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A real options model for the transferability value of telecommunications licenses

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Abstract Licenses for telecommunications services are awarded with a number of side obligations and commitments for the licensee. Under such obligations the licensee is typically not allowed to transfer its license to another operator. Such prohibition may cause heavy inconveniences for customers, so that its removal is strongly advocated and already a reality in many cases. Its removal adds value to the original license and may then constitute a valuable option (the transferability option). A method is here proposed to assess such value, by using the framework of real options. The method is applied in a variety of settings and shows that the value of the option depends superlinearly on the reselling price and the market volatility, and linearly or sub-linearly on the expiry time of the option.

Keywords Licensing · Regulation · Auctions · Telecommunications economics

1 Introduction

The licensing process for wireless telecommunications is, in most cases, conducted by issuing an auction. In such auctions, the final price is set by the interaction among the competing service providers, and the resulting price is expected to reflect the value of the investment (a form of value-based pricing, driven by competition). Other assignment processes may be of the bureaucratic or beauty-contest type [4]. Whatever the license-awarding process, licenses are typically assigned with associated constraints on performance and with binding commitments on the side of the license holder. The so-called performance bonds may consist, e.g., in achieving a specified degree of coverage of the territory and/or of the population within a given timeframe. Additionally, the license holder is typically required to commit itself to accomplish the investment objectives set in the tender while not being allowed to resell the license. However, the industrial sector may be hit by a general economic recession, or by a specific fall of the expected demand for services, which may significantly alter the business perspectives as expressed at the time of the tender submitted by the service provider. In the presence of such a downturn, the license holder may be compelled to defer its investment or even to withdraw from it [11, 26]. In this case, the constraints and the commitments associated to the license may make the licensee's situation even harder, since they compel the service provider to proceed with its investment plans in the face of the crisis. However, if the licensing agreement does not allow reselling the license, the crisis of the single company may transform into a damage for the customers, who will not be able to access the service or will be denied the advantages of a fuller competition. In fact, in such cases, the authority governing the licensing process may either reassign the idle spectrum (previously assigned to the crisis-hit license holder) to one of the other

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license holders or to restart the assignment procedure; both ways may take a long time to complete. Allowing license reselling would alleviate that problem [34], since it would allow the license holder to get rid of the license-associated burdens and obtain cash at the same time well ahead of officially admitting its state of crisis. On the other hand, allowing the reselling option right in the assignment conditions would add to the value of the licenses. And such value increase is expected to be incorporated in the price at which licenses are assigned. Since the amount of money earned by the government through license assignment is a relevant issue in the decisions on the assignment process, the assessment of the value added by the reselling option is an important task.

The task of assessing the value of licenses, namely UMTS licenses, has been addressed in the past [3, 15, 32]. Such efforts are reviewed in Section 4. The attention was, however, focused on the investment itself rather than on the side conditions of the licensing contract. As far as the authors know, no evaluation has been conducted for the reselling permission.

In this paper, we aim specifically at the reselling option and provide a method to assess the value that such option adds to the license. We focus on the (semi)permanent transfer of licenses, rather than on the dynamic access to limited portions of spectrum as envisaged in [25], which is now made possible by spectrum sensing techniques in cognitive networks [6, 31]. We show that the value of the reselling option is strongly affected (in a nonlinear fashion) both by the prospective reselling price and by the expected variability of the market conditions after the license is assigned (embodied by the investment volatility). The option's value grows, instead, rather linearly with the expiry time of the option. We adopt the tool of real options to accomplish our assessment, as reported in Section 5, and use the numerical method described in Section 6. Since the value of the option results to be a function of a number of parameters, we finally show in Section 7 the impact of each of them.

2 Licenses for telecommunications services

A telecommunications license authorizes an entity to provide telecommunications services or operate telecommunications facilities. In this section, we briefly review the main ways in which licenses are awarded and highlight some shortcomings of current assignment procedures, which advocate for the introduction of transferability rights.

Licenses also generally define the terms and conditions of such authorizations and describe the major rights and obligations of a telecommunications operator. The terms "license," "concession," and "franchise" may be defined differently in the laws of different countries. However, these terms generally refer to the same basic concept, i.e., a legal document granted or approved by the regulator, or by another government authority, that defines the rights and obligations of a telecommunications service provider. For the sake of simplicity, in this paper, we will use the term "license" in its general meaning.

Several mechanisms have been put into place to assign telecommunications licenses. They can be summed up in the following three classes:

1. General authorization (requires compliance with basic terms and obligations)
2. Beauty contest (evaluation of technical quality of submission)
3. Auction

The first mechanism is non-competitive and applies to cases where the resource to be awarded is not scarce (e.g., fixed wireline infrastructures). The latter two ones are, instead, competitive mechanisms, where the number of licenses to be awarded is limited and typically much less than the number of competitors (though some degenerate cases may happen where the number of licenses to be awarded is equal to or even larger than the number of competitors). Basically, all wireless services (such as UMTS, WiFi, and WiMax) should be awarded with competitive mechanisms, since they assign varying degrees of rights on a scarce resource such as the radio spectrum. In particular, licenses for 3G mobile services in Europe were assigned either by beauty contests or by auctions [4].

