.2125	108	0	169 959	138	10 908	26 343.645
19	2500	2500	49	300	300	79	5800	42 914.6475	109	56 000	70 532.985	139	500	33 991.8
20	1600	1600	50	7200	7200	80	700	85 659.336	110	40 400	5948.565	140	500	33 991.8
21	1600	1600	51	1840	40 616.1515	81	1000	1000	111	19 000	80 730.525	121	7000	67 983.6
22	2000	2000	52	1200	1200	82	2000	2000	112	23 000	254 493.85	122	500	500
23	400	400	53	5	123 234	83	250 000	250 000	113	660	8497.95			
24	8800	8800	54	20	25 493 850	84	2000	2000	114	849 000	849 000			
25	500	22 107.5255	55	10	34 841 595	85	6000	6000	115	54 000	72 232.575			
26	4500	4500	56	120 000	33 991.8	86	3500	39 940.365	116	1800	28 893.03			
27	360	360	57	48 000	49 288.11	87	18 410	115 164.784	117	26 000	100 275.81			
28	2900	20 719 378.55	58	110 000	21 244.875	88	51 750	133 159.2815	118	810	57 786.06			
29	4000	4000	59	17 500	16 995.9	89	37 730	96 656.158	119	2140	76 481.55			
30	60 380	25 706.425	60	800 000	800 000	90	22 000	33 142.005	120	113 000	10 197.54			

TABLE 6. Warehouse items classification based on the traditional ABC analysis.

No.	Annual turnover	Class	No.	Annual turnover	Class	No.	Annual turnover	Class	No.	Annual turnover	Class	No.	Annual turnover	Class
28	60 086 197 795	A	13	1 079 669 850	B	34	280 432 350	C	121	65 774 133	C	62	6 798 360	C
88	6 890 992 818	A	105	1 065 642 930	B	94	275 333 580	C	107	63 947 073.75	C	113	5 608 647	C
1	4 163 995 500	A	132	1 038 497 079	B	45	265 136 040	C	75	61 695 420	C	74	5 098 770	C
56	4 079 016 000	A	101	957 209 088	B	79	248 904 955.5	C	85	60 000 000	C	73	5 098 770	C
109	3 949 847 160	A	2	911 354 400	B	110	240 322 026	C	22	60 000 000	C	72	5 098 770	C
115	3 900 559 050	A	131	865 605 436	B	67	236 730 576	C	80	59 961 535.2	C	71	5 098 770	C
89	3 646 836 841	A	42	848 312 025	B	126	225 195 675	C	116	52 007 454	C	84	4 400 000	C
18	3 497 400 000	A	90	729 124 110	B	36	225 000 000	C	20	48 000 000	C	33	4 272 407.835	C
135	3 428 701 878	A	134	679 880 189.3	B	24	220 000 000	C	118	46 806 708.6	C	76	2 136 000	C
102	3 387 775 751	A	133	593 496 828	C	3	182 400 000	C	19	42 500 000	C	66	2 100 000	C
117	2 607 171 060	A	112	586 358 550	C	65	175 953 115.5	C	128	40 790 160	C	69	2 039 508	C
60	2 400 000 000	A	9	565 541 350	C	37	170 000 000	C	4	36 985 809	C	47	1 960 000	C
57	2 365 829 280	B	99	530 272 080	C	139	169 959 000	C	21	34 800 000	C	98	1 500 000	C
58	2 336 936 250	B	100	526 363 023	C	93	169 959 000	C	52	34 800 000	C	70	713 827.8	C
44	2 185 046 125	B	40	514 128 500	C	119	163 670 517	C	35	33 991 800	C	53	616 170	C
87	2 120 183 673	B	54	509 877 000	C	12	160 611 255	C	32	28 893 030	C	68	382 407.75	C
130	1 951 299 279	B	114	509 400 000	C	125	157 212 075	C	124	27 193 440	C	6	345 400	C
122	1 912 038 750	B	92	483 873 273	C	83	150 000 000	C	39	27 000 000	C	31	276 000	C
17	1 585 717 470	B	141	475 885 200	C	86	139 791 277.5	C	41	24 150 000	C	49	210 000	C
96	1 583 515 780	B	63	471 636 225	C	78	117 271 710	C	27	18 000 000	C	29	200 000	C
30	1 552 153 942	B	77	424 897 500	C	26	112 500 000	C	142	16 995 900	C	14	15 000	C
111	1 533 879 975	B	8	424 897 500	C	10	110 000 000	C	140	16 995 900	C	108	0	C
103	1 487 141 250	B	5	395 899 510.1	C	104	107 040 178.2	C	50	15 120 000	C			
11	1 398 429 520	B	38	390 737 660	C	64	92 543 130	C	81	14 000 000	C			
48	1 346 075 280	B	127	386 656 725	C	43	92 543 130	C	129	13 307 789.7	C			
61	1 320 000 000	B	55	348 415 950	C	15	88 000 000	C	82	12 000 000	C			
16	1 318 739 603	B	95	311 024 970	C	7	76 758 040	C	25	11 053 762.75	C			
97	1 223 704 800	B	46	304 863 956.3	C	91	76 481 550	C	106	10 537 458	C			
137	1 173 902 564	B	59	297 428 250	C	51	74 733 718.76	C	23	10 000 000	C			
120	1 152 322 020	B	138	287 356 479.7	C	136	74 170 107.6	C	123	7 988 073	C			

