Non–locality and quantum theory: new experimental evidence

Luigi Accardi   Massimo Regoli

Centro Vito Volterra

Università degli Studi di Roma "Tor Vergata", Roma, Italy
email: accardi@volterra.mat.uniroma2.it, WEB page: http://volterra.mat.uniroma2.it

Abstract

Starting from the late 60's many experiments have been performed to verify the violation Bell's inequality by Einstein–Podolsky–Rosen (EPR) type correlations. The idea of these experiments being that: (i) Bell's inequality is a consequence of locality, hence its experimental violation is an indication of non locality; (ii) this violation is a typical quantum phenomenon because any classical system making local choices (either deterministic or random) will produce correlations satisfying this inequality.

Both statements (i) and (ii) have been criticized by quantum probability on theoretical grounds (not discussed in the present paper) and the experiment discussed below has been devised to support these theoretical arguments. We emphasize that the goal of our experiment is not to reproduce classically the EPR correlations but to prove that there exist perfectly local classical dynamical systems violating Bell's inequality.

The conclusions of the present experiment are:
 (I) no contradiction between quantum theory and locality can be deduced from the violation of Bell's inequality.
(II) The Copenhagen interpretation of quantum theory becomes quite reasonable and not metaphyisic if interpreted at the light of the chameleon effect.
(III) One can realize quantum entanglement by classical computers.

In our experiment a common source (a computer) generates pseudo–random points in the unit disk (analogue of singlet pairs) and sends them, one by one, to two spatially separated experimenters. Each of them, given such a point, asks an independent local binary question (analogue of a local measurement of spin in one direction). The answers, coded as ± 1, are obtained by locally calculating the value of a function depending only on the local question and on the common points in the disk. The algorithm to calculate these answers is completely local (no computer knows which question has been asked to the other one) but devised so to guarantee entanglement: if the same question is posed, opposite

answers will be given. The questions are parametrized by vectors in the unit circle and we prove that three such vectors can be chosen so that the corresponding correlations violate Bell's inequality. In section (7) we prove that our experiment also provides a classical analogue of the type of logical (i.e. independent of statistics) incompatibilities pointed out by Greenberger, Horne and Zeilinger.

(1) Introduction

The experiment described in the present paper has been performed in the attempt to clarify a question that has accompanied quantum theory since its origins: *is it true that the theory of relativity, quantum mechanics and a realistic interpretation of natural phenomena are mutually contradictory?*

In the past 30 years the answer to this question, accepted by the majority of physicists has been: *yes they are and this can be proved by theory and confirmed by experiment.* Furthermore, since the experimentally confirmed EPR type correlations are ... *necessarily nonlocal in character* ... [GrHoZe93], the experiments also solve the contradiction in favor of quantum theory by showing that the basic pillar of relativity theory is violated in nature. Usually this statement is tempered by the clause that quantum nonlocality cannot be used to send superluminal signals. This amounts to downgrade the locality principle from a law of nature (no action at distance exists) to a principle of telecommunications theory (the existing actions at distance cannot be used to build up a superluminal television). Of course, if this downgrade is the only reasonable way to interpret the experimental data, we have to accept it. However, given the relevance of the issue, the question whether it effectively is the only reasonable way to interpret the experimental data, naturally arises. In several papers starting from [Ac81a], (cf. [Ac97] for a general presentation, [Ac99], [AcRe99a] for more recent results) the arguments, relating Bell's inequality to locality, have been criticised on a theoretical ground. In the present paper we will substantiate these theoretical arguments with an experiment.

Before describing our experiment let us briefly review the usual proof of the contradiction between relativity, quantum mechanics and realism, which is based on a combination of Einstein, Podolsky, Rosen (EPR) type arguments with Bell's inequality and goes as follows. In the EPR type experiments a source emits pairs of systems with the following properties:

i) the members of each pair are distinguishable after separation and we denote them 1 and 2. After emission the two become spatially separated, say: 1 goes to the left, 2 goes to the right.

ii) for each member $j = 1, 2$ of each pair we can measure a family of observables (spin)

$$\hat{S}_a^{(j)} \quad ; \qquad j = 1, 2$$

parametrized by an index set T ($a \in T$). To fix the ideas let us say that T is the unit circle so that each a is a unit vector in the plane.

iii) each observable $\hat{S}_a^{(j)}$ ($j = 1, 2; a \in T$) can take only two values: ± 1 and, if $a \neq b$, the two observables

$$\hat{S}_a^{(j)} ; \qquad \hat{S}_b^{(j)} \qquad ; \qquad j = 1, 2$$

cannot be simultaneously measured on the same system

iv) The *singlet condition* is satisfied, i.e. if the same observable is measured on both particles, then the results are opposite.

According to EPR the values of the spins must be *pre–determined* otherwise, if one assumes that *these values are created* by a choice of nature at the act of measurement,

3

the only way to explain the strict validity of the singlet condition would be to postulate a mechanism of instantaneous action at distance through which a particle *instantaneously knows* which choice nature is going to do on its distant partner so that the two choices can be *matched* into the singlet law. The pre–determination of the result is called a *realism condition*. Thus the negation of the realism condition in the sense of EPR, would imply a nonlocality effect. We anticipate that, in our experiment, a crucial role will be played by a distinction between the EPR realism , or *ballot box realism*, on which the whole of classical statistics is based, from what might be called the *chameleon realism* which seems to be more appropriate when dealing with quantum systems.

Before explaining this distinction let us shortly outline the role of Bell's inequality: *if the values of the spins must be pre–determined, then there must be a hidden parameter* λ *which determines these values.* In other words the spin variables must be functions $S_a^{(j)}$ of this hidden parameter λ:

$$\hat{S}_a^{(j)} = S_a^{(j)}(\lambda) \qquad ; \qquad (j = 1, 2; a \in T)$$

and the statistical fluctuations of the spins reflect a statistical distribution of this unknown parameter. The nature of the hidden parameter and of its statistical distribution are both unknown to us, but Bell proves that, whatever this nature may be, the correlations

$$\langle \hat{S}_a^{(1)} \hat{S}_b^{(2)} \rangle$$

must satisfy a certain inequality that now brings his name.

The quantity of experimental data accumulated in EPR type experiments is now quite remarkable and the common consensus on the interpretation of these data is that they exhibit a violation of Bell's inequality. A survey of the situation up to 1988 was published in *Nature* by J. Maddox [Mdx88] and a multiplicity of results in the past 12 years have further confirmed the experimental violation of this inequality by a variety of quantum systems.