In addition to their institutional role in the assignment of operating authorizations, licenses also have a business role, since they provide certainty for investors and money lenders, and with it the confidence that is required to invest the millions or billions of dollars needed to install or upgrade a telecommunications infrastructure. On the other hand, licenses should also provide certainty for consumers and the regulator, namely that the licensed operator would be able to fulfill its obligations and bring to completion the infrastructure building. However, licenses do not provide just rights, but come with a set of obligations as well, among which are typically

– *Performance bonds*, which oblige the telecommunications provider to comply with a number of

deadlines concerning the fulfillment of technical requirements (e.g., population or territory coverage)
– *Non-transferability* of license

The rationale behind the imposition of such constraints is related to the willingness to compel service providers to make full use of the license they have been awarded. Without these constraints, it is thought, customers could be left without service when none of the service providers actually goes through the investment plan associated to the license. Actually, these constraints may turn out to be counterproductive. If the market conditions are good, the service providers will need no spur to carry on their projects. But, if the operator is in dire straits, the failure to comply with obligations will lead to license revocation, and consequently to a delay in infrastructure realization and service offer, which, in the end, represents a negative consequence for customers. Actually, in 3G spectrum assignments in Western countries, 27% of the licenses were still idle at the end of 2005 [11]. The reasons for such behavior on the side of service providers could be either the persistent lack of market perspectives at that time (with service providers delaying their investment waiting for clearer signs from the market) or the actual presence of financial difficulties. Two cases, both related to the assignment of UMTS licenses in Italy, were particularly poignant. The auction had started with a number of licenses to award just lower (by one) than the number of competitors, a situation which should have to be avoided in the first place. The operator BLU (a consortium formed, among the others, by the provider British Telecom and the TV company Mediaset) retreated during auction operations, leaving the number of competitors exactly equal to the number of licenses to be awarded, and so de facto terminating the auction. The consequence was that the license prices for the remaining players were much lower than the expectations, and rumours of collusion broke out [4]. Shortly after, the operator IPSE, a consortium (formed, among the others, by the providers Telefonica Moviles and Sonera) that had been awarded a license, gave it back, further impoverishing the offer scenario and lowering the competition among the remaining players [26, 35].

Such cases constitute clear evidence of the limitations deriving from forbidding license transferrals. Including in the auction conditions the possibility to transfer the license (transferring at the same time the associated rights and obligations to the buyer) would expedite crisis resolution and the entry into service. We can also envisage that relaxing the constraint on license transferability would add value to the license itself. In fact, it would amount to add a real option to the set of rights and obligations associated to the license.

3 Spectrum trading

As hinted in the previous section, allowing license assignees to resell their licenses would add value to the license itself. Such possibility, deployed on a wide scale, creates a secondary market for licenses. In the literature, such possibility is usually named *spectrum trading*. Its opportunity is justified in [34] with mainly qualitative arguments. However, in the same paper, a simple quantitative argument in favor of spectrum trading is reported, which shows that the social welfare increases when the license holder sells the license at a price higher than the original license cost but lower than the present license value as assessed by the prospective buyer. In that case, the license holder can make a profit by selling the license. However, during the license's life, its value may move downward as well as upward for a number of reasons:

– Entry of a new operator in the market
– Changes in customers' preferences
– Availability of new technologies

But the previous simple line of reasoning applies also when the license evaluation has lowered. In fact, if the license holder cannot sell its license and its business prospects are doomed, the license value is certainly below its acquisition cost and may be zero for all practical purposes. The tradability of the license provides an added value to its ownership. Such added value is to be reflected in the license acquisition cost, which should be higher than that applicable when the transferability option does not hold. The government can therefore benefit of this added value by earning more in the auctioning event, a welcome opportunity since the maximization of revenues for the government is considered as one of the main objectives in the licensing process [4]. Spectrum tradability is generally meant to imply that all the rights and obligations originally associated to the license are transferred jointly with the license itself, among which the most relevant ones are:

– Set of frequencies assigned
– Geographical area of validity of license
– Time period of validity
– Set of applications allowed by license
– Degree of protection granted by license assigners with respect to other users (in the same or adjoining frequency bands)

– Obligation not to interfere with other spectrum users (again in the same or adjoining frequency bands)

At present, spectrum trading, put forward, e.g., in [5] and [30], has already been introduced in the national legislations of many countries [1, 10, 18, 22, 24, 28, 29]. However, its economical benefits for the auctioneer (i.e., the government) have not been quantified yet.