TABLE 7. Summary of the traditional ABC analysis.

Class	Percentage of items	Percentage of items value	Services level (%)	The value of each class ($)
A	8.45	65.82	93	102 038 493 853.37
B	19	24.33	90	37 722 210 667
C	72.55	9.85	80	152 689 277 792
Total	100	100	100	155 029 632 312.37
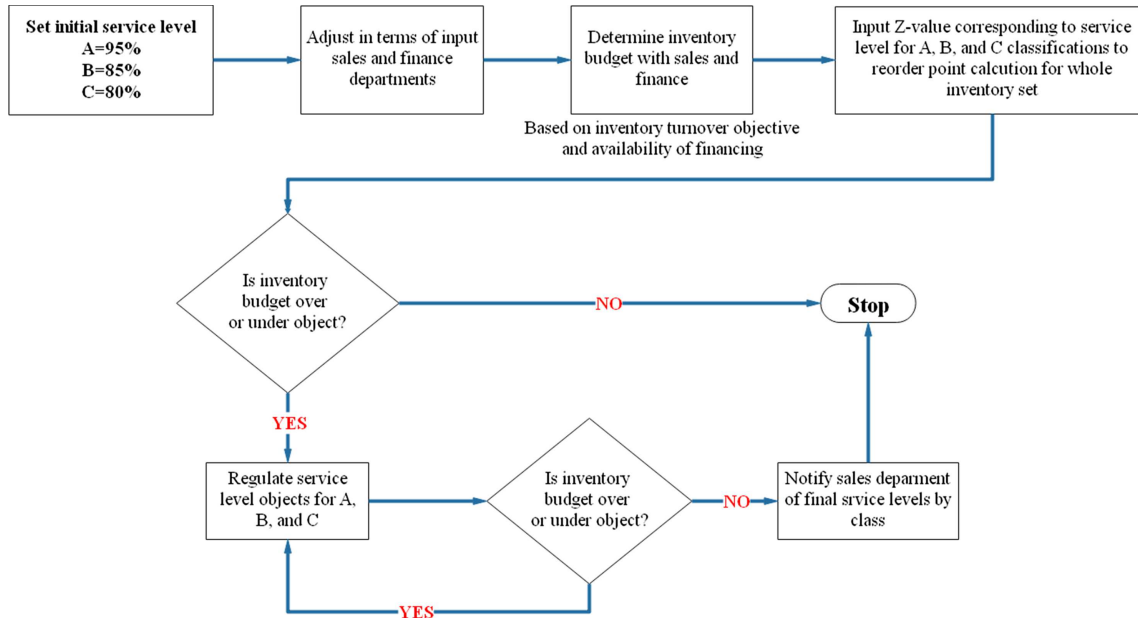
FIGURE 3. The iterative ABC procedure for inventory grouping [35].

iterative method is used to adjust the service levels for the inventory groups, as shown in Figure 3. This method starts with the desired service level for each group according to the experience of the decision-makers; *e.g.*, 95%, 85% and 80% for class A, B, and C, respectively. Since this decision is irrelevant to the inventory budget, the primary service level may result in a budget deficit [35]. Therefore, the decision-maker should review the service level before achieving a reasonable and feasible solution.

