Delta Hedging in Discrete Time under Stochastic Interest Rate

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Abstract

We propose a methodology based on the Laplace transform to compute the variance of the hedging error due to time discretization for financial derivatives when the interest rate is stochastic. Our approach can be applied to any affine model for asset prices and to a very general class of hedging strategies, including Delta hedging. We apply it in a two-dimensional market model, obtained by combining the models of Black-Scholes and Vasicek, where we compare a strategy that correctly takes into account the variability of interest rates to one that erroneously assumes that they are deterministic. We show that the differences between the two strategies can be very significant. The factors with stronger influence are the ratio between the standard deviations of the equity and that of the interest rate, and their correlation. The methodology is also applied to study the Delta hedging strategy for an interest rate option in the Cox-Ingersoll and Ross model, measuring the variance of the hedging error as a function of the frequency of the rebalancing dates. We compare the results obtained to those coming from a classical Monte Carlo simulation.

Keywords:
Laplace transform, incomplete markets, Delta hedging, contingent claim, stochastic interest rates

1. Introduction

Most of the mathematical models for arbitrage pricing in continuous time assume that markets are always open and that trading is performed continuously in time. Although it is obvious that such an assumption does not hold in practice, the pricing formulas and the hedging strategies valid in the case of continuous trading are usually also adopted in everyday practical situations. Our goal is to propose a methodology to evaluate the impact of trading in discrete time when hedging strategies are constructed under a continuous time assumption.

The object of our investigation is the ex-ante assessment of the performances of dynamic trading strategies. Probably, the most notable instance of such problem is measuring the hedging error of a strategy, based on a liquid assets, that tries to hedge a future liability. Problems of such kind arise when replicating either a claim using futures contracts, or a payoff of a derivative security with a Delta hedging strategy based on the underlying asset, and in any case when a dynamic strategy is adopted. Ex-ante, a possible way to measure the performance of a strategy is by evaluating expected value and variance of its hedging error. This is usually done by approximations or by Monte Carlo simulations. The approach we propose, based on Laplace transforms, allows to efficiently perform such computations for a very general class of models. This paper is the third one of a series of studies that addressed such an issue in different settings. Our previous works on this subject, to whom we refer for a deeper introduction to the problem, are Angelini and Herzel [1, 2], the first dealing with market models based on Lèvy processes, the second where the more general class of affine processes are considered.

We consider a market model driven by continuous time affine processes, in which, by definition, the conditional characteristic function is an exponential of an affine function of the state variables (see Duffie et al. [5] for a formal definition and properties of affine models). In this framework, Angelini and Herzel [1, 2] provide semi-closed formulas for the efficient computation of expected value and variance of the hedging error for a quite general class of strategies, called "affine", that includes the popular Delta hedging strategy. Such formulas are obtained by using a Laplace transform approach, that is based on the idea of writing the payoff of the contingent claim as an inverse Laplace transform, introduced by Hubalek et al. [7] in the context of variance-optimal hedging. An important feature of the result is that one can study different type of misspecification. For instance, it is possible to analyze the performance of the standard Black-Scholes Delta strategy when the underlying asset is driven by a process which is not log-normal, like in a stochastic volatility model.

In our previous contributions we made the simplifying assumption of deterministic interest rates. In the present work, we extend the analysis to the case of stochastic interest rates. Such an extension gives us the opportunity to study the hedging problem in a more general and realistic model. For example, we can measure the effect of assuming that the interest rate is deterministic when in fact it is stochastic. As an example, we consider a simple two-dimensional affine model, where the underlying evolves according to the Black-Scholes dynamics, while the short-term interest rate follows the process of the Vasicek model, and the stock and the interest rate may be correlated. This is a particular case of a model considered in van Haastrecht et al. [9] to price long-term derivatives. Within this model, we implement two types of Delta strategies: the correct strategy that takes into account the randomness of the interest rate, which may be called the model Delta, and the plain Black-Scholes Delta with deterministic rate. We show that the differences between the two strategies may be very relevant. The most important factors are the correlation and the ratio between the volatility of the risky asset and that of the interest rate. Therefore, the standard Black-Scholes Delta-hedging strategy, still widely used by practitioners, may be not appropriate because it may lead to a variance of the error much higher, in relative terms, to that produced by the correct Delta, especially when the volatility of the interest rates is comparable to that of the stock. It is noteworthy to observe that the relatively poor performances of the Black-Scholes Delta are peculiar of the present setting. In fact, Angelini and Herzel [2] showed that if the interest rates are deterministic but the volatility is stochastic, then the Black-Scholes Delta often outperforms the model Delta. We conclude with a study of the Delta hedging for an interest rate option in the Cox, Ingersoll and Ross model ([3]), providing numerical illustrations for the cases of objective measures different from the risk-neutral measures used to implement the strategy. In that setting we are also able to provide a further numerical validation of the precision of our algorithm, by comparing its results to those obtained by simulations.