4 License valuation

Though we are interested in the value added to a license by removing the transferability ban, it is relevant to deal with the valuation of the license in the absence of the flexibility provided by the transferability option. Due to the large amount of money involved in the building of a wireless infrastructure, it is expected that the companies taking part in the spectrum auctions spent a great effort in assigning a monetary value to the license they were running for. Due to the strategic importance of the whole process, it is also expected that such information has stayed proprietary and has not been disclosed in the scientific literature. However, we can gain some insight from the prices that the companies have paid to get their 3G license. In this section, we analyze some of the data resulting from the analysis of completed auctioning processes with the aim of putting a value on spectrum licenses.

First, we expect the value of a license to depend on a number of factors, some of which are mutually interrelated. Among them, we can name the expected demand, prices (this quantity affects the first item), expiry date of licenses, technology obsolescence, and openness of the market (e.g., as embodied by the number of actual and future competitors). Since the ultimate value of the license is linked to the net revenues it will generate for the company owning it, we can envisage that the license's value is expressed by its associated stream of cash flows. Each cash flow incorporates the overall effect of the factors listed above. An overall value for the license can therefore be obtained by computing the net present value of those cash flows. Of course, no company has made public its evaluation of the 3G business associated to the license. On the other hand, we can observe the prices that the companies have paid for the licenses. Though we expect that values should be reflected in prices, in 3G auctions, prices varied widely, with differences not easily justifiable on the basis of national peculiarities. In order to account for the different size of the market across Europe, we can consider the price per head of population. In beauty contests, such price varied from 0 (Finland and Sweden, where licenses were given for free) to 168 €(France) [4]. An even larger range was observed in auctions, where the price per head of population varied from 46 €(Greece) to 642 €(UK) [4]. Such differences may be explained either by gross errors in the valuation conducted by companies or in inefficiencies embedded in the auction design, namely, the inability to extract the full value of licenses during the bidding process. As a support for the former hypothesis (valuation error), it has been put forward that the large fraction of idle licenses feeds the suspicion of overbidding, i.e., that prices reached in auctions were higher than those compatible with the actual market structure [11]. A case supporting instead the latter hypothesis (prices well below actual values) is provided by the auction for 3G licenses in Italy where the anomalous situation already described in Section 2 brought the auction to a premature conclusion. For that case, Scandizzo and Ventura have proposed a model for the value of the license, based on real options [32]. In that model, the cash flows generated by revenues are modeled through a geometric Brownian motion. The option in that case is embedded in the possibility to build the network and exploit the license rights. Real options were also used in another attempt to assess the value of UMTS licenses in [3] and [15], in the midst of the observations after the first wave of license assignments.

5 Financial and real options

The valuation of the selling option embedded in the license relies on methods first devised in the financial market. In this section, we review the basic definitions and show how the concepts defined in that context translate to the evaluation of options pertaining to projects (which are given the name of real options).

A financial option is the right to sell (or to buy) a financial asset at prescribed conditions. The financial asset is named the underlying and may be, for example, a set of shares (the quantity that can be sold or bought is set in the option contract). That right has to be exercised either at a given date in the future (this variety is called a *European* option) or within a given date (*American* option). The price (per unit of the financial asset) at which the option allows to sell (or buy) is set at the time the option is bought and is named *exercise price* or *strike price* (in the following, we indicate it by the quantity E). The most basic examples of options are the *call* (right to buy) and the *put* (right to sell) options. While the value of the underlying is known at the time the option is bought (we denote it by S_0), it is then subject to random changes during the option's life. We will see that the path followed by that value is conveniently modeled by a stochastic process. At the time T of expiry of the option, the value of the underlying will be the random S_T (unknown at the time the option is purchased). If we are considering a European call option, we will exercise the option if $E < S_T$, since the asset is (at the deadline for exercising the option) valued more than the price we have to pay for it; in that case, the unit payoff is the difference between the current value (which is the price at which we could sell the asset) and the exercise price (the price we are entitled to pay if we want to buy the asset), i.e., $S_T - E$. In the reverse case, i.e., if the current value is lower than the exercise price, it does not pay to exercise the option, since we would pay a price (E) larger then the current value (S_T), so that the option's value is null. The payoff (i.e., the gain for the option's owner) for both cases may then be expressed by the single formula $V_T = \max(S_T - E, 0)$. Things are reversed in a put option, where the unit payoff at the expiry date is instead $V_T = \max(E - S_T, 0)$. In the case of the American option, the comparison between the underlying's value and the exercise price (in order to decide whether to exercise the option or not) is carried out continually during the option's life. The main problem with options is the correct evaluation of their value (i.e., their fair price) at the option's buying time.