According to Bell, given the realism condition (as defined above), the *vital assumption* in the deduction of his inequality is a locality condition:

… The vital assumption [2] [i.e. the EPR paper [EPR35]] is that the result B for particle 2 does not depend on the setting $\vec{a}$ of the magnet for particle 1, nor A on $\vec{b}$… [Be64]

A theory which satisfies both assumptions is called a *local realistic theory*. With this terminology Bell's result is often reformulated in the form: *a theory that violates Bell's inequality must be either non–local or non–realistic.*

Since, by the original EPR argument, a non–realistic theory is necessarily non–local, it follows that in any case the violation of Bell's inequality implies non–locality. This is Bell's original thesis and, even if balanced by a multiplicity of subtle differences on minor points, from the huge literature now available on this topic (cf. for example [Red87], [Be87], [CuMcM89], [Maud94], ...), a substantial consensus emerges with its main points which one may summarize in the scheme:

  – locality implies Bell's inequality
  – quantum mechanical (EPR type) experimental correlations violate Bell's inequality

– therefore quantum mechanics is non–local

– furthermore, no classical system can violate Bell's inequality by local choices.

Therefore if we can construct a classical system (even deteministic and macroscopic) which, by means of purely local choices produces a set of statistical correlations violating Bell's inequality, this will be an experimental proof of the fact that

(i) locality does not imply Bell's inequality

(ii) the experimental violation of Bell's inequality achieved by the quantum mechanical (EPR type) correlations is not an indication of non–locality.

(iii) the experimental violation of Bell's inequality by purely local experiments is not a typically quantum phenomenon

It is since [Ac81a] known that, if $\hat{S}_a^{(1)}$, $\hat{S}_b^{(2)}, \ldots$ are functions defined on the same probability space then the inequality (4) below must be fulfilled even if we postulate a strong non locality condition (e.g. that $\hat{S}_a^{(1)}$ **depends** on the values assumed by $\hat{S}_b^{(2)}$, cf. [Ac81]).

Therefore, if we want to violate (4) we have to find a local mechanism of generation of the answers so that each pair $(\hat{S}_a^{(1)}, \hat{S}_b^{(2)}), \ldots$ have a joint probability distribution, but this is not true for triples. Our goal in this paper is to construct such a system.

The goal of the present experiment is not to build a hidden variable model for the quantum mechanical singlet correlations (although a variant of it may play this role), but to construct a classical system showing that the three conditions of pre–determination, locality and singlet condition are not incompatible with a violation of Bell's inequality. Consequently the violation of these inequalities in the EPR–type experiment cannot be interpreted as an incompatibility of the three above mentioned conditions with quantum theory.

(2) Description of the experiment

Our experiment describes a classical situation analogous, but not identical, to the one which takes place in the EPR type experiments. The classical system considered is made of three computers:

(2.1) one, playing the role of the source of the singlet pairs, will produce pseudo–random points in the unit disk

$$p_1, p_2, \ldots, p_N \tag{1}$$

Each point p_j plays the role of a singlet pair, therefore in the following we shall speak indifferently of *the point* p_j or of *the singlet pair* p_j. The choice of the pseudo–random generator is not relevant for the results, provided the algorithm has a reasonably good performance. The algorithm we use has been taken from [PrTe93]. An order of $N = 50.000$ points is already sufficient to obtain acceptable results. Increasing the number of points gives a higher precision in the computation of the areas, but does not change the order of magnitude of the violation of the inequality. This also shows that the violation we obtain cannot be attributed to round–off errors in the measurements of the areas.

(2.2) the other two computers play the role of the two measurement apparata. The three computers are separated (e.g. they may be in different rooms or different countries, . . .).

(2.3) The role of the experimentalists will be played by two persons, one for each of the "measuring computers". Given a point p_j (a singlet pair), operator 1 makes a local independent choice of a unit vector a in the plane (analogue of the direction of the magnetic field) and activates a programme in computer 1 which computes the value of a function $S_a^{(1)}(p_j)$, depending only on the local choice a and on the point (singlet pair) p_j. Similarly operator 2 computes the value $S_b^{(2)}(p_j)$, depending only on her own local choice b, but (as in all EPR experiments) on the same singlet pair p_j. We shall say that each operator *asks a (local) question to the system (computer)*

(2.4) the values of the functions $S_a^{(1)}$, $S_b^{(2)}$ (answers) can only be $+1$ or -1.

– A priori, for any given point (singlet pair) p_j, experimenter 1 (resp. 2) can choose among infinitely many questions to be asked to system 1 (resp. 2). However only one question at a time can be asked to any single system.

(2.5) The calculation of the values $S_a^{(1)}(p_j)$, $S_b^{(2)}(p_j)$ is purely local, however the programme has been devised so that if, by chance, the same question is asked to both systems then they will give opposite answers (singlet condition).

(2.6) Finally we introduce, in our classical model, a strong form of the disturbance effect of a measurement of an observable on the other observables of the same system, incompatible with the given one, by requiring that, if the observable $\hat{S}_x$ is measured, then both types of particles instantaneously change the value of all the other observables from $\hat{S}_y$ $(y \neq x)$ to $-\hat{S}_y$ (if I measure the weight of a chameleon in a closed box, its color need not to be the same I would have found if I would have measured it on a leaf). This additional prescription plays a role in the deduction of the Bell inequality (section (5)), but is not necessary for the Zeilinger–Harne–Greenberger type contradiction of section (7).

It is clear from our rules that the answers are:

i) *pre–determined*, i.e. each system of the pair knows a priori what his/her answer will be if any question, determined by an input point p_j and a local choice a or b, will be asked.

ii) *local*, i.e. if, for a given point p_j, operator 1 makes the local choice a, the answer of system 1 depends **only** on p_j and a. In particular it *does not depend* on which question b has been asked to system 2. The same is true exchanging, in the above statement, the roles of 1 and 2.

iii) *entangled*, i.e. the singlet condition is satisfied.