2. The Computational Algorithm

Let us consider the problem of hedging a European contingent claim with maturity T, whose payoff H is represented as an inverse Laplace transform:

$$H = \int_{\mathcal{C}} e^{zy_T} \Pi(dz), \tag{2.1}$$

where $\mathcal{C} = R + i\mathbb{R}$, with $R \in \mathbb{R}$, Π is a finite complex measure on $\mathcal{C}$ and $y_T = \ln(S_T)$, where S is the price of a risky asset. The log-return $y = \ln(S)$ of the underlying asset and a short term stochastic interest rate r are components of a multi-dimensional affine process X, whose other components may include stochastic volatility, dividend yields, etc. The simplest example of such a model is obtained by taking the Black-Scholes dynamics for the underlying and a short rate model for the interest rate, like the Vasicek model ([10]). In this case one can also consider a non zero correlation between stock and interest rate. We will use this model for applications in Section 3.1. If the Cox, Ingersoll and Ross model ([3]) is used for the interest rate, the resulting two-dimensional model would be affine if and only if the correlation is zero. A model that includes stochastic volatility as well as stochastic interest rate is studied in van Haastrecht et al. [9]. Pan [8] studied a four-dimensional affine model combining stochastic volatility, interest rates and dividend yield.

Let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{0 \leq t < \infty}, P)$ be a filtered probability space satisfying the usual technical conditions. We interpret P as the physical or objective probability measure. Let us consider an affine time-homogeneous Markov process X defined in a state space $D \subset \mathbb{R}^d$ and write its conditional characteristic function as

$$\phi(u, X_t, t, s) = E_t\left[e^{u \cdot X_s}\right] = e^{\alpha(u,t,s) + \beta(u,t,s) \cdot X_t}, \tag{2.2}$$

where $u \in i\mathbb{R}^d$, $t, s \in [0, T]$ with $t \leq s$, E_t denotes the expected value conditional on $\mathcal{F}_t$ and $\cdot$ the scalar product. The functions $\alpha(u, t, s)$ and $\beta(u, t, s)$ go from $i\mathbb{R}^d \times \mathbb{R}_+ \times \mathbb{R}_+$ to $\mathbb{C}$ and to $\mathbb{C}^d$ respectively, and satisfy a system of Riccati equations whose general form is given in Duffie et al. [5, Equations (2.5) and (2.6)]. We suppose that the functions $\alpha(u, t, T)$ and $\beta(u, t, T)$ can be analytically extended to an open convex domain U containing $0 \in \mathbb{C}^d$ for all $t \in [0, T]$. In this paper we skip technical conditions on the domain U (see Angelini and Herzel [2] for a thorough analysis on this point).

We also assume that X is affine under a pricing measure Q. Conditions for a process to be affine under both measures P and Q are given by Duffie et al. [5]. We consider the discounted conditional characteristic function

$$\begin{aligned} \psi(u, X_t, t, s) &= E_t^Q\left[\exp\left(-\int_t^s r_\tau d\tau\right) e^{u \cdot X_s}\right] \\ &= e^{\bar{\alpha}(u,t,s) + \bar{\beta}(u,t,s) \cdot X_t} \end{aligned} \tag{2.3}$$

The functions $\bar{\alpha}(u,t,s)$ and $\bar{\beta}(u,t,s)$ solve a system of Riccati equations depending on the risk-neutral dynamics of X. Setting $u=0$ in (2.3) we get the discount factor between time t and s

$$\begin{aligned} P(t,s) &= E_t^Q\left[\exp\left(-\int_t^s r_\tau d\tau\right)\right] \\ &= e^{\bar{\alpha}(0,t,s)+\bar{\beta}(0,t,s)\cdot X_t} \end{aligned}$$