We can similarly apply the tools developed for financial options to investment projects, where the underlying is the project itself. In that case, the value of the underlying is the net present value (NPV) of the project in the absence of the flexibility embodied by the option. This assumption is known as the *marketed asset disclaimer* (MAD) [7]. If we consider a potentially infinite lifetime for the project and use the notation C_t to indicate the net cash flow at the time t, the NPV of the project is

$$S_0 = \sum_{t=0}^{\infty} \frac{C_t}{(1+k)^t}, \tag{1}$$

where k is the yearly expected return rate on the investment. Some examples of real options are the following [2]:

- Deferring
- Stopping and resuming
- Outsourcing
- Exploring through pilot projects or prototypes
- Altering the scale of the project (either contract or expand)
- Abandoning the project
- Leasing

6 A binomial model for the transferability option

As stated in Section 5, the main problem lies in assigning a value to options. In our case, this task amounts to evaluate the transferability option. In this section, we derive a numerical procedure for this purpose.

As a model for the stochastic process for the investment project, we have opted for the binomial model, first introduced by Cox et al. [8]. In addition to its simplicity, it can boast a wide diffusion in the context of the evaluation of real options [7]. Hereafter, we follow the same arguments used in [17], modifying the expressions for the case of a put option rather than a call option. In this model, we adopt a discrete-time approximation to the actual process. We indicate the time at which the option has to be underwritten by 0; the option's expiry date is then T. The time to expiry is divided into N time intervals of duration $\delta T = T/N$. We expect the approximation to be better as the subdivision gets finer, i.e., as the number of intervals grows. We index the intervals by $i = 0, 1, \ldots, N$, so that the value of the project at the end of the j-th interval is S_j. As already stated, the initial value of the project S_0 is the NPV of the project in the absence of any flexibility, i.e., in the absence of the selling option, and can be obtained by Eq. 1. It is to be noted that the timescale employed in that expression has no relationship to that adopted in the binomial model: while a yearly interval is typical to evaluate the NPV, a duration δ as small as desired can be used for the evaluation of the option. While we suppose to know S_0 (though it is actually estimated), the values S_i, $i > 0$, are random quantities, described by a binomial process. Over any single time interval, we suppose that the project value can move from its initial value to one of two alternative values

$$S_{j+1} = \begin{cases} uS_j \\ dS_j \end{cases} \tag{2}$$

where the two up and down factors u and d are such that $0 < d < 1 < u$. We indicate by p the probability that the value of the project grows over a single time interval, i.e.,

$$p = \mathbb{P}\left[S_{j+1} = uS_j\right]. \tag{3}$$

Since, over the time T, we have N such time intervals, we end up with 2^N possible values for the project at the option expiry date. In order to account for all the possible values, we indicate by s_i^j, $i = 0, 1, \ldots, j$, and $j = 0, 1, \ldots, N$ the $i+1$-th smallest value of the project at the j-th time interval (then s_0^j is the lowest possible value at the j-th stage). Through the binomial model in an approximation of the actual movement of the underlying, we can fine-tune this approximation by increasing the number of intervals N and by fitting the model to the observed volatility of the underlying. So far, we have left the up and down factors undetermined. If we now equate the variance σ^2 of the underlying after one time period to that resulting from the model, we obtain

$$\begin{aligned} u &= 1 + \sigma\sqrt{\delta T}, \\ d &= 1 - \sigma\sqrt{\delta T}. \end{aligned} \tag{4}$$

If we neglect higher-order terms, now we have $u = 1/d$, and the possible alternative values of the project at the expiry date reduce to $N + 1$. The exact order of the up and down turns during the option's life is therefore irrelevant for the value of the project at the option's expiry date ($ud = du = 1$); that value is determined just by the number of ups and downs. The associated process tree is then recombining; a sample is shown in Fig. 1.

Then we have $S_i^j = S_0 u^i d^{j-i}$, which greatly simplifies when we recall that $ud = 1$. At each stage, we know the possible values of the project. In particular, at the expiry date, we are able to compare the values of the project at that date with the strike price of the option.