According to Figure 3, in addition to being tedious and time-consuming, the inventory management policies proposed by this approach are often undesirable [43]. A solution obtained from the ABC method in Figure 3 is presented in Table 8.

The procedure of setting service levels on the basis of item classes.

Based on the calculations made to classify the items in stock and the results obtained, the inventory manager seeks to answer the following questions:

– Is the current A–B–C plan optimal, or does the plant have to manage groups of more than three considering their different service levels to maximize profit?
– Are the annual sales a reliable principle of group assignment for each SKU?
– How to optimally allocate inventory budget to each SKU?
– Should the factory remove some of the SKUs from its inventory? If so, which SKUs?

TABLE 8. Optimal service level and inventory grouping solution obtained by the formulation.

Group with service level (%)	No. of SKUs (%)	Inventory spending ($)	Gross profit ($)	ROI
99	16(14.82)	524 775 050.5	2 329 017 000	4.44
95	5(4.63)	606 107 955.4	2 261 408 000	3.74
90	7(6.49)	681 122 713.1	2 168 154 000	3.19
87	3(2.78)	744 877 892.1	2 074 900 000	2.78
80	4(3.71)	821 236 088.2	1 981 646 000	2.42
75	4(3.71)	1 051 414 416	1 748 511 000	1.67
70	6(5.56)	659 843 788.3	1 585 317 000	2.41
60	16(14.82)	783 066 205.5	1 352 182 000	1.73
50	10(9.26)	823 893 136.2	1 119 047 000	1.36
40	9(8.34)	906 697 438.5	885 912 400	0.98
30	8(7.41)	620 087 203.9	652 777 600	1.06
	20(18.52)	0	0	–

In order to answer these questions, the model proposed in the present study is implemented for the factory's warehouse in question. At the time of implementation, a total of 108 contingency groups are included. They comprise 99 groups with service levels from 1% to 99% (up 1%) and nine groups with service levels from 99.1% to 99.9% (up 0.1%). The purpose of considering nine additional groups is to granulate the continuous service space, as higher service levels are typically allocated to more important SKUs. The proposed formulation has been solved using the CPLEX solver in the GAMS software and the suggested Lagrangean relaxation algorithm, which was previously known as the superior algorithm. An optimal solution offered by the formulation recommends 12 groups of inventories instead of three groups and generates 8.53% more profit than the traditional ABC classification solution. Table 8 shows the group size, service level, inventory costs, and the other costs of the 12 groups. The return on investment (ROI) has been calculated and reported in the last column as a measurement criterion to quantify the profit to hold the corresponding inventory group.

The optimal grouping of the inventory and the service levels associated with it is significantly different from that in the traditional ABC method. In this classification, the highest level of service is 99%, which accounts for only 14.82% of the total 140 items but has the highest gross profit. However, its profits are far less than 78% of the programs offered by the traditional ABC. In addition, while the ABC solution accounts for 72.55% of the SKU in class C, the optimal solution of the present study has placed approximately 37.07% of the items in some medium service level groups, ranging from 60% to 90%. The optimal solution of the present study is almost always devoted to items with a higher ROI and a higher level of service, which seems reasonable. This suggests that, when a firm has a limited budget, ROI may be a better choice as a categorization criterion than sales alone. This is like the case in the traditional ABC. In addition, the MILP model in this research serves as a relevant factor for each SKU by identifying 20 items with a zero-service level and inventory.