We define the normalized price of the underlying

$$\bar{S}_t = S_t/P(t,T)$$

and its increment

$$\Delta\bar{S}_k = \bar{S}_{t_k} - \bar{S}_{t_{k-1}}$$

Given a finite and fixed set of dates from time 0 until maturity T, $0 = t_0 < t_1 < \ldots < t_N = T$, we let $\vartheta = (\vartheta_{t_k})$, for $k = 0,\ldots,N-1$, be a stochastic process representing a trading strategy. The random variable ϑ_{t_k} represents the number of shares of S held from time t_k up to time t_{k+1}. We assume that it depends only on the information available at time t_k, i.e. that it is $\mathcal{F}_{t_k}$-measurable. Assuming that the portfolio starts with an initial cash endowment c and that all portfolio readjustments are invested in, or borrowed from, the money market account, the final value of the strategy ϑ is

$$G_T(\vartheta) = c/P(0,T) + \sum_{k=1}^{N} \vartheta_{t_{k-1}}\Delta\bar{S}_k. \tag{2.4}$$

The hedging error of the strategy is then given by

$$\varepsilon(\vartheta,c) = H - G_T(\vartheta). \tag{2.5}$$

We consider strategies of the following affine form:

$$\vartheta_{t_k} = \int_{\mathcal{C}} e^{a(z,t_k)+b(z,t_k)\cdot X_{t_k}}\Pi(dz), \tag{2.6}$$

for all $k=0,\ldots,N-1$, where $a(z,t_k)$ and $b(z,t_k)$ are functions from $\mathbb{C}\times\mathbb{R}_+$ to $\mathbb{C}$ and to $\mathbb{C}^d$ respectively. We skip here technical conditions on the functions $a(z,t_k)$ and $b(z,t_k)$, referring to Angelini and Herzel [2] for details and comments on the assumed form of the strategy.

From (2.3) and using (the conditional version of) Fubini's Theorem, it is possible to obtain an integral representation for the value at time t of a European claim with payoff expressed as in (2.1) (see Angelini and Herzel [2, Section 3]). By differentiating such a representation, it is possible to compute the sensitivities of the pricing formula with respect to the factors of the model. In particular, the Delta is obtained by differentiating with respect to S and the "Rho" by differentiating with respect to the interest rate. It is easy to see that those are examples of strategies of the form (2.6).

The hedging error (2.5) of a strategy of the form (2.6) for a contingent claim whose payoff can be written as (2.1), has the integral representation

$$\begin{aligned}\varepsilon(\vartheta, c) &= -c/P(0,T) + \\ &+ \int_{\mathcal{C}} \left(e^{zy_T} - \sum_{k=1}^{N} e^{a(z,t_{k-1})+b(z,t_{k-1})\cdot X_{t_{k-1}}} \Delta \bar{S}_k \right) \Pi(dz).\end{aligned} \tag{2.7}$$

The following result extends the main theorem of Angelini and Herzel [2] to the case of stochastic interest rates:

Theorem 2.1. *Let H be a contingent claim satisfying condition (2.1), ϑ be strategy of the form as in (2.6), and c be the initial capital, then*

$$E[\varepsilon(\vartheta, c)] = \int_{\mathcal{C}} e(z)\Pi(dz) - c/P(0,T) \tag{2.8}$$

and

$$E[\varepsilon(\vartheta, 0)^2] = \int_{\mathcal{C}} \int_{\mathcal{C}} (v_1(w,z) - v_2(w,z) - v_3(w,z) + v_4(w,z))\Pi(dw)\Pi(dz) \tag{2.9}$$

The functions $e(z)$, $v_i(w,z)$, $i = 1,2,3,4$ can be explicitly computed.

The proof, as well as the expressions for the functions $e(z)$, $v_i(w,z)$, $i = 1,2,3,4$, are a generalization to the case of stochastic interest rates to those of Theorem 3.1 in Angelini and Herzel [2] and can be obtained from the authors upon request.

Theorem 2.1 states that the expected value and the variance of the hedging error may be represented respectively as a one-dimensional and a two-dimensional inverse Laplace transforms. It may be used to study the effects of model misspecification or trader personal views, in terms of hedging strategies and parameters, on the performance of the hedge. This because the claim, the model and the strategy are completely independent from each other. As an example, in Section 3.1, we will consider the use of a plain Black-Scholes Delta, implemented considering a deterministic interest rate, in a two-dimensional model where instead the interest rate is stochastic. This is an example of misspecified strategy and it will be compared with the correct model Delta, which takes into account the randomness of interest rate.