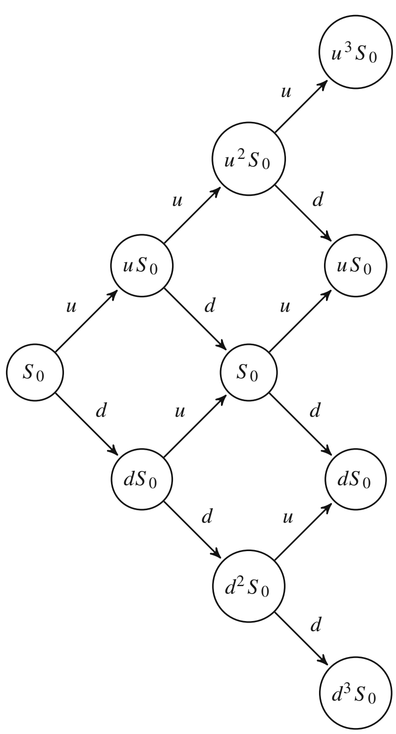

Fig. 1 Event tree for a three-stage binomial process

When the project's value lies below the exercise price, it is worth exercising the put option. We are then able to associate a decision (concerning the exercise of the option) to each of the project's possible outcomes, and to compute the possible values of the option at the expiry date. We now need to go back to the tree root in order to obtain the value of the option at time 0. For the purpose of evaluating the option, we first consider a single period, namely, the first period. We define a synthetic portfolio that includes the option itself plus some quantity Δ of the project (the risky underlying). If we indicate the value of the option at time 0 by V_0, at the same time 0, our portfolio's value is $V_0 + \Delta S_0$. After the first period, following the binary evolution of the project, the option will be worth one of two values: either V^+ (when the project's value has increased) or V^-. It is to be noted that the value of a put option moves in the direction opposite to the underlying: if the underlying's value grows, the option's value decreases; then, for a put option $V^- > V^+$. At the same time, the portfolio will be valued either $V^+ + \Delta u S_0$ or $V^- + \Delta d S_0$. At this point, we can apply the risk-neutral valuation principle. According to this principle, we can assume a risk-neutral approach when evaluating an option. The resulting prices will be valid not just under the risk-neutral hypothesis, but in general terms. In our case, the risk-neutral condition is equivalent to consider that the value of the portfolio stays unchanged independently of the occurrence of an up or down movement. By equating the two possible outcomes (the portfolio values), we achieve risk neutralization if the quantity Δ is

$$\Delta = \frac{V^- - V^+}{(u-d)S_0}, \tag{5}$$

i.e., the ratio between the spread in the option values and the spread in the project's values. Having eliminated the risk, we can now apply the no-arbitrage condition, i.e., equating the value of the portfolio to what would be obtained by investing in a risk-free activity (at the risk-free rate r). The resulting equation is

$$V^+ + \Delta u S_0 = V^- + \Delta d S_0 = (V_0 + \Delta S_0)(1 + r\delta T), \tag{6}$$

whose solution provides the value of the option

$$V_0 = \frac{p^* V^+ + (1-p^*)V^-}{1 + r\delta T}, \tag{7}$$

where

$$p^* = \frac{1}{2} + \frac{r\sqrt{\delta T}}{2\sigma} \tag{8}$$

is the so-called risk-neutral probability. In fact, the resulting option value can be seen as the expected value of the option under the risk-neutral probability, discounted at the risk-free rate. If we now extend the procedure to a multi-stage binomial process and go back to the process tree, we can use Eq. 7 to derive the possible values of the option for each possible outcome. In fact, the option is worth zero if it is not going to be exercised and the difference between the exercise price and the project's value otherwise. Going backward by using the discounting equation (Eq. 7) for each couple of adjacent outcomes, we can derive the value of the option at the inner stages of the process tree. With respect to the simple procedure outlined so far, we have to consider that we are dealing with an American option. Namely, since we can exercise the option at any time, we must always compare the value resulting from keeping the option (provided by the discounting equation) with the exercise price; the value of the option will be the maximum between these two values. Here, we adopt for the option's values a notation similar to that adopted for the project's value: the $i+1$-th possible value (with increasing order) of the option at the j-th time interval is V_i^j. In the end, the equation providing the option value is then

$$V_i^j = \max\left(\frac{p^* V_{i+1}^{j+1} + (1-p^*)V_i^{j+1}}{1 + r\delta T}, E - S_i^j\right). \tag{9}$$

In the previous expressions, we have to provide values for two parameters in order to solve for the option value, namely, the volatility σ and the risk-free rate. As to the latter, its value is easily determined, typically as the interest rate paid by non-defaultable securities, such as short-term government bonds. Instead, determining the volatility is quite harder. In fact, we should refer to the volatility of the project's value. If we look at the NPV expression (Eq. 1), the value of the project is subject to uncertainty because of the uncertainty in any of the NPV components, namely, uncertainty on the cash flows deriving from uncertainty in demand, cost, and price (with a strong inter-dependence between demand and price), and uncertainty on the cost of capital (which, in turn, depends on changes in the perceived risk). As to the latter, valuations are typically carried out specifically for mobile operators, see, e.g., [23, 27, 33]. In view of the difficulties in accounting separately for each of these factors and examining their joint influence on the project's value, often a proxy for the project is used. Typically, a choice is made to consider a traded asset, for which a price is known with continuity, such as the stocks of a company operating in the same sector (and exhibiting the same level of risk as the project of interest). We report hereafter some of the approaches that appeared in the literature, with the resulting estimates for the yearly volatility. The main approach has consisted in considering the historical price movements of the stocks, either of a single company or of an industrial sector. Examples of the first choice are represented by [12], where the upgrading of a mobile network from 2.5G to 3G technology was analyzed (with an estimated value $\sigma = 0.18$), and [14], where the investments in the wireless industry were evaluated using the estimate $\sigma = 0.3768$, and finally, by [3], where the return rate of France Telecom is used to evaluate the value of UMTS licenses, with an estimated range $\sigma = 0.4 - 0.6$. An example of the second choice is instead [13], where the value of the US Telecom Index is employed to evaluate the option of building new transmission capacity, resulting in $\sigma = 0.31$. A similar approach is adopted in [19], where the price of the product (i.e., bandwidth), rather than of the company, is used to evaluate decisions on the purchase of bandwidth. An alternative source of information is represented by accounting statements, as in [32], where the volatility of cash flows based on the quarterly reports of a mobile company is used to evaluate ex-post the value of UMTS licenses. Finally, a completely different approach is taken in [9], where the volatility in traffic demand is estimated, on the basis of daily measurements, and employed to evaluate the price of capacity, with an estimate $\sigma = 0.95$. As we can see, largely different values have been obtained. In the following, we do not opt for a particular approach, but rather leave the volatility as a parameter, observing its influence on the value of the reselling option.