6.1. Sensitivity analysis

In this section, an analysis of sensitivity is performed to evaluate the effect of changing the main parameters of the formulation on the objective results. In particular, the management costs of each group, β_j and limited budget (D) are examined for their impacts on the optimal inventory grouping solutions. The goal is to achieve managerial insights for professionals to describe service level and inventory grouping strategies better. Initially, β_j is considered for each group in the interval of $(500, 2500)$ with an increase of 200 and the budget for D in the range of $(1 500 000, 3 500 000)$ with a rise of $200 000$.

As represented in Figure 4, when the management cost (overhead cost) for each group grows, the number of inventory groups to categorize decreases. Such a relationship seems nonlinear, so it can be concluded that, with an increase in the costs of inventory management for each group, the optimal number of inventory groups is
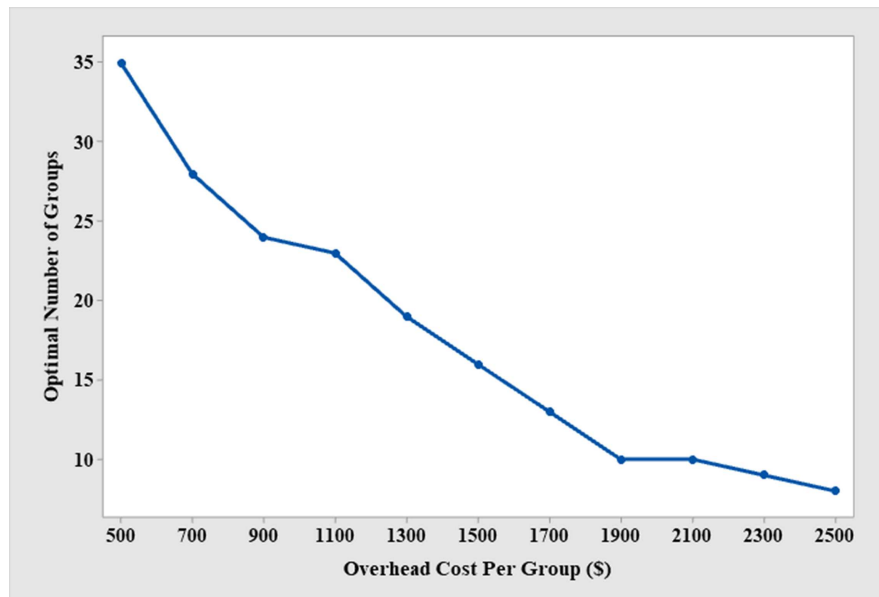
FIGURE 4. The optimal number of groups in terms of increasing overhead cost.

significantly reduced. The non-linear relationship between the management costs for each group and the optimal number of groups justifies the need for a decision support optimization formulation; it would be difficult to determine the optimal number of inventory groups without the solution presented through the mathematical model proposed in this paper.

According to the effect of inventory budget, Figure 5 illustrates the relationship between the inventory budget and the optimal number of groups. The constant management cost for each group is considered at the levels at $500, 1500 and 2000. As it can be seen in the figure, the more inventory funds, the fewer inventory groups. When the inventory budget is low, the optimal number of inventory groups increases. As a suggestion, when there is too much capital to invest in an inventory, the optimal number of groups is three (based on the traditional ABC method). However, in this case, it should be considered that the ABC approach may be undesirable because it does not optimally allocate SKUs to inventory groups. Figure 5 also represents that the relationship between the inventory budget and the optimal number of groups is similar for several management costs in each group but has different indices of sensitivity. When management costs are low, the optimal number of groups is more sensitive to the inventory budget. Therefore, a slight decrease in the inventory budget may cause to a substantial growth in the optimal number of inventory groups. It can also be concluded that selecting more inventory groups when the attainable budget is low is desirable. When a large inventory budget is accessible, however, there will be fewer inventory groups to categorize items.

The relationship between the inventory budget and the optimal number of inventory groups is very important, given the low and high management costs for each group. The sensitivity of the number of inventory groups to the inventory budget proves the value of proposed optimization formulation when the budget is low. This is because, in such cases, there is a greater motivation to allocate budget better by varying their service level.