Formulas (2.8) and (2.9) can be evaluated numerically through numerical inversion of one-dimensional and two-dimensional Laplace transform. For more details on this as well as numerical integration schemes and tests we refer to Angelini and Herzel [1, 2]. In Section 3.2, we provide a test for the algorithm by comparing the results computed with it to those obtained via a Monte Carlo simulation in the Cox, Ingersoll and Ross model.

3. Applications

In this section we apply our results to analyze the performances of Delta hedging strategies first to the case of an option written on a risky asset, e.g. a stock, in a two dimensional model where the interest rate is stochastic, and then to the case of an option written on a zero coupon bond in a model where the only source of risk comes from a stochastic interest rate.

3.1. Options on Equities

To study the case of an option written on a risky asset when the interest rate is stochastic, we use a simple two-factor model. In this model, the state variable $X = (y, r)$ has two components, $y = \ln(S)$ and r, the stochastic short-term interest rate. The respective dynamics under the risk-neutral measure are

$$\begin{aligned} dy_t &= (r_t - \frac{1}{2}\sigma_y^2)dt + \sigma_y dW_t^1, && (3.10) \\ dr_t &= \kappa(\theta - r_t)dt + \sigma_r dW_t^2 && (3.11) \end{aligned}$$

We suppose that the two Brownian motions are correlated with correlation coefficient ρ. This is a combination of the Black-Scholes model with the Vasicek model for the short term interest rate ([10]). In the degenerate case where $\sigma_r = 0$, the short rate process is deterministic, and, if in addition $\theta = r_0$, one recovers the Black-Scholes model with constant rate.

This model is affine and one can write the Riccati Equations (see Duffie et al. [5, Equations (2.5) and (2.6)]) for $\alpha(u,t,T)$, $\beta(u,t,T)$ of (2.2) and $\bar{\alpha}(u,t,T)$ and $\bar{\beta}(u,t,T)$ of (2.3). The differential equations for the two components of the $\bar{\beta}$ and β functions are particularly simple to solve. The functions α and $\bar{\alpha}$ are then determined by a straightforward integration. We do not report the results here because they are a particular case of those obtained in van Haastrecht et al. [9]. For simplicity, in our numerical analysis we suppose that the objective measure and the risk-neutral measure coincide. Notice that if we replaced the Vasicek dynamics for the short rate model with the CIR dynamics, the model would be affine if and only if the correlation between stock and interest rate is zero. Hence, the model would be much less flexible, if analytic tractability were required.

We consider two interesting hedging strategies: the first one is the model Delta, in which we use the model's $\bar{\alpha}$ and $\bar{\beta}$, the second one is the standard Black-Scholes Delta, for which $\bar{\alpha}$ and $\bar{\beta}$ are given according to that model and the drift would be given by a deterministic risk-free rate. Of course in the model, the risk-free rate will change with time, and the hedger has to insert a value for it at each rebalancing date t_k. To do so, the hedger will naturally extrapolate it from the price of a riskless bond as follows:

$$\tilde{r}_{t_k} = -\frac{\log(P(t_k,T))}{T-t_k} = -\frac{\bar{\alpha}(0,t_k,T)+\bar{\beta}(0,t_k,T)r_{t_k}}{T-t_k}$$

This is an example of a misspecified strategy, as it ignores the randomness of interest rates. Since this is a strategy widely used in practice, it is of interest to compare its hedging performances to those of the first strategy.

Let us consider a European call option written on the risky asset S, with maturity $T = 0.5$ years and strike $K = S_0 = 100$. For our analysis, we fix once and for all some of the parameter of the model: the initial rate $r_0 = 0.05$, the drift parameters $\theta = r_0$ and $\kappa = 0.05$, and the volatility of the underlying $\sigma_y = 0.3$. We also fix the number of rebalancing dates to be $N = 12$ (roughly twice a month), but the results are analogous for different values of N, the only difference being the level of the variance of hedging errors, which obviously decreases with N.