7 Sensitivity of the transferability option value

Following the description of the binomial model adopted in the evaluation of the transferability option, in this section, we employ that model to assess the value of the option under a variety of conditions. In Section 6, we have seen that the option's value depends on a number of parameters, partly related to the characteristics of the reselling option (e.g., the strike price and the expiry date of the option) and partly due to the riskiness associated to the project (embodied by the volatility). Our aim is now to understand the role played by those parameters and the impact on the overall value of the option. Hence, we now provide the results of a sensitivity analysis, focusing on the following parameters:

– Reselling price of the license (strike price of the option)

– Limit date to resell the option (expiry date of the option)
– Volatility

Though the evaluation process can be implemented quite straightforwardly according to the description of Section 6, we have employed the code provided in [16] for its high efficiency. In all the computations reported in this section, we have adopted a subdivision of the time to expiry in $N = 256$ intervals, which is the value suggested in [16]. Though this number is deemed to be sufficiently large to warrant good accuracy, further details on the convergence speed of the binomial model can be found in [20] and [21]. In addition, all the figures obtained for the option value have been normalized to the NPV of the investment project.

We start with considering the price at which the license holder resells its license. Since the reselling option represents a parachute alternative when the investment project is not performing as well as expected, we limit our analysis to the case where the reselling price (the strike price, if we stick to the option terminology) is lower than the present value of the project, as evaluated at the time of license assignment. Lower reselling prices reflect more critical situations for the license holder. In Fig. 2, we can observe the changes in the option value for a wide range of possible values for the reselling price and three possible values for the yearly volatility (corresponding roughly to low, medium, and high volatility scenarios). The other parameters are kept constant; namely, we assume that the license holder can resell it within 3 years from the license assignment date, and that the risk-free rate is 5%. In the low-volatility scenario, the project value is expected to remain fairly stable during the option's lifetime; the resulting option value is quite negligible, excepting very high reselling prices, since the license holder would gain nothing from reselling the license at prices lower than the current value of the project. As the volatility increases, so does the value of the option. In both the medium- and high-volatility scenarios, the value of the option grows more than linearly with the reselling price.

Next, we evaluate the impact of the limit date to resell the license. In Figs. 3 and 4, we report the value of the option in two scenarios of medium and high volatility (we consider $\sigma = 0.25$ and $\sigma = 0.5$ as the two corresponding values of the volatility). For each volatility scenario, we consider three possible reselling prices for the license, respectively equal to 30%, 50%, and 70% of the NPV of the project, and the same value $r = 0.05$ of the risk-free rate as adopted previously. In the medium volatility scenario, we see that the option has negligible value when the reselling price is low,

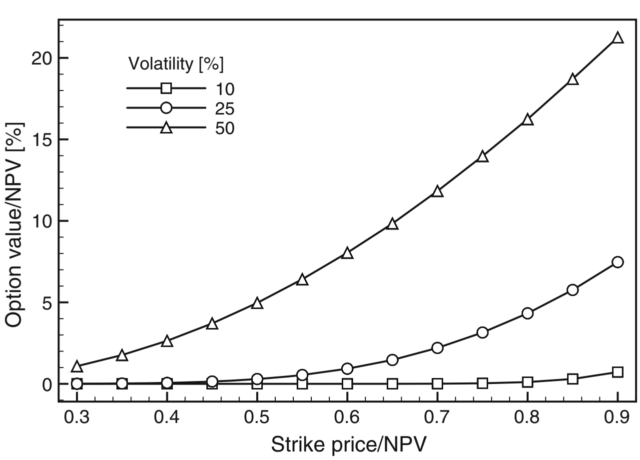

Fig. 2 Impact of the license reselling price on the option's value

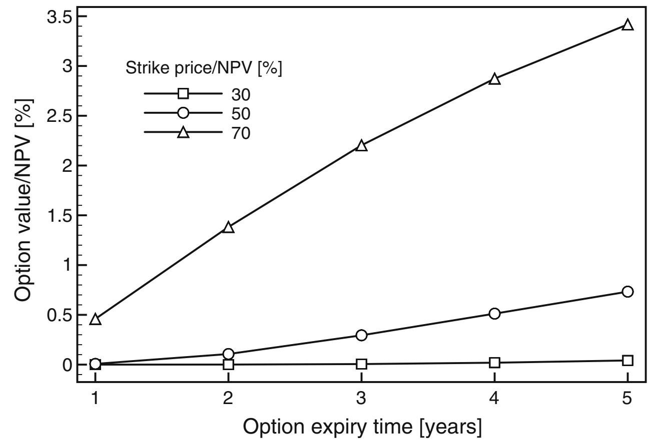

Fig. 3 Impact of the expiry time on the reselling option's value (medium volatility)

