1 Introduction

Semiconductor nanowires (NWs) have attracted wide interest in recent years, thanks to their potential use in a variety of nanoelectronic and photonic devices. Silicon and germanium NWs offer particularly good compatibility with established silicon-based microelectronics and their use in appliances, ranging from bipolar and field-effect transistors to nanoscale sensors and nonvolatile memory devices, has been demonstrated.1-6

Si and Ge NWs are currently grown through various synthetic methods, bottom-up techniques or top-down approaches. The most used methodology is the vapor–liquid–solid (VLS) mechanism, in which a catalytic nanoparticle, normally Au, is used to promote the decomposition of an appropriate gaseous precursor. The obtained wires have single crystal nature and grow along a well-defined crystalline orientation. The diameter of the NWs depends on the orientation: the smallest (a few nanometers) are obtained, usually, for the (110) direction; the largest for the (111) direction; and intermediate ones, about 10 nm, for the (112) direction.5-8 Many experimental9-12 and theoretical13-19 studies have been carried out on these materials. In spite of the progress made, significant uncertainty remains concerning their fundamental physical properties, in part due to the difficulty in correlating the experimental observations with the electronic properties, and due to the use of simplified models in theoretical calculations. Regarding the latter, studies at both the semi-empirical and ab initio levels20-25 have aimed to interpret the electronic and optical data in the visible-UV region, and succeeded to demonstrate the important role played by excitons in the optical properties of these one-dimensional systems.23,24

Among the experimental techniques, electron energy loss spectroscopy combined with scanning transmission microscopy (EELS-STEM) has been developed into a powerful means to study electronic excitations in nanostructures. Interesting and particular features have been observed in the EELS spectra of semiconductor NWs (including Si and Ge NWs) as well as in C, BN, and WS2 nanotubes,26-33 and are believed to derive from the reduced dimensionality of these systems. Nevertheless, few theoretical studies have addressed the interpretation of EELS spectra of nanostructures. Most of these have been based on the continuum dielectric theory, which neglects the quantum confinement effects on the electronic structure, and as such, can only be considered a reasonable approach for large nanostructures. Only one theoretical study, about zero and one-dimensional Si nanostructures, is present in the literature which calculates EELS spectra based on atomistic calculations, albeit within the parametrized tight-binding approach.34 It is noticeable that this work fails to reproduce the low energy peak observed in experiments of very thin Si nanostructures. The aim of the present work is to provide a first-principles theoretical analysis of the dynamical dielectric response and EELS spectra for Si and Ge NWs. The main goal is to elucidate, using the available EELS measurements, the connection between the NW structural parameters and the observed spectra of these nanomaterials.

2 Theoretical approach

2.1 Ab initio response function

Density-functional theory within the local-density approximation (DFT-LDA) calculations of Si and Ge wires was carried...
out by means of a plane-wave code (ABINIT\textsuperscript{35}) and norm-conserving pseudopotentials.\textsuperscript{36} Kinetic energy cutoffs of 20 and 30 Ry, for Si and Ge NWs, respectively, have been used. A vacuum region of 1.5 nm has been used to simulate the isolated wires. A one-dimensional \( 8 \times 1 \times 1 \) grid is used for self-consistent DFT simulations, while the sampling is increased to a \( 16 \times 1 \times 1 \) grid for the non-self-consistent runs needed to calculate the dielectric functions. Convergence of the results has been carefully checked with respect to the kinetic energy cut-off, the \( k \)-point sampling, and the vacuum size within the simulation cell. All the wires under study were oriented along the [100] direction. See ref. 23, 25 and 37 for further details of the NW geometries and calculation methods.

Surface dangling bonds were saturated with hydrogen atoms in order to eliminate states within the electronic gaps. The geometrical structure of the relaxed ground-state configuration of each wire has been obtained by solving self-consistently the one-particle Kohn–Sham equations (KS),\textsuperscript{38,39} and the obtained KS eigenvectors and eigenvalues were used in the calculation of the full dielectric matrix.\textsuperscript{40} This was done within the linear response theory using the random phase approximation (RPA) and including the so-called local-field effects (LFEs).\textsuperscript{41}

In this approach – usually referred to as the time-dependent Hartree–Fock method – the density response function is the crucial quantity to be calculated. It connects the induced electron density to the external potential and satisfies the integral equation: \( \chi = \chi^0 + \rho \varphi_T \). Here all the correlation effects are neglected, whereas the LFEs resulting from the microscopic part of the density variation of the Hartree potential are retained. For the nanostructures under study (or any 1-D system, in fact), it has already been shown that the inclusion of LFEs is essential to describe the experimentally observed “depolarization effect”, in which the component of the dielectric tensor perpendicular to the wire axis (the radial component) is strongly depressed with respect to the parallel component.\textsuperscript{23,25,42}