We are interested in analyzing the effect on the variance of the hedging error of the correlation coefficient ρ, that we will let vary in the set $[-0.8, -0.6, -0.3, 0, 0.3, 0.6, 0.8]$, and of the relation between the volatility σ_y and that of the interest rate σ_r. For this, we let σ_r assume the values

$[0.01, 0.05, 0.1, 0.15, 0.2, 0.3, 0.4, 0.5]$. In particular, we will compare the variances of the hedging error of the two Delta strategies described above. Figure 1 shows the increasing effect of the correlation ρ on the variance of the hedging error for both strategies. When the volatility of the interest rate is small $\sigma_r = 0.01$ (top panel) the two strategies perform in a similar way, while for higher volatilities, respectively $\sigma_r = 0.15$ and 0.3 (middle and bottom panel), the two strongly differ. In relative terms, the variance of the hedging error of the Black-Scholes Delta is higher than that of the model Delta going from less than 1%, for $\sigma_r = 0.01$, to 10-16%, for $\sigma_r = 0.15$, up to 27-65%, for $\sigma_r = 0.3$. The range of values for each σ_r is due to the different correlation coefficients and it is higher for values near 0, as it is clear from the figure. In Figure 2 we illustrate, for three different values of ρ the impact on the variance of the interest rate volatility compared to that of the underlying. On the x-axis we indeed represent the ratio σ_r/σ_y. Notice that, for $\rho = -0.6$ and in general for negative correlations, the variance of the model Delta decreases with σ_r, while for zero and positive values increases.

3.2. Interest Rate Options

In this section we consider an option written on a zero coupon bond and we apply Theorem 2.1 to the case of affine short rate models, where the process X is the one-dimensional process of the short rate. The functions $\alpha(u, t, T)$ and $\beta(u, t, T)$ in (2.2) and $\bar{\alpha}(u, t, T)$ and $\bar{\beta}(u, t, T)$ in (2.3) may be computed explicitly in some important cases as the models by Cox, Ingersoll and Ross [3] or Vasicek [10], and their expressions can be found in Filipović [6, Section 10.3.2.1, 10.3.2.2]. We consider here the case of Cox, Ingersoll and Ross model. The dynamics of the short rate are given by

$$dr_t = \kappa(\theta - r_t)dt + \sigma\sqrt{r_t}dW_t \tag{3.12}$$

The dynamics (3.12) are given as under the objective measure P and we assume the market price of risk $q = -\pi/\sigma\sqrt{r}$ to get the dynamics under a martingale measure Q, so that the drift of (3.12) under Q is $\kappa\theta - (\kappa - \pi)r_t$.

Let us now consider a European claim H maturing at date T_1, written on the zero coupon bond with maturity $T_2 > T_1$. Hence,

$$S_t = P(t, T_2) = e^{\bar{\alpha}(0,t,T_2)+\bar{\beta}(0,t,T_2)r_t}$$

and $y_t = \ln(S_t) = \bar{\alpha}(0, t, T_2) + \bar{\beta}(0, t, T_2)r_t$ is an affine function of $X_t = (r_t)$.

In this case, the representation in (2.1) is

$$
\begin{aligned}
H &= \int_{\mathcal{C}} e^{zy_{T_1}} \Pi(dz) = \\
&= \int_{\mathcal{C}} e^{\bar{\alpha}(0,T_1,T_2)z+\bar{\beta}(0,T_1,T_2)zr_{T_1}} \Pi(dz).
\end{aligned}
$$

With straightforward calculations, one can still represent the price of the claim at time t as an inverse Laplace transform. Therefore, its derivatives with respect to the price of the bond, i.e. the Delta of the option, may be represented as in (2.6). For the Delta, one has

$$
\begin{aligned}
a(z,t) &= \ln\left(\frac{\bar{\beta}(\bar{\beta}(0,T_1,T_2)z,t,T_1)}{\bar{\beta}(0,t,T_2)}\right) + \\
&+ \bar{\alpha}(0,T_1,T_2)z + \bar{\alpha}(\bar{\beta}(0,T_1,T_2)z,t,T_1) - \bar{\alpha}(0,t,T_2) \\
b(z,t) &= \bar{\beta}(\bar{\beta}(0,T_1,T_2)z,t,T_1) - \bar{\beta}(0,t,T_2).
\end{aligned}
$$

As a numerical illustration, we take parameters estimated in Duan and Simonato [4], namely $\kappa = 0.1644$, $\theta = 0.0648$, $\sigma = 0.0438$ and we set $r_0 = 0.06$. In Figure 3, we show the results of our method as a function of the number of trading intervals $N = [1,3,6,12,24,48,60,72,80]$ for an at-the-money forward European option with maturity $T_1 = 1$ year written on a zero coupon bond with maturity $T_2 = 2$ years. We consider zero, positive (0.1237) and negative (-0.1237) risk premia π. In the last two cases then, the objective measure differs from the risk-neutral measure. In the top panel we show the variance of the error of Delta hedging. Since the figures are small for such an option, to get a more precise idea about the cost of hedging, we also compare the results with the price of the option. In the middle panel we then show the ratio between the standard deviation of the hedging error and the price of the option, which is quite relevant, in particular in the case of a positive risk premium, while a negative risk premium lowers the cost of hedging. The bottom panel depicts the ratio between the expected value of the hedging error and the cost of the option. We observe that the expected value of the hedging error is zero when risk premium is zero (i.e. when the objective measure and the risk-neutral measure coincide), while it is positive (negative) in case of a positive (negative) risk premium.

