



CONTROL OF COUPLING MASS BALANCE ERROR IN A PROCESS-BASED NUMERICAL MODEL OF SURFACE-SUBSURFACE FLOW INTERACTION

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KEY POINTS

- Sources of mass balance error in a process-based hydrological model of surface-subsurface flow interaction are investigated to improve the model's coupling scheme
- These sources of mass balance error are identified by using a set of dimensionless indices and the analysis of temporal and spatial patterns of error
- A time step control based on a degree of coupling index is proposed and the interpolation algorithm used to pass exchange variables of surface-subsurface flow interaction is improved

1 INTRODUCTION

Several process-based hydrologic models have been developed in recent years to describe the surface-subsurface flow interaction (e.g., *Kampf & Burges, 2007; Furman, 2008; Camporese et al., 2010; Maxweel et al., 2014*). The coupling scheme implemented in distributed hydrologic models can be asynchronous (or sequential), sequential iterative, or full coupled as presented, for instance, in *Ivanov et al. (2004), Camporese et al. (2010), and Kollett & Maxwell (2006)*, respectively. Mathematically, the coupling can be described by using a first order exchange approach, pressure continuity, or a boundary switching procedure as described in *Panday & Huyakorn (2004), Kollet & Maxwell (2006), and Camporese et al. (2010)*. Intercomparison projects have studied the differences between several hydrologic models on synthetic test cases in terms of hydrologic response of drainage basins, as for instance *Maxwell et al. (2014)*. The performance of a process-based model can be also measure by the mass balance error since it can identify and track the discrepancy of model results from perfect closure of water mass balance (*Freeze & Harlan, 1969*).

An in-depth analysis of the sources of mass balance error is presented by considering the CATHY (CATchment HYdrology) model described in *Camporese et al. (2010)*, which features sequential iterative coupling with staggered nodal points at the land surface interface. In CATHY, local contributions to surface flow propagation are computed using a cell-centered scheme based on the grid digital elevation model describing the land surface topography (*Orlandini & Moretti, 2009*), while subsurface flow propagation is described using a mesh of tetrahedra built by subdividing the grid cells of the digital elevation model into triangles (*Paniconi & Putti, 1994*). This design facilitates a flexible description of surface and subsurface flow processes that are inherently different. However, since the discretized computational domains at the land-atmosphere interface require node-to-cell and cell-to-node interpolation algorithms to pass from surface to subsurface variables, and vice versa, the coupling scheme can be a source of error. A second source of coupling error in the CATHY model is in the sequential iterative procedure that solves the surface flow routing with a time-explicit scheme and the subsurface equation with a time-implicit, iterative scheme. Due to these different temporal schemes, exchange fluxes between the surface and subsurface compartments are not completely synchronous, and errors can be generated.

The aim of this work is identify, track and reduce the mass balance errors of the CATHY model given by the coupling algorithm between the surface and subsurface water flow domains. Temporal and spatial patterns of mass balance errors are studied by using dimensionless global indices to help the monitoring of hydrologic response of drainage basins: the saturation index and the degree of coupling index. The numerical performance of CATHY model has been improved by introducing a new time step size adaptation scheme and a new interpolation algorithm to pass the exchange variables between the surface and subsurface compartments.

2 SURFACE-SUBSURFACE FLOW MODELING

The CATHY model simulates surface and subsurface water flows by coupling a 3-D solver of Richards' equation for variably saturated porous media with a path-based grid network solver of the diffusion wave equation. The general scheme for a drainage basin model in CATHY is defined by the coupled surface-subsurface domain Ω and its subdomains: the surface Ω_{sw} and subsurface Ω_{ssw} compartments. CATHY implements a sequential iterative coupling scheme that solves in cascade the surface and subsurface water flow. A boundary condition switching procedure manages the exchange of information between Ω_{sw} and Ω_{ssw} by partitioning the atmospheric forcing flow rate into a volumetric flow rate to the surface and to the subsurface (Camporese et al., 2010).

2.1 Mass balance errors

The water mass balance equations are solved by considering the control volume Ω for the whole drainage basin, the control volume Ω_{sw} for the surface compartment, and the control volume Ω_{ssw} for the subsurface compartment. The analysis is developed by integrating over time the fluxes so that the cumulative global water balance error can be defined in volumetric terms at every time level. The cumulative mass balance error due to the coupling scheme can be computed as the residual between the global mass error and the sum of subsurface and surface errors. The analysis is supported by the definition of saturation index and degree of coupling index.