The macroscopic dielectric function is obtained as

\[
e_{\text{M}}(\omega) = \lim_{q \to 0} \frac{1}{[1 + v(q)\chi(q, \omega)]_{G \to G_0 = 0}}.
\]

In principle, for a more accurate description of the dielectric function, both self-energy and excitonic effects should be included in the calculation. However, computation of EELS spectra requires a wide energy range, which would imply treating a huge number of transitions at this level of theory, hence making the calculation intractable or at least very cumbersome. It has been shown in previous studies that, in Si and Ge[100]-oriented NWs, these effects actually compensate quite well.\textsuperscript{23,25} Furthermore, in the EELS spectra of bulk semiconducting compounds, excitonic effects play a minor role with respect to the local-field effects.\textsuperscript{43}

2.2 Simulation of EELS

To calculate the energy loss probability we follow the approach proposed in ref. 44–46, which describes electrons impinging upon a nanowire at a given impact parameter \( b \), in a non-penetrating geometry. Under the assumption that the thickness of the nano-object is a much more important parameter than its actual geometry, the loss probability for the NWs can be calculated from the following expression for an anisotropic slab:\textsuperscript{46}

\[
P(\alpha, b) = \int_0^\infty dk C_k(\alpha, b) \text{Im} \gamma_k(\omega),
\]

where \( K \) is the modulus of the transferred momentum, \( C_k(\omega) \approx e^{-Kd} \) is a kinematic factor and

\[
\gamma_k(\omega) = -\frac{1 - e_\parallel e_\perp}{1 + \sqrt{e_\parallel e_\perp}} \frac{\sinh (Kd/\sqrt{\lambda})}{\cosh (Kd/\sqrt{\lambda}) - 1 - \sqrt{e_\parallel e_\perp} e^{-Kd/\sqrt{\lambda}}}. \tag{2}
\]

Here \( e_\perp(\omega) \) is the radial (axial) component of the dielectric function of the isolated nanowire, \( \lambda = e_\perp/e_{\parallel} \), and \( d \) is the NW diameter. Finally, since the studied NWs are very thin, and the transferred momenta (available from the experiments) are very small, \( Kd \ll 1 \) and \( \gamma_k(\omega) \) can be reduced to

\[
\frac{Kd}{4} \left[ -\frac{1}{e_\parallel(\omega)} + e_\parallel(\omega) \right].
\]

Hence, the loss resonances occur at the maxima of \( \text{Im}[\epsilon_\parallel(\omega)] \) and of \( \text{Im}[\frac{1}{\epsilon_\parallel(\omega)}] \).

It is important to point out that, within a repeated cell approach as used here, the output of the simulation is the dielectric response of a periodic lattice or a supercell of parallel NWs. What is required for the EELS calculation (eqn (2)), however, is the knowledge of the dielectric response of the isolated nanostructure. As stressed by several authors,\textsuperscript{47,48} depolarization effects related to the long-range electrostatic interactions between NWs in different cells have to be eliminated, whereas the so-called “surface depolarization” effects, arising from the solid-vacuum interface of an isolated NW, must be retained in the calculation.

The axial (\( x_z \)) and radial (\( x_\perp \)) components of the polarizability of the isolated NW can be extracted from the corresponding components of the dielectric response of the NW lattice, as obtained in a supercell (SC) calculation \( [\epsilon^{\text{SC}}_z, \epsilon^{\text{SC}}_\perp] \) in the following way. For the axial component, the relation is straightforward:

\[
x_z = (\Omega^{\text{SC}}/4\pi)[(\epsilon^{\text{SC}}_z - 1)/(\epsilon^{\text{SC}}_\perp + 1)]
\]

Once these long-range effects have been removed, the effective dielectric function of the truly isolated nanowire, both in the axial and radial direction, is obtained from:\textsuperscript{48,49}

\[
\epsilon_{z,\perp} = 1 + \frac{4\pi}{\Omega^{\text{NW}}_{X} C_{0,\perp}}
\]

where \( \Omega^{\text{NW}} \) is the cross sectional area of the nanowire, and \( e_{z,\perp} \) still fully contains the effects of (local) surface depolarization and quantum confinement.