Finally, we asses the validity of our algorithm through a Monte Carlo simulation, by computing the 95% confidence bands for the simulated value of

N	MC 95% c.i.	2-dim alg
1	$(8.0940, 8.3215)$	8.1626
3	$(5.1330, 5.2772)$	5.1637
6	$(3.7902, 3.8967)$	3.8879
12	$(2.9089, 2.9906)$	2.9762
24	$(2.3431, 2.4089)$	2.3518

Table 1: Standard deviation of the hedging error when the number of trading dates is $N = [1, 3, 6, 12, 24]$. The two-dimensional inversion algorithm is validated through 95% confidence intervals from Monte Carlo simulations with 10000 paths. The Table reports, from left to right, the number of trading intervals N, the 95% confidence interval and the value returned by the two-dimensional inversion algorithm. The strategy is the model Delta for an at-the money forward call option with maturity $T_1 = 1$ year written on a zero coupon bond with maturity $T_2 = 10$ years and notional 10^3.

the standard deviation of the hedging error. We remark that, with the model at hand, this can be implemented via an exact simulation of the dynamics of the short rate. For the test, we set the risk premium $\pi = 0$ and consider an at-the-money forward European option with maturity $T_1 = 1$ year written on a bond with maturity $T_2 = 10$ and notional 10^3. We report the results in Table 1 for number of trading dates $N = [1, 3, 6, 12, 24]$: the standard deviation of the Delta hedging error computed with our two-dimensional algorithm is shown in the third column and the 95% confidence intervals in a simulation with 10000 paths in the second column. The values with our method always fall within the confidence bands, hence confirming the precision of the numerical implementation of the algorithm.

4. Conclusions

We generalized a result of Angelini and Herzel [2] to the case of stochastic interest rates, which allows to consider more general and realistic models. We apply the result in a two-dimensional model to study the hedging performance of Delta strategies. We show that the Black-Scholes hedging strategy, which neglects the stochasticity of interest rates, performs poorly with respect to the model Delta strategy, especially when the volatility of the interest rate is comparable to that of the risky asset. The methodology is based on Laplace transforms, a powerful computational tool that is also very well suited for dealing with problems of quantitative finance.