Here is an account of how the results of the proposed MILP mathematical model and the ABC method change when the inventory budget is increased. The effects are presented in Figure 6. As the budget grows from $1.5 to $6.5 million, the net profit initially rises rapidly, but, as the budget increases, the rate of change decreases and shows a less return on the investment. It can be concluded that, as the inventory budget increases, the net profit decreases rapidly.
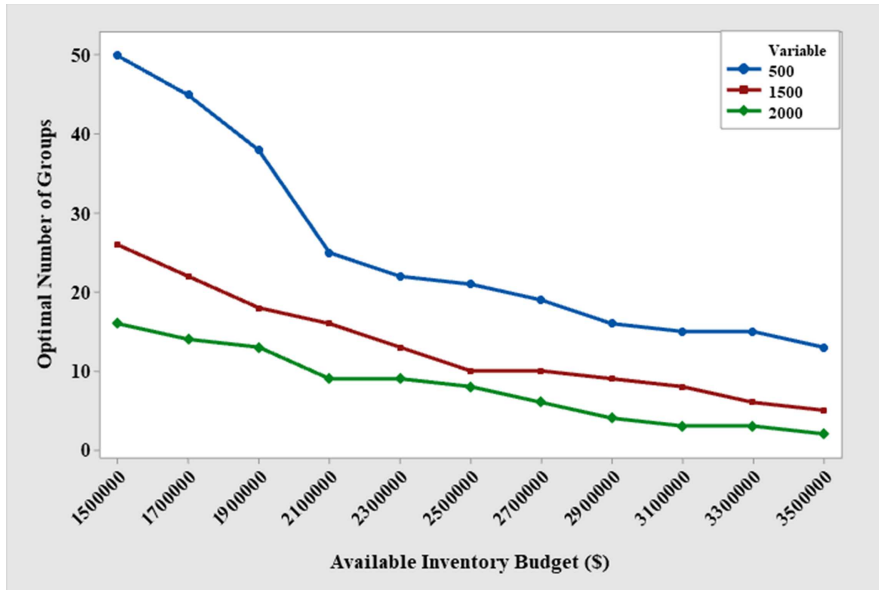
FIGURE 5. The optimal number of groups in terms of growth in the budget (Fixed overhead cost).
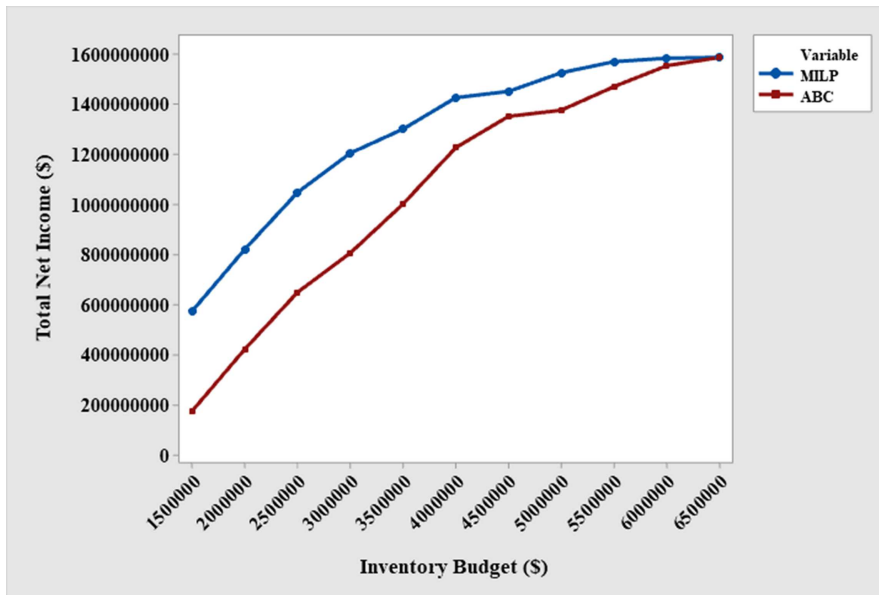

