Chameleon effect, the range of values hypothesis and reproducing the EPR-Bohm correlationss

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Abstract

We present a detailed analysis of assumptions that J. Bell used to show that local realism contradicts QM. We find that Bell's viewpoint on realism is nonphysical, because it implicitly assume that observed physical variables coincides with ontic variables (i.e., these variables before measurement). The real physical process of measurement is a process of dynamical interaction between a system and a measurement device. Therefore one should check the adequacy of QM not to "Bell's realism," but to adaptive realism (chameleon realism). Dropping Bell's assumption we are able to construct a natural representation of the EPR-Bohm correlations in the local (adaptive) realistic approach.

1 Introduction

1.1 "No-go" theorems

During the last 70 years the understanding of QM was highly improved by wide debate on various "no–go" theorems, e.g., von Neumann, Kochen–Specker, Bell, see [1], [2]. The latter one really beats all records on publications, citations, discussions and controversies, see [3]–[5] for recent debates. We emphasize that as any mathematical theorem a "no–go" theorem is based on a number of mathematical assumptions. And adequacy of a mathematical assumption to physical reality should be the subject of very careful investigation. For example, J. Bell criticized strongly some assumptions of the von Neumann, Jauch-Piron, and Gleason "no–go" theorems [2]. Some assumptions of Bell's theorem were also strongly criticized, see e.g. [3]–[18].

1.2 Probabilistic and quantum contextualities

In particular, it was pointed out that the proof of Bell's inequality is based on the implicit use of a single Kolmogorov probability space, see Accardi [7]–[9], Khrennikov [11]–[14], Hess and Philipp [17]. We can call such an assumption *probabilistic non–contextuality.* By *probabilistic contextuality* we understand dependence of probability on experimental settings. This notion differs essentially from the conventional notion of quantum contextuality [2].

We recall that *quantum contextuality* is defined as follows: the result of measurement of an observable a depends on another measurement on an observable b, although these two observables commute with each other [2]. It should be emphasized that property of locality is a special case of quantum non-contexuality.

We now compare conventional quantum contextuality and probabilistic one. In some special cases one can obtain probabilistic contextuality from quantum contextuality. However, probabilistic contextuality need not be induced by the quantum one:

The probability distribution can be dependent on both (commuting) observables even if the result of measurement of an observable a does not depend on another measurement on observable b.

2 Realistic models violating Bell's inequality

Many authors studied different probabilistically contextual models which violate Bell's inequality. In particular, the efficiency of detectors loophole as well as more general the fair sampling loophole, see e.g. [19]- [25] for these loopholes, are just special forms of probabilistic contextuality. In the latter cases different Kolmogorov spaces correspond to different ensembles of particles created through selections corresponding to various experimental settings. By choosing observables a, b and c, d in the EPR-Bohm framework[1] we select two different sub-ensembles $\Lambda_{a,b}$ and $\Lambda_{c,d}$. Fair sampling assumption means that restrictions of the probability P (originally defined on the complete space Λ of hidden variables) onto sub-ensembles $\Lambda_{a,b}$ and $\Lambda_{c,d}$ coincide:

$$P|_{\Lambda_{a,b}} = P|_{\Lambda_{c,d}} \tag{1}$$

This is a special case of probabilistic non-contextuality. And unfair sampling means that the coincidence condition (1) is violated for some experimental settings. This is a special case of probabilistic contextuality.

We remark that in the probabilistic contextual approach one can derive generalizations of Bell's inequality which are not violated for quantum covariations [11]–[14].

2.1 Physical origin of probabilistic contextuality

Thus mathematically everything is clear: by dropping the assumption on probabilistic non–contextuality and assuming that different experimental settings induce different probability spaces it is possible to violate Bell's inequality. But the physical origin of probabilistic contextuality is a problem of huge complexity. In all conventional models probabilistic contextuality is induced either by quantum contextuality or by losses of particles[2] On the other hand, we do not know any natural physical explanation of quantum contextuality, besides nonlocality.

[1] Here a and b as well as c and d are orientations of two spatially separated polarization beam splitters.

[2] E.g., efficiency of detectors, fair sampling, and time–window loopholes [19]–[25], induce losses of particles: a part of the original ensemble should disappear. We agree that losses of particles is the important problem. However, we do not think that this is the essence of Bell's argument. We agree with experimenters that such losses of particles can be considered merely as a technological problem. One of the authors would like to thank Alain Aspect and Gregor Weihs for discussions on this problem during Växjö conferences.