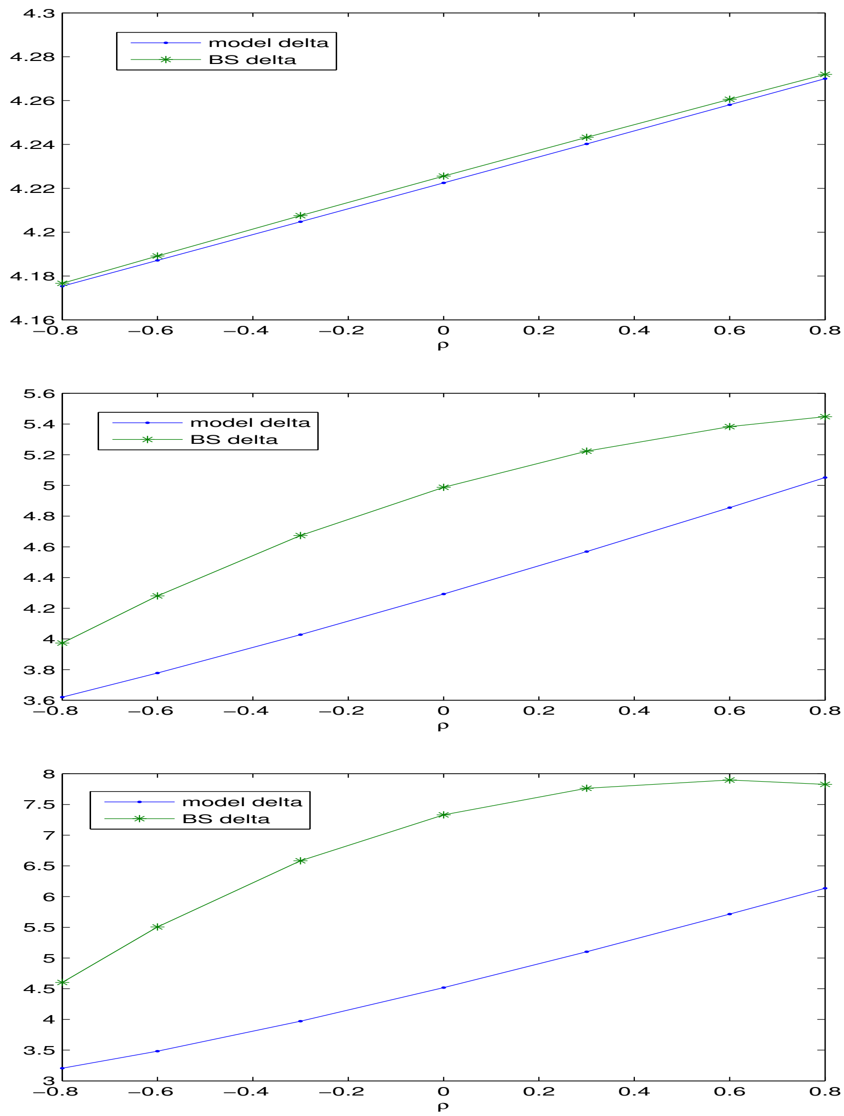

Figure 1: Variances of model Delta and Black-Scholes Delta hedging strategies for a European call option with maturity $T_1 = 0.5$ years written on the risky asset as a function of the correlation ρ for $\sigma_r = 0.01$ (Top), $\sigma_r = 0.15$ (middle) and $\sigma_r = 0.3$ (bottom). The volatility of the underlying is $\sigma_y = 0.3$ and the number of rebalancing dates is $N = 12$.