2.2 Numerical experiments

Two synthetic test cases, a sloping plane and a tilted v-catchment (Maxwell et al., 2014), and a real case study, the Enza River drainage basin, were used to analyze coupling mass balance errors and to test the proposed indices, and modified algorithms. The synthetic test cases were forced with a single rainfall period followed by a drainage or evaporation period, whereas the Enza River case study was subjected to a seasonal forcing composed of two wet periods alternated by two dry seasons. The influence of different time step sizes and horizontal mesh size resolutions has been investigated.

3 RESULTS

The mass balance behavior is investigated by monitoring the simulated temporal and spatial patterns of subsurface, surface, and coupling errors as shown in Figure 1 for the sloping plane test case and in Figure 2 for the tilted v-catchment drainage basin. In both test cases the rainfall occurs between 0 and 3.33 h, while the recession phase has no atmospheric fluxes for the sloping plane and an evaporative demand for the tilted v-catchment basin.

3.1 Errors due to time stepping scheme

The magnitude of subsurface and coupling error is in part dependent on the time step sizes used in the sequential iterative solution procedure as shown in Figure 1 (panels a and c). In particular, the coupling error is influenced by the time step size where the slope and the drop of error is proportional to time step (Figure 1, panel e). The connection between surface error and time step size is less direct, with this error influenced by transitions from unsaturated to saturated conditions and vice versa.

3.2 Errors due to the interpolation procedure

The CATHY model performs an interpolation of volumetric flow rate to the surface from nodes to cells that filters spatial patterns over two dimensions laterally without considering the effective availability of water on surface cells. This procedure can have significant effects on surface mass balance error in particular during recession limb as demonstrated in Figure 1, panels c and d. The surface error is not significantly controlled and influenced by surface routing along the drainage network, but it rises in all cells that are at the boundary between saturated and unsaturated zones. The use of high mesh resolutions reduces the size of these transition zones and thus the magnitude of the related error as demonstrated in Figure 1, panel d.

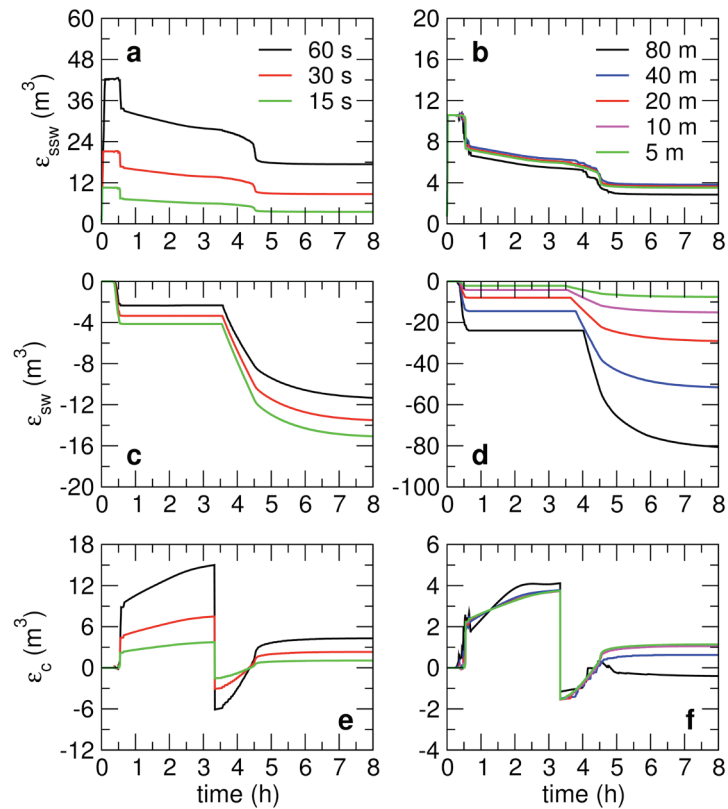


Figure 1. Sensitivity of subsurface (panels a and b), surface (panels c and d), and coupling (panels e and f) mass balance errors for the sloping plane test case with a 10 m DEM. The temporal resolution impact is shown in panels a, c, and e, while the spatial resolution impact is reported in panels b, d, and f.