We want to study the statistics of these answers. In order to do so we have to ask the same pair of questions to many, say N, pairs: $p_1, \ldots, p_N$. In particular, denoting

$$S_a^{(\nu)}(p_j) ; \quad \nu = 1, 2 ; \quad j = 1, \ldots, N \tag{2}$$

the answer given by system $\nu (= 1, 2)$ of the j–th pair to the local questions $S_a^{(\nu)}$, we can define the *empirical correlation* for any pair $(S_a^{(1)}, S_b^{(2)})$ of local questions

$$\langle S_a^{(1)} S_b^{(2)} \rangle = \frac{1}{N} \sum_{j=1}^{N} S_a^{(1)}(p_j) S_b^{(2)}(p_j) \tag{3}$$

The question we want to answer with our experiment is the following:
can the members of each pair p_j make an agreement on their answers so that the inequality

$$|\langle S_a^{(1)} S_b^{(2)} \rangle - \langle S_c^{(1)} S_b^{(2)} \rangle| \le 1 + \langle S_a^{(1)} S_c^{(2)} \rangle \tag{4}$$

is violated for some choices of the indices a, b, c?

According to Bell's analysis, if the inequality (4) is violated, then at least one of the conditions (i), (ii), (iii) must be violated (cf. section 5 for further discussion of this point). Our experiment contradicts this conclusion.

(3) Intuitive interpretation of the experiment

The present experiment describes an ensemble of pairs of classical particles which can interact with a vector observable $\vec{B}$, that we call *magnetic field*. We assume that there are two types of particles, type I (left handed) and type II (right handed) and that each pair contains exactly one particle of type I and one particle of type II.

The pairs decay, i.e. split, sequentially, one after the other, and we suppose that left handed particles always go on the left and right handed particles always go on the right, so that after a short time they become spatially separated. On the path of each type of particles there is an experimentalist and the two have coordinated their space–time reference frames so that it makes sense to say that they make simultaneous measurements

and that they orient a physical vector quantity in a common direction. Without loss of generality we can assume that:

– all the experiments take place in the same plane
– in this plane we have fixed an oriented reference frame $e = (e_1, e_2)$
– this reference frame can be parallel transported without holonomy to different points of the (affine) plane (Figure (1)).
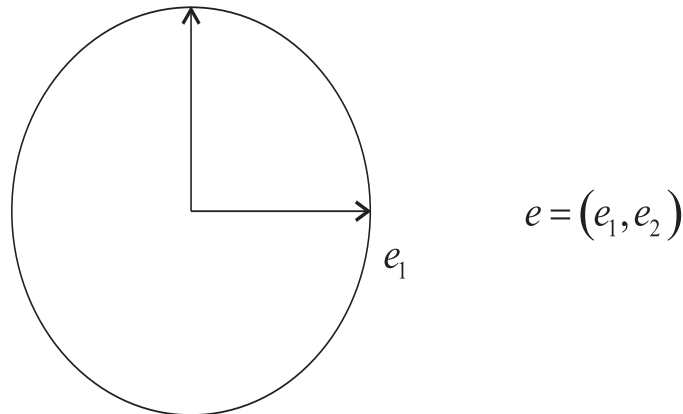

$$e = \left(e_1, e_2 \right)$$

Figure (1)

In particular, with respect to this frame, we can speak of upper and lower half–plane and of counterclockwise or clockwise measurement of the angles (Figures (2) and (3)). Moreover for any point c in the disk, we denote Rc its reflection with respect to the origin (Figure (3))
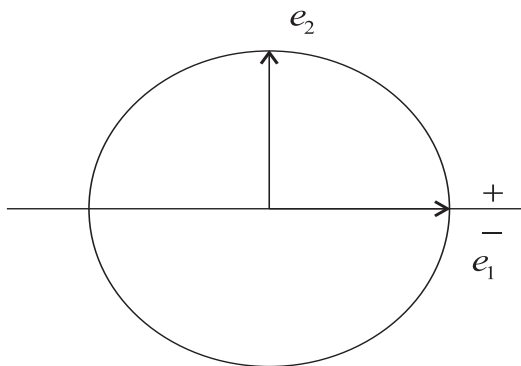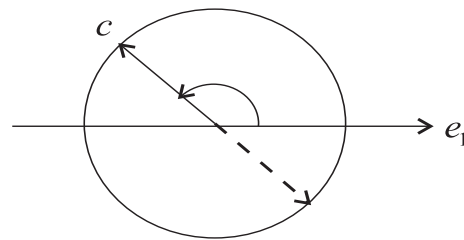
Figure (2)   *Figure* (3)

Denote D the unit disk in the plane:

$$D := \{ p = (x, y) \in \mathbb{R}^2 : x^2 + y^2 \le 1 \}$$

∂D the unit circle
$D_+ = $ (resp. ∂D_+) the upper semi–disk (resp. semi–circle)
$D_- = $ (resp. ∂D_-) the lower semi–disk (resp. semi–circle)
(cf. Figure (2)). To both types of particles we associate a family of observables (that we call *spin*) parametrized by the points on the unit circle in the plane

$$\{ \hat{S}_a : a \in \partial D \}$$

and with values ± 1 in appropriate units. The two types of particles differ because a magnetic field of unit strength in direction $a \in \partial D$ splits a beam of particles of type I according to the values of the observable $\hat{S}_a$, and a beam of particles of type II according to the values of the observable $\hat{S}_{Ra}$. More precisely: if a is any unit vector in the upper half plane (which includes the vector e_1, but not the vector Re_1) then, under the action of a magnetic field $B(a)$ in direction a, a particle of type I will deviate in the upward direction if $\hat{S}_a = +1$, in the downward direction if $\hat{S}_a = -1$

$$\text{Type I} \ \to B(a) \to \begin{cases} \uparrow, & \text{if } \hat{S}_a = + \\ \downarrow, & \text{if } \hat{S}_a = - \end{cases} \tag{1}$$

where the first arrow means *input* and the second one denotes the effect of the interaction with the magnetic field. Under the action of the same magnetic field $B(a)$, a particle of type II will deviate in the upward direction if $\hat{S}_{Ra} = +1$, in the downward direction if $\hat{S}_{Ra} = -1$

$$\text{Type II} \ \to B(a) \to \begin{cases} \uparrow, & \text{if } \hat{S}_{Ra} = + \\ \downarrow, & \text{if } \hat{S}_{Ra} = - \end{cases} \tag{2}$$

For a as above, a magnetic field in direction Ra will have the symmetric effect:

$$\text{Type I} \ \to B(Ra) \to \begin{cases} \uparrow, & \text{if } \hat{S}_{Ra} = - \\ \downarrow, & \text{if } \hat{S}_{Ra} = + \end{cases} \tag{3}$$

$$\text{Type II} \ \to B(Ra) \to \begin{cases} \uparrow, & \text{if } \hat{S}_a = - \\ \downarrow, & \text{if } \hat{S}_a = + \end{cases} \tag{4}$$

i.e. particles of type I will go up if $\hat{S}_{Ra} = -1$, down if $\hat{S}_{Ra} = +1$ and particles of type II will go up if $\hat{S}_a = -1$, down if $\hat{S}_a = +1$

In this sense the observable $\hat{S}_a$ can be understood as measuring the response of a particle (either of Type or of I Type II) to a magnetic field in direction a, *being understood that no other interaction has to take place with the same particle in the same time.* In fact what we measure is only this response and the *values* ± 1 are simply a conventional way to code these two mutually exclusive physical alternatives.