FIGURE 6. Comparison of the profit of the MILP model and the ABC approach in terms of different budgets.
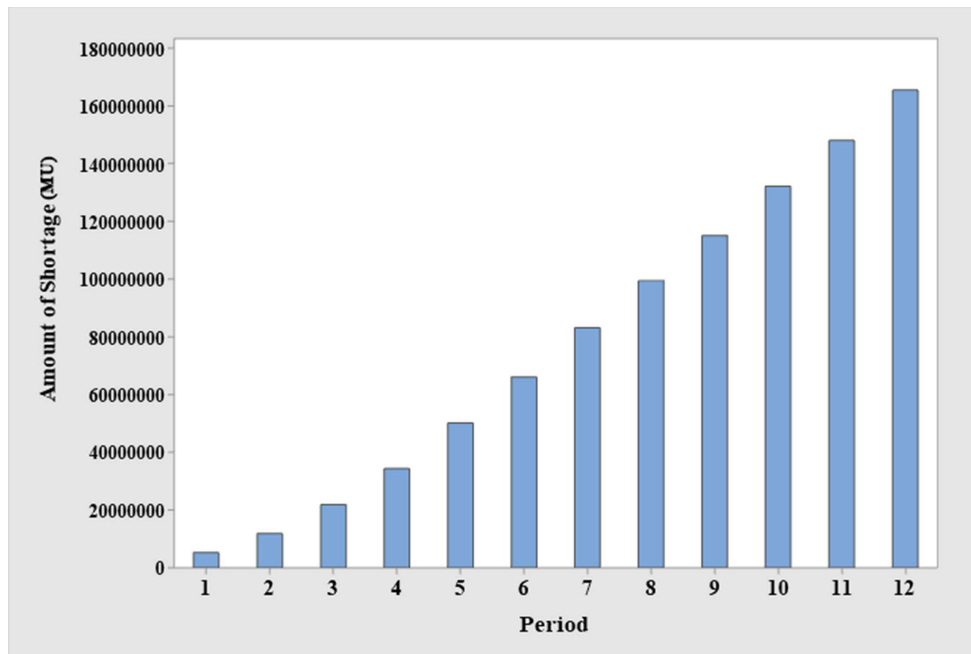
FIGURE 7. The amount of shortage based on the period.

Since the net profit curve obtained by the ABC method is always within effective bounds, the proposed mathematical model in this paper always outperforms the ABC method in solution quality. That is, when the inventory budget is lower, the optimization formulation performs considerably better than the ABC method. So, its advantage over the ABC method is when there is not much budget available. This is when it offers more acceptable results. According to the previous results reported in Figure 8, when the inventory budget is sufficient, the optimal number of groups reaches three, $i.e.$, the quality of the ABC method is close to the optimal solution, namely the MILP model. Conversely, when the inventory budget is low, the classified groups need to be more than three. So, the optimal MILP solution becomes more important than the ABC method.

As a matter of fact, the proposed mathematical model is not without shortages. To examine this issue, let's consider the product demand in the range of $[10\,000, 30\,000]$. Figures 7 and 8 show the shortage rate for each period and item respectively.

As it can be seen in Figure 7, the shortage is in a periodic ascending trend, which is normal. This is because the shortage of each preceding period is added to the shortage of the next period. As a result, the trend goes up. Figure 8 also shows the shortage trend for each item in the inventory. Generally, the chart is highly volatile because the shortage of some items is greater than that of the others.

In the following, the cost of shortage and its effect on the net profit (objective function) are discussed. To this end, let's consider the shortage cost in the range of $[150, 650]$. The results gained after running the model are provided in Figure 9.