3.3 Mass balance error control

Mass balance errors can be controlled by monitoring the degree of coupling between the surface and subsurface modules. When the coupling index is changing rapidly during the simulation, there are high variations in volumetric flow rate to the surface and subsurface compartments and consequently strong dynamics on the catchment. This information is used to improve the adaptive time stepping algorithm used in CATHY and defined as “TC” (Figure 2).

A new interpolation scheme, denoted as “IA” (Figure 2), is proposed to force the loss of surface water only on cells that have an effective availability of water storage during recession limb. In that way, the interpolation is not geometric and outflows from surface cells (due to reinfiltration of evaporation) are not passed if the surface storage is zero.

These two version of algorithm (TC and IA) are compared with the original code, defined as “O,” and the combination between TC and IA, defined as “IT,” as shown in Figure 2 for the tilted v-catchment test case. In Figure 2, the subsurface, surface, and coupling errors are reported, respectively, in panels a, c, and e, while the outflow discharge from outlet, the saturation index and the degree of coupling index are shown, respectively, in panels b, d, and f. The surface error is significantly improved with a reduction of 97% by using the new interpolation scheme as shown in Figure 2, panel c. The improvement is significant during the recession limb. The combined version of CATHY model IT has the best global performance in terms of mass balance error for the subsurface, surface and coupling errors as demonstrated in Figure 2, panels a, c, and e. The hydrologic response of drainage basin is not significantly affected by the modification in terms of outflow discharge from outlet, while the saturation index and degree of coupling index show a steeper recession phase. The results obtained from the new versions display a faster recession limb so that the surface saturation falls to zero earlier.

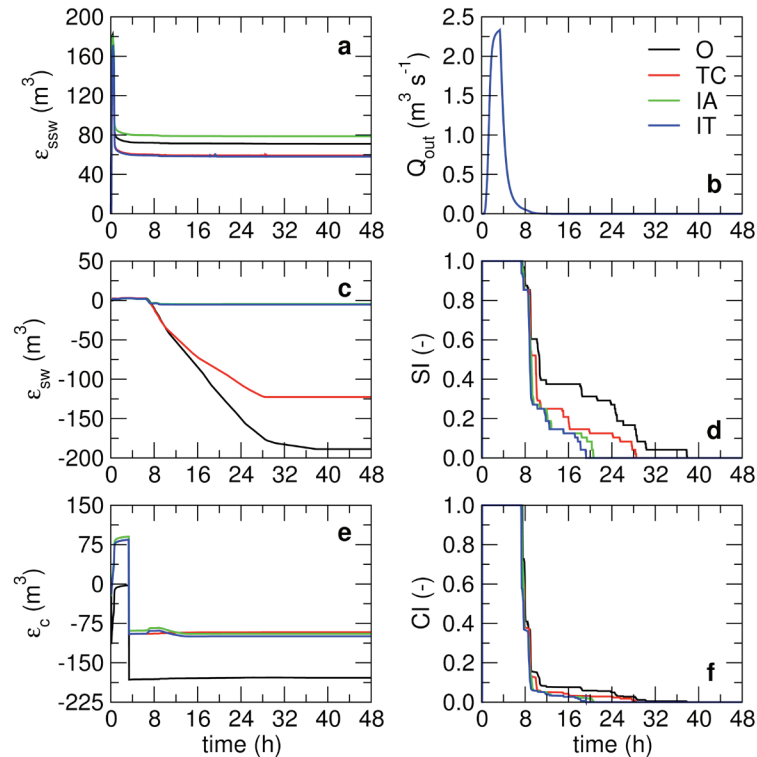


Figure 2. Comparison between the four CATHY versions (original code, O; modified time step size control, TC; new interpolation algorithm, IA; combined TC and IA algorithm, IT) for the tilted v-catchment test case with a time step size of 150 s and horizontal mesh size of 75 m. In panels different variables are compared: (a) subsurface error, (b), (c) surface error, (d) saturation index, (e) coupling error, and (f) coupling index.

4 CONCLUSIONS

An in-depth analysis of the sources of mass balance error in sequential iterative surface-subsurface flow models was performed. The different sources were investigated by isolating subsurface, surface, and coupling errors. A modified time step control scheme introduced was found to improve model performance by considering a normalized variation of the coupling index. A new interpolation algorithm, which considers the availability of surface water over the cell, was found to improve the description of subsurface-surface interaction especially during recession periods.

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