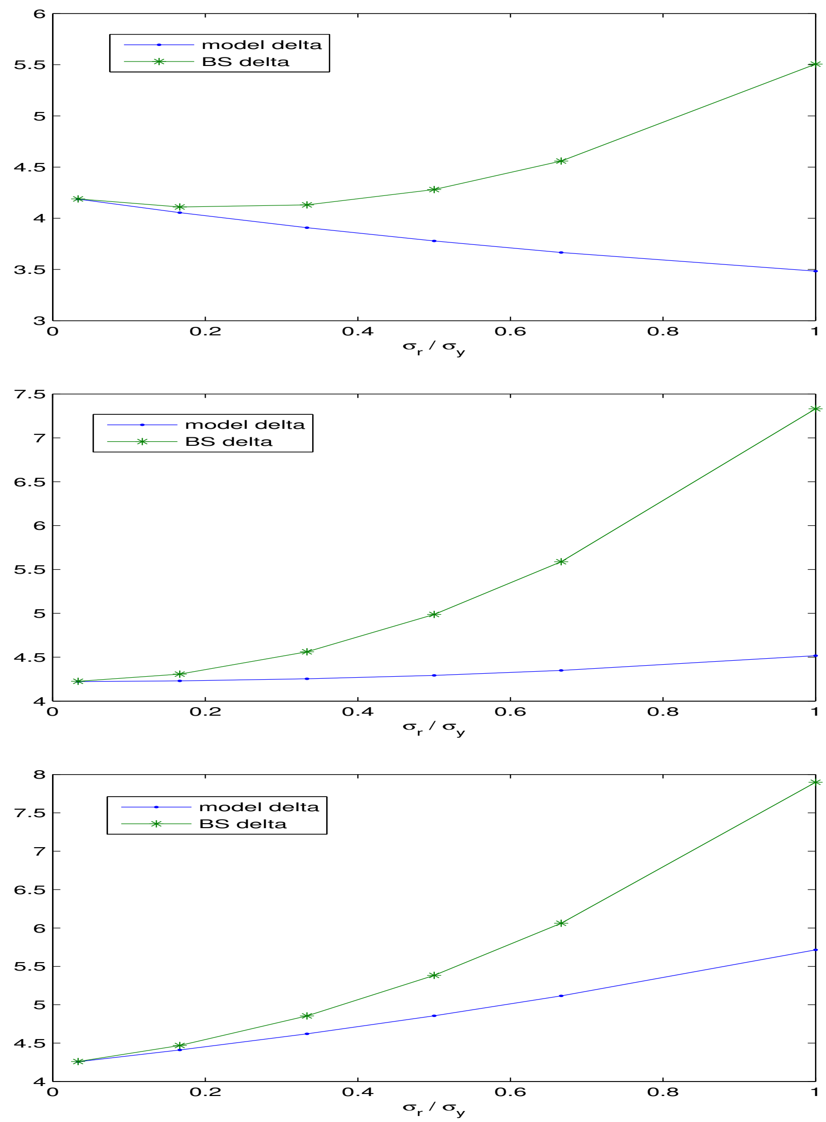

Figure 2: Variances of model Delta and Black-Scholes Delta hedging strategies for a European call option with maturity $T_1 = 0.5$ years written on the risky asset as a function of the ratio $\frac{\sigma_r}{\sigma_y}$ for $\rho = -0.6$ (Top), $\rho = 0$ (middle) and $\rho = 0.6$ (bottom). The volatility of the underlying is $\sigma_y = 0.3$ and the number of rebalancing dates is $N = 12$.

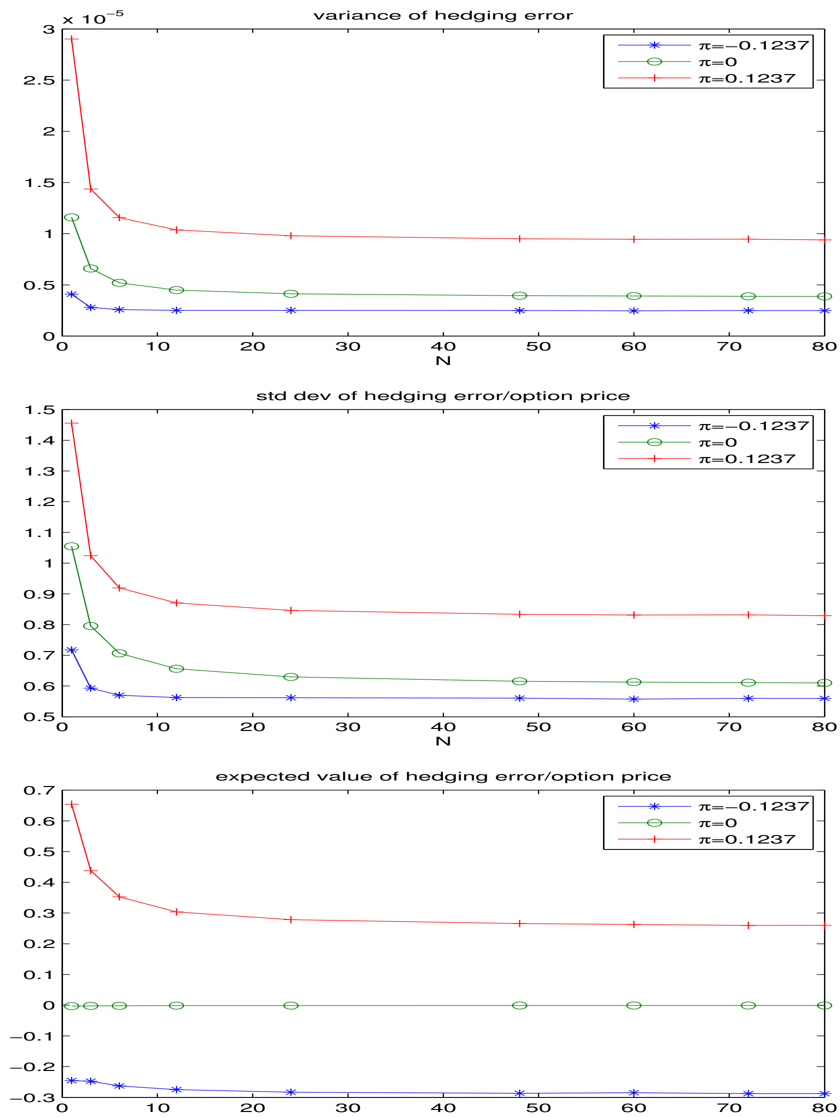

Figure 3: Variances (top), standard deviation over the price of the option (middle), expected value over the price of the option (bottom) of Delta hedging for a European call option with maturity $T_1 = 1$ year written on a zero coupon bond with maturity $T_2 = 2$ years as a function of the number of trading intervals N. CIR model with parameters $r_0 = 0.06$, $\kappa = 0.1644$, $\theta = 0.0648$, $\sigma = 0.0438$ and $\pi = [-0.1237, 0, 0.1237]$.

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