From this definition it is clear that for any two different vectors $a, b \in \partial D$ the corresponding observables $\hat{S}_a$, $\hat{S}_b$ are incompatible because it is impossible to place two different magnetic fields $B(a), B(b)$ in the same point and, even if it were possible, a single particle cannot interact *only with $B(a)$* and, at the same time, *only with $B(b)$*. A biological analogue of this situation would be when $\hat{S}_a$ denotes the color of a chameleon on a leaf and $\hat{S}_b$ its color on a log: it is self–contradictory to say that the color *is simultaneously measured only on a leaf and only on a log.* This expression of logical impossibility has to be distinguished from the usual statement of the Heisenberg principle, expressing the *physical* (but by no means logical) impossibility of simultaneously measuring, with arbitrary precision and on the same system, position and momentum.

REMARK. In standard quantum mechanics it is well known that the spin in direction a is a pseudo–scalar quantity, i.e. under proper rotations it transforms as a scalar but, under parity, according to the rule ([Sak85], chap. 4)

$$S_{Ra} = -S_a$$

9

and obviously the two hermitean matrices S_a and $-S_a$ commute. However the above considerations on the impossibility, for a single particle, to interact simultaneously with two magnetic fields oriented in opposite directions and placed in the same point also apply to spin variables and show that some care is needed when identifying the simultaneous physical measurability of two observables with the commutativity of the associated operators, at least when pseudo–scalar quantities are involved.

On two different particles any pair of observables can be simltaneously measured. Accordingly an ordered pair $(B(a), B(b))$ of magnetic fields in directions $a, b \in \partial D_+$ in the upper half plane will describe a simultaneous measurement of observable $\hat{S}_a$, for particle I, and of observable $\hat{S}_{Rb}$, for particle II, of the pair. We will also use the notation $(S_a^{(1)}, S_b^{(2)})$ for such a measurement.

Given that the two types of particles react differently to the same apparatus, we have to distinguish between the two statements:
(i) the same measurement operation is performed on both particle 1 and particle 2
(ii) the same observable is measured on both particle 1 and particle 2

For example, case (i) corresponds to use the same magnetic field $B(a)$ on both particles of the pair. In this case we know that we are measuring observable $\hat{S}_a$ on particle 1 and observable $\hat{S}_{Ra}$ on particle 2.

Case (ii) corresponds to use the magnetic field $B(a)$ for particle 1 and the magnetic field $B(Ra)$ for particle 2. In this case we know that we are measuring the same observable $\hat{S}_a$ on both particles.

With these premises we will discuss the statistics, over an esemble of pairs, of measurements of pairs of observables of the form $(S_a^{(1)}, S_b^{(2)})$. Since we are interested in the Bell's inequality we will consider three experiments of the form

$$(B(a), B(b)) \quad , \quad (B(c), B(b)) \quad , \quad (B(a), B(c)) \tag{5}$$

corresponding to the measurements of the pairs of observables

$$(S_a^{(1)}, S_b^{(2)}) \quad , \quad (S_c^{(1)}, S_b^{(2)}) \quad , \quad (S_a^{(1)}, S_c^{(2)}) \tag{6}$$

which, in terms of the original $\hat{S}$–observables can be written

$$(\hat{S}_a, \hat{S}_{Rb}) \quad , \quad (\hat{S}_c, \hat{S}_{Rb}) \quad , \quad (\hat{S}_a, \hat{S}_{Rc}) \tag{7}$$

Notice that we have *three* experiments, corresponding to the three magnetic fields

$$B(a), \quad B(b), \quad B(c) \tag{8}$$

but the observables involved cannot be reduced to three in fact, as made explicit by the notation (4) they are *four*, i.e.

$$\hat{S}_a, \ \hat{S}_{Rb}, \ \hat{S}_c, \ \hat{S}_{Rc} \tag{9}$$

due to the different dynamical reaction of particle I and particle II to the same magnetic field $B(c)$. This is an example, in a particularly simple and idealized classical situation, of the *chameleon effect* (the interaction Hamiltonian of a system with an apparatus, hence the dynamical evolution of the system , depends on the observable one wants to measure) described, for example, in [Ac99], [AcRe99a].