2.2 Chameleon effect

However, there exists a model in that probabilistic contextuality (i.e., dependence of probabilities on experimental settings) can be produced without losses of particles. Moreover, in that model probabilistic contextuality is not a consequence of the quantum contextuality and hence the model is *local.*

This is *the chameleon model* which is described in detail [7]–[9]. In these papers Bell's definition of realism was criticized and there was proposed a new approach to realistic models, namely, *adaptive realism.* In the chameleon model one could not identify results of measurements with ontic variables (i.e., preexisting before measurement). Suppose a particle has some property, say spin. At the ontic level spin is characterized by some parameter σ. *Can one assert that precisely this parameter is obtained as the result of a spin–measurement?* Definitely not! Any measurement is a complicated process of interaction of a microscopic system with a measurement device. Finally we cannot say that we obtain the ontic parameter σ, but only the observed spin, say S. We emphasize that QM as about S and not about σ (as N. Bohr pointed out in many occasions QM is not about reality as it is, but about the results of measurements).

How does the result of measurement S arise? This is the result of *dynamical process* of interaction of a system and a measurement device. In such an approach there as nothing against realism. However, this is the adaptive (or chameleon) realism (which is not at all realism of balls having once and for ever determined color).

The chameleon effect simply states that, since dynamics is determined by the variable subjected to measurement, we obtain probability distributions depending on experimental settings. Thus the chameleon approach implies probabilistic contextuality, hence, the possibility of violation of Bell's inequality. Nevertheless, dynamics of measurements can be completely local. Let a and b be two quantum observables represented by commuting operators. Then there are two different dynamical systems corresponding to the a and b-measurements, respectively. In general, they do not depend on each other. Therefore the chameleon effect induces probabilistic contextuality, but not at all quantum contextuality.

Finally, we remark that we question neither Bell's theorem as a mathematical result nor experimental violation of Bell's inequality. We question the adequacy of Bell's realistic model (which he used to confront classical and quantum physics) to the physical situation. We show that by rejecting two basic implicit assumptions in Bell's definition of a realistic model, namely

a) non-adaptive realism of observables;
b) the range coincidence hypothesis,

we can construct a model with hidden variables which reproduces precisely the EPR–Bohm correlations.

3 Forward and backward Kolmogorov equations

Our further considerations generalize the well known dynamical scheme for statistical states and variables associated with the diffusion process. Therefore we recall the standard scheme. Let $x(s)$ be a diffusion process. To simplify considerations, we consider at the beginning the state space $X = \mathbf{R}$, the real line. We set

$$p(s,x,t,y) = P(x(t) = y | x(s) = x)$$

We consider the probability measure (statistical state describing an ensemble of particles)

$$p(s,t,y) = \int p(s,x,t,y)p_0(x)dx, \tag{2}$$

where $p_0(x)$ is the density of the initial probability distribution on the state space. This probability satisfies to the forward Kolmogorov equation:

$$\frac{\partial p(s,t,y)}{\partial t} = L(p(s,t,y)) \ , \tag{3}$$

where the generator of diffusion is given by

$$L(p)(t,y) = \frac{1}{2}\frac{\partial^2}{\partial y^2}\left[\sigma^2(t,y)p(t,y)\right] - \frac{\partial}{\partial y}\left[a(t,y)p(t,y)\right] . \tag{4}$$

Here $a(t,y)$ and $\sigma(t,y)$ are the drift and diffusion coefficients, respectively. We note that in physics (3) is known as the Fokker–Planck equation. The evolution equation (3) is completed by the initial condition:

$$\lim_{t \downarrow s} p(s,t,y) = p_0(y) \tag{5}$$

Let us now consider the corresponding dynamics of functions. We set

$$f(s,\tau,x) = \int g(y)p(s,x,\tau,y)dy \tag{6}$$

Then this function satisfies to the backward Kolmogorov equation:

$$\frac{\partial f}{\partial s}(s,\tau,x) = W(f(s,\tau,x)) \tag{7}$$

where the operator W which is conjugate to the generator L is given by

$$W(f)(s,x) = -\frac{1}{2}\sigma^2(s,x)\frac{\partial^2 f(s,x)}{\partial x^2} - a(s,x)\frac{\partial}{\partial x}f(s,x)\ . \tag{8}$$

The evolution equation (7) is completed not by initial condition, but by the "final condition":

$$\lim_{s\uparrow\tau} f(s,\tau,x) = g(x)$$

We emphasize this crucial difference between the equations for statistical states (probabilities) and physical variables (functions on the configuration space). The former is *a forward equation* and the latter is *a backward equation.* By knowing a probability distribution $p_0(y)$ at the initial instance of time $s = t_0$ we can find it at any $t \geq t_0$: $p(t_0,t,y)$. By knowing a physical variable $g(y)$ at the end of evolution $t = \tau$ we can reconstruct it at the initial instance of time $t_0 : f(t_0,\tau,x)$.