According to Figure 9, as the inventory cost rises to \$250, the net inventory profit increases significantly. However, as the cost increases further, there is no change in the net profit, and the profit graph becomes steady. It is reasonably concluded that, given the high cost of inventory shortages, the profit can increase because the management has to plan to offset these costs and the potential financial losses. To offset the costs, more profit must be made. On the other hand, the low cost of shortages has no significant impact on the net profit.
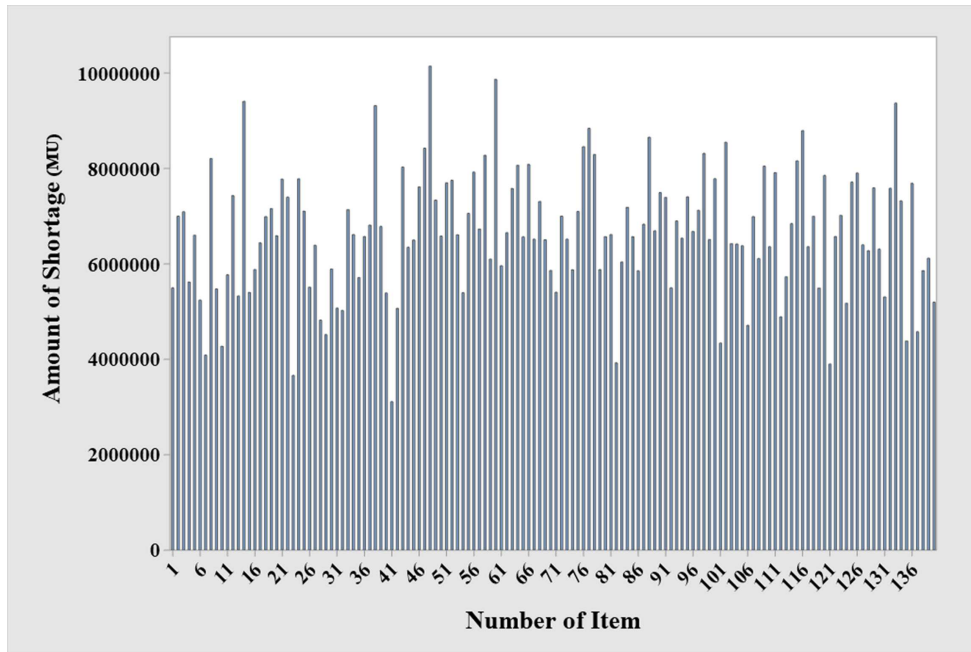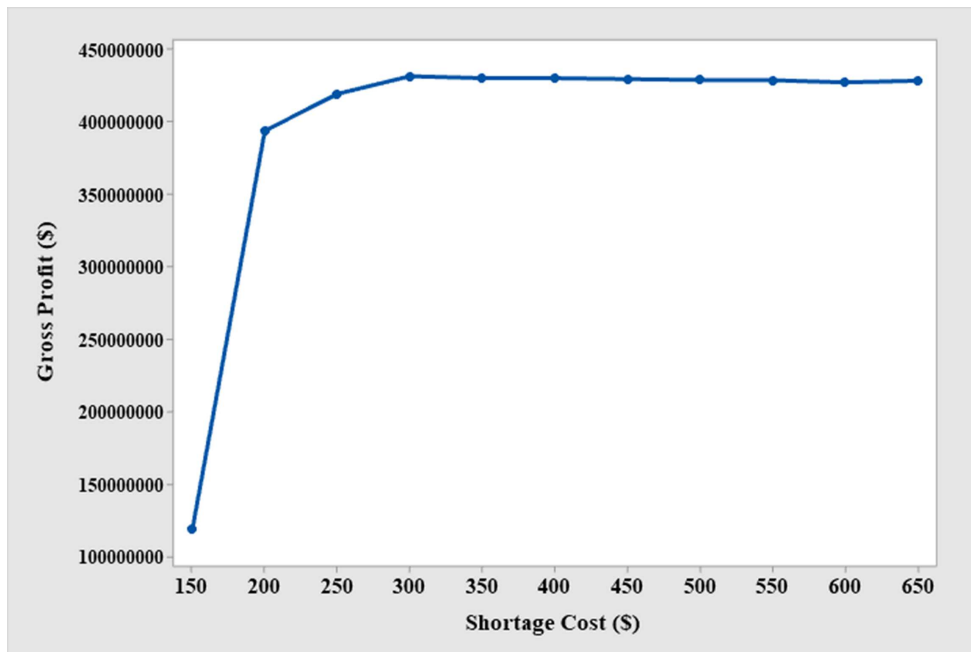
FIGURE 8. Lack of inventory items.