(4) Prescriptions for the local answers

The state space of our classical particles is the unit disk D and, for c in the unit circle, we denote $S_c^{(j)}$ the response of particle j to the magnetic field $B(c)$ as described in section (3). We will describe these responses by means of functions $S_x : p \in D \to S_x(p) = \pm 1$, parametrized by points p in the unit circle. Throughout this section, the symbol S_x shall denote such a function **and should not be confused with the observables $\hat{S}_x$ of Section (3)**: for example $S_x^{(2)}$ measures the observabales $\hat{S}_{Rx}$, but is calculated using the function $-S_{Rx}(p) = S_x(p)$. The values of the observables $S_c^{(j)}$ are distributed according to the rule described in Figures (4) and (5):
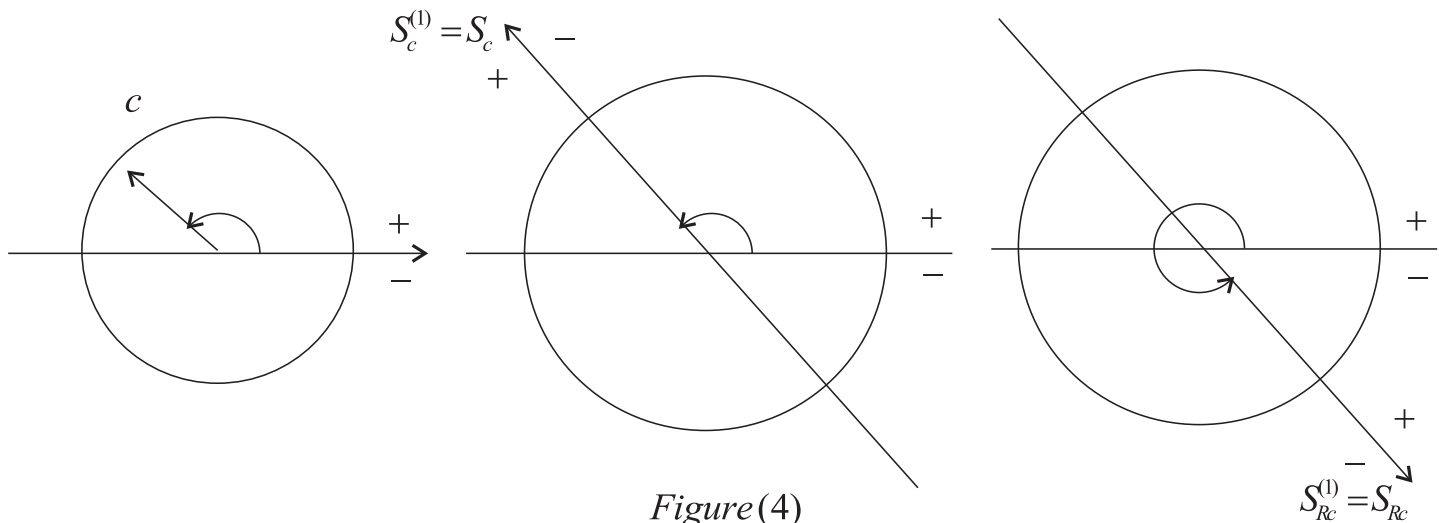
$$Figure\,(4)$$
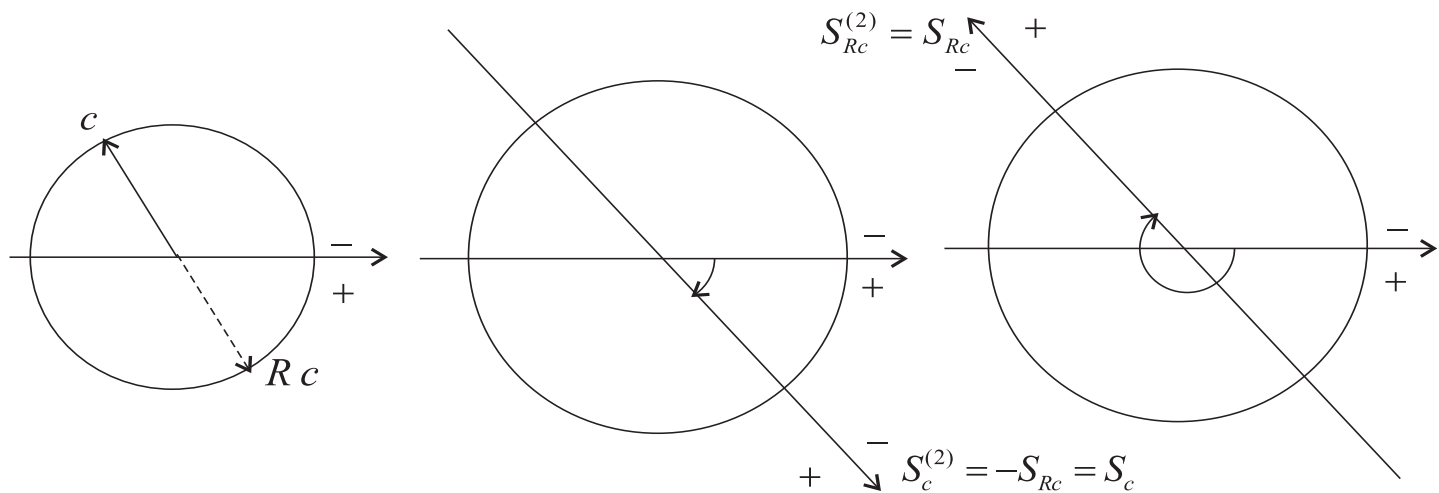
$$Figure\,(5)$$

Notice that the two rules give exactly the same function for $c = b$, i.e.

$$S_c^{(1)} = S_c = -S_{Rc} = S_c^{(2)} \tag{1}$$

As explained in the previous section, this corresponds to the measurement of S_c on particle 1 and of S_{Rc} on particle 2 while, to measure S_c on both particles we have to place a magnetic

11

field oriented in direction c for particle 1 and in direction Rc for particle 2. This gives, according to (1):

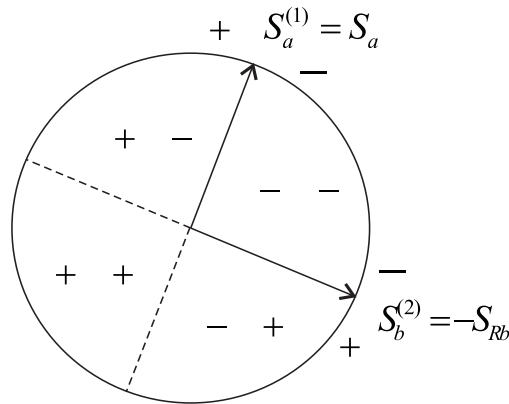$$S^{(2)}_{Rc} = -S_{R^2 c} = -S_c = -S^{(1)}_c \qquad (2)$$

which is the singlet law.

The programme to calculate the *hidden functions* S_a is the following. For $a \in \partial D_+$, making a angle $\alpha \in [0, \pi)$ (measured counterclockwise) with the x–axis (cf. Figure (6)),
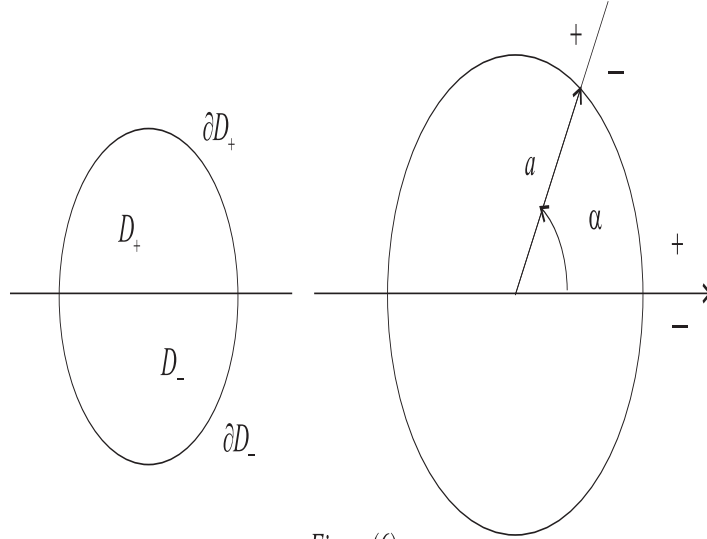
Figure (6)

the calculation of the value of $S_a =: S^{(1)}_a$ in the point p of the disk D is done as follows:
– one rotates the point p, counterclockwise, of an angle α

$$R_{-\alpha} p \quad ; \quad R_{-\alpha} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \qquad (4)$$