We remark that

$$\int f(t_0,\tau,x)p_0(x)dx = \int \left(\int g(y)p(t_0,x,\tau,y)dy\right)p_0(x)dx$$

$$= \int g(y)\left(\int p(t_0,x,\tau,y)p_0(x)dx\right)dy = \int g(y)p(t_0,\tau,y)dy.$$

Since in our further considerations we will not always be able to operate always with densities, we consider just probability measures: $p_0(dy)$, $p(s,t,dy)$ and so on. We rewrite the forward and backward Kolmogorov equations in the compact form:

$$\frac{\partial p(t_0,t)}{\partial t} = L(p(t_0,t))\ ,\quad p(t_0,t_0) = p_0\ ; \tag{9}$$

$$\frac{\partial f}{\partial s}(s,\tau) = W(f(s,\tau))\ ,\quad f(\tau,\tau) = g \tag{10}$$

We have the following conjugation condition:

$$\int f(t_0, \tau, x) p_0(dx) = \int g(y) p(t_0, \tau, dy) \tag{11}$$

or

$$\int f(t_0, \tau, x) p(t_0, t_0, dy) = \int f(\tau, \tau, x) p(t_0, \tau, dx) \tag{12}$$

We remark that only one quantity, either a probability measure or a function, is known in each side of this equality.

The Cauchy problem (9) induces the dynamical system $V_{t_0,t}$ in the space of probability measures:

$$p(t_0, t) = V_{t_0,t}(p_0) \ , \tag{13}$$

and the (backward) Cauchy problem (10) induces the dynamical system $U_{s,\tau}$ in the space of functions:

$$f(s, \tau) = U_{s,\tau}(g) \tag{14}$$

These dynamical systems are conjugate:

$$\int U_{t_0,\tau}(g)(x) p_0(dx) = \int g(x) V_{t_0,\tau}(p_0)(dx) \tag{15}$$

4 Classical statistical model with the chameleon effect

Denote by Λ the state space of physical systems under consideration. We also consider statistical states describing ensembles of systems. They are represented by probability measures on Λ. Physical variables are represented by functions $f : \Lambda \to \mathbb{R}$. The average of a variable f with respect to a statistical state p is given by

$$\langle f \rangle_p = \int_\Lambda f(\lambda) p(d\lambda) \tag{16}$$

Dynamics of a statistical state is given by a dynamical system $V_{t_0,t}$ in the space of probability measures. Dynamics of a physical variable is given by a dynamical system $U_{s,\tau}$ in the space of functions.

We no longer assume that these dynamics are generated by a diffusion (not even a Markov process). The $V_{t_0,t}$ and $U_{s,\tau}$ are two general dynamics. The only condition coupling them is the conjugation condition (15).

We emphasize that $V_{t_0,t}$ is the forward dynamics: by knowing the initial statistical state, p_0, we can find it at any instant of time t: $p(t_0, t) = V_{t_0,t}(p_0)$. In contrast, $U_{s,\tau}$ is the backward dynamics: by knowing the final physical variable $f_\tau(x) = g(x)$, we can reconstruct it for the $t = t_0$: $f(t_0, \tau, x) = U_{t_0,\tau}(g)(x)$. This was the well known story. The chameleon story starts when one wants to describe processes of measurements.

Suppose that we would like to present classical statistical (but dynamical!) description of the process of measurement of an observable a. Here a is just a label to denote a class of measurement devices. In QM we use self–adjoint operators as such labels.

In the chameleon model of measurement the basic assumption is that dynamics V and U depend on the observable a :

$$V_{t_0,t} \equiv V^a_{t_0,t}\ , \quad U_{s,\tau} \equiv U^a_{s,\tau}. \tag{17}$$

This is a very natural assumption: *any measurement device changes dynamics.* Suppose that initially there was prepared an ensemble of systems with the probability distribution $p_0(\lambda)$. Then in the process of the a–measurement $p_0(\lambda)$ evolves according to the dynamics V^a.

We assume that the process of measurement takes the finite interval of time τ. Thus at that moment the probability distribution becomes $p_\tau(\lambda)$ (which is, of course, depends on a).