3 Results and discussion

In Fig. 1 and 2 we report both the axial and radial components of the dielectric function of isolated Si and Ge NWs with different diameters, extracted from the supercell data using eqn (3). The different behaviour of the axial and radial components is
immediately evident. While a clear reduction of the static axial screening is observed while decreasing the NW diameter, the radial component $\Re\{e_r(0)\}$ remains almost constant (see top panels). The different behaviour of the two components is evident also for the imaginary parts (see bottom panels). A blue shift of $\Im\{e_\parallel(\omega)\}$ is observed due to the quantum confinement effect, which is stronger for Ge than for Si NWs, as must be expected because of the larger exciton Bohr radius in Ge. In contrast, the depolarization effects ensure that the radial components are greatly reduced in intensity, remaining close to zero below the high energy peak occurring somewhat independently of the NW size around 12 eV. In ref. 50 it was shown how the classical effective medium theory is able to explain this feature. For any 1-D isolated nano-object $\Im\{e_\parallel(\omega)\}$ will never tend to the absorption of the bulk, but will instead remain positioned at the maximum of $\Im\left[\frac{1}{\varepsilon_{\text{bulk}} + 1}\right]$, which coincides with the surface-plasmon peak position of the material, which for Si and Ge is located at around 12.8 eV and 11 eV respectively.

The absorptive parts (both parallel and radial components) of the dielectric functions were found to satisfy the bulk $f$-sum rule, almost independently of the NW size. This suggests that the electron density inside the wires is not very different from the bulk case. Furthermore, the energy (i.e. the longitudinal frequency $\omega_L$) where the corresponding real components become zero is strongly blue-shifted with respect to the plasma frequency, $\omega_{pl}$, as the nanowire diameter is decreased. In fact, considering a simple Lorentz model, for a material with an electronic gap $\omega_0$, the longitudinal frequency results to be $\omega_L^2 = \omega_0^2 + \omega_{pl}^2$. In this way the longitudinal frequencies are blue-shifted when the gap increases: this is due to quantum-confinement effects for the axial components, and to a combination of quantum-confinement and depolarization effects, for the radial ones.

Computed energy loss spectra for several Si and Ge NWs are compared with available experimental TEM-EELS data, in Fig. 3 and 4. It is evident that, in contrast to tight-binding results, our $ab\initio$ results clearly reproduce the low energy peaks observed in both materials. These low-energy features, which are associated with interband transitions, are strongly enhanced as the NW diameter is increased in both cases. This finding is consistent with the results obtained by Reed et al. and Zabala et al. who showed, using a multipolar dielectric theory, that when the transferred momentum is small the low energy part of the EELS spectra of NWs with diameters of a few nanometers is essentially proportional to $\Im\varepsilon_{\text{bulk}}$. In contrast to those studies, however, we have here fully taken into account the effects of the electronic structure, quantum confinement and depolarization arising from the reduced dimensionality. For Si NWs, the absence of a peak at around 7–9 eV in the theoretical spectra, which is instead present in the experimental curve of Fig. 3,
confirms its origin due to the Si/SiO2 interface plasmon excitation.29
As the NW size decreases, a clear blue-shift of the theoretical high-energy EELS peak – deriving from the maximum of $\text{Im} \left[ \frac{-1}{\varepsilon_1 (\omega)} \right]$, as explained above – is evident for both Si and Ge.

The apparent disagreement with the experimental curves in Fig. 3 and 4 is due to the fact that the diameters of our computed NWs are much smaller than those used in the experiments. To better demonstrate this interpretation, in Fig. 5 we report the computed energetic positions of the high energy EELS peak as a function of NW size, together with experimental measurements ascribed to the volume plasmon (at $\omega_{pl}$) as reported for Si27 and Ge28 NWs of larger diameters. For both materials we fitted the experimental data with the scaling law $\omega_{pl} + C/d^b$, where $d$ is the NW diameter. For consistency, we fix the exponents $b$ to the values previously obtained by us23 in a similar fit of the electronic gaps of the same nanowires. (It is notable that the scaling exponent for Ge NWs is very close to the best fit value ($b = 1.2$) reported in ref. 28.) The two corresponding curves are reported in Fig. 5 (solid lines). In both cases (and as noted in ref. 28 for Ge) we obtain better fits to the ensemble of experimental and theoretical data with these exponents than what is predicted (dashed lines) from a simple particle-in-a-box model with $b = 2$, often used to explain confinement in zero-dimensional nanoparticles.35,36

4 Conclusions
In conclusion, we have investigated by first-principles methods the energy loss spectra of Si and Ge NWs. The presence and strong enhancement of a direct interband transition peak, in agreement with experiments, has been found. Moreover a clear size dependence of the bulk volume plasmon peak position has been obtained due to a proper description of electronic structure, quantum confinement and surface depolarization.

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References