Figure (7)

– one chooses the value

$$S_a(p) = \pm 1 \quad \text{if} \quad R_{-\alpha} p \in D_{\pm} \qquad (5)$$

12

– The value of $S_{Ra}(p)$ is determined by the prescription

$$S_{Ra} := -S_a \tag{6}$$

Given (4) these prescriptions uniquely determine all pairs $(S_a^{(1)}, S_b^{(1)})$. The statistics of a pair $(S_a^{(1)}, S_b^{(1)})$ is uniquely determined by the joint probabilities of concordance, $(+, +)$, $(-, -)$ and discordance, $(-, +)$, $(+, -)$, which are proportional to the areas of the regions marked in Figure (7).

(5) The chameleon effect

In this section we show that, if one pretends to apply to our experiment the same type of arguments which are applied to the EPR type experiments, then one arrives to the conclusion that Bell's inequality should be satisfied and this leads to a contradiction with the experimental data.

Recall that the chameleon effect means that *the dynamical evolution of a system depends on the observable A one is going to measure*. In particular, if we measure A, the evolution of another observable B, hence its value at the time of measurement, might be quite different from what it would have been if B, and not A, had been measured. As explained in item (2.6) of section (2), in our classical model, this *disturbance effect* is not only deterministic but also known, so that we can use it in our calculations.

For a given pair (point in the unit disk) $p \in D$, we do not know a priori the values assumed by $S_x^{(j)}(p)$ (because p is a hidden parameter for the experimentalists). However whatever these values are they must satisfy the inequality

$$|S_a^{(1)}(p)S_b^{(2)}(p) - S_c^{(1)}(p)S_b^{(2)}(p)| \le 1 - S_a^{(1)}(p)S_c^{(1)}(p) \tag{1}$$

Since, in the second experiment, $S_b^{(2)}$ has been measured and $b \neq c$, we now, from item (2.6) of section (2), that particle 2 of the pair p, has changed the value of $S_c^{(2)}$ from $S_c^{(2)}(p)$ to $-S_c^{(2)}(p)$. Moreover, from section (4), we know that

$$S_c^{(2)}(p) = S_c^{(1)}(p)$$

Therefore, if we want to insert the value of $S_c^{(2)}(p)$ *that we would have found if $S_c^{(2)}$ would have been measured instead of $S_b^{(2)}$*, then we have to replace in (1), $S_c^{(1)}(p)$ by $-S_c^{(2)}(p)$ thus obtaining

$$|S_a^{(1)}(p)S_b^{(2)}(p) - S_c^{(1)}(p)S_b^{(2)}(p)| \le 1 + S_a^{(1)}(p)S_b^{(2)}(p)$$

From which Bell's inequality

$$|\langle S_a^{(1)} S_b^{(2)} \rangle - \langle S_c^{(1)} S_b^{(2)} \rangle| \le 1 + \langle S_a^{(1)} S_c^{(2)} \rangle$$

is easily deduced by taking averages.

In the following section we show that the experimental results contradict this argument.

(6) Violation of the Bell's inequality

We will now prove that there exist three directions a, b, c in the plane such that the correlations

$$\langle S_a^{(1)} S_b^{(2)} \rangle \quad , \quad \langle S_c^{(1)} S_b^{(2)} \rangle \quad , \quad \langle S_a^{(1)} S_c^{(2)} \rangle \tag{1}$$

violate the Bell inequality. To this goal we fix a to be the x–axis and consider a generic choice of both vectors c, b in the upper semi–circle. According to our rules the corresponding statistics is described by Figure (8) below:
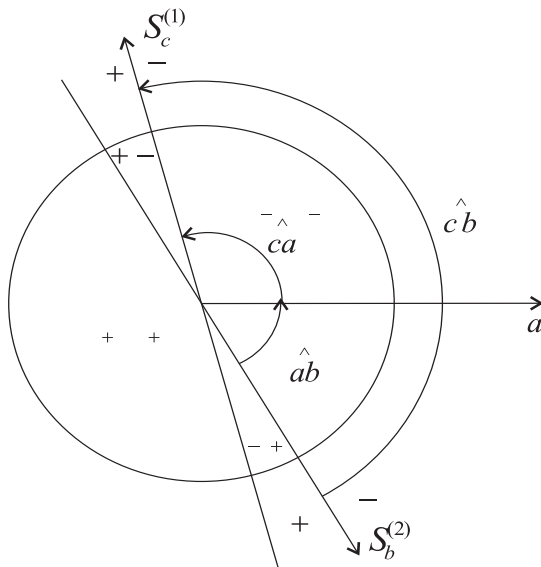

Figure (8)