The physical variable $f^a_t(\lambda)$ evolves according to the dynamics U^a. We do not know the initial (ontic) physical variable $f^a_{t_0}(\lambda)$. This is a hidden physical variable – an ontic property of systems before the a–measurement starts. In our model a particle has the ontic position, momentum, spin and so on. But it would be very naive to expect (as J. Bell did) to measure directly $f^a_{t_0}(\lambda)$. We measure the result of evolution, namely, $f^a_\tau(\lambda)$. The latter variables are the results of measurements. QM is, in fact, about such variables. But, in contrast to the chameleon model, QM does not permit the functional representation of observables.

We repeat again that dynamics for variables is a backward dynamics. Such a mathematical description is totally adequate to the physical experimental situation. We do not know the initial variable $f^a_{t_0}(\lambda)$, but only the final (observed) variables $f^a_\tau(\lambda)$.

We can reconstruct $f^a_{t_0}(\lambda)$ from the observed quantity $f^a_\tau(\lambda)$. But we never know $f^a_{t_0}(\lambda)$ from the very beginning. Therefore we are not able to construct $f^a_\tau(\lambda)$ and hence predict the result of measurement.

We have two types of averages:
(CL) The ontic ("classical") averages are given by

$$\langle f^a \rangle_{CL} \equiv \langle f^a_{t_0} \rangle_{p_0} = \int_\Lambda f^a_{t_0}(\lambda) p_0(d\lambda) \ ; \tag{18}$$

(OB) The observational averages (in particular, the quantum ones) are given by

$$\langle f^a \rangle_{OB} \equiv \langle f^a_\tau \rangle_{p^a_\tau} = \int f^a_\tau(\lambda) p^a_\tau(d\lambda) \ . \tag{19}$$

As a consequence of the conjugation condition (11), these averages coincide:

$$\langle f^a \rangle_{CL} = \langle f^a \rangle_{OB} \tag{20}$$

Thus one can either consider the average with respect to the initial probability distribution: $\langle f^a \rangle_{CL}$, but the f^a be the ontic variable and not the observed one, or the average of the observed physical variable, $\langle f^a \rangle_{OB}$, but in this case the initial probability distribution p_0 could not be used. In the latter case one should consider the probability we assure p^a_τ that depends on a.

In the special case of quantum measurements the (OB) gives the quantum average and the average (CL) can be called prequantum. In the model under consideration we assume that the quantum and prequantum averages coincide. Recently there was proposed a model, Prequantum Classical Statistical Field Theory, producing a prequantum average which coincides with the quantum one only approximately, see [26]–[29].

Finally we remark that if f^a_τ takes, e.g., the values $\{\pm 1\}$, then there will be no reasons to assume that f^a_0 takes the same values.

4.1 The range of values coincidence hypothesis

Recently it was paid attention, see [26]–[29], to another problem in Bell's definition of realism [2]. This is the *range of values coincidence problem:*

A priori there are no reasons to assume that the range of values of an ontic physical variable (say ontic spin σ) should coincide with the range of values of the corresponding observables (say measured spin S).

As was already pointed out, the process of measurement is the process of interaction of a microscopic system and a measurement device. Therefore it is not surprising that the σ can be transformed into a different value S. In fact, by its very definition σ is unobservable in principle.

Denote by λ the state of a system, the "hidden variable". Both σ and S are functions of $\lambda : \sigma = \sigma(\lambda)$, $S = S(\lambda)$. But there are no reasons to assume that

$$\text{Range } \sigma = \text{ Range } S \ . \tag{21}$$

Thus one should sharply distinguish ontic and observed variables. The condition that the observed spin $S = \pm 1$ does not imply that the ontic spin σ (which is in principle unobservable) also takes values ± 1.

4.2 Spectral postulate

We point out that we do not want to drop the standard *spectral postulate* of QM. By this postulate the range of vales of a quantum observable coincides with the spectral set of the corresponding self-adjoint operator. This postulate was confirmed by all quantum experiments and it could not be questioned. In our approach the range of values of say the observed spin S coincides with the spectral set of the corresponding quantum operator. We simply remark that there is no reasons to expect that the range of values of say the ontic spin σ should coincide with this spectral set.

4.3 Classical reproduction of the EPR-Bohm correlations

In fact, we need not to consider a new classical adaptive (chameleon) model which would give us the EPR-Bohm correlations. By taking into account the analysis of measurement process which was performed in the present paper (and especially the evident possibility of violation of the range of values coincidence hypothesis) we can now use the well known model of Accardi and Regoli [10].

Conclusion: *The common conclusion that Bell's arguments imply incompatibility of local realism and the quantum formalism is based on a rather naive understanding of coupling between ontic reality (i.e., reality as it is when nobody make measurements) and the observational reality. By considering the adaptive measurement framework (based on the chameleon effect) we showed that in fact local realism can peacefully coexist with the quantum formalism.*

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