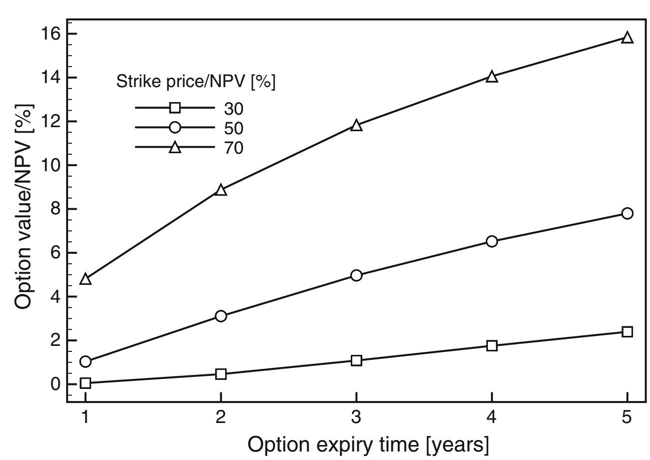

Fig. 4 Impact of the expiry time on the reselling option's value (large volatility)

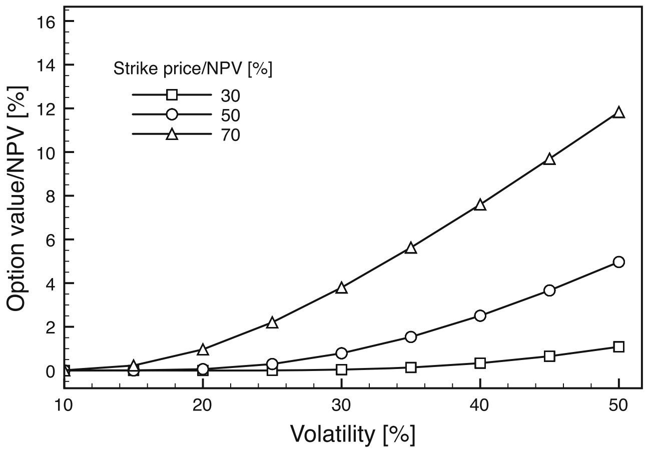

Fig. 5 Impact of the volatility on the reselling option's value

regardless of the time window allowed to resell. The effect of the reselling price is quite nonlinear: for a reselling price equal to 70% of the original project value, the option value may be even larger than 3% of the NPV if the license holder is allowed up to 5 years to make its decision. As expected, the increase of volatility largely adds to the value of the reselling option. In Fig. 4, we see that the inclusion of the reselling option in the license conditions may add value to the license even in excess of 15% of the NPV, depending on the reselling price. In both scenarios, the growth of the option value with time is, at most, linear with the option's expiry time, with a small concavity (growing with the reselling price).

Finally, we examine the direct relationship between the option's value and the volatility. In Fig. 5, we show the growth of the option value, again for three possible values of the reselling price and for a risk-free rate $r = 0.05$, with a time limit of 3 years. As can be seen, the growth is strongly nonlinear, and the presence of large reselling price acts as a booster for the option's value. On the contrary, if the reselling price is quite low (see the curve pertaining to a reselling price equal to 30% of the NPV), a large volatility scenario is not of much help to increase the option's value.

8 Conclusions

A method, based on real options, has been proposed to determine the value of the transferability (i.e., reselling) option for telecommunications licenses. The method allows to evaluate how much the value of the license increases due to the introduction of the transferability option. The method has been applied in a variety of settings as to the reselling price, the expiry time of the option, and the variability of the telecommunications market (embodied by the volatility). It has been shown that the reselling price and the volatility play the most important role, and that the option's value increases superlinearly with an increase either in the reselling price or in the volatility. On the other hand, the option's value grows no more than linearly with the expiry time, and the growth rate gets lower as the reselling price increases. In addition, to assess the value of the option when the side conditions are known (or at least estimated), we can use the method to negotiate those side conditions. In fact, two of the three parameters considered (namely, the reselling price and the expiry time, the volatility being out of control) may form part of the license itself or of a side contract. They can then be tuned to reconcile the price of the option with its exercise conditions.