FIGURE 9. The effect of the increase in the shortage cost.

7. Managerial insights

Through the implementation of the proposed model in a real case study, several managerial implications were gained on optimal inventory grouping and item control strategies. As the insights suggest, (a) when the overhead cost in a group is reduced, it is advisable to have more classified groups for the inventory, (b) the proposed model decreases ROI in the net profit and helps organizations justify the benefits of increased inventory budgets, (c) when the budget is limited, it is more likely to grow the number of inventory class groups; in contrast, when a large budget is available, it may be possible to classify items into fewer groups, which is a feature of the traditional ABC approach, and (d) if there is a high cost for inventory shortages, it makes sense to increase profits, but considering its low cost has no significant impact on the net profit.

8. Conclusion and suggestions for future research

Nowadays, inventory management and control structures are among the issues of urgency seriously addressed by the organizations that are expanding. In this research, an optimization model was developed to identify inventory groups, determine their service levels simultaneously and assign items to those groups. This approach improves inventory grouping based on the ABC analysis by providing integrated, automated, and optimized solutions. The model proposed in this study differs from other optimization models in two distinct features. Firstly, instead of minimizing inventory costs, the model examines and maximizes the company's profit. Secondly, it optimizes the trade-off between inventory costs and revenues and allocates the inventory optimally to inventory items. The objective of the mathematical model used in this study was to maximize the net profit of items in stock. Limitations such as budget as well as shortages of inventory were taken into account. Another feature that distinguishes this research from other studies in the field is the use of decomposition algorithms and their comparative examination. To evaluate the proposed solution methods, two indices including the objective function value and the CPU time were determined. Then, the mathematical model was run for 10 different numerical examples, and the results of the two suggested solutions were statistically compared through a t-test. The solutions were very close in terms of quality and time of response. To choose between them, therefore, the TOPSIS technique was applied; the Lagrangean method was chosen as the superior method. Finally, in order to work out the proposed model and the solution method, a real-world case study was used in the ceramic tile industry, which distributes thousands of products to customers annually. The model presented in this paper, compared to the traditional ABC method used in the factory, improved the factory net profit by 8.53%. Also, the proposed model helped to better manage the inventory, optimally allocate service levels to each SKU, and even determine which SKUs had to be reasonably removed from the warehouse. Subsequently, several sensitivity analyses were performed on the model, which helped the inventory managers assess the impact of inventory costs on optimal decision making and item grouping. Like the other studies, there were limitations in this paper. The most important was collecting the data needed for a case study in the ceramic tile industry. Because some of the parameters in this article were considered confidential data, the proposed company refused to provide them. Therefore, based on estimates from previous years, these parameters have been set. Also, due to management's opposition, only a small part of the selected factory was available to implement the proposed model of this article, which can be another significant limitation of this article.

Additionally, several suggestions are explained for future works as follows: (I) Consider some of the model parameters such as demand and costs as uncertain parameters and use various methods such as Soyster, robust programming, Bental and Nimeski to overcome the uncertainty of those parameters. This can bring the model closer to reality; (II) Use meta-heuristic algorithms to solve the model in larger dimensions and evaluate its performance; (III) Since taking products into different categories in some cases increases the holding cost while decreasing the cost of ordering, inventory managers can solve this problem by more detailed sensitivity analysis and scenario analysis based on different needs; (IV) Pay particular attention to what percentage of the products is consumed and how long they take to fail (especially in the case of perishable goods), considering the conditions of the country's economy and the number of products purchased from suppliers; and, (V) The model in this

paper was implemented in a case study of the ceramic tile industry, but future research can focus on any stock-based industry. For example, small and medium-sized enterprises (SMEs) that have small-scale warehouses can be investigated as case studies with this model.

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