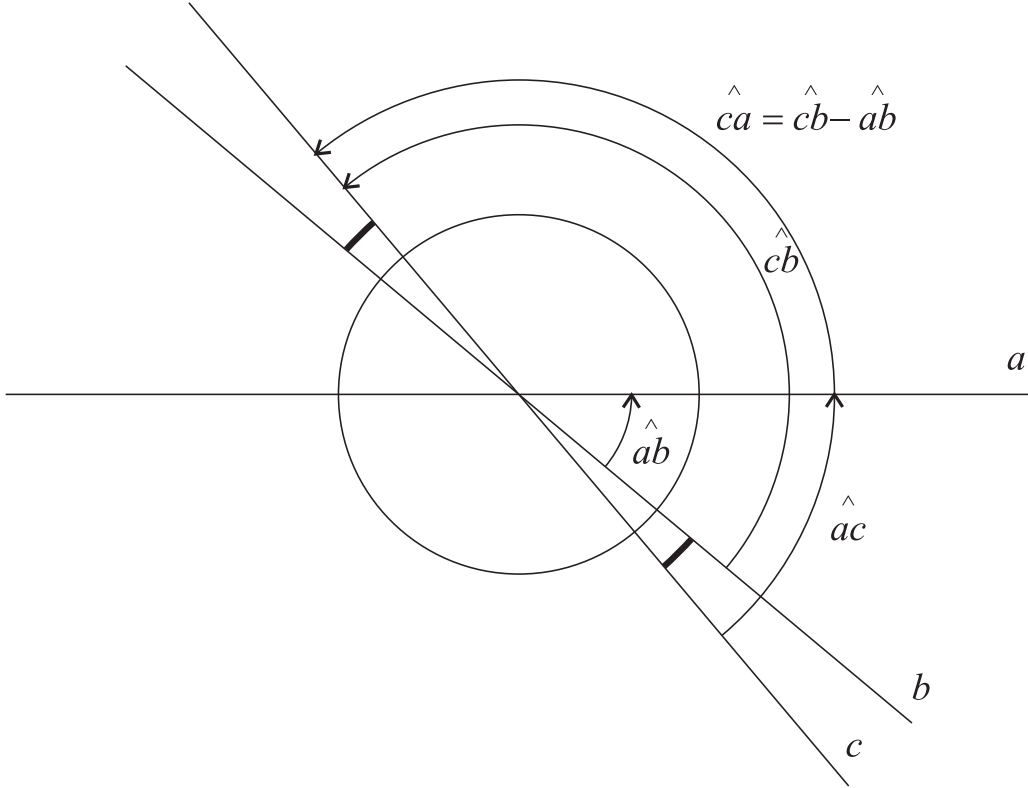Denoting $(+,+)$, $(-,-)$ the probabilities of concordance, and $(-,+)$, $(+,-)$, those of discordance, one has

$$\langle S_c^{(1)} S_b^{(2)} \rangle = 2(-,-) - 2(+,-) = 2(-,-) - 2(\frac{1}{2} - (-,-)) = -1 + 4(-,-)$$

Denoting $\widehat{cb}$ the probability of the $(-,-)$–concordance for the choice $(S_c^{(1)}, S_b^{(2)})$, this gives the general formula

$$\langle S_c^{(1)} S_b^{(2)} \rangle = -1 + 4\widehat{cb} \qquad (2)$$

valid for any choice of the vectors c, b in the upper semi–circle.

Let us now choose the three directions a, b, c in the plane as indicated in Figure (9).

$$\overset{\wedge}{ca} = \overset{\wedge}{cb} - \overset{\wedge}{ab}$$

Figure (9)

Then, according to equation (2), the two sides of the Bell inequality are respectively:

$$|\langle S_a^{(1)} S_b^{(2)} \rangle - \langle S_c^{(1)} S_b^{(2)} \rangle| = 4|\widehat{ab} - \widehat{cb}| = 4(\widehat{cb} - \widehat{ab})$$

$$1 + \langle S_a^{(1)} S_c^{(2)} \rangle = 4\widehat{ac}$$

and we are reduced to compare

$$\widehat{cb} - \widehat{ab} = \widehat{ca} \qquad \text{and} \qquad \widehat{ac}$$

But, because of our choice of the axes (cf. Figure 9), one has

$$\widehat{ca} > \widehat{ac}$$

15

Thus we conclude that

$$|\langle S_a^{(1)} S_b^{(2)} \rangle - \langle S_c^{(1)} S_b^{(2)} \rangle| > 1 + \langle S_a^{(1)} S_c^{(2)} \rangle$$

which violates the Bell inequality.

(7) A GREENBERGER–HORNE–ZEILINGER TYPE CONTRADICTION

Greenberger, Horne and Zeilinger [GrHoZe93] have constructed an example showing that the attempt to attribute simultaneous values to 2–valued observables represented by non commuting operators may lead to a contradiction independently of any statistical consideration. In this section we show that our experiment also provides a classical analogue of the construction of these authors.

From (3.1) and (3.3) we know that the answers of a particle of type I to the two (mutually exclusive) measurements $B(a)$ and $B(Ra)$ will be opposite:

$$S_a^{(1)} = -S_{Ra}^{(1)} \tag{1}$$

Exactly in the same sense we know that

$$S_a^{(2)} = -S_{Ra}^{(2)} \tag{2}$$

Moreover we know that

$$S_a^{(1)} = S_a^{(2)} \tag{3}$$

$$S_a^{(1)} = -S_{Ra}^{(2)} \tag{4}$$

Notice that, if we interpret S_a (resp. S_{Ra}) as the response of a type I particle to the only action of the magnetic field $B(a)$ (resp. $B(Ra)$) the joint event $[S_a = +]$ and $[S_{Ra} = -]$ makes no sense as a simultaneous statement on the same particle. Now let us show that, if we pretend (as done in the original Bell's argument and in all discussions of the EPR type experiments) that all these relations hold simultaneously, in the sense of joint events, then we arrive to a contradiction. Interpreting the diagrams below as explained in section (3) (both columns in each diagram are referred to the same particle), we see that relations (1) and (3.1) imply that, for any vector a in the upper half plane:

$$I \to B(a) \to \begin{cases} S_a = + \; ; & S_{Ra} = - \\ S_a = - \; ; & S_{Ra} = + \end{cases} \tag{5}$$

and relations (2) and (3.2) imply:

$$II \to B(a) \to \begin{cases} S_{Ra} = + \; ; & S_a = - \\ S_{Ra} = - \; ; & S_a = + \end{cases} \tag{6}$$

16

Relations (1) and (3.3) imply:

$$I \to B(Ra) \to \begin{cases} S_{Ra} = - \; ; & S_a = + \\ S_{Ra} = + \; ; & S_a = - \end{cases} \tag{7}$$

and relations (2) and (3.4) imply:

$$II \to B(Ra) \to \begin{cases} S_a = - \; ; & S_{Ra} = + \\ S_a = + \; ; & S_{Ra} = - \end{cases} \tag{8}$$

But, if we pretend to attribute to $S_a^{(2)}$ the values given by the second column of (6) then, because of (5) this would contradict (3) because it gives

$$S_a^{(1)} = -S_a^{(2)}$$

Similarly also (7) and (8) contradict (3) because they give

$$S_{Ra}^{(1)} = -S_{Ra}^{(2)}$$

Summing up: the functions of the "hidden parameter" $p \in D$ correctly describe the behavior of **pairs of observables referred to different particles**. In fact this behavior can be simulated on the computer with arbitrary precision. However, if we pretend to extend this descriptions to triples (or quadruples) of observables by including **pairs of observables referred to the same particle**, then we arrive to a logical contradiction, independent of any statistics.