References

1. AA,VV (2004) Study on conditions and options in introducing secondary trading of radio spectrum in the European Community. Final report for the European Commission, Analysis Consulting
2. Benaroch M (2001) Option-based management of technology investment risk. IEEE Trans Eng Manage 48:428–444
3. Bessière V, Kaestner M (2001) Quel prix pour les options UMTS? Une approche par les options réelles. Accepted Papers Series 2001-3, Montpellier University, Center for Research in Finance. http://ideas.repec.org/p/grf/mtpaps/bk001.html
4. Börgers T, Dustmann C (2003) Awarding telecom licences: the recent European experience. Econ Policy 18:215–268
5. Cave M (2002) Review of radio spectrum management. In: Cave M, Majumdar S, Vogelsang I (eds) Handbook of telecommunications economics. Elsevier, Amsterdam
6. Clancy T (2009) Dynamic spectrum access using the interference temperature model. Ann Telecommun 64(7–8):573–592. doi:10.1007/s12243-009-0098-x
7. Copeland T, Antikarov V (2001) Real options. Texere, New York
8. Cox JC, Ross SA, Rubinstein M (1979) Option pricing: a simplified approach. J Financ Econ 7(3):229–263
9. d'Halluin Y, Forsyth P, Vetzal K (2002) Managing capacity for telecommunications networks under uncertainty. IEEE/ACM Trans Netw 10:579–588
10. Foster A (2006) Introducing spectrum trading. In: Executive level training for regulators and policy-makers. Hong Kong
11. Gruber H (2006) 3G Mobile telecommunications licenses in Europe: a critical review. SSRN Preprint no. 918003
12. Harmantzis F, Tanguturi V (2004) Delay in the expansion from 2.5G to 3G wireless networks: a real options approach. In: International telecommunications society 15th Biennial conference, Berlin
13. Harmantzis F, Tanguturi V (2004) Real options analysis for GPRS network with Wi-Fi integration. In: 31st annual conference of the northeast business & economics association, New York

14. Harmantzis F, Tanguturi V (2007) Investment decisions in the wireless industry applying real options. Telecommun Policy 31:107–123
15. Herbst P, Walz U (2001) Real options valuation of highly uncertain investments: are UMTS-licenses worth their money? Tech Rep, University of Tuebingen
16. Higham D (2002) Nine ways to implement the binomial method for option valuation in MATLAB. SIAM Rev 44(4):661–677
17. Hull J (2008) Options, futures, and other derivatives, 7th edn. Prentice Hall, Englewood Cliffs
18. Kalmus P (2004) Wireless local loop lessons from the US. Tech Rep. NERA
19. Kenyon C, Cheliotis G (2001) Stochastic models for telecom commodity prices. Comput Networks 36(5/6):533–555
20. Lamberton D (1998) Error estimates for the binomial approximation of American put options. Ann Appl Probab 8(1):206–233. http://www.jstor.org/stable/2667243
21. Leisen D (1996) Pricing the American put option: a detailed convergence analysis for binomial models. Discussion Paper Serie B 366, University of Bonn, Germany. http://ideas.repec.org/p/bon/bonsfb/366.html
22. Lichtenberger E (2003) Spectrum trading in Germany, Austria and the UK: the influence of regulatory regimes and evaluation of criteria on competition in the European mobile telecommunication sector. In: ITS Conference, Helsinki
23. Marsden Jacob Associates (2004) The cost of capital for mobile operators. Report for TelstraClear
24. Martino M (2004) I principali temi regolamentari nel mercato delle reti e dei servizi radiomobili e nell'assegnazione delle frequenze. In: Seminario di studio AGCOM. Anacapri
25. Mastroeni L, Naldi M (2009) Option-based dynamic management of wireless spectrum. In: NGI 2009. Aveiro
26. Northstream (2002) 3G rollout status. Report PTS-ER-2002:22
27. Ofcom (2005) Ofcom's approach to risk in the assessment of the cost of capital. Ofcom, London
28. Ofcom (2004) A statement on spectrum trading: implementation in 2004 and beyond. Ofcom, London
29. Pogorel G (2003) Radio spectrum policy and management: a turning point? Commun Strategies 49:109–117
30. Radiocommunications Agency (2002) Implementing spectrum trading. Radiocommunications Agency, London
31. Sallent O, Agustí R, Pérez-Romero J, Giupponi L (2008) Decentralized spectrum and radio resource management enabled by an on-demand cognitive pilot channel. Ann Télécommun 63(5–6):281–294
32. Scandizzo P, Ventura M (2006) Bids for the UMTS system: An empirical evaluation of the Italian case. Telecommun Policy 30:533–551
33. Schaefer S, Giles T, Butterworth D (2003) Cost of capital for T-Mobile (uk). CRA Report No. D03367-00
34. Valletti T (2001) Spectrum Trading. Telecommun Policy 30:655–670
35. Whalley J, Curwen P (2006) Third generation new entrants in the European mobile telecommunications industry. Telecommun Policy 30:622–632