This kind of contradictions should not be considered as pathological but rather as natural manifestation of the paradoxes that might arise if one attempts to apply the conceptual schemes, elaborated having in mind a *passive* (ballot–box like) reality, to a physical situation involving an *adaptive* (chameleon like) reality. In the latter case the *value of an observable* may express the dynamical reaction to an interaction. Two incompatible interactions (e.g. *only magnetic field $B(a)$* or *only magnetic field $B(Ra)$*), might give rise to opposite physical reactions (e.g. *going up* or *going down*). If we codify this reactions with numbers, e.g. $+1$ or -1 and then we pretend to interpret these numbers as *values of observables*, in the same sense of *passive reality* (e.g. color or weight of a ball in a ballot box) then it is clear that joint events of the form $[S_a = +1]$ and $[S_{Ra} = -1]$, when referred to the same particle at the same time, are in trinsically contradictory. In fact the same particle cannot simultaneously *go up* and *go down*.

(8) CONCLUSIONS AND ACKNOWLEDGMENTS

Our experiments show that there is no contradiction between lcality, realism and quantum theory, thus confirming the results of the theoretical analysis of [Ac81],..., [AcRe99b]. The three directions in Figure (9) parametrize three experiments I (a, b), II (c, b), III (a, c), in which the experimentalists 1 and 2 locally choose a unit vector according to the scheme illustrated in Table 1.

	I	II	III
1	a	c	a
2	b	b	c

Table 1

Having fixed a to be the x–axis, as in Figure (9) this is equivalent to choose the angles $\widehat{ab}$ and $\widehat{ac}$ of Figure 9.

For the choice of the angles illustrated in Table 2 (angles are given in radiants, so the local choices consist in numbers between 0 and $3, 14$)

	I	II	III
1	0	1.989675	0
2	0.3141593	0.3141593	1.989675

Table 2

we verify that the difference between the left and the right hand side of Bell's inequality is of order

$$0.521 > 0$$

leading to a violation of the inequality. Since our scheme is obviously stable under small perturbations, we have violation of the inequality for infinitely many directions.

The present experiment has been first performed on july 10, 1999 at the Centro Volterra and first publicly performed on september 13, 1999 at the Centre for Philosophy of Natural and Social Science of the London School of Economics. Then, in the same year: on September 29 at the Steklov Institute (Moscow), in occasion of the conference dedicated to the 90–th birthday of N.N.Bogoliubov; on November 9 at the 3–rd Tohwa University International Meeting on Statistical Physics (Tohwa University, Fukuoka, Japan); in January 10, 2000 at the IV International Workshop on Stochastic Analysis and Mathematical Physics, in Santiago; on February 24, 2000 at the Department of Physics of the University of Pavia; on March 7, 2000 at the 3rd Meijo Conference on Quantum Information. It was described without performance of the experiment (because of problems with the computer) on December 17, 1999 at the conference on Quantum Probability and Infinite Dimensional Analysis, Jointly organised by Indian Statistical Institute and the Jawaharlal

Nehru Centre for Advanced Scientific Research (Bangalore, India) and on May 4, 2000 at the conference on Quantum Paradoxes in Nottingham.
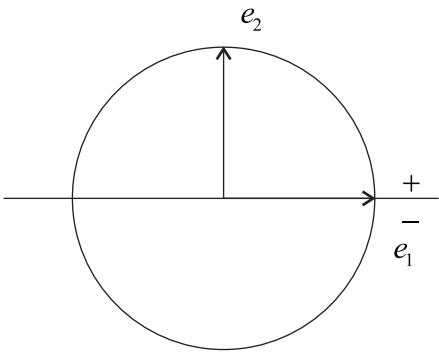
Those who want see how the experiment concretely works may consult the Volterra Center's WEB page: **http://volterra.mat.uniroma2.it** from where the programme can be down–loaded. In the same page one will also find the files corresponding to the papers mentioned in the bibliography as well as a synopsis (in English) of the book [Ac97] and other material relevant to explain in more detail the main theses of quantum probability.

BIBLIOGRAPHY

[Ac81] Luigi Accardi : "Topics in quantum probability", Phys. Rep. 77 (1981) 169-192

[Ac97] Luigi Accardi: Urne e camaleonti. Dialogo sulla realtà, le leggi del caso e la teoria quantistica. Il Saggiatore (1997). Japanese translation, Maruzen (2000), russian translation, ed. by Igor Volovich, PHASIS Publishing House (2000), english translation by Daniele Tartaglia, to appear

[Ac99] Luigi Accardi: On the EPR paradox and the Bell inequality Volterra Preprint (1998) N. 350.

[AcRe99a] Luigi Accardi, Massimo Regoli: Quantum probability and the interpretation of quantum mechanics: a crucial experiment, Invited talk at the workshop: *"The applications of mathematics to the sciences of nature: critical moments and aspetcs"*, Arcidosso June 28-July 1 (1999). To appear in the proceedings of the workshop, Preprint Volterra N. 399

[AcRe99b] Luigi Accardi, Massimo Regoli: Local realistic violation of Bell's inequality: an experiment, Conference given by the first–named author at the Dipartimento di Fisica, Università di Pavia on 24-02-2000, Preprint Volterra N. 402

[Be64] J.S. Bell: On the Einstein Podolsky Rosen Paradox Physics 1 no. 3 (1964) 195-200

[Be87] J.S. Bell: Speakable and Unspeakable in Quantum Mechanics, Cambridge, Cambridge University Press (1987)

[CuMcM89] J.T. Cushing, E. McMullin (eds.): Philosophical Consequences of Quantum Theory Reflections on Bell' s Theorem. University of Notre Dame Press, vol. II (1989)

[EPR35] A. Einstein, B. Podolsky, N. Rosen: Can quantum mechanical description of reality be considered complete? Phys. Rev. 47(1935)777-780

[GrHoZe93] A. Zeilinger, M.A. Horne, D.M. Greenberger: Multiparticle interferometry and the superposition principle. American Institute of Physics (1993)

[Maud94] Tim Maudlin: Quantum Non–locality and relativity, Blackwell (1994)

 [Mdx88] J. Maddox: New ways with Bell's inequalities. Nature 335 (1988)

 [PrTe93] Press H. William, A. Teukolsky Saul: Numerical recepees in C. The art of scientific computing, Cambridge University Press, 1993.

 [Red87] M. Redhead: Incompleteness, nonlocality and realism, Clarendon Press (1987)

 [Sak85] Sakurai Jun John: Modern Quantum Mechanics, Benjamin (1985)

Figure (2)